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# Stochastic Discount Factors under Uncertainty States: Theory and Evidence from Equity Portfolio Pricing

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**Abstract.** This study examines whether uncertainty variables improve stochastic discount factor (SDF) performance in cross-sectional asset pricing beyond the Fama–French five-factor (FF5) model. Using monthly U.S. data from January 1985 to March 2026, including FF5 factors, the VIX index, the Economic Policy Uncertainty (EPU) index, and twenty-five size–book-to-market portfolios, the paper develops an uncertainty-augmented SDF framework. Pricing performance is evaluated through time-series regressions and the Gibbons–Ross–Shanken (GRS) efficiency test. The analysis further distinguishes between persistent uncertainty states and transitory uncertainty shocks by comparing uncertainty levels and first-difference measures. The results indicate that the benchmark FF5 model continues to exhibit economically meaningful pricing errors. Incorporating uncertainty variables improves pricing efficiency, particularly for value-weighted portfolios. VIX generates the greatest improvement, reducing the FF5 GRS statistic by more than 60% and eliminating rejection of the joint pricing restrictions in several specifications. Persistent uncertainty states contribute more strongly to pricing efficiency than short-run uncertainty shocks, highlighting the importance of state-dependent pricing mechanisms in modern asset pricing.

## 1. Introduction

Modern asset pricing theory is fundamentally built upon the stochastic discount factor (SDF) framework, in which equilibrium expected returns are determined by the covariance structure between asset payoffs and aggregate sources of systematic risk. Within this framework, empirical factor models attempt to approximate the underlying pricing kernel through observable risk factors. Traditional models such as the Capital Asset Pricing Model (CAPM) and subsequent multifactor extensions therefore represent reduced-form implementations of broader equilibrium pricing relations. Among these specifications, the Fama–French five-factor model has become one of the dominant benchmark models in empirical finance because it substantially improves the cross-sectional explanation of stock returns relative to earlier linear factor structures [14].

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Despite the empirical success of multifactor asset pricing models, a large body of evidence suggests that substantial pricing errors continue to persist across portfolios, market conditions, and anomaly strategies. In particular, conventional linear factor models often perform poorly during periods characterized by elevated macro-financial uncertainty, volatility clustering, financial stress, and nonlinear market dynamics. The increasing frequency of global crises over recent decades—including the global financial crisis, the COVID-19 pandemic, monetary policy disruptions, and geopolitical conflicts—has intensified interest in uncertainty as a systematic source of equilibrium pricing variation. These developments suggest that expected returns may depend not only on standard factor exposures, but also on state-dependent uncertainty conditions influencing aggregate discount rates, risk premia, and investment opportunities.

From a theoretical perspective, intertemporal asset pricing models imply that equilibrium expected returns should depend on variables describing future economic conditions and state transitions. Merton's Intertemporal Capital Asset Pricing Model (ICAPM) formally demonstrates that investors price assets according to both contemporaneous market risk and variables governing future investment opportunities [2]. Similarly, Hansen and Jagannathan (1991) show that admissible stochastic discount factors are tightly constrained by observed return dynamics, implying that omitted state variables may generate persistent pricing errors [4]. Within this framework, uncertainty may operate as a conditioning state variable affecting equilibrium discounting through time-varying risk aversion, precautionary demand, conditional covariances, and long-run discount-rate dynamics.

Recent literature increasingly emphasizes that equilibrium risk compensation is conditional, nonlinear, and state dependent, particularly during periods of elevated macro-financial uncertainty and volatility stress [12, 13, 16, 18]. Jurado, Ludvigson, and Ng (2015) argue that macroeconomic uncertainty should be interpreted as the conditional volatility of unforecastable economic components rather than observable volatility alone [13]. Similarly, Bekaert, Engstrom, and Xu (2022) show that time-varying risk appetite and uncertainty substantially affect equilibrium expected returns and market conditions [18]. Ludvigson, Ma, and Ng (2021) further distinguish between endogenous and exogenous uncertainty dynamics, suggesting that uncertainty may evolve jointly with macroeconomic conditions and financial market regimes [16]. Together, these studies imply that uncertainty may directly modify the stochastic discount factor itself rather than merely influencing short-run return predictability.

A related strand of literature examines the role of volatility risk and variance premia in asset pricing. Bollerslev, Tauchen, and Zhou (2009) show that variance risk premia contain substantial information regarding expected stock returns and aggregate risk compensation [25]. Market-implied uncertainty measures such as the VIX index have therefore become widely interpreted as forward-looking indicators of aggregate fear, expected volatility, and market stress conditions [7]. Unlike backward-looking macroeconomic variables, implied volatility indices directly incorporate investors' expectations regarding future downside risk and financial fragility. More recent

evidence additionally suggests that uncertainty may affect equilibrium pricing kernels through latent macroeconomic uncertainty, variance risk premia, and time-varying risk appetite rather than through observable volatility alone [13, 18, 25].

The growing importance of uncertainty in financial markets is also closely related to the literature on conditional and adaptive market efficiency. Lo (2004) proposes the Adaptive Market Hypothesis, arguing that market efficiency evolves dynamically across changing economic environments rather than remaining constant through time [9]. Similarly, Adrian, Etula, and Muir (2014) demonstrate that intermediary balance-sheet constraints influence the cross-section of expected returns through state-dependent pricing mechanisms [12]. Ang, Chen, and Xing (2006) further show that downside market risk commands a significant premium in equity markets [10], while Lettau and Wachter (2007) emphasize the importance of long-run discount-rate variation and time-varying risk premia in explaining asset pricing anomalies [11]. These frameworks collectively imply that equilibrium pricing relations may vary substantially across uncertainty regimes and macro-financial states.

The uncertainty literature has additionally expanded beyond macroeconomic volatility toward broader forms of systemic and geopolitical risk. Baker, Bloom, and Davis (2016) construct the Economic Policy Uncertainty (EPU) index and show that policy-related uncertainty exerts substantial effects on investment, macroeconomic activity, and financial conditions [15]. Caldara and Iacoviello (2022) further demonstrate that geopolitical risk shocks significantly affect financial markets and investor behavior [17]. More recent studies continue to document that geopolitical fragility, macro financial asymmetries, and uncertainty-related shocks generate economically meaningful return dynamics beyond conventional factor structures [19]. At the same time, a growing literature suggests that traditional factor models remain incomplete when higher-order risks and nonlinear state dynamics become economically important. Recent studies document that skewness risk, exchange-rate-related risk, and macro-financial asymmetries continue to generate systematic return patterns beyond standard pricing structures. For example, Huynh and Khoa (2025) show that skewness risk generates economically significant pricing effects in the Thai stock market [22], while Huynh, Khoa, and An (2025) document return reversals associated with exchange-rate-related portfolio optimization in Vietnam's equity market [23]. These findings reinforce the broader argument that equilibrium pricing kernels may depend on state variables extending beyond conventional linear factor specifications.

Although previous studies establish important links between uncertainty and financial markets, several gaps remain in the literature. First, much of the existing uncertainty literature focuses primarily on return predictability, volatility forecasting, or macroeconomic transmission mechanisms rather than direct stochastic discount factor performance. Consequently, relatively limited evidence exists regarding whether uncertainty variables improve the joint cross-sectional pricing efficiency of benchmark multifactor models. Second, existing studies often treat uncertainty as a temporary shock variable rather than distinguishing between persistent uncertainty states

and transitory uncertainty innovations. This distinction may be economically important because long-run state variables affect conditional expectations, discount-rate dynamics, and intertemporal hedging demands more fundamentally than short-run disturbances. Third, the relative pricing roles of market-based uncertainty measures such as VIX and policy-related uncertainty measures such as EPU remain insufficiently understood within a unified stochastic discount factor framework. This study addresses these gaps by developing an uncertainty-augmented stochastic discount factor framework and evaluating its empirical pricing performance in the U.S. equity market. Using monthly data from January 1985 to March 2026, the analysis combines the Fama–French five factors with uncertainty variables represented by the VIX and EPU indices. The empirical analysis employs twenty-five portfolios formed on size and book-to-market characteristics and evaluates pricing performance using time-series regressions, Fama–MacBeth cross-sectional regressions [1], and the Gibbons–Ross–Shanken (GRS) efficiency test [3]. Importantly, the study additionally distinguishes between persistent uncertainty states and transitory uncertainty shocks by comparing uncertainty levels and first-difference uncertainty measures.

