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$\delta s(\Lambda, s)$-R_0 Spaces and $\delta s(\Lambda, s)$-R_1 Spaces

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Abstract. Our main purpose is to introduce the notions of $\delta s(\Lambda, s)$-R_0 spaces and $\delta s(\Lambda, s)$-R_1 spaces. Moreover, several characterizations of $\delta s(\Lambda, s)$-R_0 spaces and $\delta s(\Lambda, s)$-R_1 spaces are investigated.

1. Introduction

The concept of R_0 topological spaces was first introduced by Shanin [21]. Davis [7] introduced the concept of a separation axiom called R_1. These concepts are further investigated by Naimpally [16], Dube [11] and Dorsett [8]. Murdeshwar and Naimpally [15] and Dube [10] studied some of the fundamental properties of the class of R_1 topological spaces. As natural generalizations of the separations axioms R_0 and R_1, the concepts of semi-R_0 and semi-R_1 spaces were introduced and studied by Maheshwari and Prasad [14] and Dorsett [9]. Caldas et al. [4] introduced and investigated two new weak separation axioms called Λ_θ-R_0 and Λ_θ-R_1 by using the notions of (Λ, θ)-open sets and the (Λ, θ)-closure operator. Cammaroto and Noiri [2] defined a weak separation axiom m-R_0 in m-spaces which are equivalent to generalized topological spaces due to Lugojan [13]. Noiri [17] introduced the notion of m-R_1 spaces and investigated several characterizations of m-R_0 spaces and m-R_1 spaces. Moreover, Levine [12] introduced the concept of semi-open sets which is weaker than the concept of open sets in topological spaces. Veličko [23] introduced δ-open sets, which are stronger than open sets. Park et al. [19] have offered new notion called δ-semiopen sets which are stronger than semi-open sets but weaker than δ-open sets and investigated the relationships between several types of these open sets. Caldas et al. [5] investigated some weak separation axioms by utilizing δ-semiopen

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sets and the δ-semiclosure operator. Caldas et al. [3] investigated the notion of δ-Λ_s-semiclosed sets which is defined as the intersection of a δ-Λ_s-set and a δ-semiclosed set. Noiri [18] showed that a subset A of a topological space (X, τ) is δ-semiopen in (X, τ) if and only if it is semi-open in (X, τ_s). In [1], the present authors introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. Pue-on and Boonpok [20] introduced and studied the notions of $\delta s(\Lambda, s)$-open sets and $\delta s(\Lambda, s)$-closed sets. In this paper, we introduce the notions of $\delta s(\Lambda, s)$-R_0 spaces and $\delta s(\Lambda, s)$-R_1 spaces. Furthermore, several characterizations of $\delta s(\Lambda, s)$-R_0 spaces and $\delta s(\Lambda, s)$-R_1 spaces are discussed.

2. Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset A of a topological space (X, τ) is called *semi-open* [12] if $A \subseteq \mathrm{Cl}(\mathrm{Int}(A))$. The family of all semi-open sets in a topological space (X, τ) is denoted by $SO(X, \tau)$. A subset A^{Λ_s} [6] is defined as follows: $A^{\Lambda_s} = \cap\{U \mid U \supseteq A, U \in SO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a *Λ_s-set* [6] if $A = A^{\Lambda_s}$. A subset A of a topological space (X, τ) is called *(Λ, s)-closed* [1] if $A = T \cap C$, where T is a Λ_s-set and C is a semi-closed set. The complement of a (Λ, s)-closed set is called *(Λ, s)-open*. The family of all (Λ, s)-closed (resp. (Λ, s)-open) sets in a topological space (X, τ) is denoted by $\Lambda_s C(X, \tau)$ (resp. $\Lambda_s O(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, s)-cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, s)-open set U of X containing x. The set of all (Λ, s)-cluster points of A is called the *(Λ, s)-closure* [1] of A and is denoted by $A^{(\Lambda,s)}$. The union of all (Λ, s)-open sets of X contained in A is called the *(Λ, s)-interior* [1] of A and is denoted by $A_{(\Lambda,s)}$. A point x of X is called a *$\delta(\Lambda, s)$-cluster point* [22] of A if $A \cap [V^{(\Lambda,s)}]_{(\Lambda,s)} \neq \emptyset$ for every (Λ, s)-open set V of X containing x. The set of all $\delta(\Lambda, s)$-cluster points of A is called the *$\delta(\Lambda, s)$-closure* [22] of A and is denoted by $A^{\delta(\Lambda,s)}$. If $A = A^{\delta(\Lambda,s)}$, then A is said to be *$\delta(\Lambda, s)$-closed* [22]. The complement of a $\delta(\Lambda, s)$-closed set is said to be *$\delta(\Lambda, s)$-open*. The union of all $\delta(\Lambda, s)$-open sets of X contained in A is called the *$\delta(\Lambda, s)$-interior* [22] of A and is denoted by $A_{\delta(\Lambda,s)}$. A subset A of a topological space (X, τ) is said to be *$\delta s(\Lambda, s)$-open* [20] if $A \subseteq [A_{(\Lambda,s)}]^{\delta(\Lambda,s)}$. The complement of a $\delta s(\Lambda, s)$-open set is said to be *$\delta s(\Lambda, s)$-closed*. The family of all $\delta s(\Lambda, s)$-open (resp. $\delta s(\Lambda, s)$-closed) sets in a topological space (X, τ) is denoted by $\delta s(\Lambda, s)O(X, \tau)$ (resp. $\delta s(\Lambda, s)C(X, \tau)$). A subset N of a topological space (X, τ) is called a *$\delta s(\Lambda, s)$-neighborhood* [20] of a point $x \in X$ if there exists a $\delta s(\Lambda, s)$-open set V such that $x \in V \subseteq N$. Let A be a subset of a topological space (X, τ). A point x of X is called a *$\delta s(\Lambda, s)$-cluster point* [20] of A if $A \cap U \neq \emptyset$ for every $\delta s(\Lambda, s)$-open set U of X containing x. The set of all $\delta s(\Lambda, s)$-cluster points of A is called the *$\delta s(\Lambda, s)$-closure* [20] of A and is denoted by $A^{\delta s(\Lambda,s)}$.

