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# Essential Implicative UP-Filters and t-Essential Fuzzy Implicative UP-Filters of UP-Algebras

**Thiti Gaketem***

*Fuzzy Algebras and Decision-Making Problems Research Unit, Department of Mathematics School of Science, University of Phayao, Phayao 56000, Thailand*

**Corresponding author:* thiti.ga@up.ac.th

Abstract. For the purpose of this research, we give the new concept of essential implicative UP-filter from the concept of essential UP-filters in UP-algebras. Although we know that the concept of implicative UP-filters is UP-filters, we have an example showing that the concept of essential UP-filters does not necessarily be essential implicative UP-filters. After that, we extend the concept of essential implicative UP-filters to $t$-essential fuzzy implicative UP-filters of UP-algebras and study their relationship based on characteristic functions and upper level subsets.

## 1. Introduction

At present, research studies in logical algebras interest more, such as BCI-algebras [13], BCK-algebras [14], BCH-algebras [9], KU-algebras [23], PSRU-algebras [25], UP-algebras [10], and others. They are strongly connected with logic. For example, Iséki [13] introduced BCI-algebras in 1966 and has ties with BCI-logic being the BCI-system in combinatory logic, which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [13, 14] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. The concept of KU-algebras was introduced in 2009 by Prabpayak and Leerawat [23]. In 2017, Iampan [10] introduced the concept of UP-algebras as a generalization of KU-algebras.

To overcome these uncertainties, researchers are motivated to introduce some classical theories like the theories of fuzzy sets by Zadeh in 1965 [26]. Fuzzy set applied in many areas such as medical

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science, theoretical physics, robotics, computer science, control engineering, information science, measure theory, logic, set theory, topology etc. The study of fuzzy sets is ongoing such as: Al-Masarwah and Ahmad [1] introduced the concept of doubt bipolar fuzzy H-ideals of BCK/BCI-algebras. Essential fuzzy ideals of rings were studied by Medhi et at in 2008 [16]. In 2012, Pawar and Deore introduced the concept of essential ideals in semirings and radical class. They generalized the concept of essential ideals of rings to semirings and established radical class of semirings closed under essential extensions. Later in 2013, Medhi and Saikia [16] studied concept T-fuzzy essential ideals and proved properties of T-fuzzy essential ideals of rings. Later in 2017, Wani and Pawar [18] studied essential ideals and weakly essential ideals in ternary semirings. Amjadi [2] studied the essential ideal graph of a commutative ring in 2018. In 2019, Murugadas et al. [17] studied essential ideals in near-rings using $k$-quasi coincidence relation. In 2020, Baupradist et at. [3] studied essential ideals and essential fuzzy ideals in semigroups. Together, they studied 0-essential ideals and 0-essential fuzzy ideals in semigroups. In 2021, Gaketem and Iampan [5] introduced the concepts of essential UP-subalgebras and essential UP-ideals and the concepts of $t$-essential fuzzy UP-subalgebras and $t$-essential fuzzy UP-ideals of UP-algebras, and investigated their relationships. Moreover, Gaketem et al. [20] studied essential bi-ideals in semigroups. In the same year P. Khamrot and T. Gaketem, [7,8] studied essential ideals in bipolar fuzzy set and interval fuzzy set. Recently, R. Rittichuai et al. [24] disscussed properties of the essential ideals and essential fuzzy ideals in ternary semigroups.

This review shows that the concept of essential subsets is an important and ongoing study, but not much analysis, which has inspired the study of new concepts of essential subsets in UP-algebras. In this paper, we introduce the new concept of essential implicative UP-filters from the concept of essential UP-filters in UP-algebras and study some properties. We will show that the concept of essential UP-filters does not necessarily be essential implicative UP-filters. Finally, we extend the concept of fuzzy implicative UP-filters to $t$-essential fuzzy implicative UP-filters of UP-algebras and study their relationship based on characteristic functions and upper level subsets.

## 2. Preliminaries

Now, we discuss the concept of UP-algebras and basic properties for the study of next sections.