The paper contributes to the literature in several important ways. First, the study extends the stochastic discount factor framework by incorporating uncertainty directly as a state-dependent pricing component rather than treating uncertainty solely as a predictive macroeconomic variable. Second, the analysis contributes to the literature on conditional and adaptive asset pricing by showing that uncertainty conditions materially influence cross-sectional pricing efficiency beyond standard factor structures. Third, the study provides new evidence regarding the relative roles of market-implied uncertainty and policy-related uncertainty within equilibrium pricing kernels. Finally, the paper contributes to the broader literature on uncertainty and nonlinear pricing dynamics by demonstrating that persistent uncertainty states contribute more strongly to pricing efficiency than transitory uncertainty shocks.

The remainder of the paper is organized as follows. Section 2 develops the theoretical framework and reviews the related literature. Section 3 describes the data, variables, and empirical methodology. Section 4 presents the empirical results and robustness analyses. Section 5 discusses the economic implications of the findings, and Section 6 concludes.

## 2. Theoretical Framework

2.1. **Probability Space and Asset Returns.** Modern asset pricing theory is built on the idea that equilibrium asset prices are determined by the intertemporal trade-off between risk and expected return. In continuous-time and discrete-time dynamic economies, the stochastic discount factor provides a unified representation linking asset prices to investors' marginal rates of substitution. Following the stochastic discount factor literature developed in intertemporal asset pricing models [2, 4], consider the filtered probability space

$$(\Omega, \mathcal{F}, \mathbb{P}) \tag{2.1}$$

where $\Omega$ denotes the sample space, F is the sigma-algebra, and P is the underlying probability measure. Let

$$\{\mathcal{F}_t\}_{t\geq 0} \tag{2.2}$$

denote the filtration representing the information available to investors at time t. Asset returns are modeled as stochastic processes adapted to the filtration. Specifically, let

$$R_{i,t+1} \tag{2.3}$$

denote the gross return on asset i between periods t and t + 1, where i = 1,..., N.

In equilibrium asset pricing models, investors form expectations conditionally on the information set $F_t$. Expected returns therefore depend on the covariance structure between asset payoffs and the stochastic discount factor. This framework underlies both classical linear factor models and more general dynamic asset pricing economies.

2.2. **Uncertainty State Variables.** A central implication of intertemporal asset pricing theory is that state variables affecting future investment opportunities should enter the pricing kernel [2]. In modern financial markets, uncertainty measures may serve as economically meaningful state variables because they capture changes in aggregate expectations, risk perceptions, and market conditions. Recent literature identifies several uncertainty proxies with potential relevance for equilibrium pricing. Baker, Bloom, and Davis (2016) [15] develop the Economic Policy Uncertainty (EPU) index, measuring uncertainty related to fiscal, monetary, and regulatory policy conditions. In parallel, the Volatility Index (VIX) has become a widely used market-implied measure of expected stock market volatility and aggregate investor fear [7]. Unlike backward-looking macroeconomic variables, VIX aggregates forward-looking information embedded in option prices and therefore may contain information about future economic states and time-varying discount rates.

Motivated by this literature, define the uncertainty state vector

$$X_t = \left(VIX_t,\ EPU_t\right)^{'} \tag{2.4}$$

The state dynamics are assumed to evolve according to a general stochastic process:

$$X_{t+1} = \mu\left(X_t\right) + \Sigma\left(X_t\right)\varepsilon_{t+1} \tag{2.5}$$

where $\mu(\cdot)$ represents the conditional drift component, $\Sigma(\cdot)$ denotes the conditional volatility structure, and $\varepsilon_{t+1}$ is a vector of innovations satisfying

$$E_t\left[\varepsilon_{t+1}\right] = 0 \tag{2.6}$$

This formulation allows uncertainty states to evolve dynamically over time and influence equilibrium asset prices through the stochastic discount factor. Persistent uncertainty conditions may therefore alter expected returns by changing covariance structures and the market price of risk.

2.3. **Stochastic Discount Factor.** The stochastic discount factor provides the fundamental no-arbitrage pricing relation linking asset returns to investors' intertemporal marginal rates of substitution. Following Hansen and Jagannathan (1991) [4], let $M_{t+1}$ denote the stochastic discount factor. The Euler equation is given by

$$\mathbb{E}_t \left[ M_{t+1} R_{i,t+1} \right] = 1 \tag{2.7}$$

Equation (2.7) states that equilibrium asset prices satisfy a conditional moment restriction under the absence of arbitrage opportunities. For excess returns,

$$R^e_{i,t+1} = R_{i,t+1} - R_{f,t+1} \tag{2.8}$$

the pricing equation becomes

$$\mathbb{E}_t \left[ M_{t+1} R^e_{i,t+1} \right] = 0 \tag{2.9}$$

The stochastic discount factor, therefore, determines expected returns through covariance restrictions between returns and systematic sources of risk. Any omitted state variable affecting the pricing kernel may lead to systematic pricing errors and rejection of equilibrium restrictions.

2.4. **Linear Factor Representation.** Empirical asset pricing models commonly approximate the stochastic discount factor using linear factor structures. Fama and French (2015) [14] show that the market, size, value, profitability, and investment factors capture a substantial portion of cross-sectional return variation. Nevertheless, the persistence of pricing anomalies suggests that additional state variables may still be relevant. To incorporate uncertainty into the pricing kernel, specify the linear stochastic discount factor as

$$M_{t+1} = 1 - b^{'} F_{t+1} \tag{2.10}$$

where b is the vector of factor loadings and

$$F_t = \left( MKT_t, SMB_t, HML_t, RMW_t, CMA_t, U_t \right)^{'} \tag{2.11}$$

The vector $U_t$ represents the uncertainty component entering the pricing kernel. In the empirical analysis, $U_t$ alternatively denotes: the level of VIX, the level of EPU, changes in VIX, and changes in EPU. Under this specification, uncertainty acts as an additional state variable augmenting the benchmark Fama–French factor structure. Persistent uncertainty states may therefore affect equilibrium discounting and expected returns through the stochastic discount factor. Importantly, the uncertainty variables considered in this study are not interpreted as traded factor-mimicking portfolios. Instead, they are treated as macro-financial state variables entering the reduced-form stochastic discount factor representation. Under conditional asset pricing frameworks, state variables may influence equilibrium discounting through time-varying investment opportunities, conditional covariances, and state-dependent risk premia. Consequently, the empirical specifications should be interpreted as reduced-form approximations of a conditional stochastic discount factor economy rather than purely static linear factor models.

2.5. **Cross-Sectional Pricing Implications.** The linear factor representation generates standard cross-sectional pricing implications. Substituting Equation (2.10) into the Euler equation yields

$$\mathbb{E}\left[R_i^e\right] = Cov\left(R_i^e, F\right) b \tag{2.12}$$

Define the factor loading vector as

$$\beta_i = Cov\left(R_i, F\right)\ \Sigma_F^{-1} \tag{2.13}$$

where $\Sigma_F = Var(F)$ is the factor covariance matrix. Expected excess returns can then be expressed as

$$\mathbb{E}\left(R_i^e\right) = \beta_i^{'}\lambda \tag{2.14}$$

where $\lambda$ denotes the vector of factor risk premia. Equation (2.13) establishes the central empirical prediction of the stochastic discount factor framework: assets with greater exposure to systematic uncertainty-related risks should earn different equilibrium expected returns. If uncertainty variables improve the pricing kernel, they should reduce pricing errors and potentially command significant cross-sectional risk premia.

2.6. **Reduced-Form Estimation.** The theoretical stochastic discount factor framework motivates several reduced-form empirical procedures commonly used in asset pricing tests. First, time-series factor regressions are estimated as

$$R_{i,t} = \alpha_i + \beta_i^{'} F_t + \varepsilon_{i,t} \tag{2.15}$$

where $\alpha_{\mathrm{i}}$ denotes the pricing error associated with portfolio i. Under a correctly specified stochastic discount factor, $\alpha_{\mathrm{i}} = 0$ for all assets. Second, cross-sectional risk premia are estimated using the two-pass Fama–MacBeth procedure [1]. The cross-sectional regression is given by

$$\overline{R}_i = \gamma_0 + \beta_i^{'}\lambda + u_i \tag{2.16}$$

where $\lambda$ measures the equilibrium price of systematic risk. Third, joint pricing efficiency is evaluated using the Gibbons–Ross–Shanken test [3]. The null hypothesis is

$$H_0 : \alpha = 0 \tag{2.17}$$

The GRS framework jointly tests whether all pricing errors are statistically indistinguishable from zero. Rejection of the null hypothesis implies that the proposed factor structure fails to fully explain the cross-section of expected returns. The theoretical interpretation of uncertainty as a state-dependent component of the stochastic discount factor is closely related to several strands of modern asset pricing literature. Traditional linear factor models, including the Fama–French framework, assume that systematic risk exposures can be represented by a relatively stable set of aggregate pricing factors. However, a growing body of research argues that market conditions, volatility regimes, and macroeconomic uncertainty may alter the dynamics of expected returns and pricing kernels over time.

