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A Novel Method for Finding the Shortest Path With Two Objectives Under Trapezoidal Intuitionistic Fuzzy Arc Costs

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Abstract. The Shortest Path Problem is a core problem in network optimization, with applications in various scientific and engineering fields, such as communication, transportation, routing, scheduling, and computer networks. Many studies and algorithms have been proposed to solve the traditional shortest path problem, but they often fail to provide optimal solutions when dealing with the uncertainties and vagueness that exist in real-world situations. This study aims to address the Bi-objective Shortest Path Problem using intuitionistic fuzzy arc numbers. The main goal is to find the path that minimizes both cost and time between a given source node and destination node. To handle the complexities introduced by trapezoidal intuitionistic fuzzy numbers, an accuracy function is used. The study suggests a simple yet effective method to solve this problem and shows its efficiency through a numerical example. The research tries to offer innovative solutions for optimizing paths in scenarios where cost and time factors are important, navigating the complex landscape of uncertainty inherent in practical applications.

1. Introduction

The Shortest Path Problem represents a highly captivating research area spanning fields like engineering, robotics, networking, and transportation. Its core objective is to pinpoint the most efficient route from a source node to a destination node within a network, taking into account factors such as cost, time, safety, distance, and more. However, real-life scenarios often introduce measurement

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inaccuracies due to natural circumstances. To address this inherent uncertainty, researchers turn to fuzzy set theory, a powerful tool adept at managing the vagueness that arises.

In 1965, Lofti A. Zadeh proposed the groundbreaking Fuzzy Set (FS) theory in his seminal work [3], igniting the exploration of various properties within fuzzy set theory. Extending the generalization of fuzzy sets, Atanassov introduced the Intuitionistic Fuzzy Set (IFS) [4], characterized by its membership and non-membership functions. Notably, the degree of the non-membership function complements the membership function, enhancing its expressiveness.

While many researchers delve into the Fuzzy Shortest Path Problem, its introduction by Dubois [7] laid the foundation for subsequent investigations. Okada's work [6] analyzes labeling algorithms for fuzzy multi-criteria shortest paths. Kelin presented an algorithm [8] aiming to find the optimal path length and minimum arc weights, followed by Okada et al. [10] discussing the optimization of paths considering degrees of possibility within the arcs.

Addressing a limitation in path length determination, Chuang and Kung [9] introduced an innovative approach to discover optimal paths within network structures. Mahdavi [11] extended algorithmic techniques, implementing dynamic programming for the shortest chains while considering fuzzy distances for each edge. A comparative study for the fuzzy shortest path problem is proposed by Vidhya et al. [19].

The Intuitionistic Fuzzy Shortest Path Problem has garnered significant attention, leading to the development of various methods [12]- [14]. Arana et al. [15] proposed mixed-integer linear programming for a fully fuzzy version of the problem.

In this study, we tackle the Intuitionistic Fuzzy Shortest Path (IFSP) problem by formulating it and proposing an innovative method to optimize for minimum-maximum cost and time. The study culminates in the attainment of optimal results, contributing to this dynamic research domain.

The structure of the paper unfolds as follows: Section 2 provides an exploration of fundamental concepts and definitions pertinent to the topic. Section 3 delves into the mathematical formulation of IFSPP. Section 4 presents the novel method proposed for solving IFSPP, offering a practical approach to address the problem. In Section 5, the proposed method is put into action through the resolution of a numerical example, demonstrating its real-world applicability. The paper concludes in Section 6, summarizing the findings and providing a thoughtful conclusion to the study

2. Preliminaries

This section discusses the basic definition and the notion of fuzzy sets, intuitionistic fuzzy numbers, and Accuracy functions.

Definition 2.1. *[3] Let X be the universal set, where a fuzzy set $\tilde{F}$ in X is described as follows:*

$$\tilde{F} = \{\langle x, \mu_{\tilde{F}}(x)\rangle : x \in X\} \tag{2.1}$$

where $\mu_{\tilde{F}}(x) \in [0, 1]$ signifies the membership degree of each element $x \in X$.

