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A Three Parameter Power Nakagami Distribution: Properties and Application on the Tax Revenue Data

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ABSTRACT: Developing the probability distributions is increasing extensively over the decades but even though the newly developed distributions have elegant properties and variety of shapes which are applicable in wide areas of real-life situations and a numerous type of data sets. In this article, we introduced a three-parameter positively skewed model named Power Nakagami (PN) distribution based on power transformation. Various statistical properties of the Power Nakagami distribution are derived, including moments. Some reliability measures such as survival function, hazard function, cumulative hazard function and reversed hazard function are discussed also expressions for mills ratio, odd function, elasticity and Lorenz and Bonferroni Curve are developed. Graphical representation of probability density function, cumulative density function and reliability measures are presented. Maximum likelihood estimation is used to estimate the parameters. The distribution is fitted to real life dataset (Tax revenue) to demonstrate the comparison of the new distribution with the base distribution.

1. INTRODUCTION

A flexible lifetime distribution as a function of gamma distribution known as Nakagami distribution introduced by [6]. It is a positively skewed distribution with two parameters shape and spread and it plays a vital role in communication engineering, medicine, reliability and many areas of real-life scenarios. It often applied for modeling the fading of radio signals and model dilution of wireless signals pass through several paths.

If a random variable Y follows Nakagami distribution, then the probability density function and the cumulative density function is as follows respectively,

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$$f(y;\theta,\lambda)=\frac{2}{\Gamma(\lambda)}\left(\frac{\lambda}{\theta}\right)^{\lambda} y^{2\lambda-1}e^{-\frac{\lambda}{\theta}y^{2}} \quad x,\theta,>0 \ \lambda\geq 0.5 \tag{1.1}$$

$$F(y;\theta,\lambda)=\frac{1}{\Gamma(\lambda)}\gamma\left(\lambda,\frac{\lambda}{\theta}y^{2}\right) \tag{1.2}$$

where, λ & θ are shape and spread parameters respectively.

The length biased Nakagami distribution (LBND) was developed by [1]. They derived the posterior risk under various loss functions of the proposed distribution and have demonstrated that LBND is useful for modeling failure time The parameters of Nakagami distribution were estimated by [2] through the method of L moments utilizing simulated data and they suggested that estimates via MLM provides better outcomes as a contrast with MOM. The generalized form for the classical Nakagami distribution was proposed by [3] and they suggested it as Exponentiated Nakagami (ENAK) which is an improvement over beta-Nakagami distribution. They discussed some statistical properties and derived observed information matrix of the distribution. Applicability and flexibility of the proposed distribution are shown by utilizing the flood dataset. Inferences on exponentiated power Lindley distribution with order statistics was proposed by [11]. The cubic spline interpolation was used by [8] to obtain the closed form of the inverse cumulative distribution function of the Nakagami-m distribution. The weighted inverse Nakagami distribution with properties and applications was investigated by [10]. The inverse Nakagami distribution was studied by [12] under progressive Type-II censored data with applications on lifetime data.

Power transformation presents a more flexible model by inserting an additional new parameter. In the literature, some power transformation-based distributions are available such as Power Lomax by [9], Power Lindley by [4], power transformation of Half-Logistic distribution by [5].

2. PROPOSED DISTRIBUTION

A continuous three-parameter distribution named power transformation Nakagami distribution (PND) was formed by using the transformation $X=Y^{1/\beta}$.The pdf of the PN distribution is given by

$$f(x;\beta,\lambda,\theta)=2\beta\left(\frac{\lambda}{\theta}\right)^{\lambda}\frac{1}{\Gamma(\lambda)}x^{2\beta\lambda-1}e^{-\frac{\lambda}{\theta}x^{2\beta}} \tag{2.1}$$

where, $x,\theta,\beta>0$ and $\lambda\geq 0.5$

and the cdf of the PN distribution is as follows

$$F(x;\beta,\lambda,\theta)=\frac{1}{\Gamma(\lambda)}\gamma\left(\lambda,\frac{\lambda}{\theta}x^{2\beta}\right) \tag{2.2}$$