Lemma 2.1. [20] *For the $\delta s(\Lambda, s)$-closure of subsets A, B in a topological space (X, τ), the following properties hold:*

(1) *A is $\delta s(\Lambda, s)$-closed in (X, τ) if and only if $A = A^{\delta s(\Lambda,s)}$.*

(2) *If $A \subseteq B$, then $A^{\delta s(\Lambda,s)} \subseteq B^{\delta s(\Lambda,s)}$.*

(3) *$A^{\delta s(\Lambda,s)}$ is $\delta s(\Lambda, s)$-closed, that is, $A^{\delta s(\Lambda,s)} = [A^{\delta s(\Lambda,s)}]^{\delta s(\Lambda,s)}$.*

3. On $\delta s(\Lambda, s)$-R_0 spaces

In this section, we introduce the concept of $\delta s(\Lambda, s)$-R_0 spaces. Moreover, some characterizations of $\delta s(\Lambda, s)$-R_0 spaces are discussed.

Definition 3.1. *A topological space (X, τ) is called $\delta s(\Lambda, s)$-R_0 if for each $\delta s(\Lambda, s)$-open set U and each $x \in U$, $\{x\}^{\delta s(\Lambda,s)} \subseteq U$.*

Theorem 3.1. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is $\delta s(\Lambda, s)$-R_0.*

(2) *For each $\delta s(\Lambda, s)$-closed set F and each $x \in X - F$, there exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $F \subseteq U$ and $x \notin U$.*

(3) *For each $\delta s(\Lambda, s)$-closed set F and each $x \in X - F$, $F \cap \{x\}^{\delta s(\Lambda,s)} = \emptyset$.*

(4) *For any distinct points x, y in X, $\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$ or $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$.*

Proof. (1) ⇒ (2): Let F be a $\delta s(\Lambda, s)$-closed set and $x \in X - F$. Since (X, τ) is $\delta s(\Lambda, s)$-R_0, we have $\{x\}^{\delta s(\Lambda,s)} \subseteq X - F$. Put $U = X - \{x\}^{\delta s(\Lambda,s)}$. Thus, by Lemma 2.1, $U \in \delta s(\Lambda, s)O(X, \tau)$, $F \subseteq U$ and $x \notin U$.

(2) ⇒ (3): Let F be a $\delta s(\Lambda, s)$-closed set and $x \in X - F$. By (2), there exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $F \subseteq U$ and $x \notin U$. Since $U \in \delta s(\Lambda, s)O(X, \tau)$, $U \cap \{x\}^{\delta s(\Lambda,s)} = \emptyset$ and hence $F \cap \{x\}^{\delta s(\Lambda,s)} = \emptyset$.

(3) ⇒ (4): Let x and y be distinct points of X. Suppose that $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} \neq \emptyset$. By (3), $x \in \{y\}^{\delta s(\Lambda,s)}$ and $y \in \{x\}^{\delta s(\Lambda,s)}$. By Lemma 2.1, $\{x\}^{\delta s(\Lambda,s)} \subseteq \{y\}^{\delta s(\Lambda,s)} \subseteq \{x\}^{\delta s(\Lambda,s)}$ and hence

$$\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}.$$

(4) ⇒ (1): Let $V \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in V$. For each $y \notin V$, $V \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$ and hence $x \notin \{y\}^{\delta s(\Lambda,s)}$. Thus, $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. By (4), for each $y \notin V$, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. Since $X - V$ is $\delta s(\Lambda, s)$-closed, $y \in \{y\}^{\delta s(\Lambda,s)} \subseteq X - V$ and $\cup_{y \in X-V}\{y\}^{\delta s(\Lambda,s)} = X - V$. Thus,

$$\begin{aligned}
\{x\}^{\delta s(\Lambda,s)} \cap (X - V) &= \{x\}^{\delta s(\Lambda,s)} \cap [\cup_{y \in X-V}\{y\}^{\delta s(\Lambda,s)}] \\
&= \cup_{y \in X-V}[\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)}] \\
&= \emptyset
\end{aligned}$$

and hence $\{x\}^{\delta s(\Lambda,s)} \subseteq V$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_0. □

Corollary 3.1. *A topological space (X, τ) is $\delta s(\Lambda, s)$-R_0 if and only if for any points x and y in X, $\{x\}^{\delta p(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$ implies $\{x\}^{\delta p(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$.*

Proof. This is obvious by Theorem 3.1.

