δs(Λ, s)-R₀ Spaces and δs(Λ, s)-R₁ Spaces

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Abstract. Our main purpose is to introduce the notions of δs(Λ, s)-R₀ spaces and δs(Λ, s)-R₁ spaces. Moreover, several characterizations of δs(Λ, s)-R₀ spaces and δs(Λ, s)-R₁ spaces are investigated.

1. Introduction

The concept of R₀ topological spaces was first introduced by Shanin [21]. Davis [7] introduced the concept of a separation axiom called R₁. These concepts are further investigated by Naimpally [16], Dube [11] and Dorsett [8]. Murdeshwar and Naimpally [15] and Dube [10] studied some of the fundamental properties of the class of R₁ topological spaces. As natural generalizations of the separations axioms R₀ and R₁, the concepts of semi-R₀ and semi-R₁ spaces were introduced and studied by Maheshwari and Prasad [14] and Dorsett [9]. Caldas et al. [4] introduced and investigated two new weak separation axioms called Λθ-R₀ and Λθ-R₁ by using the notions of (Λ, θ)-open sets and the (Λ, θ)-closure operator. Cammaroto and Noiri [2] defined a weak separation axiom m-R₀ in m-spaces which are equivalent to generalized topological spaces due to Lugojan [13]. Noiri [17] introduced the notion of m-R₁ spaces and investigated several characterizations of m-R₀ spaces and m-R₁ spaces. Moreover, Levine [12] introduced the concept of semi-open sets which is weaker than the concept of open sets in topological spaces. Veličko [23] introduced δ-open sets, which are stronger than open sets. Park et al. [19] have offered new notion called δ-semiopen sets which are stronger than semi-open sets but weaker than δ-open sets and investigated the relationships between several types of these open sets. Caldas et al. [5] investigated some weak separation axioms by utilizing δ-semiopen sets.
sets and the δ-semiclosure operator. Caldas et al. [3] investigated the notion of δ-s-L_s-semiclosed sets which is defined as the intersection of a δ-s-L_s-set and a δ-s-semiclosed set. Noiri [18] showed that a subset A of a topological space (X, τ) is δ-semiopen in (X, τ) if and only if it is semi-open in (X, τ_s). In [1], the present authors introduced and investigated the concept of (Λ, s)-closed sets by utilizing the notions of Λ_s-sets and semi-closed sets. Pue-on and Boonpok [20] introduced and studied the notions of δs(Λ, s)-open sets and δs(Λ, s)-closed sets. In this paper, we introduce the notions of δs(Λ, s)-R_0 spaces and δs(Λ, s)-R_1 spaces. Furthermore, several characterizations of δs(Λ, s)-R_0 spaces and δs(Λ, s)-R_1 spaces are discussed.

2. Preliminaries

Let A be a subset of a topological space (X, τ). The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is called semi-open [12] if A ⊆ Cl(Int(A)). The family of all semi-open sets in a topological space (X, τ) is denoted by SO(X, τ). A subset A^Λ [6] is defined as follows: A^Λ = ∩{U ⊆ X,τ | U ⊇ A, U ∈ SO(X, τ)}. A subset A of a topological space (X, τ) is called a Λ_s-set [6] if A = A^Λ. A subset A of a topological space (X, τ) is called (Λ, s)-closed [1] if A = T ∩ C, where T is a Λ_s-set and C is a semi-closed set. The complement of a (Λ, s)-closed set is called (Λ, s)-open. The family of all (Λ, s)-closed (resp. (Λ, s)-open) sets in a topological space (X, τ) is denoted by Λ_sC(X, τ) (resp. Λ_sO(X, τ)). Let A be a subset of a topological space (X, τ). A point x ∈ X is called a (Λ, s)-cluster point [1] of A if A ∩ U ⊈ ∅ for every (Λ, s)-open set U of X containing x. The set of all (Λ, s)-cluster points of A is called the (Λ, s)-closure [1] of A and is denoted by A^(Λ,s). The union of all (Λ, s)-open sets of X contained in A is called the (Λ, s)-interior [1] of A and is denoted by A_(_Λ,s). A point x of X is called a δ(Λ, s)-cluster point [22] of A if A ∩ [V^(Λ,s)](Λ,s) ⊈ ∅ for every (Λ, s)-open set V of X containing x. The set of all δ(Λ, s)-cluster points of A is called the δ(Λ, s)-closure [22] of A and is denoted by A^δ(Λ,s). If A = A^δ(Λ,s), then A is said to be δ(Λ, s)-closed [22]. The complement of a δ(Λ, s)-closed set is said to be δ(Λ, s)-open. The union of all δ(Λ, s)-open sets of X contained in A is called the δ(Λ, s)-interior [22] of A and is denoted by A_δ(Λ,s). A subset A of a topological space (X, τ) is said to be δs(Λ, s)-open [20] if A ⊆ [A^(Λ,s)]^δ(Λ,s). The complement of a δs(Λ, s)-open set is said to be δs(Λ, s)-closed. The family of all δs(Λ, s)-open (resp. δs(Λ, s)-closed) sets in a topological space (X, τ) is denoted by δs(Λ, s)O(X, τ) (resp. δs(Λ, s)C(X, τ)). A subset N of a topological space (X, τ) is called a δs(Λ, s)-neighborhood [20] of a point x ∈ X if there exists a δs(Λ, s)-open set V such that x ∈ V ⊆ N. Let A be a subset of a topological space (X, τ). A point x of X is called a δs(Λ, s)-cluster point [20] of A if A ∩ U ⊈ ∅ for every δs(Λ, s)-open set U of X containing x. The set of all δs(Λ, s)-cluster points of A is called the δs(Λ, s)-closure [20] of A and is denoted by A^δs(Λ,s).