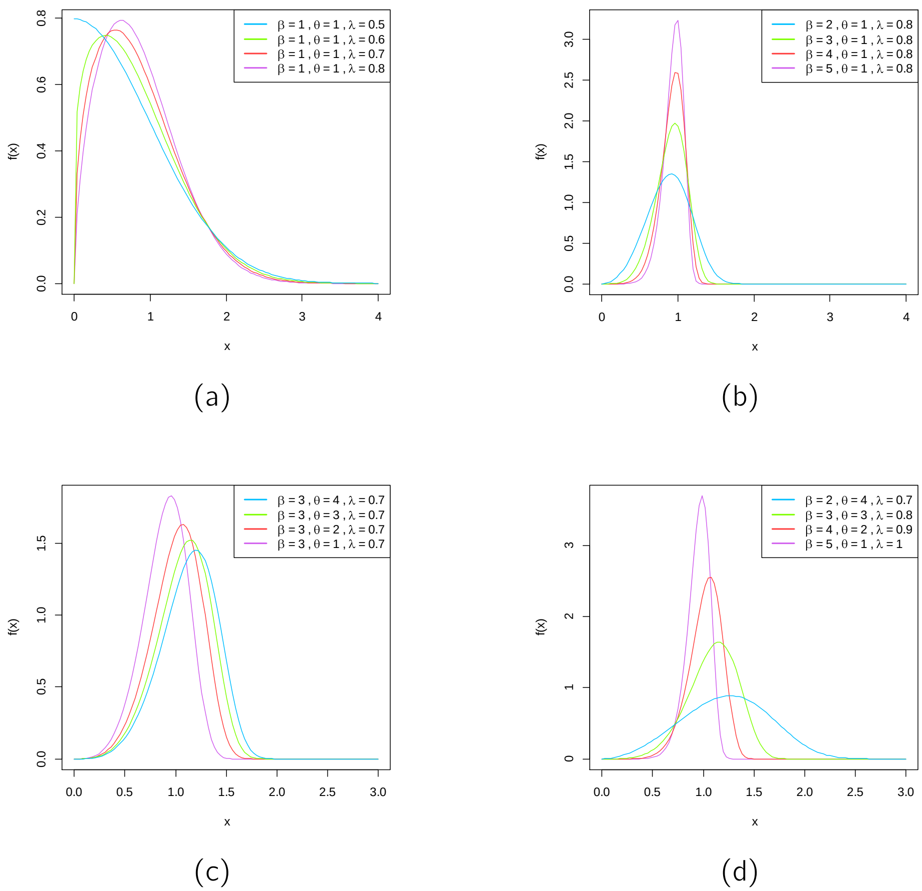

Figure 1: Probability density function plot of PN distribution for various values of β, λ and θ

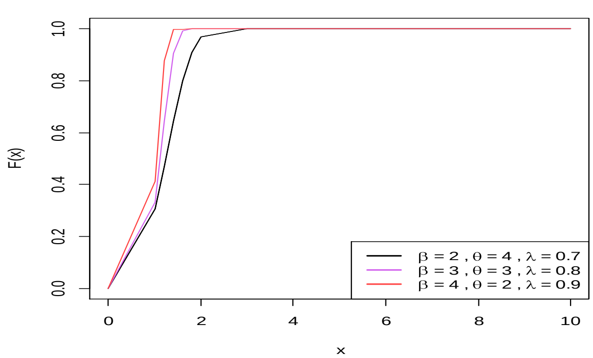

Figure 2: Cumulative distribution function plot of PN distribution for various values of β, λ and θ

3. RELIABILITY ANALYSIS

In this section, some measures of reliability of PND have been derived.

