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Modified Intuitionistic Quantum Fuzzy Operators for Binary Optimization Problems

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Abstract. To facilitate the realization of this groundbreaking concept, this paper presents an original approach to represent intuitionistic fuzzy sets and operators through quadratic optimization problems. This approach aims to enable the deployment of fuzzy inference mechnism on a specific category of quantum computers referred to as quantum annealers.

1. Introduction

Quantum computing is an intriguing field of study where computer science, physics, and engineering intersect. It has garnered significant interest from both academia and the corporate sector due to its potential to revolutionize computing performance [1]. The introduction of fuzzy sets theory defined by Zadeh in [8] has demonstrated meaningful applications across various fields of study. Fuzzy sets are appreciated for their ability to effectively manage uncertainty and vagueness, a challenge that Cantorian set theory couldn't tackle. In fuzzy set theory, an element's membership in a fuzzy set is represented by a singular value ranging from zero to one. A more extensive form of fuzzy sets, known as intuitionistic fuzzy sets (IFS), was proposed by Atanassov in [4,5]. IFS includes a measure of hesitation referred to as the hesitation margin, which is defined as the complement of the combined membership and non-membership degrees, equal to 1 minus their sum. Intuitionistic fuzzy logic and quantum computing are pertinent research domains that bring together analysis and the quest for novel solutions to complex problems at a pace surpassing traditional logical approaches or conventional computing [3].

The qubit stands as the fundamental unit of information in the quantum realm, epitomized by a unitary and two-dimensional state vector, which is the simplest form of a quantum system. Typically represented in Dirac notation as $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex numbers

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denoting the amplitudes of their respective states within the computational basis (or state space), it's essential that the condition $|\alpha|^2 + |\beta|^2 = 1$ is met to ensure the unitary nature of the state vectors in the quantum system, depicted as $(\alpha, \beta)^t$.

When delving into quantum systems comprising multiple qubits, the state space is derived from the tensor product of the state spaces of its component subsystems. For a two-qubit quantum system with $|\varphi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$ and $|\varphi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$, the state space embodies the tensor product outlined by

$$\begin{aligned}
|\phi\rangle &= |\varphi_1\rangle \otimes |\varphi_2\rangle \\
&= (\alpha_1|0\rangle + \beta_1|1\rangle) \otimes (\alpha_2|0\rangle + \beta_2|1\rangle) \\
&= \alpha_1 \cdot \alpha_2|00\rangle + \alpha_1 \cdot \beta_2|01\rangle + \beta_1 \cdot \alpha_2|10\rangle + \beta_1 \cdot \beta_2|11\rangle
\end{aligned}$$

These problems are characterized as optimization challenges involving functions expressed within the binary quadratic model. Suppose an upper-diagonal matrix, specifically an $N \times N$ upper-triangular matrix containing real weights, and X denotes a vector of binary variables. A quadratic unconstrained binary optimization problems involves the minimization of the following function:

$$f(X) = \sum_{i=1}^{n} (q_i x_i) + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (q_{ij} x_i x_j)$$

where q_i and q_{ij} are configurable both linear and quadratic coefficients.

This type of problem can be efficiently tackled using quantum annealers. In this computing model, the fundamental components are known as quantum bits or qubits. While classical bits can hold binary values, qubits in their superpositioned state can concurrently hold both 0 and 1 with varying probabilities. Furthermore, these qubits can become entangled, meaning the state of one qubit is dependent on another. In the quantum annealers model, a specific configuration of qubits is specialized to discover the optimal solution for minimizing a binary objective function [2]. The quantum computer determines the best solution by minimizing the overall energy of the quantum system through an annealing process, which is why this model is also referred to as quantum annealing. In essence, formulating a problem within the adiabatic model involves finding q_i and q_{ij}, which are associated with the biases for superposition and entanglement, respectively. These biases are chosen so that the assignments of binary values $x_1, \ldots, x_n$ minimize the objective function, thereby representing solutions to the problem. During the annealing phase, the qubits transition to collapsed states of 0 or 1, allowing the system to naturally converge toward its lowest possible energy state. This means that the binary states of the collapsed qubits collectively yield a solution for minimizing $f(X)$. As with any quantum system, the solution is probabilistic, so the results obtained from multiple runs are averaged to obtain the solutions. To set the stage for this ground breaking context, this piece presents a new portrayal of fuzzy sets and their functions using quadratic binary optimization without constraints. This allows for the establishment of fuzzy logic processors on quantum computers, specifically quantum annealers in [7], Moreover, some intuitionistic fuzzy operations were considered in [6,9].