**Lemma 2.1.** [20] For the δs(Λ, s)-closure of subsets A, B in a topological space (X, τ), the following properties hold:
Definition 3.1. A topological space \( (X, \tau) \) is called \( \delta s(\Lambda, s) \)-\( R_0 \) if for each \( \delta s(\Lambda, s) \)-open set \( U \) and each \( x \in U \), \( \{x\}^{\delta s(\Lambda, s)} \subseteq U \).

Theorem 3.1. For a topological space \( (X, \tau) \), the following properties are equivalent:

1. \( (X, \tau) \) is \( \delta s(\Lambda, s) \)-\( R_0 \).
2. For each \( \delta s(\Lambda, s) \)-closed set \( F \) and each \( x \in X - F \), there exists \( U \in \delta s(\Lambda, s)O(X, \tau) \) such that \( F \subseteq U \) and \( x \notin U \).
3. For each \( \delta s(\Lambda, s) \)-closed set \( F \) and each \( x \in X - F \), \( F \cap \{x\}^{\delta s(\Lambda, s)} = \emptyset \).
4. For any distinct points \( x, y \in X \), \( \{x\}^{\delta s(\Lambda, s)} = \{y\}^{\delta s(\Lambda, s)} \) or \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \).

Proof. (1) \( \Rightarrow \) (2): Let \( F \) be a \( \delta s(\Lambda, s) \)-closed set and \( x \in X - F \). Since \( (X, \tau) \) is \( \delta s(\Lambda, s) \)-\( R_0 \), we have \( \{x\}^{\delta s(\Lambda, s)} \subseteq X - F \). Put \( U = X - \{x\}^{\delta s(\Lambda, s)} \). Thus, by Lemma 2.1, \( U \in \delta s(\Lambda, s)O(X, \tau) \), \( F \subseteq U \) and \( x \notin U \).

(2) \( \Rightarrow \) (3): Let \( F \) be a \( \delta s(\Lambda, s) \)-closed set and \( x \in X - F \). By (2), there exists \( U \in \delta s(\Lambda, s)O(X, \tau) \) such that \( F \subseteq U \) and \( x \notin U \). Since \( U \in \delta s(\Lambda, s)O(X, \tau) \), \( U \cap \{x\}^{\delta s(\Lambda, s)} = \emptyset \) and hence \( F \cap \{x\}^{\delta s(\Lambda, s)} = \emptyset \).

(3) \( \Rightarrow \) (4): Let \( x \) and \( y \) be distinct points of \( X \). Suppose that \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} \neq \emptyset \). By (3), \( x \in \{y\}^{\delta s(\Lambda, s)} \) and \( y \in \{x\}^{\delta s(\Lambda, s)} \). By Lemma 2.1, \( \{x\}^{\delta s(\Lambda, s)} \subseteq \{y\}^{\delta s(\Lambda, s)} \subseteq \{x\}^{\delta s(\Lambda, s)} \) and hence

\[ \{x\}^{\delta s(\Lambda, s)} = \{y\}^{\delta s(\Lambda, s)}. \]

(4) \( \Rightarrow \) (1): Let \( V \in \delta s(\Lambda, s)O(X, \tau) \) and \( x \in V \). For each \( y \notin V \), \( V \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \) and hence \( x \notin \{y\}^{\delta s(\Lambda, s)} \). Thus, \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). By (4), for each \( y \notin V \), \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \). Since \( X - V \) is \( \delta s(\Lambda, s) \)-closed, \( y \in \{y\}^{\delta s(\Lambda, s)} \subseteq X - V \) and \( \cup_{y \in X - V} \{y\}^{\delta s(\Lambda, s)} = X - V \). Thus,

\[ \{x\}^{\delta s(\Lambda, s)} \cap (X - V) = \{x\}^{\delta s(\Lambda, s)} \cap [\cup_{y \in X - V} \{y\}^{\delta s(\Lambda, s)}] = \cup_{y \in X - V} \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \]

and hence \( \{x\}^{\delta s(\Lambda, s)} \subseteq V \). This shows that \( (X, \tau) \) is \( \delta s(\Lambda, s) \)-\( R_0 \). \( \square \)

Corollary 3.1. A topological space \( (X, \tau) \) is \( \delta s(\Lambda, s) \)-\( R_0 \) if and only if for any points \( x \) and \( y \) in \( X \), \( \{x\}^{\delta p(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \) implies \( \{x\}^{\delta p(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \).
Proof. This is obvious by Theorem 3.1.