The survival function of PND is given by

$$S(x) = 1 - \frac{1}{\Gamma(\lambda)} \gamma\left(\lambda, \frac{\lambda}{\theta} x^{2\beta}\right) \tag{3.1}$$

The hazard function of PND is as follows

$$h(x) = \frac{2\beta\lambda^{\lambda} x^{2\beta\lambda-1} e^{-\frac{\lambda}{\theta}x^{2\beta}}}{\theta^{\lambda}\Gamma(\lambda)\left[1 - \frac{1}{\Gamma(\lambda)}\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)\right]} \tag{3.2}$$

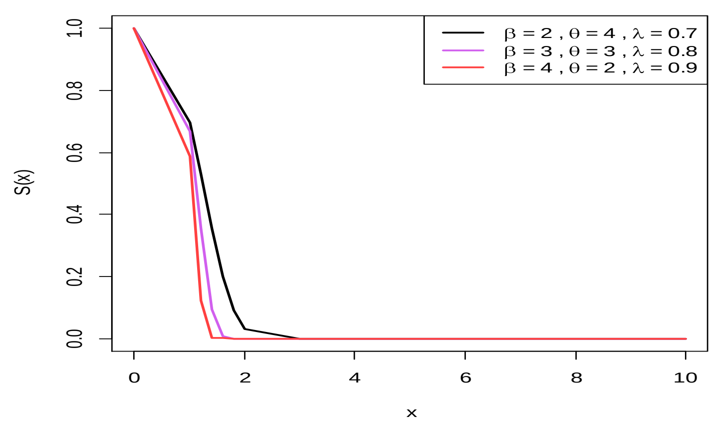

Figure 3: Survival function plot of PN distribution for various values of β, λ and θ

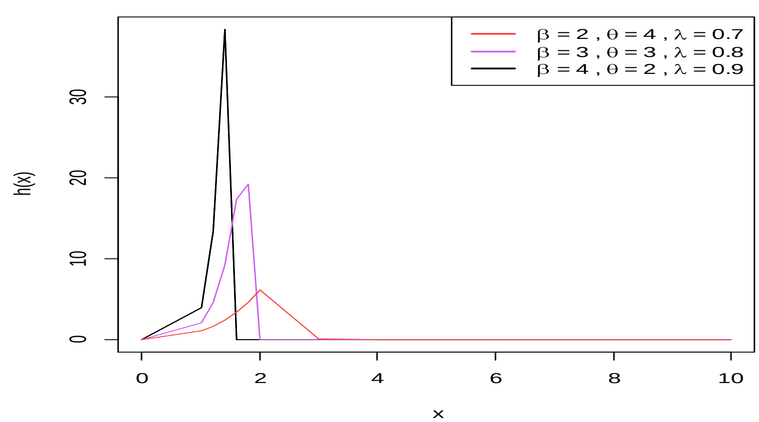

Figure 4: Hazard function plot of PN distribution for various values of β, λ and θ

The cumulative hazard function of PND is

$$H(x) = -\ln\left[1 - \frac{1}{\Gamma(\lambda)}\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)\right] \tag{3.3}$$

The reversed hazard function of PND is

$$r(x) = \frac{2\beta\lambda^{\lambda}x^{2\beta\lambda-1}e^{-\frac{\lambda}{\theta}x^{2\beta}}}{\theta^{\lambda}\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)} \tag{3.4}$$

The mills ratio of PND is given by

$$M(x) = \frac{1 - \frac{1}{\Gamma(\lambda)}\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)}{2\beta\left(\frac{\lambda}{\theta}\right)^{\lambda}\frac{1}{\Gamma(\lambda)}x^{2\beta\lambda-1}e^{-\frac{\lambda}{\theta}x^{2\beta}}} \tag{3.5}$$

The odd function of PND is

$$O(x) = \frac{\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)}{\Gamma(\lambda)\left[1 - \frac{1}{\Gamma(\lambda)}\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)\right]} \tag{3.6}$$

The elasticity of PND is given by

$$e(x) = \frac{2\beta\lambda^{\lambda}x^{2\beta\lambda}e^{-\frac{\lambda}{\theta}x^{2\beta}}}{\theta^{\lambda}\gamma\left(\lambda, \frac{\lambda}{\theta}x^{2\beta}\right)} \tag{3.7}$$

4. STATISTICAL PROPERTIES

In this section, we derived the r^{th} moments, variance, coefficient of variation, skewness, and kurtosis of the Power Transformation Nakagami distribution.