**Definition 2.1.** *[10] An algebra $\mathcal{A} := (\mathcal{A}, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra, where $\mathcal{A}$ is a nonempty set, $\cdot$ is a binary operation on $\mathcal{A}$, and $0$ is a fixed element of $\mathcal{A}$ (i.e., a nullary operation) if it satisfies the following axioms:*

$$(\text{for all } x, y, z \in \mathcal{A})((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0), \tag{2.1}$$

$$(\text{for all } x \in \mathcal{A})(0 \cdot x = x), \tag{2.2}$$

$$(\text{for all } x \in \mathcal{A})(x \cdot 0 = 0), \text{ and} \tag{2.3}$$

$$(\text{for all } x, y \in \mathcal{A})(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y). \tag{2.4}$$

From [10], we know that the concept of UP-algebras is a generalization of KU-algebras.

The binary relation $\leq$ on a UP-algebra $\mathcal{A}$ is defined as follows:

$$(\text{for all } x, y \in \mathcal{A})(x \leq y \Leftrightarrow x \cdot y = 0) \tag{2.5}$$

and the following assertions are valid (see [10, 11]).

$$(\text{for all } x \in \mathcal{A})(x \leq x), \tag{2.6}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y, y \leq z \Rightarrow x \leq z), \tag{2.7}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \Rightarrow z \cdot x \leq z \cdot y), \tag{2.8}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \Rightarrow y \cdot z \leq x \cdot z), \tag{2.9}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \cdot x, \text{ in particular, } y \cdot z \leq x \cdot (y \cdot z)), \tag{2.10}$$

$$(\text{for all } x, y \in \mathcal{A})(y \cdot x \leq x \Leftrightarrow x = y \cdot x), \tag{2.11}$$

$$(\text{for all } x, y \in \mathcal{A})(x \leq y \cdot y), \tag{2.12}$$

$$(\text{for all } a, x, y, z \in \mathcal{A})(x \cdot (y \cdot z) \leq x \cdot ((a \cdot y) \cdot (a \cdot z))), \tag{2.13}$$

$$(\text{for all } a, x, y, z \in \mathcal{A})(((a \cdot x) \cdot (a \cdot y)) \cdot z \leq (x \cdot y) \cdot z), \tag{2.14}$$

$$(\text{for all } x, y, z \in \mathcal{A})((x \cdot y) \cdot z \leq y \cdot z), \tag{2.15}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \Rightarrow x \leq z \cdot y), \tag{2.16}$$

$$(\text{for all } x, y, z \in \mathcal{A})((x \cdot y) \cdot z \leq x \cdot (y \cdot z)), \text{ and} \tag{2.17}$$

$$(\text{for all } a, x, y, z \in \mathcal{A})((x \cdot y) \cdot z \leq y \cdot (a \cdot z)). \tag{2.18}$$

**Example 2.1.** *[21] Let $U$ be a nonempty set and let $X \in \mathcal{P}(U)$ where $\mathcal{P}(U)$ means the power set of $U$. Let $\mathcal{P}_X(U) = \{A \in \mathcal{P}(U) \mid X \subseteq A\}$. Define a binary operation $\triangle$ on $\mathcal{P}_X(U)$ by putting $A \triangle B = B \cap (A^C \cup X)$ for all $A, B \in \mathcal{P}_X(U)$ where $A^C$ means the complement of a subset $A$. Then $(\mathcal{P}_X(U), \triangle, X)$ is a UP-algebra. Let $\mathcal{P}^X(U) = \{A \in \mathcal{P}(U) \mid A \subseteq X\}$. Define a binary operation $\blacktriangle$ on $\mathcal{P}^X(U)$ by putting $A \blacktriangle B = B \cup (A^C \cap X)$ for all $A, B \in \mathcal{P}^X(U)$. Then $(\mathcal{P}^X(U), \blacktriangle, X)$ is a UP-algebra.*

**Definition 2.2.** *[10, 22] A nonempty subset $S$ of a UP-algebra $\mathcal{A}$ is called*

(1) *a UP-ideal of $\mathcal{A}$ if*
   (a) *the constant $0$ of $\mathcal{A}$ is in $S$, and*
   (b) *(for all $x, y, z \in \mathcal{A}$)$(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$,*
(2) *a UP-filter of $\mathcal{A}$ if*
   (a) *the constant $0$ of $\mathcal{A}$ is in $S$, and*
   (b) *(for all $x, y \in \mathcal{A}$)$(x \in S, x \cdot y \in S \Rightarrow y \in S)$.*
(3) *an implicative UP-filter of $\mathcal{A}$ if*
   (a) *the constant $0$ of $\mathcal{A}$ is in $S$, and*

(b) *(for all* $x, y, z \in \mathcal{A})(x \cdot (y \cdot z) \in S, x \cdot y \in S \Rightarrow x \cdot y \in S)$*.*

Every implicative UP-filter is a UP-filter, but the converse is not true in general as study in the following example.