Definition 2.2. *[2] Let X denote the universal set. An intuitionistic fuzzy set (IFS), denoted as $\tilde{A}$, in X is defined as a collection of ordered triples:*

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : x \in X\} \tag{2.2}$$

Here, the functions $\mu_{\tilde{A}}(x) : X \to [0,1]$ and $\nu_{\tilde{A}}(x) : X \to [0,1]$ represent the membership and non-membership degrees, respectively. These functions are subject to the constraint that for each element $x \in X$, it satisfies $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$. For any given IFS $\tilde{A}$ and an element $x \in X$, the degree of hesitation of x towards $\tilde{A}$ is defined as $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x)$.

Definition 2.3. *[5] An intuitionistic fuzzy set (IFS) $\tilde{A} = \langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle : x \in \mathbb{R}$ over the real numbers $\mathbb{R}$ is classified as an intuitionistic fuzzy number (IFN) when it exhibits the following essential properties:*

(1) *$\tilde{A}$ is considered intuitionistic fuzzy normal if there exists $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1$ (implying $\nu_{\tilde{A}}(x_0) = 0$).*
(2) *The membership function $\mu_{\tilde{A}}(x)$ defines a convex set: $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right)$ $\forall x_1, x_2 \in \mathbb{R}$, $\lambda \in [0,1]$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$.*
(3) *The non-membership function $\nu_{\tilde{A}}(x)$ represents a concave set: $\nu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \le max\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right)$ $\forall x_1, x_2 \in \mathbb{R}$, $\lambda \in [0,1]$ for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0,1]$.*
(4) *The membership function $\mu_{\tilde{A}}$ is upper semi-continuous, and the non-membership function $\nu_{\tilde{A}}$ is semi-lower continuous.*
(5) *The support of $\tilde{A}$, denoted as $\{Supp(\tilde{A}) = x \in \mathbb{R} : \nu_{\tilde{A}}(x) < 1\}$, is bounded.*

Definition 2.4. *A Trapezoidal Intuitionistic Fuzzy Number (TrIFN) denoted as $\tilde{A}$ can be represented as $\tilde{A} = (a_1, a_2, a_3, a_4; a'1, a'2, a'3, a'4)$. In this context, it is classified as an Intuitionistic Fuzzy Number (IFN), and it is characterized by its membership and non-membership functions defined as follows:*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 < x \le a_2 \\ 1, & a_2 < x \le a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & otherwise \end{cases}$$

and

$$\nu_{\tilde{A}}(x) = \begin{cases} \frac{a'_2-x}{a'_2-a'_1}, & a'_1 < x \le a'_2 \\ 0, & a'_2 < x \le a'_3, \\ \frac{x-a'_3}{a'_4-a'_3}, & a'_3 \le x \le a'_4 \\ 1, & otherwise \end{cases}$$

where $a'_1 \le a_1 \le a'_2 \le a_2 \le a_3 \le a'_3 \le a_4 \le a'_4$

Definition 2.5. *[5] Let $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; b'_1, b'_2, b'_3, b'_4)$ is defined as $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4)$.*

Definition 2.6. *[5] For $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$, the accuracy function is defined as follows*

$$H(\tilde{A}) = \frac{(a_1 + a_2 + a_3 + a_4) + (a'_1 + a'_2 + a'_3 + a'_4)}{8} \tag{2.3}$$

Proposition 2.1. *Consider two trapezoidal intuitionistic fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; b'_1, b'_2, b'_3, b'_4)$ then,*

(1) $H(\tilde{A}) > H(\tilde{B})$ *then* $\tilde{A} \succ \tilde{B}$
(2) $H(\tilde{A}) < H(\tilde{B})$ *then* $\tilde{A} \prec \tilde{B}$
(3) $H(\tilde{A}) = H(\tilde{B})$ *then* $\tilde{A} \sim \tilde{B}$

3. Mathematical Formulation

This section introduces the mathematical model for the shortest path problem with intuitionistic fuzzy arc weights.

We consider a directed network represented by$G = (V, E)$, where V is the set of nodes with elements $1, 2, \ldots, n$, and E is the set of arcs (edges), consisting of ordered pairs (i,j), where i and j are elements of V, and i is not equal to j. This network represents the source node, denoted by 's', and the destination node, denoted by 't'. A path, denoted by P_{ij} is a sequence of arcs $\{(i, i_1), (i_1, i_2, ...(i_k, j))\}$, where the initial node of each arc matches the final node of the previous arc in the sequence.