The R^{th} moment about origin of PN distribution is

$$\mu_r' = \int_0^{\infty} x^r f(x)dx$$

$$\mu_r' = \left(\frac{\theta}{\lambda}\right)^{\frac{r}{2\beta}}\frac{\Gamma\left(\lambda + \frac{r}{2\beta}\right)}{\Gamma(\lambda)} \tag{4.1}$$

For r =1,2,3 & 4 in equation (4.1) we get first four raw moment

$$\mu_1^{'} = Mean = \left(\frac{\theta}{\lambda}\right)^{\frac{1}{2\beta}} \frac{\Gamma\left(\lambda + \frac{1}{2\beta}\right)}{\Gamma(\lambda)} \tag{4.2}$$

$$\mu_2^{'} = \left(\frac{\theta}{\lambda}\right)^{\frac{1}{\beta}} \frac{\Gamma\left(\lambda + \frac{1}{\beta}\right)}{\Gamma(\lambda)} \tag{4.3}$$

$$\mu_3^{'} = \left(\frac{\theta}{\lambda}\right)^{\frac{3}{2\beta}} \frac{\Gamma\left(\lambda + \frac{3}{2\beta}\right)}{\Gamma(\lambda)} \tag{4.4}$$

$$\mu_4^{'} = \left(\frac{\theta}{\lambda}\right)^{\frac{2}{\beta}} \frac{\Gamma\left(\lambda + \frac{2}{\beta}\right)}{\Gamma(\lambda)} \tag{4.5}$$

Variance of the PN distribution is

$$\mu_2 = Variance = \left(\frac{\theta}{\lambda}\right)^{\frac{1}{\beta}} \left[\frac{\Gamma\left(\lambda + \frac{1}{\beta}\right)}{\Gamma(\lambda)} - \frac{\left(\Gamma\left(\lambda + \frac{1}{2\beta}\right)\right)^2}{\left(\Gamma(\lambda)\right)^2} \right] \tag{4.6}$$

Third central moments of the PN distribution is given by

$$\mu_3 = \left(\frac{\theta}{\lambda}\right)^{\frac{3}{2\beta}} \left[\frac{\Gamma\left(\lambda + \frac{3}{2\beta}\right)}{\Gamma(\lambda)} - 3\frac{\Gamma\left(\lambda + \frac{1}{\beta}\right)\Gamma\left(\lambda + \frac{1}{2\beta}\right)}{\left(\Gamma(\lambda)\right)^2} + 2\frac{\left(\Gamma\left(\lambda + \frac{1}{2\beta}\right)\right)^3}{\left(\Gamma(\lambda)\right)^3} \right] \tag{4.7}$$

Forth central moments of the PN distribution is as follows

$$\mu_4 = \left(\frac{\theta}{\lambda}\right)^{\frac{2}{\beta}} \left[\frac{\Gamma\left(\lambda + \frac{2}{\beta}\right)}{\Gamma(\lambda)} - 4\frac{\Gamma\left(\lambda + \frac{3}{2\beta}\right)\Gamma\left(\lambda + \frac{1}{2\beta}\right)}{\left(\Gamma(\lambda)\right)^2} + 6\frac{\Gamma\left(\lambda + \frac{1}{\beta}\right)\left(\Gamma\left(\lambda + \frac{1}{2\beta}\right)\right)^2}{\left(\Gamma(\lambda)\right)^3} - 3\frac{\left(\Gamma\left(\lambda + \frac{1}{2\beta}\right)\right)^4}{\left(\Gamma(\lambda)\right)^4} \right] \tag{4.8}$$