**Example 2.2.** *[20] Let* $\mathcal{A} = \{0, 1, 2, 3\}$ *be semigroup with the following Cayley table:*

| $\cdot$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 |
| 1 | 0 | 0 | 1 | 2 |
| 2 | 0 | 0 | 0 | 2 |
| 3 | 0 | 0 | 0 | 0 |

*Then* $\{0\}$ *is a UP-filter of* $\mathcal{A}$*, but it is not an implicative UP-filter since* $2 \cdot (2 \cdot 3) = 0 \in \{0\}$ *and* $2 \cdot 2 = 0 \in \{0\}$*, but* $2 \cdot 3 = 2 \notin \{0\}$*.*

A *fuzzy set* $\omega$ in a nonempty set $S$ is a function from $S$ into the unit closed interval $[0, 1]$ of real numbers, i.e., $\omega : S \to [0, 1]$.

For any two fuzzy sets $\omega$ and $\varpi$ in a nonempty set $S$, we define

(1) $\omega \geq \varpi \Leftrightarrow \omega(x) \geq \varpi(x)$ for all $x \in S$.
(2) $\omega = \varpi \Leftrightarrow \omega \geq \varpi$ and $\varpi \geq \omega$.
(3) $(\omega \wedge \varpi)(x) = \min\{\omega(x), \varpi(x)\}$ for all $x \in S$.

If $K \subseteq S$, then the characteristic function $\omega_K$ of $S$ is a function from $S$ into $\{0, 1\}$ defined as follows:

$$\omega_K(x) = \begin{cases} 1 & \text{if } x \in K \\ 0 & \text{otherwise.} \end{cases}$$

**Definition 2.3.** *[22] A fuzzy set* $\omega$ *in a UP-algebra* $\mathcal{A}$ *is called a fuzzy UP-filter of* $\mathcal{A}$ *if*

(1) *(for all* $x \in \mathcal{A})(\omega(0) \geq \omega(x))$*, and*
(2) *(for all* $x, y \in \mathcal{A})(\omega(y) \geq \min\{\omega(x), \omega(x \cdot y)\})$*.*

**Example 2.3.** *From Example 2.2, we have* $\{0, 2, 3\}$ *is a UP-filter of* $\mathcal{A}$*. Then* $\omega_{\{0,2,3\}}$ *is a fuzzy UP-filter of* $\mathcal{A}$*.*

**Definition 2.4.** *A fuzzy* $\omega$ *in a UP-algebra* $\mathcal{A}$ *is called a fuzzy implicative UP-filter of* $\mathcal{A}$ *if*

(1) *(for all* $x \in \mathcal{A})(\omega(0) \geq \omega(x))$*, and*
(2) *(for all* $x, y, z \in \mathcal{A})(\omega(x \cdot z) \geq \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\})$*.*

We easily prove that if $\omega_1$ and $\omega_2$ are fuzzy implicative UP-filters of a UP-algebra $\mathcal{A}$, then $\omega_1 \wedge \omega_2$ is also a fuzzy implicative UP-filter of $\mathcal{A}$.

**Lemma 2.1.** *[22] Let* $F$ *be a nonempty subset* $\mathcal{A}$*. Then the constant* $0$ *of* $\mathcal{A}$ *is in* $F$ *if and only if* $\omega_F(0) \geq \omega_F(x)$ *for all* $x \in \mathcal{A}$*.*

**Theorem 2.1.** *Let $F$ be a nonempty subset of a UP-algebra $\mathcal{A}$. Then $F$ is an implicative UP-filter of $\mathcal{A}$ if and only if the characteristic function $\omega_F$ is a fuzzy implicative UP-filter of $\mathcal{A}$.*

*Proof.* Assume that $F$ is an implicative UP-filter of $\mathcal{A}$. Since $0 \in F$, it follows from Lemma 2.1 that $\omega_F(0) \geq \omega_F(x)$ for all $x \in \mathcal{A}$.