The main goal of the shortest path problem is to find an optimal path from source node 's' to destination node 't', considering non-negative weights$\tilde{c}_{ij}$ and $\tilde{t}_{ij}$ associated with each arc, representing the cost and time related to that arc, respectively.

The main goal of the shortest path problem involves bi-objective functions, namely, minimizing the maximum costs and minimizing the maximum travel time. This work proposes the use of intuitionistic fuzzy numbers to represent uncertain parameters, thus converting the problem into an Intuitionistic Fuzzy Shortest Path Problem (IFSPP).

An IFSPP problem with uncertainty and vaguness for the cost and time can be formulated as

$$\min_x \max \sum_{i=1}^{n} \sum_{j=1}^{n} \{\tilde{c}_{ij}, \tilde{t}_{ij}\}$$

$$\text{Subject to} \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{n} x_{ki} = \begin{cases} 1 & i = 1 \\ 0 & i \neq 1, n \\ -1 & i = n \end{cases} \tag{3.1}$$

$$x \geq 0, i, j = 1, 2....n$$

$$x \in Z^n$$

If the arc (i,j) is in the path then $x_{ij} = 1$ else $x_{ij} = 0$. Let P_{st} denotes the set of all paths from source node s to destination node t. Z^n is the set of integer vectors of dimension n.

4. Proposed Algorithm

In this section, we introduce a minimax mixed linear integer optimization problem with bi-objective weights designed to address the IFSPP. The minimax approach is a powerful technique that aims to minimize the maximum values of decision variables, primarily used to mitigate potential significant losses in worst-case scenarios.

In the context of the IFSPP, our objective is to find the most efficient path from a source node 's' to a destination node 't' within a directed graph while considering the intuitionistic fuzzy parameters, which are modeled as trapezoidal in nature. Specifically, we represent these parameters as $\tilde{c}ij$ and $\tilde{t}ij$, characterized by trapezoidal intuitionistic fuzzy numbers. These fuzzy numbers are defined by eight values, allowing us to capture the uncertainty and imprecision in the problem's parameters:

$\tilde{c}ij$ is represented as $(\tilde{c}ij, 1, \tilde{c}ij, 2, \tilde{c}ij, 3, \tilde{c}ij, 4, \tilde{c}'ij, 1, \tilde{c}'ij, 2, \tilde{c}'ij, 3, \tilde{c}'_{ij,4})$.

$\tilde{t}ij$ is represented as $(\tilde{t}ij, 1, \tilde{t}ij, 2, \tilde{t}ij, 3, \tilde{t}ij, 4, \tilde{t}'ij, 1, \tilde{t}'ij, 2,, \tilde{t}'ij, 3, \tilde{t}'_{ij,4})$. The IFSPP is thus framed as a problem where the goal is to minimize the maximum values while taking these trapezoidal intuitionistic fuzzy parameters into account.

Mathematically, we formulate this as a bi-objective integer linear problem for the MATLAB minmax solver:

$$
\begin{aligned}
\min_x \max\{\tilde{c}_{ij}, \tilde{t}_{ij}\} \\
\text{subject to:} \\
Ax = b \\
x \in Z^n
\end{aligned}
\tag{4.1}
$$

In this formulation, 'x' represents the vector of decision variables, $\tilde{c}ij$ and $\tilde{t}ij$ are the objective functions to be minimized, 'A' is the constraint matrix, 'b' is the constraint vector, and Z^n denotes the set of integer vectors with dimension 'n'.

This mathematical representation allows us to leverage the minmax solver in MATLAB to find the optimal solution that balances the two objectives (minimizing the maximum cost and minimizing the maximum time) while considering the trapezoidal intuitionistic fuzzy parameters in the IFSPP.The accuracy function, denoted as 2.3, plays a crucial role in managing trapezoidal intuitionistic fuzzy parameters, enhancing the accuracy and precision of calculations in the process.