Conversely, let \( U \in \delta s(\Lambda, s)O(\chi, \tau) \) and \( x \in U \). If \( y \notin U \), then \( U \cap \{ y \}^{\delta s(\Lambda, s)} = \emptyset \). Thus, \( x \notin \{ y \}^{\delta s(\Lambda, s)} \) and \( \{ x \}^{\delta s(\Lambda, s)} \neq \{ y \}^{\delta s(\Lambda, s)} \). By the hypothesis, \( \{ x \}^{\delta s(\Lambda, s)} \cap \{ y \}^{\delta s(\Lambda, s)} = \emptyset \) and hence \( y \notin \{ x \}^{\delta s(\Lambda, s)} \). This shows that \( \{ x \}^{\delta s(\Lambda, s)} \subseteq U \). Therefore, \((\chi, \tau)\) is \( \delta s(\Lambda, s)-R_0 \). □

**Definition 3.2.** [20] Let \( A \) be a subset of a topological space \((\chi, \tau)\). The \( \delta s(\Lambda, s)\)-kernel of \( A \), denoted by \( \delta s(\Lambda, s)Ker(A) \), is defined to be the set \( \delta s(\Lambda, s)Ker(A) = \cap \{ U \mid A \subseteq U, U \in \delta s(\Lambda, s)O(\chi, \tau) \} \).

**Lemma 3.1.** [20] For subsets \( A, B \) of a topological space \((\chi, \tau)\), the following properties hold:

1. \( A \subseteq \delta s(\Lambda, s)Ker(A) \).
2. If \( A \subseteq B \), then \( \delta s(\Lambda, s)Ker(A) \subseteq \delta s(\Lambda, s)Ker(B) \).
3. \( \delta s(\Lambda, s)Ker(\delta s(\Lambda, s)Ker(A)) = \delta s(\Lambda, s)Ker(A) \).
4. If \( A \) is \( \delta s(\Lambda, s)\)-open, \( \delta s(\Lambda, s)Ker(A) = A \).

**Lemma 3.2.** [20] For any points \( x \) and \( y \) in a topological space \((\chi, \tau)\), the following properties are equivalent:

1. \( \delta s(\Lambda, s)Ker(\{ x \}) \neq \delta s(\Lambda, s)Ker(\{ y \}) \).
2. \( \{ x \}^{\delta s(\Lambda, s)} \neq \{ y \}^{\delta s(\Lambda, s)} \).

**Lemma 3.3.** Let \((\chi, \tau)\) be a topological space and \( x, y \in \chi \). Then, the following properties hold:

1. \( y \in \delta s(\Lambda, s)Ker(\{ x \}) \) if and only if \( x \in \{ y \}^{\delta s(\Lambda, s)} \).
2. \( \delta s(\Lambda, s)Ker(\{ x \}) = \delta s(\Lambda, s)Ker(\{ y \}) \) if and only if \( \{ x \}^{\delta s(\Lambda, s)} = \{ y \}^{\delta s(\Lambda, s)} \).

**Proof.** (1) Let \( x \notin \{ y \}^{\delta s(\Lambda, s)} \). Then, there exists \( U \in \delta s(\Lambda, s)O(\chi, \tau) \) such that \( x \in U \) and \( y \notin U \). Thus, \( y \notin \delta s(\Lambda, s)Ker(\{ x \}) \). The converse is similarly shown.

(2) Suppose that \( \delta s(\Lambda, s)Ker(\{ x \}) = \delta s(\Lambda, s)Ker(\{ y \}) \) for any \( x, y \in \chi \). Since \( x \in \delta s(\Lambda, s)Ker(\{ x \}) \), \( x \in \delta s(\Lambda, s)Ker(\{ y \}) \), by (1). \( y \in \{ x \}^{\delta s(\Lambda, s)} \). By Lemma 2.1, \( \{ y \}^{\delta s(\Lambda, s)} \subseteq \{ x \}^{\delta s(\Lambda, s)} \). Similarly, we have \( \{ x \}^{\delta s(\Lambda, s)} \subseteq \{ y \}^{\delta s(\Lambda, s)} \) and hence \( \{ x \}^{\delta s(\Lambda, s)} = \{ y \}^{\delta s(\Lambda, s)} \).

Conversely, suppose that \( \{ x \}^{\delta s(\Lambda, s)} = \{ y \}^{\delta s(\Lambda, s)} \). Since \( x \in \{ x \}^{\delta s(\Lambda, s)} \), \( x \in \{ y \}^{\delta s(\Lambda, s)} \) and by (1),

\[
y \in \delta s(\Lambda, s)Ker(\{ x \})
\]

By Lemma 3.1, \( \delta s(\Lambda, s)Ker(\{ y \}) \subseteq \delta s(\Lambda, s)Ker(\delta s(\Lambda, s)Ker(\{ x \})) = \delta s(\Lambda, s)Ker(\{ x \}) \). Similarly, we have \( \delta s(\Lambda, s)Ker(\{ x \}) \subseteq \delta p(\Lambda, s)Ker(\{ y \}) \) and hence \( \delta s(\Lambda, s)Ker(\{ x \}) = \delta s(\Lambda, s)Ker(\{ y \}) \).