From a theoretical perspective, intertemporal asset pricing models imply that state variables governing future investment opportunities should enter the stochastic discount factor directly through conditional expected returns and time-varying risk premia. Hansen and Jagannathan (1991) [4] show that admissible stochastic discount factors are tightly constrained by the joint distribution of asset returns, implying that omitted state variables may generate persistent pricing errors. Similarly, Cochrane (2001) [8] argues that factor models should be interpreted as reduced-form representations of underlying stochastic discount factors rather than purely statistical decompositions of returns. The role of uncertainty in asset pricing has become increasingly important in recent literature. Baker, Bloom, and Davis (2016) [15] demonstrate that economic policy uncertainty exerts significant effects on macroeconomic and financial conditions, while Caldara and Iacoviello (2022) [17] show that geopolitical risk generates substantial disruptions in financial markets and investment behavior. More recent studies have further expanded empirical research on financial markets by examining return predictability, funding conditions, market efficiency, and financial resilience, highlighting the importance of incorporating broader market information into empirical asset-pricing frameworks [19–21, 26]. The interpretation of uncertainty as a pricing state variable is also closely related to the literature on conditional and adaptive asset pricing. Lo (2004) [9] proposes the Adaptive Market Hypothesis, which argues that market efficiency evolves dynamically across changing economic environments. Similarly, Shleifer and Vishny (1997) [5] and De Long et al. (1990) [6] emphasize that limits to arbitrage and noise-trader risk may generate persistent deviations from standard equilibrium pricing relations, particularly during periods of elevated uncertainty and market stress.

Several recent studies additionally document that higher-order risks and uncertainty-related asymmetries remain insufficiently captured by traditional factor models. Ang et al. (2006) [10] show that downside market risk commands a significant premium in equity markets, while Adrian, Etula, and Muir (2014) [12] and Lettau and Wachter (2007) [11] demonstrate that intermediary constraints and time-varying risk premia play important roles in shaping expected returns. In emerging markets, recent evidence further indicates that skewness risk, exchange-rate risk, and macro-financial asymmetries generate persistent return patterns beyond standard factor structures [22, 23].

Within this broader theoretical framework, uncertainty variables such as VIX and EPU may be interpreted as proxies for latent state variables governing aggregate discount-rate dynamics, market fragility, and investor risk aversion. Persistent uncertainty states may therefore influence the stochastic discount factor through long-run variations in expected returns, whereas uncertainty shocks may capture only temporary market disturbances. Consequently, the empirical analysis evaluates whether uncertainty-augmented stochastic discount factors improve cross-sectional pricing performance relative to the benchmark Fama–French five-factor model. In particular, the analysis investigates whether persistent uncertainty states contribute more strongly to pricing efficiency than transitory uncertainty shocks.

## 3. Methodology

3.1. **Data and Variables.** This study employs monthly U.S. financial data spanning the period from January 1985 to March 2026. The empirical analysis combines standard Fama–French asset pricing factors, portfolio returns, and uncertainty measures in order to evaluate whether uncertainty-related state variables improve the pricing performance of the stochastic discount factor framework. The benchmark asset pricing factors consist of the five Fama–French factors: the excess market return (MKT), size factor (SMB), value factor (HML), profitability factor (RMW), and investment factor (CMA). In addition, the one-month U.S. Treasury bill rate is used as the risk-free rate. These variables are obtained from the data library maintained by Kenneth R. French (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

To evaluate cross-sectional pricing performance, the analysis employs the twenty-five portfolios formed on size and book-to-market characteristics ($5 \times 5$ portfolios) from the same data source. The portfolios are constructed by independently sorting firms into five size groups and five book-to-market groups. The intersection of these sorts generates twenty-five portfolios capturing variation in firm size and valuation characteristics. Both value-weighted (VW) and equal-weighted (EW) portfolio returns are considered in order to examine whether uncertainty effects differ across weighting schemes and firm characteristics. The size breakpoints are determined using market equity, while book-to-market ratios are computed using accounting information from firms' balance sheets. Firms are first sorted into quintiles according to market capitalization and then independently sorted into quintiles according to book-to-market ratios. The resulting portfolios range from small-growth firms to large-value firms and are widely used in empirical asset pricing because they generate substantial variation in average returns and factor exposures [14]. The uncertainty variables consist of the Economic Policy Uncertainty (EPU) index and the Volatility Index (VIX). The EPU index is obtained from the Economic Policy Uncertainty database developed by Baker, Bloom, and Davis (2016) [15] (https://www.policyuncertainty.com/).

The index measures uncertainty related to economic policy conditions by combining newspaper coverage frequency, tax code provisions, and economic forecast disagreement measures. Higher values of EPU indicate elevated policy-related uncertainty. The VIX index is collected from Investing.com. VIX is widely interpreted as a market-implied measure of expected stock market volatility and aggregate investor fear [7]. Because VIX is derived from S&P 500 option prices, it aggregates forward-looking expectations regarding future market uncertainty and volatility conditions.

To distinguish between persistent uncertainty states and transitory uncertainty innovations, the analysis considers both levels and first differences of uncertainty variables. Specifically, the following transformed variables are constructed:

$$\Delta VIX_t = VIX_t - VIX_{t-1} \tag{3.1}$$

and

$$\Delta EPU_t = EPU_t - EPU_{t-1} \tag{3.2}$$

All portfolio returns are converted into excess returns by subtracting the one-month Treasury bill rate:

$$R^e_{i,t} = R_{i,t} - R_{f,t} \tag{3.3}$$

Table 1 reports the Description of Variables for the factor returns and uncertainty variables.

TABLE 1. Description of Variables

| Variable | Description |
|---|---|
| MKT | Excess market return, calculated as the return on the market portfolio minus the one-month U.S. Treasury bill rate. |
| SMB | Small-minus-big factor capturing the size effect in stock returns. |
| HML | High-minus-low factor capturing the value effect based on book-to-market ratios. |
| RMW | Robust-minus-weak profitability factor measuring the profitability premium. |
| CMA | Conservative-minus-aggressive investment factor capturing differences in firms' investment behavior. |
| RF | One-month U.S. Treasury bill rate used as the risk-free rate. |
| VIX | Chicago Board Options Exchange Volatility Index, representing market-implied expected stock market volatility and investor fear. |
| EPU | The Economic Policy Uncertainty index was developed by Baker, Bloom, and Davis (2016). |
| ΔVIX | The first difference of the VIX index represents volatility shocks. |
| ΔEPU | The first difference of the EPU index represents policy uncertainty shocks. |
| P1–P25 | Twenty-five portfolios were formed on size and book-to-market characteristics using value-weighted returns. |
| p1–p25 | Twenty-five portfolios were formed on size and book-to-market characteristics using equal-weighted returns. |

3.2. **Fama–French Five-Factor Model.** The benchmark pricing specification employed in this study is the Fama–French five-factor model proposed by Fama and French (2015) [14]. Relative to earlier factor structures, the FF5 model extends the traditional market, size, and value factors by additionally incorporating profitability and investment factors. The model therefore provides a richer representation of the systematic risks embedded in the cross-section of equity returns. The benchmark time-series specification is written as

$$R^e_{i,t} = \alpha_i + \beta_{i,MKT} MKT_t + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \beta_{i,RMW} RMW_t + \beta_{i,CMA} CMA_t + \varepsilon_{i,t} \tag{3.4}$$

The intercept term $\alpha_{\mathrm{i}}$ measures the pricing error associated with portfolio i. Under a correctly specified stochastic discount factor, the pricing errors should satisfy $\alpha_{\mathrm{i}} = 0$ for all test portfolios.