Conversely, let $U \in \delta s(\Lambda,s)O(X,\tau)$ and $x \in U$. If $y \notin U$, then $U \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. Thus, $x \notin \{y\}^{\delta s(\Lambda,s)}$ and $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. By the hypothesis, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$ and hence $y \notin \{x\}^{\delta s(\Lambda,s)}$. This shows that $\{x\}^{\delta s(\Lambda,s)} \subseteq U$. Therefore, (X,τ) is $\delta s(\Lambda,s)$-R_0. □

Definition 3.2. [20] *Let A be a subset of a topological space (X,τ). The $\delta s(\Lambda,s)$-kernel of A, denoted by $\delta s(\Lambda,s)Ker(A)$, is defined to be the set $\delta s(\Lambda,s)Ker(A) = \cap\{U \mid A \subseteq U, U \in \delta s(\Lambda,s)O(X,\tau)\}$.*

Lemma 3.1. [20] *For subsets A, B of a topological space (X,τ), the following properties hold:*

1. *$A \subseteq \delta s(\Lambda,s)Ker(A)$.*
2. *If $A \subseteq B$, then $\delta s(\Lambda,s)Ker(A) \subseteq \delta s(\Lambda,s)Ker(B)$.*
3. *$\delta s(\Lambda,s)Ker(\delta s(\Lambda,s)Ker(A)) = \delta s(\Lambda,s)Ker(A)$.*
4. *If A is $\delta s(\Lambda,s)$-open, $\delta s(\Lambda,s)Ker(A) = A$.*

Lemma 3.2. [20] *For any points x and y in a topological space (X,τ), the following properties are equivalent:*

1. *$\delta s(\Lambda,s)Ker(\{x\}) \neq \delta s(\Lambda,s)Ker(\{y\})$.*
2. *$\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$.*

Lemma 3.3. *Let (X,τ) be a topological space and $x, y \in X$. Then, the following properties hold:*

1. *$y \in \delta s(\Lambda,s)Ker(\{x\})$ if and only if $x \in \{y\}^{\delta s(\Lambda,s)}$.*
2. *$\delta s(\Lambda,s)Ker(\{x\}) = \delta s(\Lambda,s)Ker(\{y\})$ if and only if $\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$.*

Proof. (1) Let $x \notin \{y\}^{\delta s(\Lambda,s)}$. Then, there exists $U \in \delta s(\Lambda,s)O(X,\tau)$ such that $x \in U$ and $y \notin U$. Thus, $y \notin \delta s(\Lambda,s)Ker(\{x\})$. The converse is similarly shown.

(2) Suppose that $\delta s(\Lambda,s)Ker(\{x\}) = \delta s(\Lambda,s)Ker(\{y\})$ for any $x, y \in X$. Since $x \in \delta s(\Lambda,s)Ker(\{x\})$, $x \in \delta s(\Lambda,s)Ker(\{y\})$, by (1), $y \in \{x\}^{\delta s(\Lambda,s)}$. By Lemma 2.1, $\{y\}^{\delta s(\Lambda,s)} \subseteq \{x\}^{\delta s(\Lambda,s)}$. Similarly, we have $\{x\}^{\delta s(\Lambda,s)} \subseteq \{y\}^{\delta s(\Lambda,s)}$ and hence $\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$.

Conversely, suppose that $\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$. Since $x \in \{x\}^{\delta s(\Lambda,s)}$, $x \in \{y\}^{\delta s(\Lambda,s)}$ and by (1),

$$y \in \delta s(\Lambda,s)Ker(\{x\}).$$

By Lemma 3.1, $\delta s(\Lambda,s)Ker(\{y\}) \subseteq \delta s(\Lambda,s)Ker(\delta s(\Lambda,s)Ker(\{x\})) = \delta s(\Lambda,s)Ker(\{x\})$. Similarly, we have $\delta s(\Lambda,s)Ker(\{x\}) \subseteq \delta p(\Lambda,s)Ker(\{y\})$ and hence $\delta s(\Lambda,s)Ker(\{x\}) = \delta s(\Lambda,s)Ker(\{y\})$. □

Theorem 3.2. *A topological space (X,τ) is $\delta s(\Lambda,s)$-R_0 if and only if for each points x and y in X, $\delta s(\Lambda,s)Ker(\{x\}) \neq \delta s(\Lambda,s)Ker(\{y\})$ implies $\delta s(\Lambda,s)Ker(\{x\}) \cap \delta s(\Lambda,s)Ker(\{y\}) = \emptyset$.*

Proof. Let (X,τ) be $\delta s(\Lambda,s)$-R_0. Suppose that $\delta s(\Lambda,s)Ker(\{x\}) \cap \delta s(\Lambda,s)Ker(\{y\}) \neq \emptyset$. Let

$$z \in \delta s(\Lambda,s)Ker(\{x\}) \cap \delta s(\Lambda,s)Ker(\{y\}).$$

Then, $z \in \delta s(\Lambda, s)Ker(\{x\})$ and by Lemma 3.3, $x \in \{z\}^{\delta s(\Lambda,s)}$. Thus, $x \in \{z\}^{\delta s(\Lambda,s)} \cap \{x\}^{\delta s(\Lambda,s)}$ and by Corollary 3.1, $\{z\}^{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)}$. Similarly, we have $\{z\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$ and hence

$$\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)},$$

by Lemma 3.3, $\delta s(\Lambda, s)Ker(\{x\}) = \delta s(\Lambda, s)Ker(\{y\})$.