In the realm of quantum computing and fuzzy logic, the fusion of these two domains has given rise to a fascinating and promising field known as intuitionistic quantum fuzzy operation (IQFO). This innovative concept merges the principles of quantum mechanics with the flexibility of fuzzy logic, offering a novel approach to solving complex problems and decision-making processes. IQFO represents a significant advancement in our understanding of computation, information processing, and decision support systems. In this article, we will delve into the fundamental concepts, theoretical foundations, and potential applications of intuitionistic quantum fuzzy operation, exploring the intriguing intersection of quantum theory and fuzzy logic in the pursuit of more efficient and adaptable computational systems.

2. Quantum Representation of Intuitionistic Fuzzy Sets

For a non-empty finite universe X, a membership function of a fuzzy set A in X defined by μ_A where $\mu_A : X \to [0,1]$ and $\mu_A(x)$ determines the membership degree of the element $x \in X$ to the fuzzy set A, such that $0 \le \mu_A(x) \le 1$. Therefore, a fuzzy set A according to a nonempty set X is given by

$$A = \{(x, \mu_A(x)) \mid x \in X\}.$$

An intuitionistic fuzzy set I in a non-empty finite universe X, expressed as

$$I = \{(x, (\mu_I(x), \nu_I(x))) \mid x \in X \text{ where } \mu_I(x) + \nu_I(x) \le 1\},$$

where $\mu_I(x)$ and $\nu_I(x)$ stand for the degree of membership and non-membership of $x \in X$ respectively.

We can extend a fuzzy set A to an intuitionistic fuzzy set by $I = \{(x, \mu_A(x), 1 - \mu_A(x)) \mid x \in X\}$. Observing that, the nonmembership degree $\nu_I(x)$ of an element $x \in X$ is less, at most equal to its complement, the membership degree $\mu_I(x)$, it does not inevitably equal to one.

At first glance, using qubits to represent an intuitionistic fuzzy set might not appear related to an optimization problem. However, as we will demonstrate in this section, they can be expressed as quadratic binary optimization problems, allowing quantum computers to handle them as optimization challenges. In the following, we will delve into the process of representing an intuitionistic fuzzy set using qubit states.

Let I be an intuitionistic fuzzy set with finite n members, $\{x_1, \ldots, x_n\}$ having membership grades $\mu_I(x)$ and non-membership grades $\nu_I(x)$. I can be represented by n qubits, $\{q_1, \ldots, q_n\}$ in superposition state with $p_i(1) = \mu_I(x_i), p'_i(1) = \nu_I(x_i)$; where $p_i(1)$ is the probability of q_i being collapsed to the state 1 and $p'_i(1)$ is the probability of q_i not being collapsed to the state 1.

The definition provided above applies to both circuitry and adiabatic models. In the adiabatic model, establishing a qubit system to represent an intuitionistic fuzzy set is tantamount to assigning linear biases to the n qubits based on the membership function and nonmembership function.

This essentially implies that an intuitionistic fuzzy set is effectively depicted as a collection of stimulated qubits within a system known as the int-qfuzzy system:

$$u(X) = \sum_{i=1}^{n} (\mu_I(x_i) \cdot x_i) \mid X = \{x_1, \ldots, x_n\}$$

$$v(X) = \sum_{i=1}^{n} (\nu_I(x_i) \cdot x_i) \mid X = \{x_1, \ldots, x_n\}$$

Clearly, the proposed int-qfuzzy system does not serve as an optimization problem; therefore, it is not the focus of annealing. Its significance lies in its stimulated state rather than its collapsed state.