Although the FF5 model substantially improves cross-sectional pricing relative to earlier benchmarks, empirical evidence suggests that pricing errors may still persist across certain portfolios

and market conditions. This observation motivates the inclusion of additional state variables related to aggregate uncertainty. In particular, uncertainty may influence equilibrium discount rates through time-varying risk perceptions, changing investment opportunities, and state-dependent covariance structures. To examine whether uncertainty improves pricing performance, the benchmark FF5 specification is augmented with uncertainty-related state variables. The uncertainty-augmented specification is given by

$$R^e_{i,t} = \alpha_i + \beta_i' FF5_t + \delta_i U_t + \varepsilon_{i,t} \tag{3.5}$$

where $FF5_t = (MKT_t, SMB_t, HML_t, RMW_t, CMA_t)'$ and $U_t$ denotes the uncertainty component entering the pricing kernel. In the empirical analysis, $U_t$ alternatively represents the level and first difference of VIX and EPU. The level variables capture persistent uncertainty states, whereas the differenced variables represent transitory uncertainty innovations. Within the stochastic discount factor framework, the augmented specification allows uncertainty to affect equilibrium pricing through an additional state-dependent component. Persistent uncertainty conditions may alter long-run discounting behavior and covariance structures, while uncertainty shocks may primarily influence short-run risk compensation. The empirical analysis, therefore, evaluates whether uncertainty-related factors improve pricing efficiency relative to the benchmark FF5 model.

## 4. Empirical Results

4.1. **Descriptive Statistics and Correlation Structure.** Table 2 reports the descriptive statistics for the Fama–French five factors and the uncertainty variables over the full sample period from January 1985 to March 2026. Several important characteristics emerge from the data.

Table 2. Descriptive Statistics

| Variable | Mean | SD | Min | P25 | Median | P75 | Max |
|---|---|---|---|---|---|---|---|
| MKT | 0.741 | 4.37 | -17.2 | -1.92 | 1.22 | 3.41 | 13.6 |
| SMB | 0.082 | 3.05 | -15.5 | -1.83 | -0.02 | 1.83 | 18.5 |
| HML | 0.163 | 3.23 | -13.8 | -1.66 | -0.08 | 1.78 | 12.9 |
| RMW | 0.342 | 2.6 | -19 | -0.92 | 0.35 | 1.46 | 13 |
| CMA | 0.169 | 2.18 | -7.08 | -1.12 | -0.06 | 1.38 | 9.01 |
| VIX | 19.5 | 7.39 | 9.51 | 13.8 | 17.5 | 23.4 | 59.9 |
| EPU | 136 | 78 | 44.8 | 87.7 | 115 | 161 | 725 |
| ΔVIX | 0.058 | 4.73 | -19.4 | -2.2 | -0.15 | 1.95 | 22 |
| ΔEPU | 0.651 | 44.2 | -204 | -19 | -1.52 | 18.4 | 279 |

*(Notes: This table reports descriptive statistics for the Fama–French five factors and uncertainty variables over the sample period from January 1985 to March 2026. All factor returns are expressed in monthly percentage terms. ΔVIX and ΔEPU denote first differences of the corresponding uncertainty indices.)*

For the full sample, the market factor (MKT) exhibits the highest average return among the Fama–French factors, with a monthly mean of approximately 0.74%, confirming the dominant long-run contribution of market risk to cumulative asset returns. The profitability factor (RMW) also generates a positive average return of approximately 0.34% per month, while the size (SMB), value

(HML), and investment (CMA) factors display comparatively smaller average returns. In contrast, the post-2009 subsample reveals a noticeable decline in the average returns of several traditional style factors. In particular, the value factor (HML) becomes slightly negative after 2009, reflecting the prolonged underperformance of value-oriented strategies during the post-global financial crisis period. The uncertainty variables display substantially larger volatility than the traditional pricing factors. The average level of VIX equals approximately 19.5 over the full sample, with a standard deviation of 7.39 and a maximum value close to 60 during periods of extreme market stress. Similarly, EPU exhibits strong persistence and substantial dispersion, with a mean value of 136 and a standard deviation of approximately 78 over the full sample. The post-2009 period is characterized by even higher average EPU levels, suggesting that economic policy uncertainty became increasingly important following the global financial crisis and subsequent episodes of macroeconomic instability. The first differences of uncertainty variables, represented by $\Delta$VIX and $\Delta$EPU, exhibit substantially higher volatility than their corresponding levels. For example, the standard deviation of $\Delta$EPU exceeds 44 over the full sample and increases further after 2009. This behavior is consistent with the interpretation that uncertainty shocks are highly transitory and episodic, especially during periods of financial turmoil and macroeconomic disruptions.

The correlation matrix reported in Table 3 provides additional insights into the dependence structure among the variables. Several patterns are particularly noteworthy. First, VIX exhibits a moderately negative correlation with the market factor, with a correlation coefficient of approximately $-0.39$ over the full sample. This result is economically intuitive because periods of elevated market fear and expected volatility are typically associated with declining equity market performance.

Table 3. Correlation Matrix

| | MKT | SMB | HML | RMW | EPU | VIX |
|---|---|---|---|---|---|---|
| MKT | 1 | 0.23 | -0.12 | -0.316 | -0.013 | -0.387 |
| SMB | 0.23 | 1 | 0.029 | -0.435 | -0.012 | -0.065 |
| HML | -0.12 | 0.029 | 1 | 0.316 | -0.086 | -0.075 |
| RMW | -0.316 | -0.435 | 0.316 | 1 | -0.009 | 0.107 |
| EPU | -0.013 | -0.012 | -0.086 | -0.009 | 1 | 0.304 |
| VIX | -0.387 | -0.065 | -0.075 | 0.107 | 0.304 | 1 |

*(Notes: This table reports pairwise correlations among the benchmark Fama–French factors and uncertainty variables over the full sample period. The results indicate that VIX is moderately negatively correlated with the market factor, while EPU remains largely orthogonal to the traditional Fama–French factors.)*

Second, EPU remains relatively weakly correlated with the standard Fama–French factors. The correlation between EPU and the market factor is close to zero across both samples, while the correlations between EPU and the remaining style factors remain economically small. This finding indicates that policy uncertainty captures information largely orthogonal to traditional factor structures. Consequently, EPU may provide incremental state-variable information beyond the benchmark Fama–French factors. Third, the value and investment factors remain strongly

positively correlated, with the correlation between HML and CMA exceeding 0.60 in both samples. This result is consistent with the economic intuition underlying the FF5 model, as firms with high book-to-market ratios are often associated with relatively conservative investment behavior. Similarly, the profitability factor (RMW) exhibits a negative association with the size factor (SMB), suggesting that highly profitable firms tend to be concentrated among larger firms.

Figure 1 illustrates the cumulative performance of the Fama–French five factors over time. Several long-run dynamics are immediately apparent. The market factor overwhelmingly dominates cumulative performance across the sample period, especially after the global financial crisis. The profitability factor (RMW) also exhibits persistent positive cumulative growth, particularly after 2010, suggesting that profitability-based strategies became increasingly important in the post-crisis environment. By contrast, the size factor (SMB) contributes relatively little to long-run cumulative performance, while the value factor (HML) experiences substantial weakness after 2020. The investment factor (CMA) remains positive over the sample but exhibits comparatively modest long-run accumulation.

