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Neutrosophic Generalized Exponential Robust Ratio Type Estimators

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ABSTRACT. Estimators proposed under classical statistics fail if data are vague or indeterminate. Neutrosophic Statistics are the only alternative because its deal with indeterminacy. Extensive reserch has been conducted in this field because of its wide applicability. This study aimed to further develop the theory of neutosophic simple random sampling without replacement. In this study, a generalized neutrosophic exponential robust ratio-type estimator was proposed, and five of its member neutrosophic estimators were developed. Derivations of the bias and Mean Square Error were provided up to the first-order approximation. To demonstrate the high efficiency of the proposed neutrosophic estimators an empirical study on the stock price of Moderna and four simulation studies have been conducted, and the results show that the proposed neutrosophic estimators are more efficient than similar existing ratio type estimators discussed in this paper in neutrosophic as well as classical forms.

1. INTRODUCTION

Classical statistics and its methods deal with randomness but there are cases where the data at hand is indeterminate or vague or ambiguous or imprecise rather than random. In such situations estimation using classical statistical methods does not yield promising results. Fuzzy logic [1, 2] is one solution to tackle such a problem but still, it ignores indeterminacy. In such cases, neutrosophic methods are much more reliable. They deal with both randomness and more importantly with indeterminacy. Neutrosophic statistics refers to a set of data such that the data or a part of it is indeterminate and methods to analyze such a data [3].

Neutrosophic statistics is an extension of classical statistics and when the indeterminacy is zero, neutrosophic statistics coincides with classical statistics [3]. Estimation through neutrosophic

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methods is a new field and therefore it is unexplored unlike estimation problems in classical probability sampling designs where the data is determinate [4-8]. But, due to its wide applicability, it has gained much more importance than classical statistics and as a results it is being applied in various fileds for instance in decision making [9]. [10] developed a new sampling plan using neutrosophic process. [11] proposed neutrosophic analysis of variance. [12] used neutrosophic statistics in analyzing road traffic accidents. [13] proposed goodness of fit test in neutrosophic statistics. As a result filed of neutrosophic sampling has been developed and some neutrosophic ratio-type estimators has been proposed [14] and this paper is the second paper aimed at further developing the theory of neutrosophic SRSWOR or NSRSWOR sampling.

It has been observed in some sample surveys that the data collected containes some vagueness due to many factors like methodolgy used (observing blood pressure multiple times within an interval in NFHS 4 [16]) , observing daily stock price [15, 19] or daily temperature of a city [14]. All these are examples where the data contains some indeterminacy and clssical statistical measures like mean, median or standard deviation might not give results which are useful for decision making.

Thus the aim of this paper is to further develop neutrosophic probability sampling theory particulary NSRSWOR by developing various generalized neutrosophic exponential robust ratio type estimators. In Section 2, the paper presents the terminologies of neutrosophic statistics for new readers. In Section 3, existing related neutrosophic ratio-type estimators have been presented.

In Section 4, the proposed generalized neutrosophic exponential robust ratio type estimator and the five developed estimators along with their derivations of biases and MSEs are presented. In order to demonstrate the high efficiecny of the developed neutrosophic generalized neutrosophic exponential robust ratio type estimators four simulation studies have been conducted in Section 5. The results are compared with their classical MSE values as well. Results and concluding remarks on this paper are provided in Section 6 along with some future fruitful areas of research.

2. TERMINOLOGY

A simple random neutrosophic sample of size n from a classical or neutrosophic population is a sample of n individuals such that at least one of them has some indeterminacy [3, 14].

As presented in [14], a neutrosophic observation is of the form

$Z_N = Z_L + Z_U I_N$, where $I_N \in [I_L, I_U]$ and $Z_N \in [Z_l, Z_u]$.

Now consider a simple random neutrosophic sample of size $n_N \in [n_L, n_U]$ drawn from a finite population of size N and $y_N(i) \in [y_L, y_U]$ and $x_N(i)$ are $i^{th} \in [x_L, x_U]$ neutrosophic sample observation. Here the population mean of neutrosophic survey and auxiliary variable are $\bar{Y}_N \in [Y_L, Y_U]$ and $\bar{X}_N \in [X_L, X_U]$ respectively.

$C_{yN} \in [C_{yN_L}, C_{yN_U}]$ and $C_{xN} \in [C_{xN_L}, C_{xN_U}]$ are population coefficient of variation of neutrosophic survey and auxiliary variables respectively. In addition, $\rho_{xyN} \in [\rho_{xyN_L}, \rho_{xyN_U}]$, $\beta_1(x_N) \in [\beta_1(x_{N_L}), \beta_1(x_{N_U})]$ and $\beta_2(x_N) \in [\beta_2(x_{N_L}), \beta_2(x_{N_U})]$ are the correlation coefficient between the neutrosophic survey and auxiliary variables, coefficient of skewness and coefficient of kurtosis of the neutrosophic auxiliary variable respectively.

The MSE of a neutrosophic estimator is of the form, $MSE(\bar{y}_N) \in [MSE_L, MSE_U]$.

The error terms in neutrosophic statistics are:

$$\bar{e}_{yN} = \bar{y}_N - \bar{Y}_N,$$

$$\bar{e}_{xN} = \bar{x}_N - \bar{X}_N,$$

$$E\left(\bar{e}_{yN}\right) = E(\bar{e}_{xN}) = 0, \quad E\left(\bar{e}_{yN}^2\right) = \frac{N-n}{Nn}\frac{S_{yN}^2}{\bar{Y}_N^2} = \xi_{20}$$

$$E(\bar{e}_{xN}^2) = \frac{N-n}{Nn}\frac{S_{xN}^2}{\bar{X}_N^2} = \xi_{02}$$

$$E(\bar{e}_{xN}\bar{e}_{yN}) = \frac{N-n}{Nn}\frac{S_{yN}S_{xN}}{\bar{X}_N\bar{Y}_N} = \xi_{11},$$

where $\bar{e}_{yN} \in [\bar{e}_{yN_L}, \bar{e}_{yN_U}]$,

$$\bar{e}_{xN} \in [\bar{e}_{xN_L}, \bar{e}_{xN_U}],$$

$$\bar{e}_{yN}^2 \in [\bar{e}_{yN_L}^2, \bar{e}_{yN_U}^2],$$

$$\bar{e}_{xN}^2 \in [\bar{e}_{xN_L}^2, \bar{e}_{xN_U}^2].$$

3. SOME RELATED NEUTROSOPHIC ESTIMATORS

Since neutrosophic probability sampling is a new area of research handful of ratio type estimators are proposed in this Neutrosophic Simple Random Sampling Without Replacement (NSRSWOR). Tahir et al. [14] proposed the following ratio-type estimators given by

