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Weakly $p(\Lambda, p)$-Open Functions and Weakly $p(\Lambda, p)$-Closed Functions

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Abstract. Our main purpose is to introduce the concepts of weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions. Moreover, several characterizations of weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions are investigated.

1. Introduction

In 1984, Rose [10] introduced and studied the notions of weakly open functions and almost open functions. Rose and Janković [9] investigated some of the fundamental properties of weakly closed functions. In 2004, Caldas and Navalagi [5] introduced two new classes of functions called weakly preopen functions and weakly preclosed functions as generalization of weak openness and weak closedness due to [10] and [9], respectively. Moreover, Caldas and Navalagi [3] introduced and investigated the concepts of weakly semi-open functions and weakly semi-closed functions as a new generalization of weakly open functions and weakly closed functions, respectively. In 2006, Caldas et al. [4] presented the class of weakly semi-θ-openness (resp. weakly semi-θ-closedness) as a new generalization of semi-θ-openness (resp. semi-θ-closedness). In 2009, Noiri et al. [8] introduced and studied two new classes of functions called weakly b-θ-open functions and weakly b-θ-closed functions by utilizing the notions of b-θ-open sets and the b-θ-closure operator. Weak b-θ-openness (resp. b-θ-closedness) is a generalization of both θ-preopenness and weak semi-θ-openness (resp. θ-preclosedness and weak semi-θ-closedness). In [2], the present authors studied some properties of (Λ, sp)-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets, $\beta(\Lambda, sp)$-open sets and $b(\Lambda, sp)$-open sets. Srisarakham and Boonpok [11] investigated several properties of $\delta p(\Lambda, s)$-closed sets and the $\delta p(\Lambda, s)$-closure operator. The concepts of (Λ, p)-closed sets and (Λ, p)-open

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sets were studied by Boonpok and Viriyapong [1]. In this paper, we introduce the concepts of weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions. Furthermore, some properties of weakly $p(\Lambda, p)$-open functions and weakly $p(\Lambda, p)$-closed functions are discussed.

2. Preliminaries

Throughout the present paper, spaces (X, τ) and (Y, σ) (or simply X and Y) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a topological space (X, τ), $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, represent the closure and the interior of A, respectively. A subset A of a topological space (X, τ) is said to be *preopen* [7] if $A \subseteq \mathrm{Int}(\mathrm{Cl}(A))$. The complement of a preopen set is called *preclosed*. The family of all preopen sets of a topological space (X, τ) is denoted by $PO(X, \tau)$. A subset $\Lambda_p(A)$ [6] is defined as follows: $\Lambda_p(A) = \cap\{U \mid A \subseteq U, U \in PO(X, \tau)\}$. A subset A of a topological space (X, τ) is called a *Λ_p-set* [1] (*pre-Λ-set* [6]) if $A = \Lambda_p(A)$. A subset A of a topological space (X, τ) is called *(Λ, p)-closed* [1] if $A = T \cap C$, where T is a Λ_p-set and C is a preclosed set. The complement of a (Λ, p)-closed set is called *(Λ, p)-open*. The family of all (Λ, p)-open (resp. (Λ, p)-closed) sets in a topological space (X, τ) is denoted by $\Lambda_p O(X, \tau)$ (resp. $\Lambda_p C(X, \tau)$). Let A be a subset of a topological space (X, τ). A point $x \in X$ is called a *(Λ, p)-cluster point* [1] of A if $A \cap U \neq \emptyset$ for every (Λ, p)-open set U of X containing x. The set of all (Λ, p)-cluster points of A is called the *(Λ, p)-closure* [1] of A and is denoted by $A^{(\Lambda,p)}$. The union of all (Λ, p)-open sets of X contained in A is called the *(Λ, p)-interior* [1] of A and is denoted by $A_{(\Lambda,p)}$. A subset A of a topological space (X, τ) is said to be *$p(\Lambda, p)$-open* [1] (resp. *$\alpha(\Lambda, p)$-open* [13], *$r(\Lambda, p)$-open* [1]) if $A \subseteq [A^{(\Lambda,p)}]_{(\Lambda,p)}$ (resp. $A \subseteq [[A_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}$, $A = [A^{(\Lambda,p)}]_{(\Lambda,p)}$). The union of all $p(\Lambda, p)$-open sets of X contained in A is called the *$p(\Lambda, p)$-interior* of A and is denoted by $A_{p(\Lambda,p)}$. The complement of a $p(\Lambda, p)$-open (resp. $\alpha(\Lambda, p)$-open, $r(\Lambda, p)$-open) set is called *$p(\Lambda, p)$-closed* (resp. *$\alpha(\Lambda, p)$-closed*, *$r(\Lambda, p)$-closed*). The intersection of all $p(\Lambda, p)$-closed sets of X containing A is called the *$p(\Lambda, p)$-closure* of A and is denoted by $A^{p(\Lambda,p)}$. Let A be a subset of a topological space (X, τ). The *$\theta(\Lambda, p)$-closure* [1] of A, $A^{\theta(\Lambda,p)}$, is defined as follows:

$$A^{\theta(\Lambda,p)} = \{x \in X \mid A \cap U^{(\Lambda,p)} \neq \emptyset \text{ for each } (\Lambda, p)\text{-open set } U \text{ containing } x\}.$$