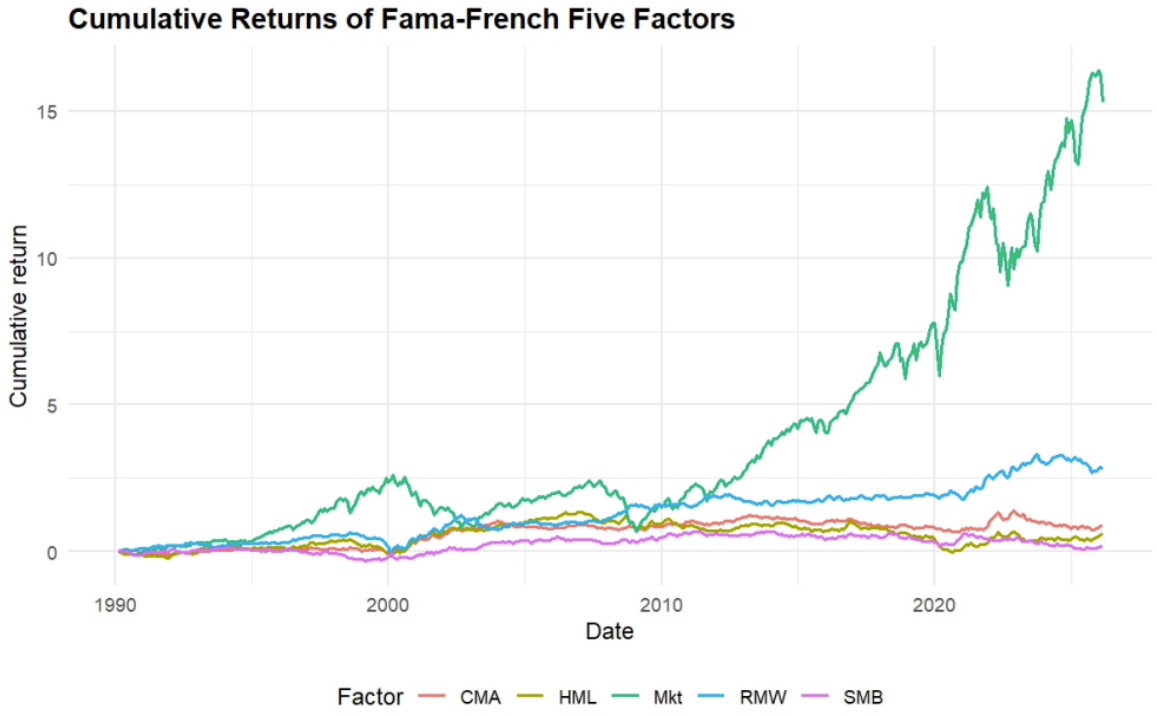



FIGURE 1. Cumulative Returns of Fama–French Five Factors

The descriptive statistics and correlation structure suggest that uncertainty variables capture dimensions of systematic risk distinct from traditional Fama–French factors. In particular, VIX appears closely related to aggregate market stress and volatility conditions, whereas EPU captures broader policy-related uncertainty that remains relatively orthogonal to standard factor exposures.

These preliminary observations provide initial support for the hypothesis that uncertainty-related state variables may improve stochastic discount factor performance in cross-sectional asset pricing.

4.2. **Baseline FF5 Performance.** Before examining the role of uncertainty-augmented models, this subsection evaluates the pricing performance of the benchmark Fama–French five-factor model. Table 4 reports the GRS statistics and pricing-error measures for the FF5 model using both value-weighted and equal-weighted portfolios over the full sample and the post-2009 subsample.

TABLE 4. Baseline Fama–French Five-Factor Model Performance

| Sample Period | Portfolio Weighting | GRS Statistic | p-value | Average Absolute Alpha | RMSE of Alpha | Adjusted $R^2$ | Significant Alphas |
|---|---|---|---|---|---|---|---|
| Full Sample | Equal-Weighted (EW) | 2.95 | 0.0000 | 0.123 | 0.172 | 0.87 | 3 |
| Full Sample | Value-Weighted (VW) | 3.12 | 0.0000 | 0.108 | 0.144 | 0.91 | 6 |
| Post-2009 | Equal-Weighted (EW) | 1.45 | 0.0876 | 0.092 | 0.161 | 0.9 | 2 |
| Post-2009 | Value-Weighted (VW) | 1.55 | 0.0558 | 0.083 | 0.12 | 0.93 | 3 |

*(Notes: This table reports the pricing performance of the Fama–French five-factor model. EW and VW denote equal-weighted and value-weighted portfolios, respectively. Avg. $|\alpha|$ denotes the average absolute pricing error, RMSE $\alpha$ denotes the root mean squared pricing error, and Sig. $\alpha$ reports the number of statistically significant alphas at the 5% level.)*

The results indicate that the benchmark FF5 model is strongly rejected over the full sample. For equal-weighted portfolios, the GRS statistic is 2.95 with a p-value close to zero. For value-weighted portfolios, the rejection is even stronger, with a GRS statistic of 3.12 and a p-value close to zero. These results imply that the joint null hypothesis of zero pricing errors is rejected for both portfolio weighting schemes. Therefore, despite its high explanatory power, the FF5 model does not fully price the cross-section of size–book-to-market portfolios over the full sample period.

The magnitude of pricing errors provides further evidence of model incompleteness. In the full sample, the average absolute alpha equals 0.123 for equal-weighted portfolios and 0.108 for value-weighted portfolios. The corresponding root mean squared pricing errors are 0.172 and 0.144, respectively. Although the adjusted $R^2$ values are high, reaching 0.87 for equal-weighted portfolios and 0.91 for value-weighted portfolios, the persistence of statistically significant pricing errors indicates that high time-series explanatory power does not necessarily imply cross-sectional pricing efficiency. The post-2009 subsample provides a somewhat different picture. The GRS statistics decline substantially to 1.45 for equal-weighted portfolios and 1.55 for value-weighted portfolios. The corresponding p-values, 0.0876 and 0.0558, indicate that the FF5 model is no longer rejected at the conventional 5% significance level, although the value-weighted result remains

close to the rejection threshold. Pricing errors also become smaller in the post-2009 period, with average absolute alphas falling to 0.092 for equal-weighted portfolios and 0.083 for value-weighted portfolios. Economically, these findings suggest that the FF5 model captures a large portion of systematic variation in portfolio returns, but its pricing performance is not uniform across sample periods. The strong full-sample rejection indicates that the standard five factors leave economically meaningful pricing errors over the longer historical period. This is consistent with the possibility that additional state variables, such as aggregate uncertainty conditions, may enter the stochastic discount factor.

The weaker rejection after 2009 suggests that the standard factor structure performs better in the more recent period, possibly because profitability and investment factors became more effective in capturing post-crisis return variation. The baseline results provide the main motivation for extending the FF5 model with uncertainty-related state variables. If uncertainty affects equilibrium discount rates or covariance structures, then augmenting the pricing kernel with VIX or EPU may reduce joint pricing errors and improve stochastic discount factor performance.

4.3. **Uncertainty-Augmented Pricing.** This subsection evaluates whether uncertainty-related state variables improve the pricing performance of the benchmark Fama–French five-factor model. In particular, the analysis compares the FF5 benchmark with uncertainty-augmented specifications incorporating VIX and EPU. Table 5 reports the pricing performance of the augmented models relative to the benchmark FF5 specification.

TABLE 5. Pricing Performance of FF5 and Uncertainty-Augmented Models

| Sample | Model | GRS | p-value | Avg. $\|\alpha\|$ | RMSE $\alpha$ | Adj. $R^2$ | Sig. $\alpha$ |
|---|---|---|---|---|---|---|---|
| Equal-weighted portfolios | | | | | | | |
| Full sample | FF5 | 2.95 | 0 | 0.123 | 0.172 | 0.87 | 3 |
| Full sample | FF5+EPU | 2.1 | 0.0017 | 0.2 | 0.248 | 0.87 | 0 |
| Full sample | FF5+VIX | 3.01 | 0 | 0.703 | 0.811 | 0.87 | 14 |
| Post-2009 | FF5 | 1.45 | 0.0876 | 0.092 | 0.161 | 0.9 | 2 |
| Post-2009 | FF5+EPU | 1.26 | 0.191 | 0.259 | 0.357 | 0.9 | 0 |
| Post-2009 | FF5+VIX | 2.71 | 0.0001 | 1.06 | 1.28 | 0.91 | 11 |
| Value-weighted portfolios | | | | | | | |
| Sample | Model | GRS | p-value | Avg. $\|\alpha\|$ | RMSE $\alpha$ | Adj. $R^2$ | Sig. $\alpha$ |
| Full sample | FF5 | 3.12 | 0 | 0.108 | 0.144 | 0.91 | 6 |
| Full sample | FF5+EPU | 1.95 | 0.0045 | 0.167 | 0.224 | 0.91 | 4 |
| Full sample | FF5+VIX | 1.15 | 0.284 | 0.239 | 0.328 | 0.91 | 2 |
| Post-2009 | FF5 | 1.55 | 0.0558 | 0.083 | 0.12 | 0.93 | 3 |
| Post-2009 | FF5+EPU | 1.09 | 0.357 | 0.188 | 0.237 | 0.93 | 1 |
| Post-2009 | FF5+VIX | 0.997 | 0.473 | 0.266 | 0.318 | 0.93 | 2 |

*(Notes: This table reports the pricing performance of uncertainty-augmented models relative to the benchmark FF5 specification. Avg. $|\alpha|$ denotes the average absolute pricing error, RMSE $\alpha$ denotes the root mean squared pricing error, and Sig. $\alpha$ reports the number of statistically significant alphas at the 5% level.)*

Several important findings emerge from the results. First, the inclusion of uncertainty variables substantially affects pricing efficiency, although the magnitude and direction of the effect

depend strongly on both the uncertainty proxy and the portfolio weighting scheme. For equal-weighted portfolios, the EPU-augmented specification improves pricing performance relative to the benchmark FF5 model. Over the full sample, the GRS statistic declines from 2.95 under the benchmark specification to 2.10 under the FF5+EPU model, corresponding to a reduction of approximately 29%. Similarly, in the post-2009 subsample, the GRS statistic decreases from 1.45 to 1.26. Although the model is not fully accepted in the full sample, the reduction in the GRS statistic suggests that policy-related uncertainty contributes additional state-variable information beyond the standard Fama–French factors. By contrast, the VIX-augmented specification performs poorly for equal-weighted portfolios.