$$\bar{y}_{R_N} = \frac{\bar{y}_N}{\bar{x}_N}\bar{X}_N, \tag{3.1}$$

$$\bar{y}_{SDr_N} = \bar{y}_N \frac{\bar{X}_N + C_{xN}}{\bar{x}_N + C_{xN}}, \tag{3.2}$$

$$\bar{y}_{SKr_N} = \bar{y}_N \frac{\bar{X}_N + \beta_2(xN)}{\bar{x}_N + \beta_2(xN)}, \tag{3.3}$$

$$\bar{y}_{USr_N} = \bar{y}_N \frac{\bar{X}_N\beta_2(xN) + C_{xN}}{\bar{x}_N\beta_2(xN) + C_{xN}}, \tag{3.4}$$

where $\bar{y}_N \in [\bar{y}_{N_L}, \bar{y}_{N_U}]$ and $y_{R_N} \in [y_{R_L}, y_{R_U}]$, $\bar{y}_{USr_N} \in [\bar{y}_{SDr_L}, \bar{y}_{SDr_U}]$, $\bar{y}_{SKr_N} \in [\bar{y}_{SKr_L}, \bar{y}_{SKr_U}]$, and $\bar{y}_{USr_N} \in [\bar{y}_{USr_L}, \bar{y}_{USr_U}]$.

Their expressions of MSEs are:

$$MSE(\bar{y}_R) = \frac{N-n}{Nn}\bar{Y}_N^2[C_{yN}^2 + C_{xN}^2 - 2C_{xN}C_{yN}\rho_{xyN}], \tag{3.5}$$

$$MSE(\bar{y}_{SDr_N}) = \frac{N-n}{Nn}\bar{Y}_N^2\left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right)C_{xN}^2 - 2\left(\frac{\bar{X}_N}{\bar{X}_N + C_{xN}}\right)C_{xN}C_{yN}\rho_{xyN}\right], \tag{3.6}$$

$$MSE(\bar{y}_{SKr_N}) = \frac{N-n}{Nn}\bar{Y}_N^2\left[C_{yN}^2 + \left(\frac{\bar{X}_N}{\bar{X}_N + \beta_2(xN)}\right)C_{xN}^2 - 2\left(\frac{\bar{X}_N}{\bar{X}_N + \beta_2(xN)}\right)C_{xN}C_{yN}\rho_{xyN}\right] \tag{3.7}$$

and

$$MSE(\bar{y}_{USr}) = \frac{N-n}{Nn}\bar{Y}_N^2\left[C_{yN}^2 + \left(\frac{\bar{X}_N\beta_2(xN)}{\bar{X}_N\beta_2(xN) + C_{xN}}\right)C_{xN}^2 - 2\left(\frac{\bar{X}_N\beta_2(xN)}{\bar{X}_N\beta_2(xN) + C_{xN}}\right)C_{xN}C_{yN}\rho_{xyN}\right] \tag{3.8}$$

where $C_{yN}^2 \in [C_{yN_L}^2, C_{yN_U}^2]$, $C_{xN}^2 \in [C_{xN_L}^2, C_{xN_U}^2]$ and $\rho_{xyN} \in [\rho_{xyN_L}, \rho_{xyN_U}]$.

4. PROPOSED NEUTROSOPHIC GENERALIZED ESTIMATORS

The aim of this article is to propose a generalized neutrosophic exponential robust ratio type estimator of finite neutrosophic population mean.

Motivated by [14], [17] and [18] we propose the following generalized neutrosophic exponential robust ratio type estimator

$$t_{p_N}^{G} = \left(o_1\bar{y}_N + o_2(\bar{X}_N - \bar{x}_N)\right)exp\left(\frac{\bar{X}_N\Omega + \Psi}{\alpha(\bar{x}_N\Omega + \Psi) + (1-\alpha)(\bar{X}_N\Omega + \Psi)} - 1\right), \tag{4.1}$$

where, o_1 and o_2 are scalars which minimizes the MSE of the proposed generalizedneutrosophic estimator t_p^G. Further, Ω and Ψ are scalaras which would assume different known population parameter values of neutrosophic auxiliary variable precisely Hodges Lehmann, Tri-mea, Mid range and coefficient of variation. It should be noted that $t_{p_N}^{G} \in [t_{p_{N_L}}^{G}, t_{p_{N_U}}^{G}]$, $o_1 \in [o_{1_L}, o_{1_U}]$, $o_2 \in [o_{2_L}, o_{2_U}]$, $\bar{X}_N \in [\bar{X}_{N_L}, \bar{X}_{N_U}]$. In order to obtain the expression of bias and Mean squared error of the proposed generalized neutrosophic estimator t_p^G, we re-write it usingerror terms defined in Section 2 and using Taylor series obtain the expression as follows

$$Bias(t_{p_N}^G) = -\bar{Y}_N + \bar{X}_N\theta o_2\xi_{02} + o_1(\bar{Y}_N + \frac{3}{2}\bar{Y}_N\theta^2\xi_{02} - \bar{Y}_N\theta\xi_{11}), \tag{4.2}$$

$$MSE\left(t_{p_N}^G\right) = -\bar{Y}_N^2 + \bar{X}_N o_2(-2\bar{Y}_N\theta + \bar{X}_N o_2)\xi_{02} + \bar{Y}_N o_1(-2\bar{Y}_N + \theta(-3\bar{Y}_N\theta + 4\bar{X}_N o_2)\xi_{02} + 2(\bar{Y}_N\theta + \bar{X}_N o_2)\xi_{11}) + \bar{Y}_N^2 o_1^2(1 + 4\theta^2\xi^2{}_{02} - 4\theta\xi_{11} + \xi_{20}). \tag{4.3}$$

Partially differentiating $MSE\left(t_{p_N}^G\right)$ $with\ respect\ to\ o_1$ and $o_2$to find their optimum values we get

$$o_{1_{opt}} = \frac{\xi_{02}(2 - \theta^2\xi_{02})}{2(-\xi^2{}_{11} + \xi_{02}(1 + \xi_{20}))} \tag{4.4}$$

$$o_{2_{opt}} = \frac{\bar{Y}_N\{2\theta^3\xi^2{}_{02} - 2\xi_{11}(-1+\theta\xi_{11}) - \theta\xi_{02}(2+\theta\xi_{11}-2\xi_{20}))}{2\bar{X}_N\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)} \quad (4.5)$$

Using these optimum values we get

$$MSE\left(t^G_{p_N\ opt}\right) = \frac{\bar{Y}_N^2\{4\xi^2{}_{11}+\xi_{02}\{\theta^4\xi^2{}_{02}-4\theta^2\xi^2{}_{11}+4(-1+\theta^2\xi_{02})\xi_{20}\}\}}{4\{\xi^2{}_{11}-\xi_{02}(1+\xi_{20})\}}, \quad (4.6)$$

where $\theta = \alpha\frac{\bar{X}_N\Omega}{\bar{X}_N\Omega+\Psi}$.