5. Numerical Example

In this section, a numerical example is solved using proposed method

Example 1 Let us conisder a numerical example of a network graph [18] with 5 nodes and 8 arcs.

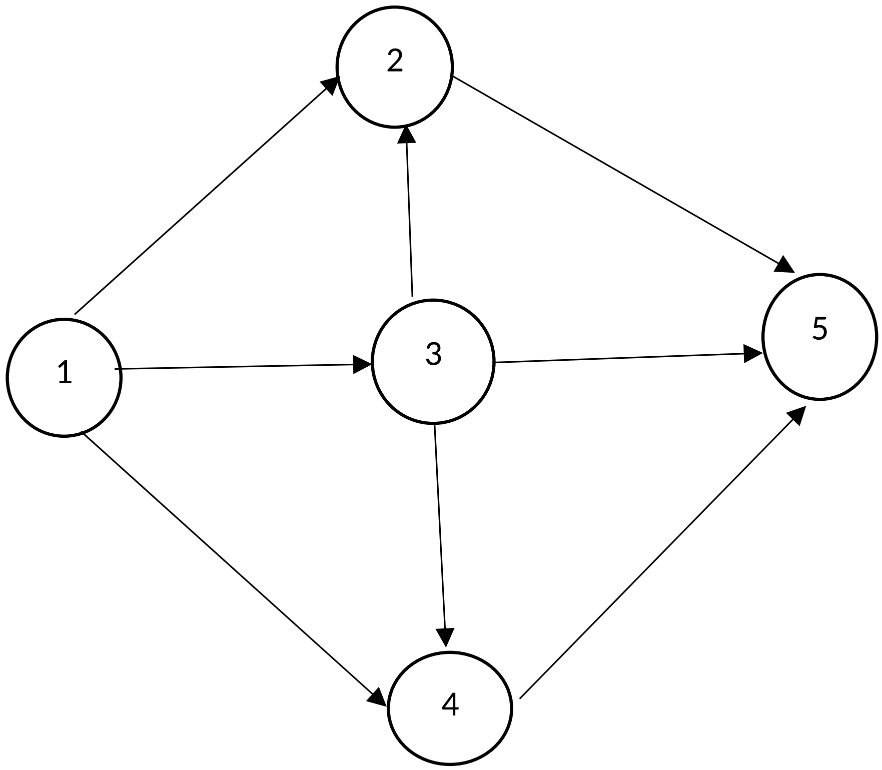

Figure 1. Graph with Trapezoidal IFSP

Table 1. Arc Values of Trapezoidal Intuitionistic Fuzzy Number

Arc	Trapezoidal Intuitionistic Fuzzy cost	Trapezoidal Intuitionistic Fuzzy Time
$1 \to 2$	(12, 14, 15, 17; 11, 13, 16, 18)	(2, 4, 5, 7; 1, 3, 6, 8)
$1 \to 3$	(8, 10, 11, 13; 7, 9, 12, 14)	(2, 4, 5, 7; 1, 3, 6, 8)
$1 \to 4$	(10, 12, 13, 15; 9, 11, 14, 16)	(3, 5, 6, 8; 2, 4, 7, 9)
$2 \to 5$	(5, 7, 8, 10; 4, 6, 9, 11)	(5, 7, 8, 10; 4, 6, 9, 11)
$3 \to 2$	(3, 5, 6, 8; 2, 4, 7, 9)	(4, 6, 7, 9; 3, 5, 8, 10)
$3 \to 5$	(11, 13, 14, 16; 10, 12, 15, 17)	(3, 5, 6, 8; 2, 4, 7, 9)
$4 \to 3$	(5, 7, 8, 10; 4, 6, 9, 11)	(1, 3, 4, 6; 0, 2, 5, 7)
$4 \to 5$	(8, 10, 11, 13; 7, 9, 12, 14)	(3, 5, 6, 8; 2, 4, 7, 9)

Using Equation 2.3 the trapezoidal intuitionistic fuzzy numbers can be converted as