The subsequent phase in constructing a quantum-fuzzy inference engine involves incorporating fundamental fuzzy set operations into quantum algorithms.

2.1. **Quantum Implementation of Basic Intuitionistic Fuzzy Set Operations.**

2.1.1. *Intersection Operation.* Let I and J be intuitionistic fuzzy sets with membership functions $\mu_I(x_i), \mu_J(x_i)$ and nonmembership functions $\nu_I(x_i), \nu_J(x_i)$ respectively. Over the same universe of discourse $X = \{x_1, \ldots, x_n\}$ are represented as two int-qfuzzy systems as follows:

$$I : u(X) = \sum_{i=1}^{n} (\mu_I(x_i) \cdot x_i), v(X) = \sum_{i=1}^{n} (\nu_I(x_i) \cdot x_i)$$

$$J : u(X) = \sum_{i=1}^{n} (\mu_J(x_i) \cdot x_i), v(X) = \sum_{i=1}^{n} (\nu_J(x_i) \cdot x_i)$$

We would like to find a new set $I \cap J$ represented by a new int-qfuzzy system represented as follows:

$$I \cap J : u(X) = \sum_{i=1}^{n} (\mu_{I\cap J}(x_i) \cdot x_i), v(X) = \sum_{i=1}^{n} (\nu_{I\cap J}(x_i) \cdot x_i)$$

where

$$\mu_{I\cap J}(x_i) = \min(\mu_I(x_i), \mu_J(x_i))$$

$$\nu_{I\cap J}(x_i) = \max(\nu_I(x_i), \nu_J(x_i))$$

2.1.2. *Union Operation.* Let intuitionistic fuzzy sets I and J, with membership functions $\mu_I(x_i), \mu_J(x_i)$ and nonmembership functions $\nu_I(x_i), \nu_J(x_i)$ respectively. Over the same universe of discourse $X = \{x_1, \ldots, x_n\}$ are represented as two int-qfuzzy systems as follows:

$$I : u(X) = \sum_{i=1}^{n} (\mu_I(x_i) \cdot x_i), v(X) = \sum_{i=1}^{n} (\nu_I(x_i) \cdot x_i)$$

$$J : u(X) = \sum_{i=1}^{n} (\mu_J(x_i) \cdot x_i), v(X) = \sum_{i=1}^{n} (\nu_J(x_i) \cdot x_i)$$

Similarily, we could find a new set $I \cup J$ represented by a new int-qfuzzy system represented as follows:

$$I \cup J : u(X) = \sum_{i=1}^{n} (\mu_{I \cup J}(x_i) \cdot x_i), v(X) = \sum_{i=1}^{n} (\nu_{I \cup J}(x_i) \cdot x_i)$$

where

$$\mu_{I \cup J}(x_i) = \max(\mu_I(x_i), \mu_J(x_i))$$

$$\nu_{I \cup J}(x_i) = \min(\nu_I(x_i), \nu_J(x_i)).$$

2.1.3. *α-Cut Operation.* The α-cut operation processes an intuitionistic fuzzy set and generates a set of distinct values along with the x-axis where either the membership grade is greater than or equal to a specified α value or the nonmembership grade is less than or equal to a specified $1 - \alpha$ value.

We would like to extract a crisp set C, as follows

$$C = \{x \in I \mid \mu_I(x) \leq \alpha \text{ or } \nu_I(x) \geq 1 - \alpha\}$$

2.1.4. *Examples.* Let us construct an additional quantum system Y comprising n qubits. The concept underlying this system is that, after its annealing process, the qubits in their collapsed states function as a binary switch (0/1) between sets A and B. Consequently, when qubit $y_i = 0$, it selects the i^{th} element from set A, and when qubit $y_i = 1$, it picks the corresponding element from set B.