From the proposed generalized neutrosophic exponential robust ratio type estimator t^G_p we have developed five generalized neutrosophic exponential robust ratio type estimators.

(i) $$t^{G_1}_{p_N} = \left(o_1\bar{y}_N + o_2(\bar{X}_N - \bar{x}_N)\right)exp\left(\frac{\bar{X}_N\mathrm{HL}+\mathrm{TM}}{\bar{x}_N\mathrm{HL}+\mathrm{TM}} - 1\right) \quad (4.7)$$

The bias and MSE_{opt} are

$$Bias\left(t^{G_1}_{p_N}\right) = -\bar{Y}_N + \bar{X}_N\theta_1 o_{2_{opt}}\xi_{02} + o_{1_{opt}}(\bar{Y}_N + \frac{3}{2}\bar{Y}_N\theta_1^2\xi_{02} - \bar{Y}_N\theta_1\xi_{11}), \quad (4.8)$$

$$MSE\left(t^{G_1}_{p_N\ opt}\right) = \frac{\bar{Y}_N^2 4\xi^2{}_{11}+\xi_{02}\theta_1^4\xi^2{}_{02}-4\theta_1^2\xi^2{}_{11}+4(-1+\theta_1^2\xi_{02})\xi_{20}}{4\xi^2{}_{11}-\xi_{02}(1+\xi_{20})}, \quad (4.9)$$

where

$$o_{1_{opt}} = \frac{\xi_{02}(2-\theta_1^2\xi_{02})}{2\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)}, \quad (4.10)$$

$$o_{2_{opt}} = \frac{\bar{Y}_N\{2\theta^3\xi^2{}_{02}-2\xi_{11}(-1+\theta_1\xi_{11})-\theta_1\xi_{02}(2+\theta_1\xi_{11}-2\xi_{20}))}{2\bar{X}_N\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)}, \quad (4.11)$$

and $\theta_1 = \frac{\bar{X}_N\mathrm{HL}}{\bar{X}_N\mathrm{HL}+\mathrm{TM}}$,

where, $t^{G_1}_{p_N} \in [t^{G_1}_{p\ L}, t^{G_1}_{p\ U}]$, $\theta_1 \in [\theta_{1_L}, \theta_{1_U}], o_{1_{opt}} \in [o_{1_{opt_L}}, o_{1_{opt_U}}]$ *and* $o_{2_{opt}} \in [o_{2_{opt_L}}, o_{2_{opt_U}}]$.

(ii) $$t^{G_2}_{p_N} = \left(o_1\bar{y}_N + o_2(\bar{X}_N - \bar{x}_N)\right)exp\left(\frac{\bar{X}_N\mathrm{TM}+\mathrm{MR}}{\bar{x}_N\mathrm{TM}+\mathrm{MR}} - 1\right) \quad (4.12)$$

The bias and MSE_opt are

$$Bias\left(t^{G_2}_{p_N}\right) = -\bar{Y}_N + \bar{X}_N\theta_2 o_{2_{opt}}\xi_{02} + o_{1_{opt}}(\bar{Y}_N + \frac{3}{2}\bar{Y}_N\theta_2^2\xi_{02} - \bar{Y}_N\theta_2\xi_{11}), \quad (4.13)$$

$$MSE\left(t^{G_2}_{p_N\ opt}\right) = \frac{\bar{Y}_N^2 4\xi^2{}_{11}+\xi_{02}\theta_2^4\xi^2{}_{02}-4\theta_2^2\xi^2{}_{11}+4\left(-1+\theta_2^2\xi_{02}\right)\xi_{20}}{4\xi^2{}_{11}-\xi_{02}(1+\xi_{20})}, \quad (4.14)$$

where

$$o_{1_{opt}} = \frac{\xi_{02}(2-\theta_2^2\xi_{02})}{2\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)}, \quad (4.15)$$

$$o_{2_{opt}} = \frac{\bar{Y}_N\{2\theta^3\ \xi^2{}_{02}-2\xi_{11}(-1+\theta_2\xi_{11})-\theta_1\xi_{02}(2+\theta_2\xi_{11}-2\xi_{20}))}{2\bar{X}_N\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)}, \quad (4.16)$$

and $\theta_2 = \frac{\bar{X}_N\mathrm{TM}}{\bar{X}_N\mathrm{TM}+\mathrm{MR}}$

where, $t^{G_2}_{p_N} \in [t^{G_2}_{p\ L}, t^{G_2}_{p\ U}]$, $\theta_2 \in [\theta_{2_L}, \theta_{2_U}], o_{1_{opt}} \in [o_{1_{opt_L}}, o_{1_{opt_U}}]$ and $o_{2_{opt}} \in [o_{2_{opt_L}}, o_{2_{opt_U}}]$.

(iii) $t_{p_N}^{G_3} = \left(o_1\bar{y}_N + o_2(\bar{X}_N - \bar{x}_N)\right)exp(\frac{\bar{X}_N \text{HL} + \text{MR}}{\bar{x}_N \text{HL} + \text{MR}} - 1)$ (4.17)