Case 1: $x \cdot (y \cdot z), x \cdot y \in F$. Then $\omega_F(x \cdot (y \cdot z)) = 1$ and $\omega_F(x \cdot y) = 1$. Since $F$ is an implicative UP-filter of $\mathcal{A}$ we have $x \cdot z \in F$. Thus $\omega_F(x \cdot z) = 1$. Hence $\omega(x \cdot z) = \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\}$.

Case 2: $x \cdot (y \cdot z) \notin F$ or $x \cdot y \notin F$. Then $\omega_F(x \cdot (y \cdot z)) = 0$ or $\omega_F(x \cdot y) = 0$. Since $F$ is an implicative UP-filter of $A$ we have $x \cdot z \in F$. Thus $\omega_F(x \cdot z) = 1$. Hence $\omega(x \cdot z) = 1 \geq \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\}$.

From cases 1 and 2, we have that $\omega_F$ is a fuzzy implicative UP-filter of $\mathcal{A}$.

Conversely, assume that $\omega_F$ is a fuzzy implicative UP-filter of $\mathcal{A}$. Since $\omega_F(0) \geq \omega_F(x)$ for all $x \in \mathcal{A}$, it follows form Lemma 2.1 that $0 \in F$. Let $x, y, z \in \mathcal{A}$ be such that $x \cdot (y \cdot z), x \cdot y \in F$. Then $\omega_F(x \cdot (y \cdot z)) = 1$ and $\omega(x \cdot y) = 1$. If $x \cdot z \notin F$, then $\omega(x \cdot z) = 0$. By assumption, $\omega_F(x \cdot z) \geq \min\{\omega(x \cdot (y \cdot z)), \omega_F(x \cdot y)\}$. Thus, $0 = \omega_F(x \cdot z) \geq \min\{\omega_F(x \cdot (y \cdot z)), \omega_F(x \cdot y) = 1\}$. It is contradiction. So, $x \cdot z \in \mathcal{A}$. Hence $F$ is an implicative UP-filter of $\mathcal{A}$. □

**Definition 2.5.** *[22] Let $\omega$ be a fuzzy set in a UP-algebra $\mathcal{A}$. For any $t \in [0, 1]$, the sets*

$$U(\omega; t) = \{x \in \mathcal{A} \mid \omega(x) \geq t\} \text{ and } U^+(\omega; t) = \{x \in \mathcal{A} \mid \omega(x) > t\}$$

*are called an upper $t$-level subset and an upper $t$-strong level subset of $\omega$, respectively.*

**Theorem 2.2.** *Let $\omega$ be a fuzzy set in a UP-algebra $\mathcal{A}$. Then the following statements hold:*

(1) *$\omega$ is a fuzzy implicative UP-filter of $\mathcal{A}$ if and only if for any $t \in [0, 1]$, $U(\omega; t)$ is an implicative UP-filter of $\mathcal{A}$ if $U(\omega; t) \neq \emptyset$,*

(2) *$\omega$ is a fuzzy implicative UP-filter of $\mathcal{A}$ if and only if for any $t \in [0, 1]$, $U^+(\omega; t)$ is an implicative UP-filter of $\mathcal{A}$ if $U^+(\omega; t) \neq \emptyset$.*

*Proof.* (1) Assume that $\omega$ is a fuzzy implicative UP-filter of $\mathcal{A}$. Let $t \in [0, 1]$ be such that $U(\omega; t) \neq \emptyset$ and let $a \in U(\omega; t)$. Then $\omega(a) \geq t$. By assumption, $\omega(0) \geq \omega(a) \geq t$. Thus $0 \in U(\omega; t)$. Let $x, y, z \in \mathcal{A}$ be such that $x \cdot (y \cdot z), x \cdot y \in U(\omega; t)$. Then $\omega(x \cdot (y \cdot z)) \geq t$ and $\omega(x \cdot y) \geq t$. Thus $t$ is a lower bound of $\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\}$. Since $\omega$ is a fuzzy implicative UP-filter of $\mathcal{A}$, we have $\omega(x \cdot z) \geq \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\}$. Thus, $\omega(x \cdot z) \geq \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\} \geq t$. So, $x \cdot z \in U(\omega; t)$. Hence $U(\omega; t)$ is an implicative UP-filter of $\mathcal{A}$.