Co-efficient of variation of the PN distribution is

$$CV(X)=\frac{\sqrt{\left(\frac{\theta}{\lambda}\right)^{\frac{1}{\beta}}\left[\frac{\Gamma\left(\lambda+\frac{1}{\beta}\right)}{\Gamma(\lambda)}-\frac{\left(\Gamma\left(\lambda+\frac{1}{2\beta}\right)\right)^{2}}{\left(\Gamma(\lambda)\right)^{2}}\right]}}{\left(\frac{\theta}{\lambda}\right)^{\frac{1}{2\beta}}\frac{\Gamma\left(\lambda+\frac{1}{2\beta}\right)}{\Gamma(\lambda)}} \tag{4.9}$$

Co-efficient of skewness of the PN distribution is

$$skewness(X)=\frac{\frac{\Gamma\left(\lambda+\frac{3}{2\beta}\right)}{\Gamma(\lambda)}-3\frac{\Gamma\left(\lambda+\frac{1}{\beta}\right)\Gamma\left(\lambda+\frac{1}{2\beta}\right)}{\left(\Gamma(\lambda)\right)^{2}}+2\frac{\left(\Gamma\left(\lambda+\frac{1}{2\beta}\right)\right)^{3}}{\left(\Gamma(\lambda)\right)^{3}}}{\left[\frac{\Gamma\left(\lambda+\frac{1}{\beta}\right)}{\Gamma(\lambda)}-\frac{\left(\Gamma\left(\lambda+\frac{1}{2\beta}\right)\right)^{2}}{\left(\Gamma(\lambda)\right)^{2}}\right]^{\frac{3}{2}}} \tag{4.10}$$

Co-efficient of kurtosis of the PN distribution is

$$kurtosis(X)=\frac{\frac{\Gamma\left(\lambda+\frac{2}{\beta}\right)}{\Gamma(\lambda)}-4\frac{\Gamma\left(\lambda+\frac{3}{2\beta}\right)\Gamma\left(\lambda+\frac{1}{2\beta}\right)}{\left(\Gamma(\lambda)\right)^{2}}+6\frac{\Gamma\left(\lambda+\frac{1}{\beta}\right)\left(\Gamma\left(\lambda+\frac{1}{2\beta}\right)\right)^{2}}{\left(\Gamma(\lambda)\right)^{3}}-3\frac{\left(\Gamma\left(\lambda+\frac{1}{2\beta}\right)\right)^{4}}{\left(\Gamma(\lambda)\right)^{4}}}{\left[\frac{\Gamma\left(\lambda+\frac{1}{\beta}\right)}{\Gamma(\lambda)}-\frac{\left(\Gamma\left(\lambda+\frac{1}{2\beta}\right)\right)^{2}}{\left(\Gamma(\lambda)\right)^{2}}\right]^{2}} \tag{4.11}$$

R^{th} moment about mean of the PN distribution is

$$E(X-\mu)^{r}=\sum_{k=0}^{r}\binom{r}{k}(-\mu)^{r-k}\left(\frac{\theta}{\lambda}\right)^{\frac{k}{2\beta}}\frac{\Gamma\left(\lambda+\frac{k}{2\beta}\right)}{\Gamma(\lambda)} \tag{4.12}$$

where, $k=1,2,3,\ldots$ and $\mu=mean$

Mode of the PN distribution is

$$Mode=\left[\frac{\theta\left(2\beta\lambda-1\right)}{2\beta\lambda}\right]^{\frac{1}{2\beta}} \tag{4.13}$$

Geometric mean of the PN distribution is

$$G.M=\prod_{i=1}^{n}\left[\left(\frac{\theta}{\lambda}\right)^{\frac{1}{2\beta n}}\frac{\Gamma\left(\lambda+\frac{1}{2\beta n}\right)}{\Gamma\left(\lambda\right)}\right] \tag{4.14}$$

Harmonic mean of the PN distribution is

$$H.M=\left[\left(\frac{\lambda}{\theta}\right)^{\frac{1}{2\beta}}\frac{\Gamma\left(\lambda-\frac{1}{2\beta}\right)}{\Gamma\left(\lambda\right)}\right]^{-1} \tag{4.15}$$