**Theorem 3.2.** A topological space \((\chi, \tau)\) is \( \delta s(\Lambda, s)-R_0 \) if and only if for each points \( x \) and \( y \) in \( \chi \),
\[
\delta s(\Lambda, s)Ker(\{ x \}) \neq \delta s(\Lambda, s)Ker(\{ y \}) \implies \delta s(\Lambda, s)Ker(\{ x \}) \cap \delta s(\Lambda, s)Ker(\{ y \}) = \emptyset.
\]

**Proof.** Let \((\chi, \tau)\) be \( \delta s(\Lambda, s)-R_0 \). Suppose that \( \delta s(\Lambda, s)Ker(\{ x \}) \cap \delta s(\Lambda, s)Ker(\{ y \}) \neq \emptyset \). Let \( z \in \delta s(\Lambda, s)Ker(\{ x \}) \cap \delta s(\Lambda, s)Ker(\{ y \}) \).
Then, \( z \in \delta s(\Lambda, s) \text{Ker}(\{x\}) \) and by Lemma 3.3, \( x \in \{z\}^{\delta s(\Lambda, s)} \). Thus, \( x \in \{z\}^{\delta s(\Lambda, s)} \cap \{x\}^{\delta s(\Lambda, s)} \) and by Corollary 3.1, \( \{z\}^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)} \). Similarly, we have \( \{z\}^{\delta s(\Lambda, s)} = \{y\}^{\delta s(\Lambda, s)} \) and hence
\[
\{x\}^{\delta s(\Lambda, s)} = \{y\}^{\delta s(\Lambda, s)},
\]
by Lemma 3.3, \( \delta s(\Lambda, s) \text{Ker}(\{x\}) = \delta s(\Lambda, s) \text{Ker}(\{y\}) \).

Conversely, we show the sufficiency by using Corollary 3.1. Suppose that \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). By Lemma 3.3, \( \delta s(\Lambda, s) \text{Ker}(\{x\}) \neq \delta s(\Lambda, s) \text{Ker}(\{y\}) \) and hence
\[
\delta s(\Lambda, s) \text{Ker}(\{x\}) \cap \delta s(\Lambda, s) \text{Ker}(\{y\}) = \emptyset.
\]
Thus, \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \). In fact, assume that \( z \in \{x\}^{\delta s(\Lambda, s)} \) implies \( x \in \delta s(\Lambda, s) \text{Ker}(\{z\}) \) and hence \( x \in \delta s(\Lambda, s) \text{Ker}(\{z\}) \cap \delta s(\Lambda, s) \text{Ker}(\{x\}) \). By the hypothesis, \( \delta s(\Lambda, s) \text{Ker}(\{z\}) = \delta s(\Lambda, s) \text{Ker}(\{x\}) \) and by Lemma 3.3, \( \{z\}^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)} \). Similarly, we have \( \{z\}^{\delta s(\Lambda, s)} = \{y\}^{\delta s(\Lambda, s)} \) and hence \( \{x\}^{\delta s(\Lambda, s)} = \{y\}^{\delta s(\Lambda, s)} \). This contradicts that \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \).

Thus, \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \). This shows that \((X, \tau)\) is \( \delta s(\Lambda, s)\)-\( R_0 \).

**Theorem 3.3.** For a topological space \((X, \tau)\), the following properties are equivalent:

1. \((X, \tau)\) is \( \delta s(\Lambda, s)\)-\( R_0 \).
2. \( x \in \{y\}^{\delta s(\Lambda, s)} \) if and only if \( y \in \{x\}^{\delta s(\Lambda, s)} \).

**Proof.** (1) \( \Rightarrow \) (2): Suppose that \( x \in \{y\}^{\delta s(\Lambda, s)} \). By Lemma 3.3, \( y \in \delta s(\Lambda, s) \text{Ker}(\{x\}) \) and hence
\[
\delta s(\Lambda, s) \text{Ker}(\{x\}) \cap \delta s(\Lambda, s) \text{Ker}(\{y\}) = \emptyset.
\]
By Theorem 3.2, \( \delta s(\Lambda, s) \text{Ker}(\{x\}) = \delta s(\Lambda, s) \text{Ker}(\{y\}) \) and hence \( x \in \delta s(\Lambda, s) \text{Ker}(\{y\}) \). Thus, by Lemma 3.3, \( y \in \{x\}^{\delta s(\Lambda, s)} \). The converse is similarly shown.

(2) \( \Rightarrow \) (1): Let \( U \in \delta s(\Lambda, s) \text{O}(X, \tau) \) and \( x \in U \). If \( y \notin U \), then \( U \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \). Thus, \( x \notin \{y\}^{\delta s(\Lambda, s)} \) and \( y \notin \{x\}^{\delta s(\Lambda, s)} \). This implies that \( \{x\}^{\delta s(\Lambda, s)} \subseteq U \). Therefore, \((X, \tau)\) is \( \delta s(\Lambda, s)\)-\( R_0 \).

**Theorem 3.4.** A topological space \((X, \tau)\) is \( \delta s(\Lambda, s)\)-\( R_0 \) if and only if for each \( x \) and \( y \) in \( X \),
\[
\{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)}
\]
implies \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \).

**Proof.** Suppose that \((X, \tau)\) is \( \delta s(\Lambda, s)\)-\( R_0 \) and \( x, y \in X \) such that \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). Then, there exists \( z \in \{x\}^{\delta s(\Lambda, s)} \) such that \( z \notin \{y\}^{\delta s(\Lambda, s)} \) (or \( z \in \{y\}^{\delta s(\Lambda, s)} \) such that \( z \notin \{x\}^{\delta s(\Lambda, s)} \)). There exists \( V \in \delta s(\Lambda, s) \text{O}(X, \tau) \) such that \( y \notin V \) and \( z \in V \); hence \( x \in V \). Therefore, \( x \notin \{y\}^{\delta s(\Lambda, s)} \).