Conversely, we show the sufficiency by using Corollary 3.1. Suppose that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. By Lemma 3.3, $\delta s(\Lambda, s)Ker(\{x\}) \neq \delta s(\Lambda, s)Ker(\{y\})$ and hence
$\delta s(\Lambda, s)Ker(\{x\}) \cap \delta s(\Lambda, s)Ker(\{y\}) = \emptyset$. Thus, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. In fact, assume that $z \in \{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)}$. Then, $z \in \{x\}^{\delta s(\Lambda,s)}$ implies $x \in \delta s(\Lambda, s)Ker(\{z\})$ and hence $x \in \delta s(\Lambda, s)Ker(\{z\}) \cap \delta s(\Lambda, s)Ker(\{x\})$. By the hypothesis, $\delta s(\Lambda, s)Ker(\{z\}) = \delta s(\Lambda, s)Ker(\{x\})$ and by Lemma 3.3, $\{z\}^{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)}$. Similarly, we have $\{z\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$ and hence $\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}$. This contradicts that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$.
Thus, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_0. □

Theorem 3.3. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is $\delta s(\Lambda, s)$-R_0.*
(2) *$x \in \{y\}^{\delta s(\Lambda,s)}$ if and only if $y \in \{x\}^{\delta s(\Lambda,s)}$.*

Proof. $(1) \Rightarrow (2)$: Suppose that $x \in \{y\}^{\delta s(\Lambda,s)}$. By Lemma 3.3, $y \in \delta s(\Lambda, s)Ker(\{x\})$ and hence

$$\delta s(\Lambda, s)Ker(\{x\}) \cap \delta s(\Lambda, s)Ker(\{y\}) \neq \emptyset.$$

By Theorem 3.2, $\delta s(\Lambda, s)Ker(\{x\}) = \delta s(\Lambda, s)Ker(\{y\})$ and hence $x \in \delta s(\Lambda, s)Ker(\{y\})$. Thus, by Lemma 3.3, $y \in \{x\}^{\delta s(\Lambda,s)}$. The converse is similarly shown.

$(2) \Rightarrow (1)$: Let $U \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in U$. If $y \notin U$, then $U \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. Thus, $x \notin \{y\}^{\delta s(\Lambda,s)}$ and $y \notin \{x\}^{\delta s(\Lambda,s)}$. This implies that $\{x\}^{\delta s(\Lambda,s)} \subseteq U$. Therefore, (X, τ) is $\delta s(\Lambda, s)$-R_0. □

Theorem 3.4. *A topological space (X, τ) is $\delta s(\Lambda, s)$-R_0 if and only if for each x and y in X,*

$$\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$$

implies $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$.

Proof. Suppose that (X, τ) is $\delta s(\Lambda, s)$-R_0 and $x, y \in X$ such that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. Then, there exists $z \in \{x\}^{\delta s(\Lambda,s)}$ such that $z \notin \{y\}^{\delta s(\Lambda,s)}$ (or $z \in \{y\}^{\delta s(\Lambda,s)}$ such that $z \notin \{x\}^{\delta s(\Lambda,s)}$). There exists $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $y \notin V$ and $z \in V$; hence $x \in V$. Therefore, $x \notin \{y\}^{\delta s(\Lambda,s)}$. Thus,

$$x \in (X - \{y\}^{\delta s(\Lambda,s)}) \in \delta s(\Lambda, s)O(X, \tau),$$

which implies $\{x\}^{\delta s(\Lambda,s)} \subseteq X - \{y\}^{\delta s(\Lambda,s)}$ and $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. The proof for otherwise is similar.

Conversely, let $V \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in V$. Suppose that $y \notin V$. Then, $x \neq y$ and $x \notin \{y\}^{\delta s(\Lambda,s)}$. Therefore, $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. By the hypothesis, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. Thus, $y \notin \{x\}^{\delta s(\Lambda,s)}$ and hence $\{x\}^{\delta s(\Lambda,s)} \subseteq V$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_0. □

Theorem 3.5. *For a topological space (X, τ), the following properties are equivalent:*

1. *(X, τ) is $\delta s(\Lambda, s)$-R_0.*
2. *For each nonempty subset A of X and each $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $A \cap V \neq \emptyset$, there exists $F \in \delta s(\Lambda, s)C(X, \tau)$ such that $A \cap F \neq \emptyset$ and $F \subseteq V$.*
3. *For each $V \in \delta s(\Lambda, s)O(X, \tau)$, $V = \cup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$.*
4. *For each $F \in \delta s(\Lambda, s)C(X, \tau)$, $F = \cap\{V \in \delta s(\Lambda, s)O(X, \tau) \mid F \subseteq V\}$.*
5. *For each $x \in X$, $\{x\}^{\delta s(\Lambda,s)} \subseteq \delta s(\Lambda, s)Ker(\{x\})$.*

Proof. (1) ⇒ (2): Let A be a nonempty subset of X and $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $A \cap V \neq \emptyset$. There exists $x \in A \cap V$. Since $x \in V \in \delta s(\Lambda, s)O(X, \tau)$, $\{x\}^{\delta s(\Lambda,s)} \subseteq V$. Put $F = \{x\}^{\delta s(\Lambda,s)}$. Then, we have $F \in \delta s(\Lambda, s)C(X, \tau)$, $F \subseteq V$ and $A \cap F \neq \emptyset$.