Table 2. Accuracy value for Trapezoidal Intuitionistic Fuzzy Number

Arc	Crisp cost	Crisp Time
$1 \to 2$	14.5	4.5
$1 \to 3$	10.5	4.5
$1 \to 4$	12.5	5.5
$2 \to 5$	6.38	7.5
$3 \to 2$	5.5	6.5
$3 \to 5$	13.5	5.5
$4 \to 3$	7.5	3.5
$4 \to 5$	10.5	5.5

using Equation 5 the IFSP problem has been modified as,

$$\begin{aligned}
&\text{minmax } 14.5x_{12} + 10.5x_{13} + 12.5x_{14} + 6.38x_{25} + 5.5x_{32} + 13.5x_{35} + 7.5x_{43} + 10.5x_{45} \\
&\text{minmax } 4.5x_{12} + 4.5x_{13} + 5.5x_{14} + 7.5x_{25} + 6.5x_{32} + 5.5x_{35} + 3.5x_{43} + 5.5x_{45} \\
&\text{such that} \\
&x_{12} + x_{13} + x_{14} = 1 \\
&-x_{12} - x_{32} + x_{25} = 0 \\
&-x_{13} - x_{34} + x_{32} + x_{35} = 0 \\
&-x_{14} + x_{34} + x_{45} = 0 \\
&-x_{25} - x_{35} - x45 = -1 \\
&x_{12}, x_{13}, x_{14}, x_{25}, x_{32}, x_{35}, x_{43}, x_{45} \geq 0
\end{aligned} \tag{5.1}$$

To solve the bi-objective integer linear problem 5.1 using MATLAB's solver-based approach, we followed a systematic procedure.

(1) First, we defined the objective functions in MATLAB as follows:
function $z =$ objfun(x)
$D = length(x)$;
$f(1) = 14.5 * x(1) + 10.5 * x(2) + 12.5 * x(3) + 6.38 * x(4) + 5.5 * x(5) + 13.5 * x(6) + 7.5 * x(7) + 10.5 * x(8)$;
$f(2) = 4.5*x(1)+4.5*x(2)+5.5*x(3)+7.5*x(4)+6.5*x(5)+5.5*x(6)+3.5*x(7)+5.5*x(8)$;
$z = [f(1), f(2)]$;
end

(2) Defining the initial point for the proposed algorithm. This point should is a vector or matrix that matches the dimension of the objective functions.
Set $x0 = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$;
(3) Defining the constraints for the problem 5.1 according to the proposed algorithm as:
$N = 8$;
$ub = ones(N, 1)$;
$lb = zeros(N, 1)$;
$intcon = 1 : N$;
$fun = @objfun$;
$Aeq = [1\ 1\ 1\ 0\ 0\ 0\ 0\ 0;$
$-1\ 0\ 0\ 1\ \ -1\ 0\ 0\ 0;$
$0\ \ -1\ 0\ 0\ 1\ 1\ \ -1\ 0;$
$0\ 0\ \ -1\ 0\ 0\ 0\ 1\ 1;$
$0\ 0\ 0\ \ -1\ 0\ \ -1\ 0\ \ -1];$
$beq = [1; 0; 0; 0; -1]$;
(4) Configure the optimization algorithm settings by specifying the options.

options = optimoptions('fminimax','Display','iter');
options = optimoptions('fminimax','plotFcn',@optimplotx,@optimplotfval);

(5) By invoking the 'fminimax' function with the provided input arguments that define the problem, we obtained the optimal solution, the corresponding objective function values at that solution, and additional relevant information.
$[x, fval, exitflag, output] = fminimax(fun, x0, [\,], [\,], Aeq, beq, lb, ub, [\,], options)$;

The optimal solution for the problem 5.1 obtained using the MATLAB fminimax optimization tool is as follows:

Decision Variables:
$x^*_{12} = 1$
$x^*_{13} = 0$
$x^*_{14} = 0$
$x^*_{25} = 1$
$x^*_{32} = 0$
$x^*_{35} = 0$
$x^*_{43} = 0$
$x^*_{45} = 0$

This represents the optimal path $1 \to 2 \to 5$, and the bi-objective Intuitionistic fuzzy cost is 20.8800, while the Intuitionistic fuzzy time is 12.0000.