Thus,
\[
x \in (X - \{y\}^{\delta s(\Lambda, s)}) \in \delta s(\Lambda, s) \text{O}(X, \tau),
\]
which implies \( \{x\}^{\delta s(\Lambda, s)} \subseteq X - \{y\}^{\delta s(\Lambda, s)} \) and \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \). The proof for otherwise is similar.
Conversely, let $V \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in V$. Suppose that $y \notin V$. Then, $x \neq y$ and $x \notin \{y\}\delta s(\Lambda, s)$. Therefore, $\{x\}\delta s(\Lambda, s) \neq \{y\}\delta s(\Lambda, s)$. By the hypothesis, $\{x\}\delta s(\Lambda, s) \cap \{y\}\delta s(\Lambda, s) = \emptyset$. Thus, $y \notin \{x\}\delta s(\Lambda, s)$ and hence $\{x\}\delta s(\Lambda, s) \subseteq V$. This shows that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. \qed

**Theorem 3.5.** For a topological space $(X, \tau)$, the following properties are equivalent:

1. $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$.
2. For each nonempty subset $A$ of $X$ and each $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $A \cap V \neq \emptyset$, there exists $F \in \delta s(\Lambda, s)C(X, \tau)$ such that $A \cap F \neq \emptyset$ and $F \subseteq V$.
3. For each $V \in \delta s(\Lambda, s)O(X, \tau)$, $V = \bigcup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$.
4. For each $F \in \delta s(\Lambda, s)C(X, \tau)$, $F = \cap\{V \delta s(\Lambda, s)O(X, \tau) \mid F \subseteq V\}$.
5. For each $x \in X$, $\{x\}\delta s(\Lambda, s) \subseteq \delta s(\Lambda, s)Ker(\{x\})$.

**Proof.** (1) $\Rightarrow$ (2): Let $A$ be a nonempty subset of $X$ and $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $A \cap V \neq \emptyset$. There exists $x \in A \cap V$. Since $x \in V \in \delta s(\Lambda, s)O(X, \tau)$, $\{x\}\delta s(\Lambda, s) \subseteq V$. Put $F = \{x\}\delta s(\Lambda, s)$. Then, we have $F \in \delta s(\Lambda, s)C(X, \tau)$, $F \subseteq V$ and $A \cap F \neq \emptyset$.

(2) $\Rightarrow$ (3): Let $V \in \delta s(\Lambda, s)O(X, \tau)$. Then, $V = \bigcup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$. Let $x$ be any point of $V$. There exists $F \in \delta s(\Lambda, s)C(X, \tau)$ such that $x \in F$ and $F \subseteq V$. Thus,

$$x \in F \subseteq \bigcup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$$

and hence $V = \bigcup\{F \in \delta s(\Lambda, s)C(X, \tau) \mid F \subseteq V\}$. \qed

(3) $\Rightarrow$ (4): The proof is obvious.

(4) $\Rightarrow$ (5): Let $x$ be any point of $X$ and $y \notin \delta s(\Lambda, s)Ker(\{x\})$. There exists $U \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in U$ and $y \notin U$; hence $\{y\}\delta s(\Lambda, s) \subseteq U = \emptyset$. By (4), $\{y\}\delta s(\Lambda, s) \subseteq U = \emptyset$ and there exists $W \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \notin W$ and $\{y\}\delta s(\Lambda, s) \subseteq W$. Therefore, $W \cap \{x\}\delta s(\Lambda, s) = \emptyset$ and $y \notin \{x\}\delta s(\Lambda, s)$. Thus, $\{x\}\delta s(\Lambda, s) \subseteq \delta s(\Lambda, s)Ker(\{x\})$.

(5) $\Rightarrow$ (1): Let $U \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in U$. Let $y \in \delta s(\Lambda, s)Ker(\{x\})$. Then, $x \in \{y\}\delta s(\Lambda, s)$ and $y \in U$. Thus, $\delta s(\Lambda, s)Ker(\{x\}) \subseteq U$ and hence $\{x\}\delta s(\Lambda, s) \subseteq U$. This shows that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$.

**Corollary 3.2.** For a topological space $(X, \tau)$, the following properties are equivalent:

1. $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$;
2. $\{x\}\delta s(\Lambda, s) = \delta s(\Lambda, s)Ker(\{x\})$ for each $x \in X$.

**Proof.** (1) $\Rightarrow$ (2): Suppose that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. By Theorem 3.5, $\{x\}\delta s(\Lambda, s) \subseteq \delta s(\Lambda, s)Ker(\{x\})$ for each $x \in X$. Let $y \in \delta s(\Lambda, s)Ker(\{x\})$. Then, $x \in \{y\}\delta s(\Lambda, s)$ and by Theorem 3.4,

$$\{x\}\delta s(\Lambda, s) = \{y\}\delta s(\Lambda, s).$$
Thus, \( y \in \{x\}^{\delta s(\Lambda, s)} \) and hence \( \delta s(\Lambda, s)\ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda, s)} \). This shows that \( \{x\}^{\delta s(\Lambda, s)} = \delta s(\Lambda, s)\ker(\{x\}) \).

(2) \(\Rightarrow\) (1): This is obvious by Theorem 3.5.