The bias and MSE_{opt} are

$$Bias\left(t_{p_N}^{G_3}\right) = -\bar{Y}_N + \bar{X}_N\theta_3 o_{2_{opt}}\xi_{02} + o_{1_{opt}}(\bar{Y}_N + \frac{3}{2}\bar{Y}_N\theta_3^2\xi_{02} - \bar{Y}_N\theta_3\xi_{11}), \quad (4.18)$$

$$MSE\left(t_{p_N}^{G_3}{}_{opt}\right) = \frac{\bar{Y}_N^2 4\xi^2{}_{11} + \xi_{02}\theta_3^4\xi^2{}_{02} - 4\theta_3^2\xi^2{}_{11} + 4\left(-1+\theta_3^2\xi_{02}\right)\xi_{20}}{4\xi^2{}_{11} - \xi_{02}(1+\xi_{20})}, \quad (4.19)$$

where

$$o_{1_{opt}} = \frac{\xi_{02}\left(2-\theta_3^2\xi_{02}\right)}{2\left(-\xi^2{}_{11} + \xi_{02}(1+\xi_{20})\right)}, \quad (4.20)$$

$$o_{2_{opt}} = \frac{\bar{Y}_N\{2\theta^3\,\xi^2{}_{02} - 2\xi_{11}(-1+\theta_3\xi_{11}) - \theta_3\xi_{02}(2+\theta_3\xi_{11} - 2\xi_{20}))}{2\bar{X}_N\left(-\xi^2{}_{11} + \xi_{02}(1+\xi_{20})\right)}, \quad (4.21)$$

and $\theta_3 = \frac{\bar{X}_N \text{HL}}{\bar{X}_N \text{HL} + \text{MR}}$

where, $t_{p_N}^{G_3} \in [t_{p\ L}^{G_3}, t_{p\ U}^{G_3}],\ \theta_3 \in [\theta_{3_L}, \theta_{3_U}], o_{1_{opt}} \in [o_{1_{opt_L}}, o_{1_{opt_U}}]\ and\ o_{2_{opt}} \in [o_{2_{opt_L}}, o_{2_{opt_U}}]$.

(iv) $t_{p_N}^{G_4} = \left(o_1\bar{y}_N + o_2(\bar{X}_N - \bar{x}_N)\right)exp(\frac{\bar{X}_N \text{C}_{\text{x}_\text{N}} + HL}{\bar{x}_N \text{C}_{\text{x}_\text{N}} + \text{HL}} - 1)$ (4.22)

The bias and MSE_{opt} are

$$Bias\left(t_{p_N}^{G_4}\right) = -\bar{Y}_N + \bar{X}_N\theta_4 o_{2_{opt}}\xi_{02} + o_{1_{opt}}(\bar{Y}_N + \frac{3}{2}\bar{Y}_N\theta_4^2\xi_{02} - \bar{Y}_N\theta_4\xi_{11}), \quad (4.23)$$

$$MSE\left(t_{p_N}^{G_4}{}_{opt}\right) = \frac{\bar{Y}_N^2 4\xi^2{}_{11} + \xi_{02}\theta_4^4\xi^2{}_{02} - 4\theta_4^2\xi^2{}_{11} + 4\left(-1+\theta_4^2\xi_{02}\right)\xi_{20}}{4\xi^2{}_{11} - \xi_{02}(1+\xi_{20})} \quad (4.24)$$

where,

$$o_{1_{opt}} = \frac{\xi_{02}\left(2-\theta_4^2\xi_{02}\right)}{2\left(-\xi^2{}_{11} + \xi_{02}(1+\xi_{20})\right)}, \quad (4.25)$$

$$o_{2_{opt}} = \frac{\bar{Y}_N\{2\theta^3\,\xi^2{}_{02} - 2\xi_{11}(-1+\theta_4\xi_{11}) - \theta_4\xi_{02}(2+\theta_4\xi_{11} - 2\xi_{20}))}{2\bar{X}_N\left(-\xi^2{}_{11} + \xi_{02}(1+\xi_{20})\right)}, \quad (4.26)$$

and $\theta_4 = \frac{\bar{X}_N \text{C}_{\text{x}_\text{N}}}{\bar{X}_N \text{C}_{\text{x}_\text{N}} + \text{HL}}$

where, $t_{p_N}^{G_4} \in [t_{p\ L}^{G_4}, t_{p\ U}^{G_4}],\ \theta_4 \in [\theta_{4_L}, \theta_{4_U}], o_{1_{opt}} \in [o_{1_{opt_L}}, o_{1_{opt_U}}]\ and\ o_{2_{opt}} \in [o_{2_{opt_L}}, o_{2_{opt_U}}]$.

(v) $t_{p_N}^{G_5} = \left(o_1\bar{y}_N + o_2(\bar{X}_N - \bar{x}_N)\right)exp(\frac{\bar{X}_N \text{C}_{\text{x}_\text{N}} + TM}{\bar{x}_N \text{C}_{\text{x}_\text{N}} + \text{TM}} - 1)$ (4.27)

The bias and MSE_{opt} are

$$Bias\left(t_{p_N}^{G_5}\right) = -\bar{Y}_N + \bar{X}_N\theta_5 o_{2_{opt}}\xi_{02} + o_{1_{opt}}(\bar{Y}_N + \frac{3}{2}\bar{Y}_N\theta_5^2\xi_{02} - \bar{Y}_N\theta_5\xi_{11}), \quad (4.28)$$

$$MSE\left(t_{p_N}^{G_5}{}_{opt}\right) = \frac{\bar{Y}_N^2 4\xi^2{}_{11} + \xi_{02}\theta_5^4\xi^2{}_{02} - 4\theta_5^2\xi^2{}_{11} + 4\left(-1+\theta_5^2\xi_{02}\right)\xi_{20}}{4\xi^2{}_{11} - \xi_{02}(1+\xi_{20})} \quad (4.29)$$

$$o_{1_{opt}} = \frac{\xi_{02}(2-\theta_5^2\xi_{02})}{2\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)}, \tag{4.30}$$

$$o_{2_{opt}} = \frac{\bar{Y}_N\{2\theta^3\,\xi^2{}_{02}-2\xi_{11}(-1+\theta_5\xi_{11})-\theta_5\xi_{02}(2+\theta_5\xi_{11}-2\xi_{20}))}{2\bar{X}_N\left(-\xi^2{}_{11}+\xi_{02}(1+\xi_{20})\right)}, \tag{4.31}$$

and $\theta_5 = \frac{\bar{X}_N C_{x_N}}{\bar{X}_N C_{x_N}+TM}$

where, $t_{p_N}^{G_5} \in [t_{p\ L}^{G_5}, t_{p\ U}^{G_5}]$, $\theta_5 \in [\theta_{5_L}, \theta_{5_U}], o_{1_{opt}} \in [o_{1_{opt_L}}, o_{1_{opt_U}}]$ *and* $o_{2_{opt}} \in [o_{2_{opt_L}}, o_{2_{opt_U}}]$.