(2) ⇒ (3): Let $V \in \delta s(\Lambda, s)O(X, \tau)$. Then, $V \supseteq \cup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$. Let x be any point of V. There exists $F \in \delta s(\Lambda, s)C(X, \tau)$ such that $x \in F$ and $F \subseteq V$. Thus,

$$x \in F \subseteq \cup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$$

and hence $V = \cup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$. □

(3) ⇒ (4): The proof is obvious.

(4) ⇒ (5): Let x be any point of X and $y \notin \delta s(\Lambda, s)Ker(\{x\})$. There exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$ and $y \notin U$; hence $\{y\}^{\delta s(\Lambda,s)} \cap U = \emptyset$.
By (4), $(\cap\{V \in \delta s(\Lambda, s)O(X, \tau) \mid \{y\}^{\delta s(\Lambda,s)} \subseteq V\}) \cap U = \emptyset$ and there exists $W \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \notin W$ and $\{y\}^{\delta s(\Lambda,s)} \subseteq W$. Therefore, $W \cap \{x\}^{\delta s(\Lambda,s)} = \emptyset$ and $y \notin \{x\}^{\delta s(\Lambda,s)}$. Thus, $\{x\}^{\delta s(\Lambda,s)} \subseteq \delta s(\Lambda, s)Ker(\{x\})$.

(5) ⇒ (1): Let $U \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in U$. Let $y \in \delta s(\Lambda, s)Ker(\{x\})$. Then, $x \in \{y\}^{\delta s(\Lambda,s)}$ and $y \in U$. Thus, $\delta s(\Lambda, s)Ker(\{x\}) \subseteq U$ and hence $\{x\}^{\delta s(\Lambda,s)} \subseteq U$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_0.

Corollary 3.2. *For a topological space (X, τ), the following properties are equivalent:*

1. *(X, τ) is $\delta s(\Lambda, s)$-R_0;*
2. *$\{x\}^{\delta s(\Lambda,s)} = \delta s(\Lambda, s)Ker(\{x\})$ for each $x \in X$.*

Proof. (1) ⇒ (2): Suppose that (X, τ) is $\delta s(\Lambda, s)$-R_0. By Theorem 3.5, $\{x\}^{\delta s(\Lambda,s)} \subseteq \delta s(\Lambda, s)Ker(\{x\})$ for each $x \in X$. Let $y \in \delta s(\Lambda, s)Ker(\{x\})$. Then, $x \in \{y\}^{\delta s(\Lambda,s)}$ and by Theorem 3.4,

$$\{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)}.$$

Thus, $y \in \{x\}^{\delta s(\Lambda,s)}$ and hence $\delta s(\Lambda,s)Ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda,s)}$. This shows that $\{x\}^{\delta s(\Lambda,s)} = \delta s(\Lambda,s)Ker(\{x\})$.

(2) ⇒ (1): This is obvious by Theorem 3.5. □

Theorem 3.6. *For a topological space (X,τ), the following properties are equivalent:*

1. *(X,τ) is $\delta s(\Lambda,s)$-R_0.*
2. *For each $F \in \delta s(\Lambda,s)C(X,\tau)$, $F = \delta s(\Lambda,s)Ker(F)$.*
3. *For each $F \in \delta s(\Lambda,s)C(X,\tau)$ and $x \in F$, $\delta s(\Lambda,s)Ker(\{x\}) \subseteq F$.*
4. *For each $x \in X$, $\delta s(\Lambda,s)Ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda,s)}$.*

Proof. (1) ⇒ (2): This obviously follows from Theorem 3.5.

(2) ⇒ (3): Let $F \in \delta s(\Lambda,s)C(X,\tau)$ and $x \in F$. By (2), $\delta s(\Lambda,s)Ker(\{x\}) \subseteq \delta s(\Lambda,s)Ker(F) = F$.

(3) ⇒ (4): Let $x \in X$. Since $x \in \{x\}^{\delta s(\Lambda,s)}$ and $\{x\}^{\delta s(\Lambda,s)}$ is $\delta s(\Lambda,s)$-closed, by (3),

$$\delta s(\Lambda,s)Ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda,s)}.$$

(4) ⇒ (1): We show the implication by using Theorem 3.3. Let $x \in \{y\}^{\delta s(\Lambda,s)}$. By Lemma 3.3,

$$y \in \delta s(\Lambda,s)Ker(\{x\}).$$

Since $x \in \{x\}^{\delta s(\Lambda,s)}$ and $\{x\}^{\delta s(\Lambda,s)}$ is $\delta s(\Lambda,s)$-closed, by (4), $y \in \delta s(\Lambda,s)Ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda,s)}$. Thus, $x \in \{y\}^{\delta s(\Lambda,s)}$ implies $y \in \{x\}^{\delta s(\Lambda,s)}$. The converse is obvious and (X,τ) is $\delta s(\Lambda,s)$-R_0. □