Conversely, suppose that for all $t \in [0, 1]$, $U(\omega; t)$ is an implicative UP-filter of $\mathcal{A}$ where $U(\omega; t) \neq \emptyset$. Let $x \in \mathcal{A}$. Then $\omega(x) \in [0, 1]$. Choose $t = \omega(x)$. Then $\omega(x) \geq t$ so $x \in U(\omega; t) \neq \emptyset$. By assumption, we have $U(\omega; t)$ is an implicative UP-filter of $A$ and so $0 \in U(\omega; t)$. Thus $\omega(0) \geq t$.

Let $x \cdot (y \cdot z), x \cdot y \in \mathcal{A}$. Then $\omega(x \cdot (y \cdot z)), \omega(x \cdot y) \in [0, 1]$. Choose $t = \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\}$. Thus $\omega(x \cdot (y \cdot z)) \geq t$ and $\omega(x \cdot y) \geq t$ so $x \cdot (y \cdot z), x \cdot y \in U(\omega; t)$. By assumption, we have $U(\omega; t)$ is an implicative UP-filter of $\mathcal{A}$ and so $x \cdot z \in U(\omega; t)$. Thus $\omega(x \cdot z) \geq t = \min\{\omega(x \cdot (y \cdot z)), \omega(x \cdot y)\}$. Hence $\omega$ is a fuzzy implicative UP-filter of $\mathcal{A}$.

(2) Similar to (i).

□

## 3. Essential implicative UP-filters and $t$-essential fuzzy implicative UP-filters

In this section, we define essential implicative UP-filters and essential fuzzy implicative UP-filters, together study their basic properties.

First, let us recall the definition of an essential UP-filter of a UP-algebra as follows:

**Definition 3.1.** *[5] A UP-ideal $B$ of a UP-algebra $\mathcal{A}$ is called an essential UP-filter of $\mathcal{A}$ if $B \cap C$ is a nonzero subset (actually, it is a nonzero UP-filter) of $A$ for every nonzero UP-filter $C$ of $\mathcal{A}$. Equivalently, $\{0\} \subset B \cap C$ for every nonzero UP-filter $C$ of $\mathcal{A}$.*

In the same way, we define an essential implicative UP-filter of a UP-algebra as follows:

**Definition 3.2.** *An implicative UP-filter $F$ of a UP-algebra $\mathcal{A}$ is called an essential implicative UP-filter of $\mathcal{A}$ if $\{0\} \subset F \cap E$ for every nonzero implicative UP-filter $E$ of $\mathcal{A}$.*

**Example 3.1.** *By Example 2.2 we have $\{0, 2, 3\}$ and $\mathcal{A}$ are essential UP-filters, and $\mathcal{A}$ is the only one essential implicative UP-filter of $\mathcal{A}$. This example shows that an essential implicative UP-filter is not necessarily an essential UP-filter.*

**Theorem 3.1.** *Let $F$ be an essential implicative UP-filter of $A$ and $F'$ be a implicative UP-filter of $\mathcal{A}$ containing $F$. Then $F'$ is also an essential implicative UP-filter of $\mathcal{A}$.*

*Proof.* Let $G$ be a nonzero implicative UP-filter of $\mathcal{A}$. Since $F$ is an essential implicative UP-filter of $\mathcal{A}$, we have $\{0\} \subset F \cap G \subseteq F' \cap G$. Hence, $F'$ is an essential implicative UP-filter of $\mathcal{A}$. □

**Theorem 3.2.** *Let $F_1$ and $F_2$ be an essential implicative UP-filters of $\mathcal{A}$ Then $F_1 \cap F_2$ is also an essential implicative UP-filter of $\mathcal{A}$.*