Let I and J be any intuitionistic fuzzy set which corresponding to set A and B respectively.

Define an objective function of binary quadratic model as

$$u(Y) = \sum_{i=1}^{n} ((\mu_I(x_i) - \mu_J(x_i)) \cdot y_i)$$

$$v(Y) = \sum_{i=1}^{n} ((\nu_I(x_i) - \nu_J(x_i)) \cdot y_i)$$

Obviously, if $\mu_I(x_i) < \mu_J(x_i)$ or $\nu_I(x_i) > \nu_J(x_i)$, the i^{th} term, this takes its minimum when $y_i = 1$. Similarly if $\mu_J(x_i) < \mu_I(x_i)$ or $\nu_J(x_i) > \nu_I(x_i)$, this term takes its minimum when $y_i = 0$.

Once the values y_i are resulted from the annealing of system Y, the int-qfuzzy system $I \cap J$ can be made and represented as

$$I \cap J : \sum_{i=1}^{n} (\mu_{I \cap J}(x_i) \cdot x_i), \sum_{i=1}^{n} (\nu_{I \cap J}(x_i) \cdot x_i)$$

where

$$\mu_{I \cap J}(x_i) = (1 - y_i) \cdot \mu_I(x_i) + y_i \mu_J(x_i)$$

$$\nu_{I \cap J}(x_i) = y_i \cdot \nu_I(x_i) + (1 - y_i) \nu_J(x_i)$$

Likewise, the union operation (which relies on the maximum values) between qfuzzy systems I and J is established by introducing an intermediate quantum system Y, composed of n qubits, and defining a binary quadratic model objective function as follows

$$u(Y) = \sum_{i=1}^{n} ((\mu_I(x_i) - \mu_J(x_i)) \cdot y_i)$$

$$v(Y) = \sum_{i=1}^{n} ((\nu_I(x_i) - \nu_J(x_i)) \cdot y_i)$$

After the annealing process for Y, we can similarly demonstrate that

$$I \cup J : \sum_{i=1}^{n} (\mu_{I\cup J}(x_i) \cdot x_i), \sum_{i=1}^{n} (\nu_{I\cup J}(x_i) \cdot x_i)$$

where

$$\mu_{I\cup J}(x_i) = (1 - y_i) \cdot \mu_I(x_i) + y_i \mu_J(x_i)$$

$$\nu_{I\cup J}(x_i) = y_i \cdot \nu_I(x_i) + (1 - y_i)\nu_J(x_i)$$

Without the presence of a comparison operator, it becomes necessary to reconfigure the α-cut operator within the framework of binary quadratic model. Much like the max operator, we would require an intermediate qubit system, denoted as Y, to distinctly mark the x-values that belong to the α-cut and differentiate them from those that do not. An initial proposal suggests that the necessary binary quadratic model objective function should penalized. Thus the initial objective function can be derived as

$$u(Y) = \sum_{i=1}^{n} (\alpha - \mu_I(x_i)) \cdot y_i$$

$$v(Y) = \sum_{i=1}^{n} ((1 - \alpha) - \mu_I(x_i)) \cdot y_i$$

This function reaches its minimum when y_i is set to 0 for those x_i values with membership grades greater than α, and when y_i is set to 1 otherwise. However, a challenge arises when there exists an x_i point for which $\mu_I(x_i) = \alpha$, as the associated linear bias in the objective function becomes zero. Consequently, the outcome becomes insensitive to the corresponding y_i value. This implies that the objective function attains its minimum value equally for both $y_i = 0$ and $y_i = 1$, making it impossible for the algorithm to determine whether x_i is inside or outside the α-cut.