Definition 3.3. [20] *Let (X,τ) be a topological space and $x \in X$. A subset $\langle x\rangle_{\delta s(\Lambda,s)}$ is defined as follows: $\langle x\rangle_{\delta s(\Lambda,s)} = \delta s(\Lambda,s)Ker(\{x\}) \cap \{x\}^{\delta s(\Lambda,s)}$.*

Theorem 3.7. *A topological space (X,τ) is $\delta s(\Lambda,s)$-R_0 if and only if $\langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)}$ for each $x \in X$.*

Proof. Let $x \in X$. By Corollary 3.2, $\delta s(\Lambda,s)Ker(\{x\}) = \{x\}^{\delta s(\Lambda,s)}$. Thus,

$$\langle x\rangle_{\delta s(\Lambda,s)} = \delta s(\Lambda,s)Ker(\{x\}) \cap \{x\}^{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)}.$$

Conversely, let $x \in X$. By the hypothesis,

$$\{x\}^{\delta s(\Lambda,s)} = \langle x\rangle_{\delta s(\Lambda,s)} = \delta s(\Lambda,s)Ker(\{x\}) \cap \{x\}^{\delta s(\Lambda,s)} \subseteq \delta s(\Lambda,s)Ker(\{x\}).$$

It follows from Theorem 3.5 that (X,τ) is $\delta s(\Lambda,s)$-R_0. □

4. On $\delta s(\Lambda,s)$-R_1 spaces

We begin this section by introducing the notion of $\delta s(\Lambda,s)$-R_1 spaces.

Definition 4.1. *A topological space (X,τ) is said to be $\delta s(\Lambda,s)$-R_1 if for each x and y in X such that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$, there exist disjoint $\delta s(\Lambda,s)$-open sets U and V such that $\{x\}^{\delta s(\Lambda,s)} \subseteq U$ and $\{y\}^{\delta s(\Lambda,s)} \subseteq V$.*

Theorem 4.1. *A topological space (X, τ) is $\delta s(\Lambda, s)$-R_1 if and only if for each x and y in X such that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$, there exist $\delta s(\Lambda, s)$-closed sets F and K such that $x \in F$, $y \notin F$, $y \in K$, $x \notin K$ and $X = F \cup K$.*

Proof. Let x and y be any points in X with $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. Then, there exist disjoint

$$U, V \in \delta s(\Lambda, s)O(X, \tau)$$

such that $\{x\}^{\delta s(\Lambda,s)} \subseteq U$ and $\{y\}^{\delta s(\Lambda,s)} \subseteq V$. Now, put $F = X - V$ and $K = X - U$. Then, F and K are $\delta s(\Lambda, s)$-closed sets of X such that $x \in F$, $y \notin F$, $y \in K$, $x \notin K$ and $X = F \cup K$.

Conversely, let x and y be any points in X such that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. Then,

$$\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset.$$

In fact, if $z \in \{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)}$, then $\{z\}^{\delta s(\Lambda,s)} \neq \{x\}^{\delta s(\Lambda,s)}$ or $\{z\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. In case $\{z\}^{\delta s(\Lambda,s)} \neq \{x\}^{\delta s(\Lambda,s)}$, by the hypothesis, there exists a $\delta s(\Lambda, s)$-closed set F such that $x \in F$ and $z \notin F$. Then, $z \in \{x\}^{\delta s(\Lambda,s)} \subseteq F$. This contradicts that $z \notin F$. In case $\{z\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$, similarly, this leads to the contradiction. Thus, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$, by Corollary 3.1, (X, τ) is $\delta s(\Lambda, s)$-R_0. By the hypothesis, there exist $\delta s(\Lambda, s)$-closed sets F and K such that $x \in F$, $y \notin F$, $y \in K$, $x \notin K$ and $X = F \cup K$. Put $U = X - K$ and $V = X - F$. Then, $x \in U \in \delta s(\Lambda, s)O(X, \tau)$ and $y \in V \in \delta s(\Lambda, s)O(X, \tau)$. Since (X, τ) is $\delta s(\Lambda, s)$-R_0, we have $\{x\}^{\delta s(\Lambda,s)} \subseteq U$, $\{y\}^{\delta s(\Lambda,s)} \subseteq V$ and also $U \cap V = \emptyset$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_1. □

Definition 4.2. *Let A be a subset of a topological space (X, τ). The $\theta\delta s(\Lambda, s)$-closure of A, $A^{\theta\delta s(\Lambda,s)}$, is defined as follows:*
$A^{\theta\delta s(\Lambda,s)} = \{x \in X \mid A \cap U^{\delta s(\Lambda,s)} \neq \emptyset$ for each $U \in \delta s(\Lambda, s)O(X, \tau)$ containing $x\}$.