A subset A of a topological space (X, τ) is called *$\theta(\Lambda, p)$-closed* [1] if $A = A^{\theta(\Lambda,p)}$. The complement of a $\theta(\Lambda, p)$-closed set is said to be *$\theta(\Lambda, p)$-open*. A point $x \in X$ is called a *$\theta(\Lambda, p)$-interior point* [12] of A if $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$ for some $U \in \Lambda_p O(X, \tau)$. The set of all $\theta(\Lambda, p)$-interior points of A is called the *$\theta(\Lambda, p)$-interior* [12] of A and is denoted by $A_{\theta(\Lambda,p)}$.

Lemma 2.1. [12] *For subsets A and B of a topological space (X, τ), the following properties hold:*

(1) $X - A^{\theta(\Lambda,p)} = [X - A]_{\theta(\Lambda,p)}$ *and* $X - A_{\theta(\Lambda,p)} = [X - A]^{\theta(\Lambda,p)}$.
(2) *A is $\theta(\Lambda, p)$-open if and only if $A = A_{\theta(\Lambda,p)}$.*
(3) $A \subseteq A^{(\Lambda,p)} \subseteq A^{\theta(\Lambda,p)}$ *and* $A_{\theta(\Lambda,p)} \subseteq A_{(\Lambda,p)} \subseteq A$.
(4) *If $A \subseteq B$, then $A^{\theta(\Lambda,p)} \subseteq B^{\theta(\Lambda,p)}$ and $A_{\theta(\Lambda,p)} \subseteq B_{\theta(\Lambda,p)}$.*
(5) *If A is (Λ, p)-open, then $A^{(\Lambda,p)} = A^{\theta(\Lambda,p)}$.*

3. On Weakly $p(\Lambda, p)$-Open Functions

In this section, we introduce the concept of weakly $p(\Lambda, p)$-open functions. Moreover, some characterizations of weakly $p(\Lambda, p)$-open functions are discussed.

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $p(\Lambda, p)$-open if $f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$ for each (Λ, p)-open set U of X.*

Theorem 3.1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is weakly $p(\Lambda, p)$-open;*
(2) *$f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{p(\Lambda,p)}$ for every subset A of X;*
(3) *$[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{p(\Lambda,p)})$ for every subset B of Y;*
(4) *$f^{-1}(B^{p(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$ for every subset B of Y;*
(5) *for each $x \in X$ and each (Λ, p)-open set U of X containing x, there exists a $p(\Lambda, p)$-open set V of Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$;*
(6) *$f(K_{(\Lambda,p)}) \subseteq [f(K)]_{p(\Lambda,p)}$ for each (Λ, p)-closed set K of X;*
(7) *$f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$ for each (Λ, p)-open set U of X;*
(8) *$f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$ for each $p(\Lambda, p)$-open set U of X;*
(9) *$f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$ for each $\alpha(\Lambda, p)$-open set U of X.*

Proof. (1) ⇒ (2): Let A be any subset of X and $x \in A_{\theta(\Lambda,p)}$. Then, there exists a (Λ, p)-open set U of X such that $x \in U \subseteq U^{(\Lambda,p)} \subseteq A$. Then, $f(x) \in f(U) \subseteq f(U^{(\Lambda,p)}) \subseteq f(A)$. Since f is weakly $p(\Lambda, p)$-open, $f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)} \subseteq [f(A)]_{p(\Lambda,p)}$. It implies that $f(x) \in [f(A)]_{p(\Lambda,p)}$. This shows that $x \in f^{-1}([f(A)]_{p(\Lambda,p)})$. Thus, $A_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{p(\Lambda,p)})$ and hence $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{p(\Lambda,p)}$.

(2) ⇒ (1): Let U be any (Λ, p)-open set of X. As $U \subseteq [U^{(\Lambda,p)}]_{\theta(\Lambda,p)}$ implies

$$\begin{aligned} f(U) &\subseteq f([U^{(\Lambda,p)}]_{\theta(\Lambda,p)}) \\ &\subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}. \end{aligned}$$

Thus, f is weakly $p(\Lambda, p)$-open.

(2) ⇒ (3): Let B be any subset of Y. Then by (2), $f([f^{-1}(B)]_{\theta(\Lambda,p)}) \subseteq B_{p(\Lambda,p)}$. Thus,

$$[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{p(\Lambda,p)}).$$

(3) ⇒ (2): Let A be any subset of X. By (3), we have $A_{\theta(\Lambda,p)} \subseteq [f^{-1}(f(A))]_{\theta(\Lambda,p)} \subseteq f^{-1}([f(A)]_{p(\Lambda,p)})$ and hence $f(A_{\theta(\Lambda,p)}) \subseteq [f(A)]_{p(\Lambda,p)}$.