5. EMPIRICAL STUDY

In this section we have conducted an empirical study to demonstrate the high efficiency of the developed estimators. This study, is conducted using daily stock price of Moderna. The rationale behind taking the stock price as a neutrosophic data is the fact that the daily stock price ranges between a high and a low values each day. Pin pointing the point estimate of the daily stock price will not give a reliable estimate. Thus, we have taken it as a neutrosophic dataset. In this empirical study, daily stock price of Moderna has been considered form 1-September-2020 to 1-September-2021 [20] (N=253). The neutrosophic survey variable y_N i.e., varying price of the stock on each day where $y_N \in [y_l, y_u]$ (y_u is the highest price of the stock on each day and y_l is the lowest price of the stock each day).

6. SIMULATION STUDY

In this section we have conducted four simulation studies to demonstrate the high efficiency of the proposed generalized neutrosophic robust type exponential ratio estimator over similar existing ratio estimators discussed in this article. The comparison has been made on the basis of neutrosophic MSEs and neutrosophic REs.

6.1 Simulation study-1

The following algorithm is used in R language to perform the simulation study:

(i) Nutrosophic auxiliary variable x_N has been generated from Neutrosophic normal distribution NN([0.7, 1.1], 1.2) i.e., the neutrosophic auxiliary variable x has single indetermincay where population mean μ_X is indeterminate. Thus $x_N \in [x_{N_L}, x_{N_U}]$.

(ii) Neutrosophic survey variable is generated using the model $y_N = x_N - 7e$ such that $y_N \in [y_{N_L}, y_{N_U}]$ where $e \sim N(0,1)$.

(iii) For sample sizes $n_1 \in [60,60]$, $n_2 \in [65,65]$, $n_3 \in [70,70]$ and $n_4 \in [75,75]$ various values of neutrosophic estimates are obtained with 20000 iterations.

(iv) For each neutrosophic sample size used, neutrosophic MSEs and RES have been obtained and presented in Tables.

(v) Values of estimates have been calculated under classical statistics as well and their MSEs and REs are tabulated in Tables 1-4.

Table 1: Data statistics for empirical study

$S_{yU}^2 = 9624$, $S_{yL}^2 = 8111$, $C_{yU}^2 = 0.3124$, $C_{yL}^2 = 0.3022$, $S_{xU}^2 = 8743$, $S_{xL}^2 = 8965$,
$C_{xU}^2 = 0.3055$, $C_{xL}^2 = 0.3092$, $N = 253$, $n = 160$, $TM_U = 150$, $HL_U = 153.44$,
$MR_U = 270.76$, $\beta_2 xU = 1.06$, $TM_L = 153$, $HL_L = 153.96$, $MR_L = 269.4$, $\beta_2(x_L) = 1.01$
$\rho_{yUxU} = 0.99$, $\rho_{yLxL} = 0.99$.

Table 2: Neutrosophic MSE of the estimators

Estimators	MSE $\boldsymbol{MSE}[\bar{y}_{*L}, \bar{y}_{*U}]$		Relative Eficiency $\boldsymbol{RE}[\bar{y}_{*L}, \bar{y}_{*U}]$	
$\bar{y}_{R_N}$	0.1038	0.1209	1	1
$\bar{y}_{SDr_N}$	0.1021	0.1223	1.01665	0.988553
$\bar{y}_{SKr_N}$	0.1009	0.124	1.028741	0.975
$\bar{y}_{USr_N}$	0.1222	0.1021	0.849427	1.184133
$t_{p_N}^{G_2}$	0.0835	0.116	1.243114	1.042241
$t_{p_N}^{G_2}$	0.0836	0.116	1.241627	1.042241
$t_{p_N}^{G_3}$	0.0836	0.1156	1.241627	1.045848
$t_{p_N}^{G_4}$	0.0868	0.1193	1.195853	1.013412
$t_{p_N}^{G_5}$	0.0869	0.119	1.194476	1.015966

*Denotes appropriate estimator

Table 3: Neutrosophic MSEs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $MSE(L,U)$		$\bar{y}_{SDr_N}$ $MSE(L,U)$		$\bar{y}_{SKr_N}$ $MSE(L,U)$		$\bar{y}_{USr_N}$ $MSE(L,U)$		$t_{p_N}^{G_1}$ $MSE(L,U)$		$t_{p_N}^{G_2}$ $MSE(L,U)$		$t_{p_N}^{G_3}$ $MSE(L,U)$		$t_{p_N}^{G_4}$ $MSE(L,U)$		$t_{p_N}^{G_5}$ $MSE(L,U)$	
$[60,60]$	0.79376	0.71468	0.68268	0.69029	0.92986	0.75644	0.67749	0.71476	0.36048	0.49557	0.37457	0.49494	0.49492	0.49492	0.38379	0.49608	0.38321	0.49603
$[65,65]$	0.70738	0.64959	0.62434	0.63017	0.82481	0.68538	0.62051	0.65438	0.33174	0.46315	0.34235	0.46272	0.46271	0.46271	0.34892	0.46352	0.34851	0.46349
$[70,70]$	0.6408	0.58546	0.56655	0.57157	1.02672	0.61103	0.56208	0.59378	0.30331	0.43304	0.31307	0.43287	0.43287	0.43287	0.31948	0.43323	0.31906	0.43321
$[75,75]$	0.58023	0.53854	0.52302	0.52725	0.64351	0.55999	0.52060	0.54858	0.28057	0.40533	0.28759	0.40525	0.40525	0.40525	0.29178	0.40546	0.29152	0.40545

Table 4: Neutrosophic REs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $RE(L,U)$		$\bar{y}_{SDr_N}$ $RE(L,U)$		$\bar{y}_{SKr_N}$ $RE(L,U)$		$\bar{y}_{USr_N}$ $RE(L,U)$		$t_{p_N}^{G_1}$ $RE(L,U)$		$t_{p_N}^{G_2}$ $RE(L,U)$		$t_{p_N}^{G_3}$ $RE(L,U)$		$t_{p_N}^{G_4}$ $RE(L,U)$		$t_{p_N}^{G_5}$ $RE(L,U)$	
$[60,60]$	1	1	1.162712	1.035319	0.085363	0.944794	1.171619	0.999888	2.201953	1.442137	2.119123	1.443973	2.121842	1.444031	2.068214	1.440655	2.071345	1.440800
$[65,65]$	1	1	1.133004	1.030817	0.857628	0.947781	1.139998	0.99268	2.132333	1.402548	2.066248	1.403851	2.068363	1.403881	2.027342	1.401428	2.029727	1.401519
$[70,70]$	1	1	1.131056	1.024301	0.624123	0.958153	1.140051	0.985988	2.11269	1.351977	2.046827	1.352508	2.048921	1.352508	2.005759	1.351384	2.0084	1.351446
$[75,75]$	1	1	1.109384	1.021413	0.901664	0.961696	1.114541	0.981698	2.06804	1.328646	2.01756	1.328908	2.019104	1.328908	1.988587	1.32822	1.990361	1.328253