**Theorem 3.6.** For a topological space \((X, \tau)\), the following properties are equivalent:

1. \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_0\).
2. For each \(F \in \delta s(\Lambda, s)C(X, \tau)\), \(F = \delta s(\Lambda, s)\ker(F)\).
3. For each \(F \in \delta s(\Lambda, s)C(X, \tau)\) and \(x \in F\), \(\delta s(\Lambda, s)\ker(\{x\}) \subseteq F\).
4. For each \(x \in X\), \(\delta s(\Lambda, s)\ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda, s)}\).

**Proof.** (1) \(\Rightarrow\) (2): This obviously follows from Theorem 3.5.

(2) \(\Rightarrow\) (3): Let \(F \in \delta s(\Lambda, s)C(X, \tau)\) and \(x \in F\). By (2), \(\delta s(\Lambda, s)\ker(\{x\}) \subseteq \delta s(\Lambda, s)\ker(F) = F\).

(3) \(\Rightarrow\) (4): Let \(x \in X\). Since \(x \in \{x\}^{\delta s(\Lambda, s)}\) and \(\{x\}^{\delta s(\Lambda, s)}\) is \(\delta s(\Lambda, s)\)-closed, by (3),
\[
\delta s(\Lambda, s)\ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda, s)}.
\]

(4) \(\Rightarrow\) (1): We show the implication by using Theorem 3.3. Let \(x \in \{y\}^{\delta s(\Lambda, s)}\). By Lemma 3.3,
\[
y \in \delta s(\Lambda, s)\ker(\{x\}).
\]
Since \(x \in \{x\}^{\delta s(\Lambda, s)}\) and \(\{x\}^{\delta s(\Lambda, s)}\) is \(\delta s(\Lambda, s)\)-closed, by (4), \(y \in \delta s(\Lambda, s)\ker(\{x\}) \subseteq \{x\}^{\delta s(\Lambda, s)}\).

Thus, \(x \in \{y\}^{\delta s(\Lambda, s)}\) implies \(y \in \{x\}^{\delta s(\Lambda, s)}\). The converse is obvious and \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_0\). \(\square\)

**Definition 3.3.** [20] Let \((X, \tau)\) be a topological space and \(x \in X\). A subset \(\langle x \rangle^{\delta s(\Lambda, s)}\) is defined as follows:
\[
\langle x \rangle^{\delta s(\Lambda, s)} = \delta s(\Lambda, s)\ker(\{x\}) \cap \{x\}^{\delta s(\Lambda, s)}.
\]

**Theorem 3.7.** A topological space \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_0\) if and only if \(\langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)}\) for each \(x \in X\).

**Proof.** Let \(x \in X\). By Corollary 3.2, \(\delta s(\Lambda, s)\ker(\{x\}) = \{x\}^{\delta s(\Lambda, s)}\).

Conversely, let \(x \in X\). By the hypothesis,
\[
\{x\}^{\delta s(\Lambda, s)} = \langle x \rangle^{\delta s(\Lambda, s)} = \delta s(\Lambda, s)\ker(\{x\}) \cap \{x\}^{\delta s(\Lambda, s)} \subseteq \delta s(\Lambda, s)\ker(\{x\})).
\]

It follows from Theorem 3.5 that \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_0\). \(\square\)

4. On \(\delta s(\Lambda, s)\)-\(R_1\) spaces

We begin this section by introducing the notion of \(\delta s(\Lambda, s)\)-\(R_1\) spaces.

**Definition 4.1.** A topological space \((X, \tau)\) is said to be \(\delta s(\Lambda, s)\)-\(R_1\) if for each \(x\) and \(y\) in \(X\) such that \(\{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)}\), there exist disjoint \(\delta s(\Lambda, s)\)-open sets \(U\) and \(V\) such that \(\{x\}^{\delta s(\Lambda, s)} \subseteq U\) and \(\{y\}^{\delta s(\Lambda, s)} \subseteq V\).
**Theorem 4.1.** A topological space $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$ if and only if for each $x$ and $y$ in $X$ such that \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \), there exist $\delta s(\Lambda, s)$-closed sets $F$ and $K$ such that $x \in F$, $y \notin F$, $y \in K$, $x \notin K$ and $X = F \cup K$.

**Proof.** Let $x$ and $y$ be any points in $X$ with \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). Then, there exist disjoint $U, V \in \delta s(\Lambda, s)O(X, \tau)$ such that \( \{x\}^{\delta s(\Lambda, s)} \subset U \) and \( \{y\}^{\delta s(\Lambda, s)} \subset V \). Now, put $F = X - V$ and $K = X - U$. Then, $F$ and $K$ are $\delta s(\Lambda, s)$-closed sets of $X$ such that $x \in F$, $y \notin F$, $y \in K$, $x \notin K$ and $X = F \cup K$.