In the full sample, the GRS statistic slightly increases from 2.95 to 3.01 after including VIX. The deterioration becomes substantially stronger in the post-2009 subsample, where the GRS statistic rises sharply to 2.71 with a p-value close to zero. The alpha-based diagnostics also deteriorate considerably. For example, the average absolute alpha increases from 0.092 under the benchmark FF5 model to more than 1.06 under the FF5+VIX specification in the post-2009 equal-weighted sample. The number of statistically significant alphas simultaneously increases from 2 to 11. These results suggest that VIX does not effectively capture the cross-sectional variation associated with smaller firms, which receive greater weight in equal-weighted portfolios. The evidence for value-weighted portfolios is markedly different. In this case, both uncertainty variables substantially improve pricing performance relative to the benchmark FF5 model. The strongest improvement is observed for the FF5+VIX specification. Over the full sample, the GRS statistic declines dramatically from 3.12 to 1.15, representing a reduction of more than 63%. More importantly, the corresponding p-value increases to 0.284, implying that the joint null hypothesis of zero pricing errors can no longer be rejected. Similar improvements remain visible in the post-2009 subsample, where the GRS statistic falls below one.

The FF5+EPU specification also improves pricing performance for value-weighted portfolios, although the improvement is somewhat weaker than that observed for VIX. Over the full sample, the GRS statistic declines from 3.12 to 1.95, while the post-2009 value decreases from 1.55 to 1.09. These results suggest that policy-related uncertainty contributes additional information to the stochastic discount factor, although its impact appears more moderate than the market-based uncertainty captured by VIX. Economically, the results imply that uncertainty affects the stochastic discount factor through heterogeneous channels. The strong performance of VIX in value-weighted portfolios suggests that market-implied uncertainty is particularly relevant for larger firms and aggregate market conditions. Because value-weighted portfolios are dominated by large-cap firms, VIX may capture broad market-wide discount-rate variation that is not fully reflected in the benchmark FF5 factors. The asymmetric performance between value-weighted and equal-weighted portfolios further suggests that uncertainty transmission operates primarily through large-cap firms rather than smaller firms. Since value-weighted portfolios place greater emphasis on firms with large market capitalization, the strong pricing improvements generated

by VIX indicate that uncertainty-related discount-rate variation is concentrated among aggregate market leaders and systematically important firms.

In contrast, the weaker performance of VIX in equal-weighted portfolios indicates that volatility-related uncertainty may be less effective in explaining return variation among smaller firms. The results for EPU reveal a different pattern. Although the EPU-augmented model does not generate improvements as large as those associated with VIX in value-weighted portfolios, it provides relatively stable improvements across both weighting schemes and sample periods. This finding suggests that policy uncertainty may capture broader macroeconomic state conditions that remain more evenly distributed across firms of different sizes. The evidence indicates that uncertainty variables contain economically meaningful information beyond the benchmark Fama–French factor structure. More importantly, the results suggest that different forms of uncertainty influence the stochastic discount factor through distinct mechanisms. Market-implied uncertainty appears most relevant for pricing efficiency in large-cap portfolios, whereas policy-related uncertainty contributes more stable cross-sectional information across weighting schemes.

4.4. **States versus Shocks.** This subsection distinguishes between persistent uncertainty states and transitory uncertainty shocks within the stochastic discount factor framework. In particular, the analysis compares the pricing performance of uncertainty levels, represented by $VIX_t$ and $EPU_t$, with the corresponding first differences, $\Delta VIX_t$ and $\Delta EPU_t$. The objective is to determine whether equilibrium asset pricing is driven primarily by long-run uncertainty conditions or by short-run uncertainty innovations. Table 6 reports the pricing performance of the uncertainty-augmented models using both level and differenced uncertainty variables.

Several important patterns emerge from the comparison between uncertainty states and shocks. First, the pricing performance of uncertainty levels is generally stronger than that of uncertainty shocks, particularly for value-weighted portfolios. In the full-sample value-weighted specification, the FF5+VIX model produces a GRS statistic of 1.15 with a statistically insignificant p-value of 0.284, while the FF5+$\Delta$VIX specification yields a substantially larger GRS statistic of 2.89 and strongly rejects the joint pricing restrictions. A similar pattern appears for EPU. The level specification reduces the GRS statistic to 1.95, whereas the differenced specification produces a GRS statistic exceeding 3.0. These findings suggest that persistent uncertainty conditions contribute more strongly to the stochastic discount factor than short-run uncertainty shocks. Economically, the result is consistent with the interpretation that long-run uncertainty states affect equilibrium discount rates, expected cash flows, and intertemporal investment opportunities. By contrast, transitory uncertainty innovations may primarily capture temporary market disturbances that do not systematically alter long-run pricing kernels. The distinction between states and shocks becomes even clearer in the post-2009 subsample. Both FF5+VIX and FF5+EPU generate substantial improvements in value-weighted pricing performance, with GRS statistics close to one and statistically insignificant p-values. In contrast, the corresponding differenced specifications exhibit weaker pricing efficiency and higher rejection rates.

TABLE 6. States versus Shocks in Asset Pricing

| Sample | Model | GRS | p-value | Avg. \|alpha\| | Sig. alpha |
|---|---|---|---|---|---|
| Equal-weighted portfolios | | | | | |
| Full sample | FF5+EPU | 2.1 | 0.0017 | 0.2 | 0 |
| Full sample | FF5+ΔEPU | 2.97 | 0.000 | 0.123 | 3 |
| Full sample | FF5+VIX | 3.01 | 0.000 | 0.703 | 14 |
| Full sample | FF5+ΔVIX | 2.52 | 0.0001 | 0.105 | 2 |
| Post-2009 | FF5+EPU | 1.26 | 0.191 | 0.259 | 0 |
| Post-2009 | FF5+ΔEPU | 1.46 | 0.0835 | 0.096 | 1 |
| Post-2009 | FF5+VIX | 2.71 | 0.0001 | 1.06 | 11 |
| Post-2009 | FF5+ΔVIX | 1.24 | 0.208 | 0.12 | 2 |
| Value-weighted portfolios | | | | | |
| Full sample | FF5+EPU | 1.95 | 0.0045 | 0.167 | 4 |
| Full sample | FF5+ΔEPU | 3.13 | 0.000 | 0.107 | 6 |
| Full sample | FF5+VIX | 1.15 | 0.284 | 0.239 | 2 |
| Full sample | FF5+ΔVIX | 2.89 | 0.000 | 0.098 | 5 |
| Post-2009 | FF5+EPU | 1.09 | 0.357 | 0.188 | 1 |
| Post-2009 | FF5+ΔEPU | 1.59 | 0.044 | 0.086 | 3 |
| Post-2009 | FF5+VIX | 0.997 | 0.473 | 0.266 | 2 |
| Post-2009 | FF5+ΔVIX | 1.36 | 0.128 | 0.094 | 1 |

*(Notes: This table compares the pricing performance of uncertainty levels and uncertainty shocks. The level variables represent persistent uncertainty states, while the first differences represent transitory uncertainty innovations.)*

The evidence, therefore, indicates that persistent uncertainty regimes became increasingly relevant in the post-crisis environment. The equal-weighted results also support the importance of distinguishing between states and shocks, although the patterns are somewhat more heterogeneous. The differenced volatility specification, FF5+ΔVIX, performs substantially better than the level specification among equal-weighted portfolios. Over the post-2009 sample, the GRS statistic declines from 2.71 under FF5+VIX to 1.24 under FF5+ΔVIX. This result suggests that smaller firms may respond more strongly to short-run volatility shocks than to persistent uncertainty conditions. Nevertheless, the overall evidence suggests that persistent uncertainty states generally contribute more strongly to pricing efficiency than transitory uncertainty shocks, particularly for value-weighted portfolios. However, volatility shocks (ΔVIX) appear to outperform volatility levels among equal-weighted portfolios, especially in the post-2009 period.