*Proof.* Let $C$ be a nonzero implicative UP-filter of $\mathcal{A}$. Since $F_1$ and $F_2$ are implicative UP-filters of $\mathcal{A}$, we have $F_1 \cap F_2$ is a implicative UP-filter of $\mathcal{A}$. Since $F_2$ is an essential implicative UP-filter of $\mathcal{A}$, we have $F_2 \cap C$ is a nonzero implicative UP-filter of $\mathcal{A}$. Since $F_1$ is an essential implicative UP-filter of $\mathcal{A}$, we have $\{0\} \subset F_1 \cap (F_2 \cap C) = (F_1 \cap F_2) \cap C$. Hence, $F_1 \cap F_2$ is an essential implicative UP-filter of $\mathcal{A}$. □

**Definition 3.3.** *Let $t \in [0, 1)$. A fuzzy implicative UP-filter $\omega$ of $\mathcal{A}$ is called a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$ if there exists a nonzero element $x_\varpi \in \mathcal{A}$ such that $t < (\omega \wedge \varpi)(x_\varpi)$ for every nonzero fuzzy implicative UP-filter $\varpi$ of $\mathcal{A}$.*

**Theorem 3.3.** *Let $\omega$ be a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$ and $\omega'$ be a fuzzy implicative UP-filter of $\mathcal{A}$ such that $\omega \leq \omega'$. Then $\omega'$ is also a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$.*

*Proof.* Let $\varpi$ be a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. Since $\omega$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$, there exists a nonzero element $x_\varpi \in \mathcal{A}$ such that $t < (\omega \wedge \varpi)(x_\varpi) \leq (\omega' \wedge \varpi)(x_\varpi)$. Hence, $\omega'$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$. □

**Theorem 3.4.** *Let $\omega$ be a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$. Then $t < \omega(0)$.*

*Proof.* Let $\varpi$ be a fuzzy set in $\mathcal{A}$ defined by $\varpi(x) = 1$ for all $x \in \mathcal{A}$. Then we can easily prove that $\varpi$ is a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. Then there exists a nonzero element $x_\varpi \in \mathcal{A}$ such that $t < (\omega \wedge \varpi)(x_\varpi) = \min\{\omega(x_\varpi), \varpi(x_\varpi)\} = \min\{\omega(x_\varpi), 1\} = \omega(x_\varpi) \leq \omega(0)$. □

**Theorem 3.5.** *Let $\omega_1$ and $\omega_2$ be $t$-essential fuzzy implicative UP-filters of $\mathcal{A}$. Then $\omega_1 \wedge \omega_2$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$.*

*Proof.* Let $\varpi$ be a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. Since $\omega_1$ and $\omega_2$ are fuzzy implicative UP-filters of $\mathcal{A}$, we have $\omega_1 \wedge \omega_2$ is a fuzzy implicative UP-filter of $\mathcal{A}$.
Since $\omega_2$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$, we have $\omega_2 \wedge \varpi$ is a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. Since $\omega_1$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$, there exists a nonzero element $x_\varpi \in \mathcal{A}$ such that $t < (\omega_1 \wedge (\omega_2 \wedge \varpi))(x_\varpi) = ((\omega_1 \wedge \omega_2) \wedge \varpi)(x_\varpi)$. Hence, $\omega_1 \wedge \omega_2$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$. □

**Theorem 3.6.** *If $F$ is an essential implicative UP-filter of a nonzero UP-algebra $\mathcal{A}$, then characteristic function $\omega_F$ is a $0$-essential fuzzy implicative UP-filters of $\mathcal{A}$.*

*Proof.* Suppose that $F$ is an essential implicative UP-filter of $\mathcal{A}$. Then $F$ is an implicative UP-filter of $A$. Thus $\omega_F$ is a fuzzy implicative UP-filter of $\mathcal{A}$. Let $\varpi$ be a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. By Theorem 2.2 2, we have that $U^+(\varpi; 0)$ is a nonzero implicative UP-filter of $\mathcal{A}$. Then there exists a nonzero element $x_\varpi \in F \cap U^+(\varpi; 0)$. Thus $0 \leq \varpi(x_\varpi) = \min\{1, \varpi(x_\varpi)\} = \min\{\omega_F(x_\varpi), \varpi(x_\varpi)\} = (\omega_F \wedge \varpi)(x_\varpi)$. Hence, $\omega_F$ is a $0$-essential fuzzy implicative UP-filter of $\mathcal{A}$. □