Table 5: Classical MSEs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	0.70002	0.67756	0.68675	0.67328	0.39037	0.38957	0.38955	0.39044	0.39042
65	0.63624	0.61897	0.62596	0.61597	0.34903	0.34853	0.34853	0.34907	0.34906
70	0.57653	0.56183	0.56783	0.55915	0.32683	0.32636	0.32635	0.32688	0.57341
75	0.53054	0.51869	0.52345	0.5168	0.31015	0.30982	0.30982	0.31018	0.31017

Table 6: Classical REs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	1	1.03314	1.01932	1.03971	1.79322	1.79690	1.79699	1.79290	1.79299
65	1	1.02790	1.01642	1.03290	1.82288	1.82549	1.82549	1.82267	1.82272
70	1	1.02616	1.01532	1.03108	1.76400	1.76654	1.76660	1.76373	1.00544
75	1	1.02284	1.01354	1.02658	1.71059	1.71241	1.71241	1.71043	1.71048

6.2 Simulation study-2

The following algorithm is used in R language to perform the simulation study:

(i) Nutrosophic auxiliary variable x_N has been generated from Neutrosophic normal distribution NN([0.7, 1.1], 1.2) i.e., the neutrosophic auxiliary variable x has single indetermincay where population mean μ_X is indeterminate. Thus $x_N \in [x_{N_L}, x_{N_U}]$.

(ii) Neutrosophic survey variable is generated using the model $y_N = x_N - 6e$ such that $y_N \in [y_{N_L}, y_{N_U}]$ where $e \sim N(0,1)$.

(iii) For sample sizes $n_1 \in [60,60]$, $n_2 \in [65,65]$, $n_3 \in [70,70]$ and $n_4 \in [75,75]$ various values of neutrosophic estimates are obtained with 20000 iterations.

(iv) For each neutrosophic sample size used, neutrosophic MSEs and RES have been obtained and presented in Tables.

(v) Values of estimates have been calculated under classical statistics as well and their MSEs and REs are tabulated in Tables 5-8.

Table 7: Neutrosophic MSEs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $MSE(L,U)$		$\bar{y}_{SDr_N}$ $MSE(L,U)$		$\bar{y}_{SKr_N}$ $MSE(L,U)$		$\bar{y}_{USr_N}$ $MSE(L,U)$		$t_{p_N}^{G_1}$ $MSE(L,U)$		$t_{p_N}^{G_2}$ $MSE(L,U)$		$t_{p_N}^{G_3}$ $MSE(L,U)$		$t_{p_N}^{G_4}$ $MSE(L,U)$		$t_{p_N}^{G_5}$ $MSE(L,U)$	
$[60,60]$	0.58317	0.52507	0.50408	0.50940	7.06050	0.55537	0.50234	0.53718	0.28861	0.42083	0.29898	0.42056	0.29863	0.42056	0.30594	0.42109	0.30549	0.42107
$[65,65]$	0.51971	0.47725	0.4612	0.46516	0.60191	0.50290	0.46063	0.49209	0.26721	0.39468	0.27514	0.39456	0.27487	0.39455	0.28036	0.39484	0.28003	0.39483
$[70,70]$	0.47079	0.43013	0.41825	0.42180	0.74261	0.44856	0.41663	0.44617	0.24573	0.37008	0.25288	0.37014	0.25264	0.37014	0.25790	0.70110	0.25756	0.37011
$[75,75]$	0.42629	0.39566	0.38630	0.38914	0.47267	0.41108	0.38624	0.41245	0.22828	0.34735	0.23318	0.34747	0.23302	0.34748	0.23623	0.34735	0.23604	0.34734

Table 8: Neutrosophic REs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $RE(L,U)$		$\bar{y}_{SDr_N}$ $RE(L,U)$		$\bar{y}_{SKr_N}$ $RE(L,U)$		$\bar{y}_{USr_N}$ $RE(L,U)$		$t_{p_N}^{G_1}$ $RE(L,U)$		$t_{p_N}^{G_2}$ $RE(L,U)$		$t_{p_N}^{G_3}$ $RE(L,U)$		$t_{p_N}^{G_4}$ $RE(L,U)$		$t_{p_N}^{G_5}$ $RE(L,U)$	
$[60,60]$	1	1	1.156900	1.030760	0.082596	0.945442	1.160907	0.977456	2.020616	1.247701	1.950532	1.248502	1.952818	1.248502	1.906158	1.246931	1.908966	1.24699
$[65,65]$	1	1	1.126865	1.025991	0.863435	0.948996	1.128259	0.969843	1.94495	1.209207	1.888893	1.209575	1.890748	1.209606	1.853724	1.208717	1.855908	1.208748
$[70,70]$	1	1	1.125619	1.019749	0.633967	0.958913	1.129995	0.964050	1.915883	1.162262	1.861713	1.162074	1.863482	1.16E-05	1.825475	0.613507	1.827885	1.162168
$[75,75]$	1	1	1.103521	1.016755	0.901877	0.962489	1.103692	0.959292	1.867400	1.139082	1.828159	1.138688	1.829414	1.138655	1.804555	1.139082	1.806007	1.139114

Table 9: Classical MSEs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	0.51430	0.49854	0.50479	0.49612	0.33024	0.32978	0.32978	0.33028	0.33027
65	0.46744	0.45557	0.46018	0.45409	0.30102	0.30078	0.30077	0.30104	0.30103
70	0.42357	0.41335	0.41736	0.41196	0.2837	0.28345	0.28345	0.28373	0.28372
75	0.38978	0.38173	0.3848	0.38095	0.26904	0.26891	0.26891	0.26905	0.26905

Table 10: Classical REs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	1	1.031612	1.01884	1.036644	1.557352	1.559525	1.559525	1.557164	1.557211
65	1	1.026055	1.015776	1.029399	1.552854	1.554093	1.554144	1.55275	1.552802
70	1	1.024725	1.014879	1.028182	1.493021	1.494338	1.494338	1.492863	1.492916
75	1	1.021088	1.012942	1.023179	1.448781	1.449481	1.449481	1.448727	1.448727

6.3 Simulation study-3

The following algorithm is used in R language to perform the simulation study:

(i) Nutrosophic auxiliary variable x_N has been generated from Neutrosophic normal distribution NN([0.75, 1.1], 1.2) i.e., the neutrosophic auxiliary variable x has single indetermincay where population mean μ_X is indeterminate. Thus $x_N \in [x_{N_L}, x_{N_U}]$.