Conversely, let $x$ and $y$ be any points in $X$ such that \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). Then, 

\[
\{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset.
\]

In fact, if $z \in \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)}$, then \( \{z\}^{\delta s(\Lambda, s)} \neq \{x\}^{\delta s(\Lambda, s)} \) or \( \{z\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). In case \( \{z\}^{\delta s(\Lambda, s)} \neq \{x\}^{\delta s(\Lambda, s)} \), by the hypothesis, there exists a $\delta s(\Lambda, s)$-closed set $F$ such that $x \in F$ and $z \notin F$. Then, $z \in \{x\}^{\delta s(\Lambda, s)} \subset F$. This contradicts that $z \notin F$. In case \( \{z\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \), similarly, this leads to the contradiction. Thus, \( \{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset \), by Corollary 3.1. $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. By the hypothesis, there exist $\delta s(\Lambda, s)$-closed sets $F$ and $K$ such that $x \in F$, $y \notin F$, $y \in K$, $x \notin K$ and $X = F \cup K$. Put $U = X - K$ and $V = X - F$. Then, $x \in U \in \delta s(\Lambda, s)O(X, \tau)$ and $y \in V \in \delta s(\Lambda, s)O(X, \tau)$. Since $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$, we have \( \{x\}^{\delta s(\Lambda, s)} \subset U \), \( \{y\}^{\delta s(\Lambda, s)} \subset V \) and also $U \cap V = \emptyset$. This shows that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$. 

**Definition 4.2.** Let $A$ be a subset of a topological space $(X, \tau)$. The $\theta \delta s(\Lambda, s)$-closure of $A$, $A^{\theta \delta s(\Lambda, s)}$, is defined as follows:

\[
A^{\theta \delta s(\Lambda, s)} = \{x \in X \mid A \cap U^{\delta s(\Lambda, s)} \neq \emptyset \text{ for each } U \in \delta s(\Lambda, s)O(X, \tau) \text{ containing } x\}.
\]

**Lemma 4.1.** If a topological space $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$, then $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$.

**Proof.** Let $U \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in U$. If $y \notin U$, then $U \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset$ and $x \notin \{y\}^{\delta s(\Lambda, s)}$. This implies that \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). Since $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$, there exists $V \in \delta s(\Lambda, s)O(X, \tau)$ such that \( \{y\}^{\delta s(\Lambda, s)} \subset V \) and $x \notin V$. Thus, $V \cap \{x\}^{\delta s(\Lambda, s)} = \emptyset$ and hence $y \notin \{x\}^{\delta s(\Lambda, s)}$. Therefore, $\{x\}^{\delta s(\Lambda, s)} \subset U$. This shows that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. 

**Theorem 4.2.** A topological space $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$ if and only if \( \langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\theta \delta s(\Lambda, s)} \) for each $x \in X$.

**Proof.** Let $(X, \tau)$ be $\delta s(\Lambda, s)$-$R_1$. By Lemma 4.1, $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$ and by Theorem 3.7,

\[
\langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)} \subset \{x\}^{\theta \delta s(\Lambda, s)}
\]

for each $x \in X$. Thus, \( \langle x \rangle^{\delta s(\Lambda, s)} \subset \{x\}^{\theta \delta s(\Lambda, s)} \) for each $x \in X$. In order to show the opposite inclusion, suppose that $y \notin \langle x \rangle^{\delta s(\Lambda, s)}$. Then, \( \langle x \rangle^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). Since $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$, by Theorem 3.7, \( \{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)} \). Since $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$, there exist disjoint $\delta s(\Lambda, s)$-open sets $U$
and $V$ of $X$ such that $\{x\}^{\delta s(\Lambda, s)} \subseteq U$ and $\{y\}^{\delta s(\Lambda, s)} \subseteq V$. Since $\{x\} \cap V^{\delta s(\Lambda, s)} \subseteq U \cap V^{\delta s(\Lambda, s)} = \emptyset$, $y \not\in \{x\}^{\delta s(\Lambda, s)}$. Thus, $\{x\}^{\delta s(\Lambda, s)} \subseteq \langle x \rangle^{\delta s(\Lambda, s)}$ and hence $\{x\}^{\delta s(\Lambda, s)} = \langle x \rangle^{\delta s(\Lambda, s)}$.

Conversely, suppose that $\{x\}^{\delta s(\Lambda, s)} = \langle x \rangle^{\delta s(\Lambda, s)}$ for each $x \in X$. Then,

$$\langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)} \supseteq \{x\}^{\delta s(\Lambda, s)} \supseteq \langle x \rangle^{\delta s(\Lambda, s)}$$

and $\langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)}$ for each $x \in X$. By Theorem 3.7, $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. Suppose that

$$\{x\}^{\delta s(\Lambda, s)} \neq \{y\}^{\delta s(\Lambda, s)}.$$ 

Thus, by Corollary 3.1, $\{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset$. By Theorem 3.7, $\langle x \rangle^{\delta s(\Lambda, s)} \cap \langle y \rangle^{\delta s(\Lambda, s)} = \emptyset$ and hence $\{x\}^{\delta s(\Lambda, s)} \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset$. Since $y \not\in \{x\}^{\delta s(\Lambda, s)}$, there exists a $\delta s(\Lambda, s)$-open set $U$ of $X$ such that $y \in U \subseteq U^{\delta s(\Lambda, s)} \subseteq X - \{x\}$. Let $V = X - U^{\delta s(\Lambda, s)}$, then $x \in V \in \delta s(\Lambda, s)O(X, \tau)$. Since $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$, $\{y\}^{\delta s(\Lambda, s)} \subseteq U$, $\{x\}^{\delta s(\Lambda, s)} \subseteq V$ and $U \cap V = \emptyset$. This shows that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$. \qed

**Corollary 4.1.** A topological space $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$ if and only if $\{x\}^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)}$ for each $x \in X$.