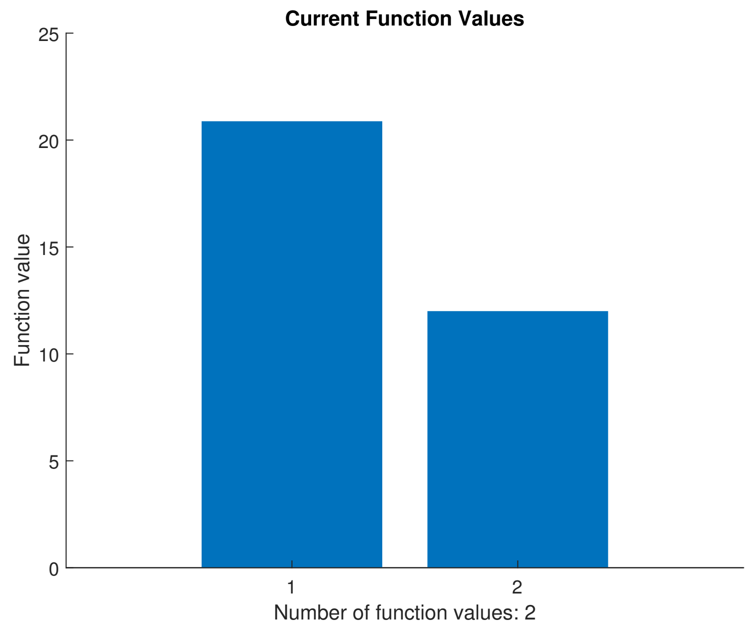

Figure 2.

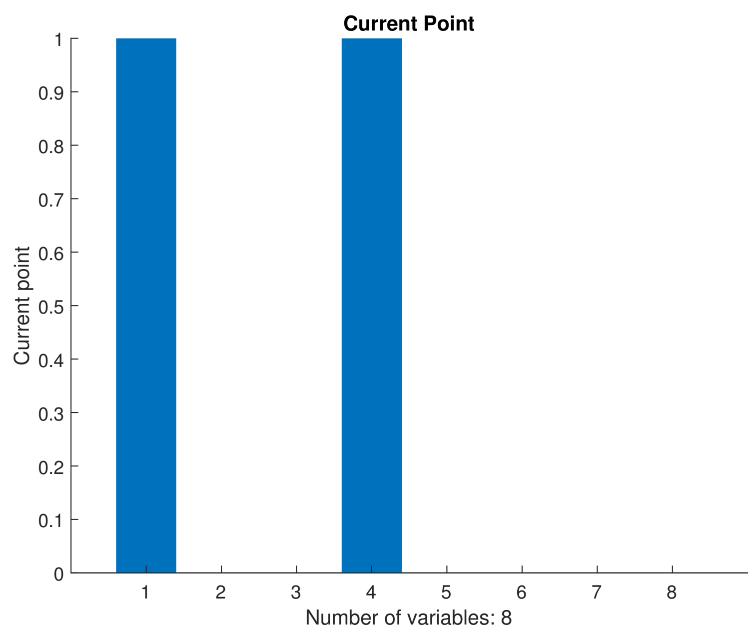

Figure 3.

Example 2 Conisder a numerical example with 6 nodes and 7 arcs with trapezoidal intuitionistic fuzzy numbers