(ii) Neutrosophic survey variable is generated using the model $y_N = x_N - 7e$ such that $y_N \in [y_{N_L}, y_{N_U}]$ where $e \sim N(0,1)$.

(iii) For sample sizes $n_1 \in [60,60]$, $n_2 \in [65,65]$, $n_3 \in [70,70]$ and $n_4 \in [75,75]$ various values of neutrosophic estimates are obtained with 20000 iterations.

(iv) For each neutrosophic sample size used, neutrosophic MSEs and RES have been obtained and presented in Tables .

(v) Values of estimates have been calculated under classical statistics as well and their MSEs and REs are tabulated in Tables 9-1.

Table 11: Neutrosophic MSEs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $MSE(L,U)$		$\bar{y}_{SDr_N}$ $MSE(L,U)$		$\bar{y}_{SKr_N}$ $MSE(L,U)$		$\bar{y}_{USr_N}$ $MSE(L,U)$		$t_{p_N}^{G_1}$ $MSE(L,U)$		$t_{p_N}^{G_2}$ $MSE(L,U)$		$t_{p_N}^{G_3}$ $MSE(L,U)$		$t_{p_N}^{G_4}$ $MSE(L,U)$		$t_{p_N}^{G_5}$ $MSE(L,U)$	
$[60,60]$	0.56715	0.52507	0.50377	0.5094	0.64524	0.55537	0.50262	0.53718	0.30052	0.42083	0.30811	0.42056	0.30785	0.42056	0.31202	0.42109	0.31173	0.42107
$[65,65]$	0.50815	0.47725	0.46082	0.46516	0.55396	0.5029	0.46063	0.49209	0.2788	0.39468	0.28427	0.39456	0.28405	0.39455	0.28699	0.39484	0.28679	0.39483
$[70,70]$	0.46026	0.43013	0.41799	0.4218	5.3525	0.44856	0.41683	0.44617	0.25763	0.37008	0.26279	0.37014	0.26262	37014	0.26538	0.37011	0.26519	0.37011
$[75,75]$	0.41836	0.39566	0.38598	0.38914	0.5705	0.41108	0.38646	0.41245	0.23982	0.34735	0.24343	0.34747	0.24331	0.34748	0.24543	0.34735	0.2451	0.34734

Table 12: Neutrosophic REs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $RE(L,U)$		$\bar{y}_{SDr_N}$ $RE(L,U)$		$\bar{y}_{SKr_N}$ $RE(L,U)$		$\bar{y}_{USr_N}$ $RE(L,U)$		$t_{p_N}^{G_1}$ $RE(L,U)$		$t_{p_N}^{G_2}$ $RE(L,U)$		$t_{p_N}^{G_3}$ $RE(L,U)$		$t_{p_N}^{G_4}$ $RE(L,U)$		$t_{p_N}^{G_5}$ $RE(L,U)$	
$[60,60]$	1	1	1.125811	1.030762	0.878975	0.945442	1.128387	0.977456	1.887229	1.247701	1.840739	1.248502	1.842293	1.248502	1.817672	1.246931	1.819363	1.24699
$[65,65]$	1	1	1.102708	1.025991	0.917304	0.948996	1.103163	0.969843	1.822633	1.209207	1.787561	1.209575	1.788946	1.209606	1.770619	1.208717	1.771854	1.208748
$[70,70]$	1	1	1.101127	1.019749	0.08599	0.958913	1.104191	0.96405	1.786516	1.162262	1.751437	1.162074	1.75257	1.16E-05	1.734343	1.162168	1.735586	1.162168
$[75,75]$	1	1	1.08389	1.016755	0.733322	0.962489	1.082544	0.959292	1.744475	1.139082	1.718605	1.138688	1.719453	1.138655	1.7046	1.139082	1.706895	1.139114

Table 13: Classical MSEs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	0.51298	0.4984	0.50418	49604	0.33895	0.33853	0.33852	0.33888	0.33887
65	0.46636	0.45541	0.45966	0.454	0.30994	0.30979	0.30972	0.3099	0.30989
70	0.4227	0.41324	0.41695	0.41189	0.2924	0.29217	0.29216	0.29236	0.29235
75	0.38903	0.38161	0.38443	0.38087	0.27718	0.27707	0.27707	0.27716	0.27716

Table 14: Classical REs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	1	1.029254	1.017454	1.03E-05	1.513439	1.515316	1.515361	1.513751	1.513796
65	1	1.024044	1.014576	1.027225	1.504678	1.505407	1.505747	1.504873	1.504921
70	1	1.022892	1.013791	1.026245	1.445622	1.44676	1.44681	1.44582	1.44587
75	1	1.019444	1.011966	1.021425	1.403528	1.404086	1.404086	1.40363	1.40363

6.4 Simulation study-4

The following algorithm is used in R language to perform the simulation study:

(i) Nutrosophic auxiliary variable x_N has been generated from Neutrosophic normal distribution NN([0.65, 1.1], 1.2) i.e., the neutrosophic auxiliary variable x has single indetermincay where population mean μ_X is indeterminate. Thus $x_N \in [x_{N_L}, x_{N_U}]$.

(ii) Neutrosophic survey variable is generated using the model $y_N = x_N - 6e$ such that $y_N \in [y_{N_L}, y_{N_U}]$ where $e \sim N(0,1)$.

(iii) For sample sizes $n_1 \in [60, 60]$, $n_2 \in [65, 65]$, $n_3 \in [70, 70]$ and $n_4 \in [75, 75]$ various values of neutrosophic estimates are obtained with 20000 iterations.

(iv) For each neutrosophic sample size used, neutrosophic MSEs and RES have been obtained and presented in Tables.