Table 4.1: For various values of β and θ when λ=0.7 mean and mode for PND

β	θ	Mean	Mode
	2	0.9990028	1.06402
4	4	1.08942	1.160323
	8	1.188021	1.265341
	2	0.996182	1.055379
5	4	1.067681	1.131127
	8	1.144313	1.212312
	2	0.995096	1.048331
6	4	1.054267	1.110668
	8	1.116957	1.176712

Table 4.2: For various values of β and θ when λ=0.9 mean and mode for PND

β	θ	Mean	Mode
	2	1.019706	1.070314
4	4	1.111998	1.167186
	8	1.212642	1.272825
	2	1.013566	1.059224
5	4	1.086313	1.135248
	8	1.164281	1.216729
	2	1.010067	1.050919
6	4	1.070129	1.11341
	8	1.133762	1.179617

Table 4.3: For various values of β and θ when λ=0.7 variance, co-efficient of variation, skewness and kurtosis for PND

β	θ	Variance	COV	Skewness	Kurtosis
	2	0.03508632	0.1644412	-0.5502302	-2425.468
4	4	0.0417249	0.1507933	-0.5502302	-2425.468
	8	0.04961955	0.138278	-0.5502302	-2425.468
	2	0.02321527	0.1377067	-0.6787926	-5479.924
5	4	0.02666734	0.1284849	-0.6787926	-5479.924
	8	0.03063273	0.1198806	-0.6787926	-5479.924
	2	0.01653862	0.1184105	-0.7709212	-10752.24
6	4	0.01856397	0.1117647	-0.7709212	-10752.24
	8	0.02083735	0.1054918	-0.7709212	-10752.24

5. INEQUALITY MEASURES

In this section, we discuss Lorenz and Bonferroni curve. The Lorenz Curve of the PN distribution is

$$L(x)=\frac{\Gamma(\lambda)\gamma\left(\lambda+\frac{1}{2\beta},\frac{\lambda}{\theta}x^{2\beta}\right)}{\left(\frac{\theta}{\lambda}\right)^{\frac{1}{2\beta}}} \tag{5.1}$$

The Bonferroni Curve is as follows

$$B(x)=\frac{\left(\Gamma(\lambda)\right)^{2}\gamma\left(\lambda+\frac{1}{2\beta},\frac{\lambda}{\theta}x^{2\beta}\right)}{\left(\frac{\theta}{\lambda}\right)^{\frac{1}{2\beta}}\gamma\left(\lambda,\frac{\lambda}{\theta}x^{2\beta}\right)} \tag{5.2}$$

6. ORDER STATISTICS

The distribution of jth order statistic of PND is as follows

$$f_{X_{(j)}}(x)=\frac{n!}{(j-1)!(n-j)!}2\beta\left(\frac{\lambda}{\theta}\right)^{\lambda}\frac{1}{\Gamma(\lambda)}x^{2\beta\lambda-1}e^{-\frac{\lambda}{\theta}x^{2\beta}}\left[\frac{1}{\Gamma(\lambda)}\gamma\left(\lambda,\frac{\lambda}{\theta}x^{2\beta}\right)\right]^{j-1}\left[1-\frac{1}{\Gamma(\lambda)}\gamma\left(\lambda,\frac{\lambda}{\theta}x^{2\beta}\right)\right]^{n-j} \tag{6.1}$$

where, $j=1,2,3.....,n$ and $0\le x_j\le\infty$

Largest order statistic of the PN distribution by substituting j=n in equation (6.1) is

$$f_{X_{(n)}}(x)=2n\beta\left(\frac{\lambda}{\theta}\right)^{\lambda}\frac{1}{\Gamma(\lambda)}x^{2\beta\lambda-1}e^{-\frac{\lambda}{\theta}x^{2\beta}}\left[\frac{1}{\Gamma(\lambda)}\gamma\left(\lambda,\frac{\lambda}{\theta}x^{2\beta}\right)\right]^{n-1},0\le x_n\le\infty \tag{6.2}$$