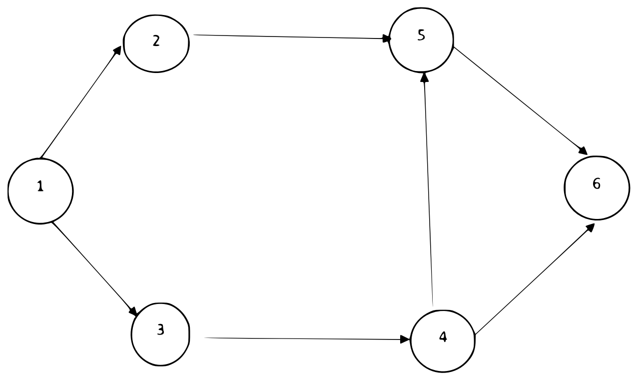

Figure 4. Graph with Trapezoidal Intuitionistic Fuzzy Number

Table 3. Arc Values of Trapezoidal Intuitionistic Fuzzy Number

Arc	Intuitionistic Fuzzy cost	Intuitionistic Fuzzy Time
$1 \to 2$	(4, 7, 6, 8; 6, 8, 7, 9)	(3, 4, 5, 6; 5, 8, 9, 11)
$1 \to 3$	(5, 10, 11, 12; 6, 11, 13, 14)	(5, 7, 9, 11; 7, 9, 10, 11)
$2 \to 5$	(3, 4, 7, 8; 4, 5, 6, 7)	(4, 7, 6, 9; 7, 10, 11, 14)
$3 \to 4$	(6, 7, 8, 9; 7, 8, 9, 10)	(3, 6, 9, 10; 4, 5, 8, 10)
$4 \to 5$	(4, 5, 6, 8; 10, 11, 12, 13)	(6, 9, 10, 11; 7, 11, 13, 15)
$4 \to 6$	(5, 7, 10, 12; 7, 9, 11, 12)	(7, 9, 11, 12; 10, 13, 14, 17)
$5 \to 6$	(3, 5, 6, 8; 5, 8, 9, 11)	(5, 8, 10, 12; 6, 7, 9, 11)

Using Equation 2.3 the trapezoidal intuitionistic fuzzy numbers can be converted as

Table 4. Accuracy value for Trapezoidal Intuitionistic Fuzzy Number

Arc	Intuitionistic Fuzzy cost	Intuitionistic Fuzzy Time
(1, 2)	6.86	6.38
(1, 3)	10.25	8.63
(2,5)	5.5	8.5
(3,4)	8	6.88
(4,5)	8.62	10.25
(4,6)	9.13	11.63
(5,6)	6.86	8.5

$$
\begin{aligned}
&\min\ \max\ 6.86x_{12} + 10.25x_{13} + 5.5x_{25} + 8x_{34} + 8.62x45 + 9.13x_{46} + 6.86x_{56} \\
&\min\ \max\ 6.38x_{12} + 8.63x_{13} + 8.5x_{25} + 6.88x_{34} + 10.25x_{45} + 11.63x_{46} + 8.5x_{56} \\
&\text{such that} \\
&x_{12} + x_{13} = 1 \\
&-x_{12} + x_{25} = 0; \\
&-x13 + x_{34} = 0; \\
&-x_{34} + x45 + x_{46} = 0; \\
&-x_{25} - x_{45} + x_{56} = 0; \\
&-x_{56} - x_{46} = -1; \\
&x_{12}, x_{13}, x_{25}, x_{34}, x_{45}, x_{46}, x_{56} \geq 0
\end{aligned}
\tag{5.2}
$$

To solve the bi-objective integer linear problem 5.2 using MATLAB's solver-based approach, we followed a systematic procedure.

(1) First, we defined the objective functions in MATLAB as follows:
$functionz = objfun2(x)$
$D = length(x);\ f(1) = 6.86 * x(1) + 10.25 * x(2) + 5.5 * x(3) + 8 * x(4) + 8.62 * x(5) + 9.13 * x(6) + 6.86 * x(7);$
$f(2) = 6.38*x(1)+8.63*x(2)+8.5*x(3)+6.88*x(4)+10.25*x(5)+11.63*x(6)+8.5*x(7);$
$z = [f(1), f(2)];$
end

(2) Defining the initial point for the proposed algorithm. This point should is a vector or matrix that matches the dimension of the objective functions.
Set $x0 = [0\ 0\ 0\ 0\ 0\ 0\ 0];$

(3) Defining the constraints for the problem 5.2 according to the proposed algorithm as:
$Aeq = [1\ 1\ 0\ 0\ 0\ 0\ 0;$
$-1\ 0\ 1\ 0\ 0\ 0\ 0;$
$0\ \ -1\ 0\ 1\ 0\ 0\ 0;$
$0\ 0\ 0\ \ -1\ 1\ 1\ 0;$
$0\ 0\ \ -1\ 0\ \ -1\ 0\ 1;$
$0\ 0\ 0\ 0\ 0\ \ -1\ \ -1];$
$beq = [1; 0; 0; 0; 0; -1];$
$N = 7;$

$ub = ones(N, 1);$
$lb = zeros(N, 1);$
$intcon = 1 : N;$
$x0 = [0000000];$
$fun = @objfun2;$