**Theorem 3.7.** *Let $F$ be a UP-filter of a nonzero UP-algebra $\mathcal{A}$. If the characteristic function $\omega_F$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$, then $F$ is an essential implicative UP-filter of $\mathcal{A}$.*

*Proof.* Assume that $\omega_F$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$. Then $\omega_F$ is a fuzzy implicative UP-filter of $\mathcal{A}$. Thus by Theorem 2.1, we have that $F$ is an implicative UP-filter of $\mathcal{A}$. Let $F'$ be a nonzero UP-filter of $\mathcal{A}$. Thus by Theorem 2.1, we have $\omega_{F'}$ is a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. Since $\omega_F$ is a $t$-essential fuzzy implicative UP-filter of $A$, there exists a nonzero element $x_{\omega_{F'}} \in \mathcal{A}$ such that $t < (\omega_F \wedge \omega_{F'})(x_{\omega_{F'}}) = \min\{\omega_F(x_{\omega_{F'}}), \omega_{F'}(x_{\omega_{F'}})\}$. This implies that $x_{\omega_{F'}} \in F \cap F'$, that is, $\{0\} \subset F \cap F'$. Hence, $F$ is an essential implicative UP-filter of $\mathcal{A}$. □

The following theorem shows the relationship between the upper level subset of a t-essential fuzzy UP filter and an essential implicative UP-filter.

**Theorem 3.8.** *Let $\omega$ be a fuzzy set in a nonzero UP-algebra $\mathcal{A}$. Then the following statements hold:*

(1) *if $\omega$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$, then $U^+(\omega;t)$ is an essential implicative UP-filter of $\mathcal{A}$,*

(2) *if $\omega$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$, then $U(\omega;t)$ is an essential implicative UP-filter of $\mathcal{A}$.*

*Proof.* (i) Assume that $\omega$ is a $t$-essential fuzzy UP-filter of $\mathcal{A}$. By Theorem 3.4, we have $0 \in U^+(\omega;t)$. Since $\omega$ is a fuzzy implicative UP-filter of $\mathcal{A}$ and by Theorem 2.2 2, we have $U^+(\omega;t)$ is an implicative UP-filter of $\mathcal{A}$. Let $E$ be a nonzero UP-filter of $\mathcal{A}$. By Theorem 2.1, we have that $\omega_E$ is a nonzero fuzzy implicative UP-filter of $\mathcal{A}$. By assumption, there exists a nonzero element $x_{\omega_E} \in \mathcal{A}$ such that $t < (\omega \wedge \omega_E)(x_{\omega_E})$. Thus $t < \omega(x_{\omega_E})$ and $\omega_E(x_{\omega_E}) = 1$, that is, $x_{\omega_E} \in U^+(\omega;t) \cap E$. Thus $\{0\} \subset U^+(\omega;t) \cap E$. Hence, $U^+(\omega;t)$ is an essential implicative UP-filter of $\mathcal{A}$.

(ii) Assume that $\omega$ is a $t$-essential fuzzy implicative UP-filter of $\mathcal{A}$. By Theorem 3.4, we have $0 \in U(\omega;t)$. Since $\omega$ is a fuzzy UP-filter of $\mathcal{A}$ and by Theorem 2.2 1, we have that $U(\omega;t)$ is an implicative UP-filter of $\mathcal{A}$. By assumption, $U^+(\omega;t)$ is an essential implicative UP-filter of $\mathcal{A}$ and $U^+(\omega;t) \subseteq U(\omega;t)$. By Theorem 3.1, we have that $U(\omega;t)$ is an essential implicative UP-filter of $\mathcal{A}$. □

## 4. Conclusion

In this paper, we have introduced the concept of essential implicative UP-filters from the concept of essential UP-filters in UP-algebras and provided some properties of essential implicative UP-filters. We have shown that the concept of essential implicative UP-filters does not necessarily be essential UP-filters. Finally, we have studied the concept of $t$-essential fuzzy implicative UP-filters of UP-algebras, which is related to the concept of essential implicative UP-filters. In future work, we can extend the study of essential comparative UP-filters and $t$-essential fuzzy comparative UP-filters of UP-algebras to the concept of (see the definition of comparative UP-filters from [12], which is a generalization of UP-filters).

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