Smallest order statistic of the PN distribution by substituting j=1 in equation (6.1) is

$$f_{X_{(1)}}(x)=2n\beta\left(\frac{\lambda}{\theta}\right)^{\lambda}\frac{1}{\Gamma(\lambda)}x^{2\beta\lambda-1}e^{-\frac{\lambda}{\theta}x^{2\beta}}\left[1-\frac{1}{\Gamma(\lambda)}\gamma\left(\lambda,\frac{\lambda}{\theta}x^{2\beta}\right)\right]^{n-1},0\le x_1\le\infty \tag{6.3}$$

7. MAXIMUM LIKELIHOOD ESTIMATION

Let $X_1, X_2, X_3, \ldots\ldots, X_n$ be a n random sample from Power Nakagami distribution. Then the Likelihood function (L) of equation (2.1) is given by

$$L(\beta,\lambda,\theta)=\left[\frac{2\beta}{\Gamma(\lambda)}\left(\frac{\lambda}{\theta}\right)^{\lambda}\right]^{n}\prod_{i=1}^{n}x_i^{2\beta\lambda-1}+e^{-\frac{\lambda}{\theta}\prod_{i=1}^{n}x_i^{2\beta}} \tag{7.1}$$

$$L(\beta,\lambda,\theta)=n\ln 2+n\ln\beta-n\ln\Gamma(\lambda)+n\lambda\ln\lambda-n\lambda\ln\theta+(2\beta\lambda-1)\sum_{i=1}^{n}\ln x_i-\frac{\lambda}{\theta}\sum_{i=1}^{n}x_i^{2\beta} \tag{7.2}$$

Now, differentiate log-likelihood w.r.t β, λ and θ respectively, we attain

$$\frac{\partial\ln L(x;\beta,\lambda,\theta)}{\partial\beta}=\frac{n}{\beta}+2\lambda\sum_{i=1}^{n}\ln x_i-\frac{\lambda}{\theta}\sum_{i=1}^{n}x_i^{2\beta}\ln(x_i) \tag{7.3}$$

$$\frac{\partial\ln L(x;\beta,\lambda,\theta)}{\partial\lambda}=-\frac{n\Gamma(\lambda)'}{\Gamma(\lambda)}+\frac{n\lambda}{\Gamma(\lambda)}+n\ln\lambda-n\ln\theta+2\beta\sum_{i=1}^{n}\ln(x_i)-\frac{1}{\theta}\sum_{i=1}^{n}x_i^{2\beta} \tag{7.4}$$

$$\frac{\partial\ln L(x;\beta,\lambda,\theta)}{\partial\theta}=\frac{n\lambda}{\theta}+\frac{\lambda}{\theta^2}\sum_{i=1}^{n}x_i^{2\beta} \tag{7.5}$$

8. SIMULATIONS

Here, we examine the behavior of estimates derived by the method of Maximum likelihood estimation from PND, through simulations study. The performance of the parameters estimated by MLE is measured on the basis of their biases.

For this purpose, we generate, n = 25, 50, 100, 300, 500 samples by 1000 simulations from Power Nakagami Distribution (2.1) using Mathematica 11. Table 8.1 and 8.2 are presenting the consistent and efficient performance of the estimates and it is evident that, as the sample size increases these estimated values of the parameters are close to the trues values of the parameters.