In contrast, differenced uncertainty variables appear to capture temporary innovations that are less persistent and less strongly connected to long-run discount-rate dynamics. From a theoretical perspective, these findings support the interpretation that uncertainty enters the stochastic discount factor primarily through persistent state-dependent components rather than purely transitory shocks. This conclusion is consistent with intertemporal asset pricing theory, where equilibrium expected returns depend not only on contemporaneous innovations but also on state variables governing future investment opportunities and long-run risk conditions. More generally, the evidence suggests that persistent uncertainty states affect the stochastic discount factor primarily through conditional expectations and long-run discount-rate dynamics, whereas transitory

uncertainty shocks mainly influence short-run return innovations. Consequently, state variables are more likely to alter equilibrium pricing kernels in a systematic and persistent manner, while shocks may generate only temporary deviations in realized returns.

4.5. **Robustness.** This subsection evaluates the robustness of the main findings using alternative benchmark asset pricing models and subsample analysis. In particular, the analysis examines whether the pricing contribution of uncertainty variables remains economically meaningful when the benchmark specification changes from the CAPM to the Fama–French three-factor (FF3) and five-factor (FF5) models. Table 7 reports the GRS statistics for the alternative benchmark specifications.

Table 7. Robustness Tests Using Alternative Benchmark Models

| Period | Model | Type | Factors | GRS | p-value |
|---|---|---|---|---|---|
| Full sample (VW portfolios) | CAPM | Benchmark | 1 | 3.76 | 0.0000 |
| Full sample (VW portfolios) | CAPM+EPU | Augmented | 2 | 2.23 | 0.0007 |
| Full sample (VW portfolios) | CAPM+VIX | Augmented | 2 | 1.48 | 0.0668 |
| Full sample (VW portfolios) | FF3 | Benchmark | 3 | 3.69 | 0.0000 |
| Full sample (VW portfolios) | FF3+EPU | Augmented | 4 | 2.08 | 0.0019 |
| Full sample (VW portfolios) | FF3+VIX | Augmented | 4 | 1.13 | 0.304 |
| Full sample (VW portfolios) | FF5 | Benchmark | 5 | 3.12 | 0.0000 |
| Full sample (VW portfolios) | FF5+EPU | Augmented | 6 | 1.95 | 0.0045 |
| Full sample (VW portfolios) | FF5+VIX | Augmented | 6 | 1.15 | 0.284 |
| Post-2009 (VW portfolios) | CAPM | Benchmark | 1 | 1.68 | 0.0284 |
| Post-2009 (VW portfolios) | CAPM+EPU | Augmented | 2 | 1.2 | 0.249 |
| Post-2009 (VW portfolios) | CAPM+VIX | Augmented | 2 | 1.3 | 0.163 |
| Post-2009 (VW portfolios) | FF3 | Benchmark | 3 | 1.58 | 0.0474 |
| Post-2009 (VW portfolios) | FF3+EPU | Augmented | 4 | 1.17 | 0.27 |
| Post-2009 (VW portfolios) | FF3+VIX | Augmented | 4 | 1.1 | 0.346 |
| Post-2009 (VW portfolios) | FF5 | Benchmark | 5 | 1.55 | 0.0558 |
| Post-2009 (VW portfolios) | FF5+EPU | Augmented | 6 | 1.09 | 0.357 |
| Post-2009 (VW portfolios) | FF5+VIX | Augmented | 6 | 0.997 | 0.473 |

*(Notes: This table reports robustness tests using alternative benchmark models for value-weighted portfolios. CAPM, FF3, and FF5 respectively denote the Capital Asset Pricing Model, the Fama–French three-factor model, and the Fama–French five-factor model.)*

Several important findings emerge from the robustness analysis. First, the pricing contribution of uncertainty variables remains economically meaningful across all benchmark specifications. Both VIX and EPU substantially reduce the GRS statistic relative to the corresponding benchmark models, suggesting that uncertainty captures systematic pricing information beyond traditional factor structures. For the full sample, the greatest improvements are generated by the VIX-augmented specifications. Under the CAPM benchmark, the inclusion of VIX reduces the GRS statistic from 3.76 to 1.48, corresponding to a reduction of more than 60%. Similarly, under the FF3 specification, the GRS statistic declines from 3.69 to 1.13, representing a reduction of approximately 69%. Most importantly, the FF3+VIX and FF5+VIX models both produce statistically insignificant GRS tests, implying that the joint null hypothesis of zero pricing errors can no longer

be rejected. The EPU-based specifications also improve pricing performance, although the magnitude of the improvement is generally smaller than that associated with VIX. For example, the FF5+EPU specification reduces the GRS statistic from 3.12 to 1.95 in the full sample, corresponding to a reduction of approximately 37%. Nevertheless, the null hypothesis remains rejected at conventional significance levels. These results suggest that policy uncertainty contributes economically relevant information to the stochastic discount factor, but its pricing contribution is weaker than the market-implied uncertainty captured by VIX.

An important pattern emerges when comparing the marginal contribution of uncertainty variables across benchmark models. The relative improvement generated by uncertainty variables tends to decline as the benchmark factor structure becomes richer. This pattern suggests that part of the pricing information associated with uncertainty overlaps with the systematic variation already captured by multifactor models. At the same time, the persistence of substantial GRS improvements under the FF5 specification indicates that uncertainty variables are not fully spanned by traditional pricing factors. In particular, the FF5+VIX specification consistently produces the strongest pricing performance across both sample periods. In the post-2009 subsample, the GRS statistic falls below one and becomes strongly insignificant, suggesting that market-implied uncertainty became increasingly important in the post-crisis environment. The subsample analysis further reinforces the main conclusions. Across all benchmark specifications, the post-2009 period exhibits substantially smaller GRS statistics than the full sample. This finding indicates that standard factor models perform better after the global financial crisis.

Nevertheless, uncertainty variables continue to improve pricing efficiency even in the more recent period, particularly for value-weighted portfolios. Economically, the robustness results support the interpretation that uncertainty acts as an additional state-dependent component of the stochastic discount factor. Market-implied uncertainty, represented by VIX, appears particularly effective in capturing aggregate discount-rate variation among large-cap firms. Policy uncertainty, represented by EPU, provides broader macroeconomic information that complements traditional factor structures but generates more moderate improvements in pricing efficiency. Overall, the robustness analysis confirms that the main findings are not driven by a particular benchmark specification or sample period. The pricing role of uncertainty remains economically meaningful across CAPM, FF3, and FF5 frameworks, providing further support for the uncertainty-augmented stochastic discount factor interpretation.

## 5. Discussion

5.1. **Economic Interpretation of Uncertainty-Augmented Pricing.** The empirical results provide strong evidence that uncertainty variables contain economically meaningful information beyond the traditional Fama–French factor structure. Across multiple benchmark specifications, uncertainty-augmented stochastic discount factor models substantially improve cross-sectional pricing performance, particularly for value-weighted portfolios. These findings support the inter-

pretation that uncertainty acts as a state-dependent component of the pricing kernel rather than merely a predictive variable for returns or volatility.

From a theoretical perspective, the results are consistent with intertemporal asset pricing theory in which equilibrium expected returns depend on variables governing future investment opportunities and time-varying discount rates [2,4]. In particular, persistent uncertainty states may alter investor risk aversion, precautionary demand, and covariance structures between asset returns and aggregate consumption or wealth dynamics. Under this interpretation, uncertainty variables enter the stochastic discount factor because they capture shifts in macro-financial conditions affecting equilibrium pricing relations. The findings also reinforce the view that traditional linear factor models remain incomplete. Although the FF5 model exhibits high explanatory power in time-series regressions, the GRS tests continue to reject the joint null hypothesis of zero pricing errors in several benchmark specifications. This distinction is economically important because high adjusted R2 values do not necessarily imply cross-sectional pricing efficiency. The evidence, therefore, suggests that standard factor structures may omit state variables associated with market-wide uncertainty conditions.