(v) Values of estimates have been calculated under classical statistics as well and their MSEs and REs are tabulated in Tables 13-16

Table 15: Neutrosophic MSEs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $MSE(L,U)$		$\bar{y}_{SDr_N}$ $MSE(L,U)$		$\bar{y}_{SKr_N}$ $MSE(L,U)$		$\bar{y}_{USr_N}$ $MSE(L,U)$		$t_{p_N}^{G_1}$ $MSE(L,U)$		$t_{p_N}^{G_2}$ $MSE(L,U)$		$t_{p_N}^{G_3}$ $MSE(L,U)$		$t_{p_N}^{G_4}$ $MSE(L,U)$		$t_{p_N}^{G_5}$ $MSE(L,U)$	
$[60,60]$	0.60649	0.52507	0.50434	0.50940	160621	0.55537	0.50209	0.53718	0.28066	0.42083	0.29939	0.42056	0.29870	0.42056	0.31687	0.42109	0.31575	0.42107
$[65,65]$	0.53563	0.47725	0.46151	0.46516	0.95151	0.50290	0.46012	0.49209	0.26131	0.39468	0.28742	0.39456	0.28619	0.39455	0.32693	0.39484	0.32387	0.39483
$[70,70]$	0.48835	0.43013	0.41848	0.42180	0.58047	0.44856	0.41644	0.44617	0.23605	0.37008	0.25087	0.37014	0.25018	0.37014	0.27917	0.37011	0.27641	0.37011
$[75,75]$	0.43770	0.39566	0.38660	0.38914	3.32296	0.41108	0.38605	0.41245	0.21814	0.34735	0.22565	0.34747	0.22539	0.34748	0.23187	0.34735	0.23148	0.34734

Table 16: Neutrosophic REs of all the neutrosophic estimators

Sample Size	$\bar{y}_{R_N}$ $RE(L,U)$		$\bar{y}_{SDr_N}$ $RE(L,U)$		$\bar{y}_{SKr_N}$ $RE(L,U)$		$\bar{y}_{USr_N}$ $RE(L,U)$		$t_{p_N}^{G_1}$ $RE(L,U)$		$t_{p_N}^{G_2}$ $RE(L,U)$		$t_{p_N}^{G_3}$ $RE(L,U)$		$t_{p_N}^{G_4}$ $RE(L,U)$		$t_{p_N}^{G_5}$ $RE(L,U)$	
$[60,60]$	1	1	1.202542	1.030762	3.78E-06	0.945442	1.207931	0.977456	2.160942	1.247701	2.025752	1.248502	2.030432	1.248502	1.914003	1.246931	1.920792	1.24699
$[65,65]$	1	1	1.160603	1.025991	0.562926	0.948996	1.164109	0.969843	2.049788	1.209207	1.863579	1.209575	1.871589	1.209606	1.638363	1.208717	1.653843	1.208748
$[70,70]$	1	1	1.166961	1.019749	0.841301	0.958913	1.172678	0.96405	2.068841	1.162262	1.946626	1.162074	1.951995	1.162074	1.749293	1.162168	1.76676	1.162168
$[75,75]$	1	1	1.132178	1.016755	0.13172	0.962489	1.133791	0.959292	2.00651	1.139082	1.93973	1.138688	1.941967	1.138655	1.887696	1.139082	1.890876	1.139114

Table 17: Classical MSEs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	0.51574	0.49868	0.50544	0.4962	0.3217	0.32121	0.3212	0.32189	0.32188
65	0.46836	0.45572	0.46072	0.45418	0.29226	0.292	0.29199	0.29238	0.29237
70	0.42453	0.41346	0.4187	0.41203	0.27515	0.27487	0.27486	0.27526	0.27525
75	0.3906	0.38186	0.3852	0.38103	0.26101	0.26086	0.26085	0.26108	0.26107

Table 18: Classical REs of all the neutrosophic estimators

Sample size	$\bar{y}_{R_N}$	$\bar{y}_{SDr_N}$	$\bar{y}_{SKr_N}$	$\bar{y}_{USr_N}$	$t_{p_N}^{G_1}$	$t_{p_N}^{G_2}$	$t_{p_N}^{G_3}$	$t_{p_N}^{G_4}$	$t_{p_N}^{G_5}$
60	1	1.03421	1.020378	1.039379	1.603171	1.605616	1.605666	1.602224	1.602274
65	1	1.027736	1.016583	1.031221	1.602546	1.603973	1.604028	1.601888	1.601943
70	1	1.026774	1.013924	1.030338	1.542904	1.544476	1.544532	1.542287	1.542343
75	1	1.022888	1.014019	1.025116	1.496494	1.497355	1.497412	1.496093	1.49615

7. DISCUSSION AND CONCLUSION

Vagueness or indetermincacy is usually observed in the collected data. Instead of using Fuzzy logic to deal with such a data set it would be more easier and cost resource efficient to use neutrosophic statistical tools. Due to its need and wide applicability research in neutrosophic statistics has been regorously carried out. This paper aims at further developing the existing theory of Neutrosophic Simple Random Sampling Without Replacement (NSRSWOR). In this paper, a generalized neutrosophic exponential robust ratio type estimator $t_{p_N}^{G}$ has been presented using some known population parameters of neutrosophic auxiliary variables.

From the proposed generalized neutrosophic exponential robust ratio type estimator, five generalized neutrosophic exponential robust ratio type estimators $t_{p_N}^{G_1}$-$t_{p_N}^{G_5}$ have been developed using known population parameter values of auxiliary variables viz., Hodges Lehmann, Tri mean, Mid range and coefficient of variation. The high efficiency of the developed neutrosophic estimators have been demonstrated using an empirical and four simulation studies the results of which are presented in Tables 1-18.

In the empirical study on daily stock price, we can see that the proposed estimators provide a lower MSE indicating high efficiency that the similar existing neutrosophic estimators. In simulation studies, it is clear from the results that the developed neutrosophic estimators $t_{p_N}^{G_1}$-$t_{p_N}^{G_5}$ $from$ the proposed generalized neutrosphic estimator $t_{p_N}^{\mathrm{G}}$ provide a much lower MSE as compared to the similar existing neutrosophic ratio type estimators discussed in this paper (Table [3-4], Table [7-8], Table [11-12] and Table [15-16]). The results of the neutrosophic estimators estimators have also been compared with their classical values (Table [5-6], Table [9-10], Table [13-14] and Table [17-18]). It can be seen that the classical values of MSE falls in the indetermancy neutrosophic MSE intervals implying that when the data contains some indetermincay, neutrosophic estimators should be used. Further, it can be seen that, proposed neutrosophic estimators $t_{p_N}^{G_1}$-$t_{p_N}^{G_5}$ provide lowest MSE in neutrosophic as well as classical form and thus it is advised to use the proposed neutrosophic estimators $t_{p_N}^{G_1}$-$t_{p_N}^{G_5}$ when the data at hand is neutrosophic.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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