Table 8.1. MLE estimates and their biases in parenthesis are calculated at various sample sizes

$\boldsymbol{\beta = 2, \theta = 3 \ and \ \lambda = 0.7}$

Parameters	n=25	n=50	n=100	n=300	n=500
$\hat{\beta}$	1.9570	1.8817	2.0187	2.0116	2.0104
	(−0.0430)	(−0.1183)	(0.0187)	(0.0116)	(0.0104)
$\hat{\theta}$	4.0171	3.2433	3.2843	3.2179	3.0974
	(1.0171)	(0.2434)	(0.2843)	(0.2179)	(0.0973)
$\hat{\lambda}$	6.7167	1.3232	0.8409	0.7571	0.718
	(6.0167)	(0.6232)	(0.1409)	(0.0571)	(0.018)

Table 8.2. MLE estimates and their biases in parenthesis are calculated at various sample sizes

$\boldsymbol{\beta = 5, \theta = 2 \ and \ \lambda = 1}$

Parameters	n=25	n=50	n=100	n=300	n=500
$\hat{\beta}$	5.8887	5.8198	5.157	5.1105	5.0586
	(0.8886)	(0.8198)	(0.157)	(0.1105)	(0.0586)
$\hat{\theta}$	3.0424	2.7229	2.3121	2.0384	2.0519
	(1.0424)	(0.7229)	(0.3121)	(0.0384)	(0.0519)
$\hat{\lambda}$	2.1274	1.4813	1.2078	1.0954	1.0187
	(1.1274)	(0.4813)	(0.2078)	(0.0953)	(0.0187)

9. APPLICATION

In this section, the Power Nakagami Distribution is fitted on real life data set to assess the usefulness and flexibility.

We consider the dataset of Egypt monthly actual taxes revenue (in 1000 million Egyptian pounds) for the year 2006 to 2010 which consists of 59 observations. This data is taken from [7].

5.9, 20.4, 14.9, 16.2, 17.2, 7.8, 6.1, 9.2, 10.2, 9.6, 13.3, 8.5, 21.6, 18.5, 5.1,6.7, 17, 8.6, 9.7, 39.2, 35.7, 15.7, 9.7, 10, 4.1, 36, 8.5, 8, 9.2, 26.2, 21.9,16.7, 21.3, 35.4, 14.3, 8.5, 10.6, 19.1, 20.5, 7.1, 7.7, 18.1, 16.5, 11.9, 7, 8.6,12.5, 10.3, 11.2, 6.1, 8.4, 11, 11.6, 11.9, 5.2, 6.8, 8.9, 7.1, 10.8.

Table 9.1. Descriptive statistics of the tax revenue dataset for PND

Data	Statistics					
	Mean	Median	Standard Deviation	Variance	Skewness	Kurtosis
taxes revenue (in 1000 million Egyptian pounds)	13.49	10.60	8.051496	64.82658	1.608296	5.256002

Table 9.2. Estimated values of the parameters of the tax revenue dataset

Model	ML Estimates	-2Log L	AIC	BIC	CAIC
Nakagami Distribution	$\hat{m} = 0.9842$ $\hat{\sigma} = 239.5052$	395.4555	399.4555	398.9972	399.6697
Power Nakagami distribution	$\hat{\lambda} = 69.4788$ $\hat{\beta} = 0.1143$ $\hat{\theta} = 1.7681$	380.717	386.717	392.0295	387.1534

Table 9.2 results indicate that the new distribution (Power Nakagami distribution) is more flexible as compared to the base distribution as it gives smaller values of AIC, BIC and CAIC.

10. CONCLUSION

Nakagami distribution plays a vital role in communication engineering, modeling fading. In this study we generalized the Nakagami distribution by using power transformation. Derived some statistical properties including moments, geometric mean, harmonic mean, mode, coefficient of variation, skewness and kurtosis as well as discussed reliability measures such as survival function, hazard function, cumulative hazard function, mills ratio, odd function and elasticity function. Developed the expressions of Lorenz and Bonferroni curves. Provide numerical results of mean, variance, skewness and kurtosis and conclude as we increase parameter λ & θ mean and mode increases on the other hand when we increase β mean and mode decrease. Graphical representation of hazard function indicates first increasing then decreasing trend. For the dataset (Tax revenue) results suggest that the proposed distribution (PTN) is more flexible as compared to the Nakagami distribution.

Conflicts of Interest: The author declares that there are no conflicts of interest regarding the publication of this paper.

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