Lemma 4.1. *If a topological space (X, τ) is $\delta s(\Lambda, s)$-R_1, then (X, τ) is $\delta s(\Lambda, s)$-R_0.*

Proof. Let $U \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in U$. If $y \notin U$, then $U \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$ and $x \notin \{y\}^{\delta s(\Lambda,s)}$. This implies that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. Since (X, τ) is $\delta s(\Lambda, s)$-R_1, there exists $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $\{y\}^{\delta s(\Lambda,s)} \subseteq V$ and $x \notin V$. Thus, $V \cap \{x\}^{\delta s(\Lambda,s)} = \emptyset$ and hence $y \notin \{x\}^{\delta s(\Lambda,s)}$. Therefore, $\{x\}^{\delta s(\Lambda,s)} \subseteq U$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_0. □

Theorem 4.2. *A topological space (X, τ) is $\delta s(\Lambda, s)$-R_1 if and only if $\langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)}$ for each $x \in X$.*

Proof. Let (X, τ) be $\delta s(\Lambda, s)$-R_1. By Lemma 4.1, (X, τ) is $\delta s(\Lambda, s)$-R_0 and by Theorem 3.7,

$$\langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)} \subseteq \{x\}^{\theta\delta s(\Lambda,s)}$$

for each $x \in X$. Thus, $\langle x\rangle_{\delta s(\Lambda,s)} \subseteq \{x\}^{\theta\delta s(\Lambda,s)}$ for each $x \in X$. In order to show the opposite inclusion, suppose that $y \notin \langle x\rangle_{\delta s(\Lambda,s)}$. Then, $\langle x\rangle_{\delta s(\Lambda,s)} \neq \langle y\rangle_{\delta s(\Lambda,s)}$. Since (X, τ) is $\delta s(\Lambda, s)$-R_0, by Theorem 3.7, $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. Since (X, τ) is $\delta s(\Lambda, s)$-R_1, there exist disjoint $\delta s(\Lambda, s)$-open sets U

and V of X such that $\{x\}^{\delta s(\Lambda,s)} \subseteq U$ and $\{y\}^{\delta s(\Lambda,s)} \subseteq V$. Since $\{x\} \cap V^{\delta s(\Lambda,s)} \subseteq U \cap V^{\delta s(\Lambda,s)} = \emptyset$, $y \notin \{x\}^{\theta\delta s(\Lambda,s)}$. Thus, $\{x\}^{\theta\delta s(\Lambda,s)} \subseteq \langle x\rangle_{\delta s(\Lambda,s)}$ and hence $\{x\}^{\theta\delta s(\Lambda,s)} = \langle x\rangle_{\delta s(\Lambda,s)}$.

Conversely, suppose that $\{x\}^{\theta\delta s(\Lambda,s)} = \langle x\rangle_{\delta s(\Lambda,s)}$ for each $x \in X$. Then,

$$\langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)} \supseteq \{x\}^{\delta s(\Lambda,s)} \supseteq \langle x\rangle_{\delta s(\Lambda,s)}$$

and $\langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)}$ for each $x \in X$. By Theorem 3.7, (X,τ) is $\delta s(\Lambda,s)$-R_0. Suppose that

$$\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}.$$

Thus, by Corollary 3.1, $\{x\}^{\delta s(\Lambda,s)} \cap \{y\}^{\delta s(\Lambda,s)} = \emptyset$. By Theorem 3.7, $\langle x\rangle_{\delta s(\Lambda,s)} \cap \langle y\rangle_{\delta s(\Lambda,s)} = \emptyset$ and hence $\{x\}^{\theta\delta s(\Lambda,s)} \cap \{y\}^{\theta\delta s(\Lambda,s)} = \emptyset$. Since $y \notin \{x\}^{\theta\delta s(\Lambda,s)}$, there exists a $\delta s(\Lambda,s)$-open set U of X such that $y \in U \subseteq U^{\delta s(\Lambda,s)} \subseteq X - \{x\}$. Let $V = X - U^{\delta s(\Lambda,s)}$, then $x \in V \in \delta s(\Lambda,s)O(X,\tau)$. Since (X,τ) is $\delta s(\Lambda,s)$-R_0, $\{y\}^{\delta s(\Lambda,s)} \subseteq U$, $\{x\}^{\delta s(\Lambda,s)} \subseteq V$ and $U \cap V = \emptyset$. This shows that (X,τ) is $\delta s(\Lambda,s)$-R_1. □

Corollary 4.1. *A topological space (X,τ) is $\delta s(\Lambda,s)$-R_1 if and only if $\{x\}^{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)}$ for each $x \in X$.*

Proof. Let (X,τ) be a $\delta s(\Lambda,s)$-R_1 space. By Theorem 4.2, we have

$$\{x\}^{\delta s(\Lambda,s)} \supseteq \langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)} \supseteq \{x\}^{\delta s(\Lambda,s)}$$

and hence $\{x\}^{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)}$ for each $x \in X$.