**Proof.** Let $(X, \tau)$ be a $\delta s(\Lambda, s)$-$R_1$ space. By Theorem 4.2, we have

$$\{x\}^{\delta s(\Lambda, s)} \supseteq \langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)} \supseteq \{x\}^{\delta s(\Lambda, s)}$$

and hence $\{x\}^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)}$ for each $x \in X$.

Conversely, suppose that $\{x\}^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)}$ for each $x \in X$. First, we show that $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. Let $U \in \delta s(\Lambda, s)O(X, \tau)$ and $x \in U$. Let $y \not\in U$. Then, $U \cap \{y\}^{\delta s(\Lambda, s)} = U \cap \{y\}^{\delta s(\Lambda, s)} = \emptyset$. Thus, $x \not\in \{y\}^{\delta s(\Lambda, s)}$. There exists $V \in \delta s(\Lambda, s)O(X, \tau)$ such that $x \in V$ and $y \not\in V^{\delta s(\Lambda, s)}$. Since

$$\{x\}^{\delta s(\Lambda, s)} \subseteq V^{\delta s(\Lambda, s)},$$

$y \not\in \{x\}^{\delta s(\Lambda, s)}$. This shows that $\{x\}^{\delta s(\Lambda, s)} \subseteq U$ and hence $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_0$. By Theorem 3.7,

$$\langle x \rangle^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)} = \{x\}^{\delta s(\Lambda, s)}$$

for each $x \in X$. Thus, by Theorem 4.2, $(X, \tau)$ is $\delta s(\Lambda, s)$-$R_1$. \qed

**Definition 4.3.** A topological space $(X, \tau)$ is said to be:

(a) $\delta s(\Lambda, s)$-$T_0$ if for any pair of distinct points in $X$, there exists a $\delta s(\Lambda, s)$-open set containing one of the points but not the other;

(b) $\delta s(\Lambda, s)$-$T_1$ if for any pair of distinct points $x$ and $y$ in $X$, there exist $\delta s(\Lambda, s)$-open sets $U$ and $V$ of $X$ such that $x \in U$, $y \not\in U$ and $y \in V$, $x \not\in V$;

(c) $\delta s(\Lambda, s)$-$T_2$ if for any pair of distinct points $x$ and $y$ in $X$, there exist $\delta s(\Lambda, s)$-open sets $U$ and $V$ of $X$ such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

**Lemma 4.2.** For a topological space $(X, \tau)$, the following properties are equivalent:
(1) \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_1\).

(2) For each \(x \in X\), \(\{x\}\) is \(\delta s(\Lambda, s)\)-closed.

(3) \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_0\) and \(\delta s(\Lambda, s)\)-\(T_0\).

Proof. (1) \(\Rightarrow\) (2): Let \(x\) be any point of \(X\). Let \(y\) be any point of \(X\) such that \(y \neq x\). There exists a \(\delta s(\Lambda, s)\)-open sets \(U\) of \(X\) such that \(y \in U\) and \(x \notin U\). Thus, \(y \notin \{x\}\) and hence \(\{x\}\) is \(\delta s(\Lambda, s)\)-closed.

(2) \(\Rightarrow\) (3): The proof is obvious.

(3) \(\Rightarrow\) (1): Let \(x\) and \(y\) be any distinct points of \(X\). Since \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_0\), there exists a \(\delta s(\Lambda, s)\)-open sets \(U\) of \(X\) such that either \(x \in U\) and \(y \notin U\) or \(x \notin U\) and \(y \in U\). In case \(x \in U\) and \(y \notin U\), we have \(x \in \{x\}\) and \(\{y\}\) \(\delta s(\Lambda, s)\)-closed. Since the proof of the other is quite similar, \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_1\).

\[\square\]

**Theorem 4.3.** For a topological space \((X, \tau)\), the following properties are equivalent:

(1) \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_2\).

(2) \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_1\) and \(\delta s(\Lambda, s)\)-\(T_1\).

(3) \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_1\) and \(\delta s(\Lambda, s)\)-\(T_0\).

Proof. (1) \(\Rightarrow\) (2): Since \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_2\), \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_1\). Let \(x\) and \(y\) be any points of \(X\) such that \(\{x\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\). Thus, by Lemma 4.2, \(\{x\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\). and hence \(\{x\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\). There exist disjoint \(\delta s(\Lambda, s)\)-open sets \(U\) and \(V\) of \(X\) such that \(\{x\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\). This shows that \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(R_1\).

(2) \(\Rightarrow\) (3): The proof is obvious.

(3) \(\Rightarrow\) (1): Let \((X, \tau)\) be \(\delta s(\Lambda, s)\)-\(R_1\) and \(\delta s(\Lambda, s)\)-\(T_0\). By Lemma 4.1 and 4.2, \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_1\) and every singleton is \(\delta s(\Lambda, s)\)-closed. Let \(x\) and \(y\) be any points of \(X\). Then, \(\{x\}\) \(\delta s(\Lambda, s)\) \(\neq\) \(\{y\}\) \(\delta s(\Lambda, s)\). and there exist disjoint \(\delta s(\Lambda, s)\)-open sets \(U\) and \(V\) of \(X\) such that \(x \in U\) and \(y \in V\). This shows that \((X, \tau)\) is \(\delta s(\Lambda, s)\)-\(T_2\).

\[\square\]

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**References**