(4) Configure the optimization algorithm settings by specifying the options.

options = optimoptions('fminimax','Display','iter');
options = optimoptions('fminimax','plotFcn',@optimplotx,@optimplotfval);

(5) By invoking the 'fminimax' function with the provided input arguments that define the problem, we obtained the optimal solution, the corresponding objective function values at that solution, and additional relevant information.

$$[x, fval, exitflag, output] = fminimax(fun, x0, [\,], [\,], Aeq, beq, lb, ub, [\,], options);$$

The optimal solution for the problem 5.2 obtained using the MATLAB fminimax optimization tool is as follows:
The optimal solution is solved for the above equation5.2 using Matlab fminimax optimization tool

Decision Variables:
$x^*_{12} = 1$
$x^*_{13} = 0$
$x^*_{25} = 1$
$x^*_{34} = 0$
$x^*_{45} = 0$
$x^*_{46} = 0$
$x^*_{56} = 1$

This represents the optimal path $1 \to 5 \to 6$ and the Bi-objective Intuitionistic fuzzy cost 19.2200 and Intuitionistic fuzzy time 23.3800.

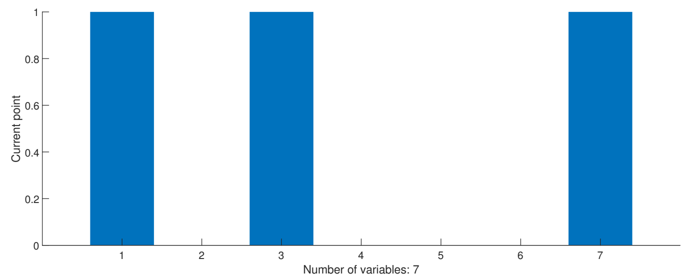

Figure 5.

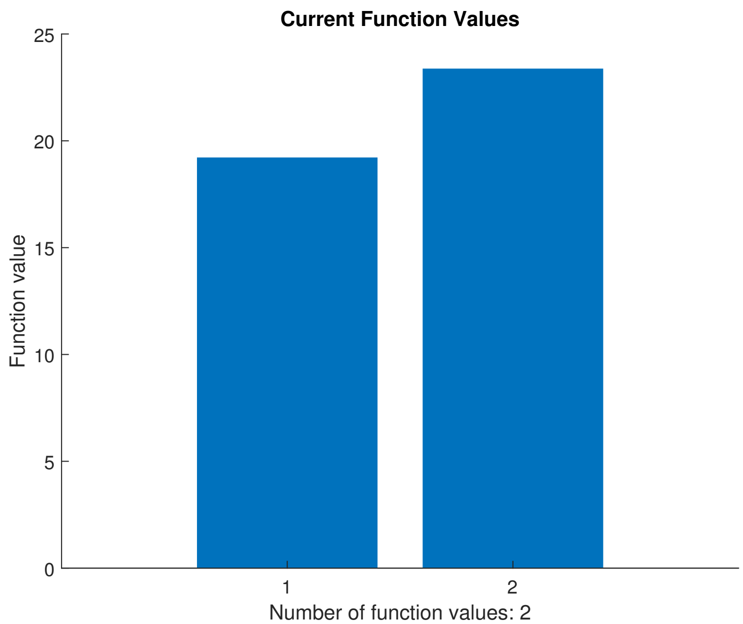

Figure 6.

6. Conculsion

This study focuses on the examination of the Minimax Bi-objective Shortest Path Problem within the context of intuitionistic fuzzy arc weights. A novel method has been introduced to address the minimax objectives while considering intuitionistic fuzzy weights. To facilitate this process, an accuracy function has been applied, effectively converting trapezoidal intuitionistic fuzzy weights into precise, well-defined weights. The problem has been transformed into a minmax integer linear programming model, setting the stage for systematic optimization. To validate the proposed method, a numerical example has been successfully tackled using the Matlab fminimax optimization tool, yielding optimal results. Looking ahead, our research endeavors extend to exploring a fully fuzzy minmax integer linear programming model in various fuzzy environments. This expansion promises to introduce valuable insights and further advance the field.

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