More broadly, the results contribute to the growing literature emphasizing conditional and state-dependent asset pricing mechanisms. Previous studies show that downside risk, intermediary constraints, and macro-financial asymmetries influence expected returns through nonlinear or time-varying channels [10–12,24,25]. The present evidence extends this literature by showing that uncertainty variables also improve stochastic discount factor performance within reduced-form empirical pricing frameworks.

5.2. **Why VIX Dominates in Pricing Efficiency.** One of the central findings of the paper is that VIX generates substantially stronger pricing improvements than EPU for value-weighted portfolios. In particular, the FF5+VIX specification reduces the GRS statistic dramatically and, in several cases, eliminates rejection of the joint pricing restrictions altogether. This result suggests that market-implied uncertainty captures dimensions of systematic risk not fully reflected in traditional Fama–French factors. Several mechanisms may explain the dominant role of VIX. First, VIX is inherently forward-looking because it is derived from option-implied volatility expectations rather than backward-looking macroeconomic indicators.

Consequently, VIX aggregates investors' expectations regarding future market volatility, downside risk, and macro-financial stress conditions [7]. In dynamic equilibrium settings, these expectations are directly linked to discount-rate variation and therefore may enter the stochastic discount factor more naturally than policy-related uncertainty indices. Second, VIX appears closely related to aggregate market conditions. The empirical correlation analysis shows that VIX is negatively associated with market returns, while changes in VIX are strongly negatively correlated with contemporaneous market movements. This evidence suggests that VIX captures systematic market-wide stress conditions influencing large-cap firms and broad market portfolios. Because value-weighted portfolios are dominated by larger firms, VIX may be particularly

effective in explaining aggregate pricing dynamics within these portfolios. The asymmetric performance between value-weighted and equal-weighted portfolios further suggests that uncertainty transmission operates primarily through large-cap firms rather than smaller firms. Since value-weighted portfolios place greater emphasis on firms with large market capitalization, the strong pricing improvements generated by VIX indicate that uncertainty-related discount-rate variation is concentrated among aggregate market leaders and systematically important firms. Third, the results support the interpretation that financial-market-based uncertainty measures contain richer pricing information than macroeconomic uncertainty proxies.

Unlike EPU, which focuses primarily on policy-related uncertainty, VIX directly reflects real-time investor expectations embedded in financial markets. This distinction may explain why VIX consistently produces stronger pricing improvements across alternative benchmark models. At the same time, the relatively stable performance of EPU across both equal-weighted and value-weighted portfolios suggests that policy uncertainty captures broader macroeconomic state conditions that remain partially orthogonal to standard pricing factors. Therefore, the evidence does not imply that EPU is unimportant; rather, it suggests that market-implied uncertainty and policy-related uncertainty influence equilibrium pricing through different channels.

5.3. **Persistent States versus Transitory Shocks.** An important contribution of the paper is the distinction between persistent uncertainty states and transitory uncertainty shocks. The empirical evidence consistently indicates that uncertainty levels contribute more strongly to pricing efficiency than first-difference uncertainty shocks, particularly for value-weighted portfolios. This result has important theoretical implications. In intertemporal asset pricing models, persistent state variables affect equilibrium pricing because they alter future investment opportunities, conditional expected returns, and long-run discount-rate dynamics [2]. Long-run uncertainty conditions may therefore influence the stochastic discount factor through persistent changes in covariance structures and investor preferences. By contrast, short-run uncertainty innovations may primarily represent temporary market disturbances with weaker implications for long-run equilibrium pricing relations.

The distinction between states and shocks also helps explain the heterogeneous empirical results across portfolio weighting schemes. For value-weighted portfolios, uncertainty levels produce the largest reductions in GRS statistics, suggesting that large-cap firms are primarily affected by persistent macro-financial conditions. In contrast, equal-weighted portfolios respond more strongly to changes in VIX, implying that smaller firms may be more sensitive to short-run volatility shocks and episodic market disruptions. Importantly, the findings indicate that pricing efficiency and risk compensation are not identical concepts. Uncertainty levels appear more effective in improving stochastic discount factor efficiency, whereas uncertainty shocks may play a more important role in explaining short-run return variation and conditional risk premia. This distinction helps reconcile why differenced uncertainty variables sometimes generate economically meaningful cross-sectional coefficients while simultaneously providing weaker joint pricing performance.

5.4. **Contribution to the Literature.** This paper contributes to the literature in several important ways. First, the study extends the stochastic discount factor framework by explicitly incorporating uncertainty variables as state-dependent pricing components. While previous uncertainty studies primarily focus on return predictability, volatility forecasting, or macroeconomic transmission mechanisms, the present analysis embeds uncertainty directly into the cross-sectional asset pricing framework through reduced-form stochastic discount factor representations. Second, the paper contributes to the literature on conditional and adaptive asset pricing. Traditional linear factor models generally assume relatively stable pricing relations over time. However, the empirical evidence suggests that uncertainty conditions materially affect equilibrium pricing efficiency. This finding is consistent with adaptive and state-dependent market frameworks in which expected returns vary across changing economic environments [9]. Third, the study contributes to the growing literature on uncertainty and higher-order risks in financial markets. Recent studies document that skewness risk, exchange-rate risk, geopolitical risk, and macro-financial asymmetries generate return patterns beyond traditional factor structures [19–23]. The present results extend this literature by showing that uncertainty states themselves may serve as economically meaningful stochastic discount factor components.

5.5. **Practical Implications.** The findings also generate several practical implications for portfolio management, risk modeling, and financial regulation. For portfolio managers, the results suggest that uncertainty conditions contain useful information for evaluating systematic risk exposures and portfolio pricing efficiency. In particular, market-implied uncertainty measures such as VIX may improve factor-based portfolio allocation and risk-management strategies, especially for large-cap equity portfolios. For empirical asset pricing research, the evidence indicates that static factor structures may be insufficient during periods of elevated macro-financial uncertainty. Incorporating state-dependent uncertainty variables may therefore improve model specification and reduce pricing errors in cross-sectional applications. For policymakers and regulators, the findings imply that financial-market uncertainty contains information relevant for aggregate market stability and equilibrium pricing conditions. Because persistent uncertainty states appear more economically important than temporary shocks, prolonged episodes of elevated uncertainty may have long-run implications for capital allocation and market efficiency.

## 6. Conclusion

This study develops an uncertainty-augmented stochastic discount factor framework and examines its implications for cross-sectional asset pricing in the U.S. equity market. Using monthly data from January 1985 to March 2026 and the twenty-five Fama–French portfolios formed on size and book-to-market characteristics, the analysis evaluates whether uncertainty variables improve the pricing performance of the benchmark Fama–French five-factor model. The empirical evidence yields several important findings. First, the benchmark FF5 model continues to exhibit economically meaningful pricing errors, particularly over the full sample period, suggesting that

the traditional factor structure remains incomplete. Second, augmenting the pricing kernel with uncertainty variables substantially improves pricing efficiency, although the magnitude of the improvement depends on both the uncertainty measure and the portfolio weighting scheme. In particular, VIX generates the greatest improvements for value-weighted portfolios and, in several specifications, eliminates rejection of the joint pricing restrictions under the GRS test. EPU also contributes positively to pricing performance, although its effects are generally more moderate and more stable across weighting schemes. Third, the results highlight the importance of distinguishing between persistent uncertainty states and transitory uncertainty shocks. The evidence indicates that uncertainty levels contribute more strongly to stochastic discount factor efficiency than first-difference uncertainty innovations, especially for value-weighted portfolios. This finding is consistent with intertemporal asset pricing theory in which long-run state variables influence equilibrium discount rates and expected returns through time-varying investment opportunities and covariance structures.

The study contributes to the literature by integrating uncertainty variables directly into the stochastic discount factor framework and linking theoretical asset pricing mechanisms with reduced-form empirical tests such as Fama–MacBeth regressions and GRS efficiency tests. The results suggest that uncertainty represents an economically meaningful state-dependent component of equilibrium pricing kernels and should therefore play a more central role in future dynamic asset pricing research.

**Conflicts of Interest:** The authors declare that there are no conflicts of interest regarding the publication of this paper.

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