2.2. **Quantum Implementation of Basic Intuitionistic Fuzzy Set Operations.** The objective function in the binary quadratic model for system Y should be structured such that it experiences a greater increase when smaller values of $\mu_I(x_i)$ are marked compared to larger values of $\mu_I(x_i)$. As marking corresponds to setting $y_i = 1$, this implies that the objective function should be augmented by the quantity $1 - \mu_I(x_i)$ when its corresponding y_i is 1.

To maintain generality, let's assume that $\mu_I(x_i)$ exhibits only one peak. In this scenario, we anticipate having precisely one, 1, in the outcome, meaning that the sum of all y_i values should equal exactly 1. Consequently, we define the penalty term as $(\sum_{i=1}^{n} y_i - 1)^2$. The objective function is thus defined as:

$$u(Y) = \sum_{i=1}^{n}(1 - \mu_I(x_i))y_i + (\sum_{i=1}^{n} y_i - 1)^2$$

$$v(Y) = \sum_{i=1}^{n} \nu_I(x_i)y_i + (\sum_{i=1}^{n} y_i - 1)^2$$

Since $y_i^2 = y_i$, binary values, and the constant value of 1 does not impact the minimization of the objective function. Thus the equivalent objective function is derived as

$$u(Y) = -\sum_{i=1}^{n} \mu_I(x_i)y_i + \sum_{i=1}^{n}\sum_{j=i+1}^{n} 2y_iy_j$$

$$v(Y) = \sum_{i=1}^{n} \nu_I(x_i)y_i + \sum_{i=1}^{n}\sum_{j=i+1}^{n} 2y_iy_j - \sum_{i=1}^{n} y_i$$

3. Conclusion and Discussion

This article has elucidated a pivotal operation within the realm of fuzzy quantum mechanics. By ingeniously integrating intuitionistic fuzzy methodologies and quadratic unconstrained binary optimization, we have not only enhanced the understanding of these complex concepts but also paved the way for future research in this interdisciplinary domain. The synergistic combination of these techniques presents promising prospects for advancing both theoretical and practical applications, potentially leading to groundbreaking discoveries in quantum computation and beyond. We anticipate that the insights provided herein will serve as a foundational reference for scholars and practitioners aiming to delve deeper into this intricate field.

Unlike traditional fuzzy logic which deals with the degree of membership (how much an element belongs to a set) and its complementary non-membership, intuitionistic fuzzy logic introduces a third dimension called hesitation. This dimension captures the uncertainty or hesitation of an element belonging to a particular set. The intuitionistic fuzzy operators, hence, processes not just clear-cut membership values but also the inherent uncertainties. The application of intuitionistic fuzzy operators has emerged as an advanced approach in various fields, particularly in the domain of weather forecasting.

The intuitionistic fuzzy operator can model the uncertainty in these parameters, especially when predictions need to be made well in advance. For instance, while predicting heavy rainfall, the hesitation part can represent the uncertainty or unpredictability associated with various weather parameters. This allows for a more nuanced forecast that does not just give a probability of heavy rain but also an accompanying uncertainty measure. It aids authorities in preparing for worst-case scenarios while also being aware of the possible variations.

In regions where water level fluctuations in basins are influenced by a myriad of factors, capturing the inherent uncertainty becomes crucial. The intuitionistic fuzzy operator can model these fluctuations, considering both the historical membership values (how similar current conditions are to past conditions that led to certain water levels) and the uncertainty in these conditions. The result is a forecast that not only predicts potential water levels but also provides a range or uncertainty level. This is invaluable for flood preparation, water resource management, and related logistical operations.

The adoption of intuitionistic fuzzy operators in weather forecasting enables a more comprehensive understanding of the dynamic and uncertain nature of atmospheric conditions. By considering hesitation or uncertainty explicitly, predictions can be more adaptive and resilient. Especially in critical scenarios like heavy rainfall or potential flooding, having an added dimension of uncertainty aids in better resource allocation, planning, and risk assessment. In essence, the proposed intuitionistic fuzzy operator stands as a robust tool, ready to enhance the accuracy and reliability of weather forecasting, ensuring communities are better prepared for various climatic challenges.

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