Conversely, suppose that $\{x\}^{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)}$ for each $x \in X$. First, we show that (X,τ) is $\delta s(\Lambda,s)$-R_0. Let $U \in \delta s(\Lambda,s)O(X,\tau)$ and $x \in U$. Let $y \notin U$. Then, $U \cap \{y\}^{\delta s(\Lambda,s)} = U \cap \{y\}^{\theta\delta s(\Lambda,s)} = \emptyset$. Thus, $x \notin \{y\}^{\theta\delta s(\Lambda,s)}$. There exists $V \in \delta s(\Lambda,s)O(X,\tau)$ such that $x \in V$ and $y \notin V^{\delta s(\Lambda,s)}$. Since

$$\{x\}^{\delta s(\Lambda,s)} \subseteq V^{\delta s(\Lambda,s)},$$

$y \notin \{x\}^{\delta s(\Lambda,s)}$. This shows that $\{x\}^{\delta s(\Lambda,s)} \subseteq U$ and hence (X,τ) is $\delta s(\Lambda,s)$-R_0. By Theorem 3.7,

$$\langle x\rangle_{\delta s(\Lambda,s)} = \{x\}^{\delta s(\Lambda,s)} = \{x\}^{\theta\delta s(\Lambda,s)}$$

for each $x \in X$. Thus, by Theorem 4.2, (X,τ) is $\delta s(\Lambda,s)$-R_1. □

Definition 4.3. *A topological space (X,τ) is said to be:*

(a) *$\delta s(\Lambda,s)$-T_0 if for any pair of distinct points in X, there exists a $\delta s(\Lambda,s)$-open set containing one of the points but not the other;*
(b) *$\delta s(\Lambda,s)$-T_1 if for any pair of distinct points x and y in X, there exist $\delta s(\Lambda,s)$-open sets U and V of X such that $x \in U$, $y \notin U$ and $y \in V$, $x \notin V$;*
(c) *$\delta s(\Lambda,s)$-T_2 if for any pair of distinct points x and y in X, there exist $\delta s(\Lambda,s)$-open sets U and V of X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Lemma 4.2. *For a topological space (X,τ), the following properties are equivalent:*

(1) *(X, τ) is $\delta s(\Lambda, s)$-T_1.*

(2) *For each $x \in X$, $\{x\}$ is $\delta s(\Lambda, s)$-closed.*

(3) *(X, τ) is $\delta s(\Lambda, s)$-R_0 and $\delta s(\Lambda, s)$-T_0.*

Proof. (1) $\Rightarrow$ (2): Let x be any point of X. Let y be any point of X such that $y \neq x$. There exists a $\delta s(\Lambda, s)$-open sets U of X such that $y \in U$ and $x \notin U$. Thus, $y \notin \{x\}^{\delta s(\Lambda,s)}$ and hence $\{x\}^{\delta s(\Lambda,s)} = \{x\}$. This shows that $\{x\}$ is $\delta s(\Lambda, s)$-closed.

(2) $\Rightarrow$ (3): The proof is obvious.

(3) $\Rightarrow$ (1): Let x and y be any distinct points of X. Since (X, τ) is $\delta s(\Lambda, s)$-T_0, there exists a $\delta s(\Lambda, s)$-open sets U of X such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$. In case $x \in U$ and $y \notin U$, we have $x \in \{x\}^{\delta s(\Lambda,s)} \subseteq U$ and hence $y \in X - U \subseteq X - \{x\}^{\delta s(\Lambda,s)}$. Since the proof of the other is quite similar, (X, τ) is $\delta s(\Lambda, s)$-T_1. □

Theorem 4.3. *For a topological space (X, τ), the following properties are equivalent:*

(1) *(X, τ) is $\delta s(\Lambda, s)$-T_2.*

(2) *(X, τ) is $\delta s(\Lambda, s)$-R_1 and $\delta s(\Lambda, s)$-T_1.*

(3) *(X, τ) is $\delta s(\Lambda, s)$-R_1 and $\delta s(\Lambda, s)$-T_0.*

Proof. (1) $\Rightarrow$ (2): Since (X, τ) is $\delta s(\Lambda, s)$-T_2, (X, τ) is $\delta s(\Lambda, s)$-T_1. Let x and y be any points of X such that $\{x\}^{\delta s(\Lambda,s)} \neq \{y\}^{\delta s(\Lambda,s)}$. Thus, by Lemma 4.2, $\{x\} = \{x\}^{\delta s(\Lambda,s)} = \{y\}^{\delta s(\Lambda,s)} = \{y\}$ and there exist disjoint $\delta s(\Lambda, s)$-open sets U and V of X such that $\{x\}^{\delta s(\Lambda,s)} = \{x\} \subseteq U$ and $\{y\}^{\delta s(\Lambda,s)} = \{y\} \subseteq V$. This shows that (X, τ) is $\delta s(\Lambda, s)$-R_1.

(2) $\Rightarrow$ (3): The proof is obvious.

(3) $\Rightarrow$ (1): Let (X, τ) be $\delta s(\Lambda, s)$-R_1 and $\delta s(\Lambda, s)$-T_0. By Lemma 4.1 and 4.2, (X, τ) is $\delta s(\Lambda, s)$-T_1 and every singleton is $\delta s(\Lambda, s)$-closed. Let x and y be any distinct points of X. Then,

$$\{x\}^{\delta s(\Lambda,s)} = \{x\} \neq \{y\} = \{y\}^{\delta s(\Lambda,s)}$$

and there exist disjoint $\delta s(\Lambda, s)$-open sets U and V of X such that $x \in U$ and $y \in V$. This shows that (X, τ) is $\delta s(\Lambda, s)$-T_2. □

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