(3) ⇒ (4): Let B be any subset of Y. Using (3), we have

$$\begin{aligned} X - [f^{-1}(B)]^{\theta(\Lambda,p)} &= [X - f^{-1}(B)]_{\theta(\Lambda,p)} \\ &= [f^{-1}(Y - B)]_{\theta(\Lambda,p)} \\ &\subseteq f^{-1}([Y - B]_{p(\Lambda,p)}) \\ &= f^{-1}(Y - B^{p(\Lambda,p)}) \\ &= X - f^{-1}(B^{p(\Lambda,p)}) \end{aligned}$$

and hence $f^{-1}(B^{p(\Lambda,p)}) \subseteq [f^{-1}(B)]^{\theta(\Lambda,p)}$.

(4) ⇒ (3): Let B be any subset of Y. Using (4), we have

$$X - f^{-1}(B_{p(\Lambda,p)}) \subseteq X - [f^{-1}(B)]_{\theta(\Lambda,p)}$$

and hence $[f^{-1}(B)]_{\theta(\Lambda,p)} \subseteq f^{-1}(B_{p(\Lambda,p)})$.

(1) ⇒ (5): Let $x \in X$ and U be any (Λ,p)-open set of X containing x. Since f is weakly $p(\Lambda,p)$-open, $f(x) \in f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$. Put $V = [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$. Then, V is $p(\Lambda,p)$-open in Y containing $f(x)$ such that $V \subseteq f(U^{(\Lambda,p)})$.

(5) ⇒ (1): Let U be any (Λ,p)-open set of X and $y \in f(U)$. It follows from (5) that $V \subseteq f(U^{(\Lambda,p)})$ for some $p(\Lambda,p)$-open set V of Y containing y. Thus, $y \in V \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$ and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$. This shows that f is weakly $p(\Lambda,p)$-open.

(1) ⇒ (6): Let K be any (Λ,p)-closed set of X. Then, $K_{(\Lambda,p)}$ is (Λ,p)-open in X. Thus by (1), $f(K_{(\Lambda,p)}) \subseteq [f([K_{(\Lambda,p)}]^{(\Lambda,p)})]_{p(\Lambda,p)} \subseteq [f(K^{(\Lambda,p)})]_{p(\Lambda,p)} = [f(K)]_{p(\Lambda,p)}$.

(6) ⇒ (7): Let U be any (Λ,p)-open set of X. Then, we have $U^{(\Lambda,p)}$ is (Λ,p)-closed in X and by (6), $f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$.

(7) ⇒ (8): Let U be any $p(\Lambda,p)$-open set of X. Then, we have $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$. By (7),

$$\begin{aligned} f(U) &\subseteq f([U^{(\Lambda,p)}]_{(\Lambda,p)}) \\ &= f([[[U^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)}) \\ &\subseteq [f([[U^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)})]_{p(\Lambda,p)} \\ &\subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}. \end{aligned}$$

(8) ⇒ (9): This is obvious since every $\alpha(\Lambda,p)$-open set is $p(\Lambda,p)$-open.

(9) ⇒ (1): Let U be any (Λ,p)-open set of X. Then, U is $\alpha(\Lambda,p)$-open in X. By (9), we have

$$f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$$

and hence f is weakly $p(\Lambda,p)$-open. □

Theorem 3.2. *Let $f : (X,\tau) \to (Y,\sigma)$ be a bijective function. Then, the following properties are equivalent:*

(1) *f is weakly $p(\Lambda,p)$-open;*
(2) *$[f(U)]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every (Λ,p)-open set U of X;*
(3) *$[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$ for every (Λ,p)-closed set K of X.*

Proof. (1) ⇒ (3): Let K be any (Λ,p)-closed set of X. By (1), we have

$$\begin{aligned} f(X-K) &= Y - f(K) \\ &\subseteq [f([X-K]^{(\Lambda,p)})]_{p(\Lambda,p)} \end{aligned}$$

and hence $Y - f(K) \subseteq Y - [f(K_{(\Lambda,p)})]^{p(\Lambda,p)}$. Thus, $[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$.

(3) ⇒ (2): Let U be any (Λ,p)-open set of X. Since $U^{(\Lambda,p)}$ is (Λ,p)-closed and $U \subseteq [U^{(\Lambda,p)}]_{(\Lambda,p)}$. Thus by (3), $[f(U)]^{p(\Lambda,p)} \subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$.

(2) ⇒ (3): Let K be any (Λ, p)-closed set of X. Since $K_{(\Lambda,p)}$ is (Λ, p)-open in X and by (2),

$$\begin{aligned}[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} &\subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)}) \\ &\subseteq f(K^{(\Lambda,p)}) \\ &= f(K).\end{aligned}$$

(3) ⇒ (1): Let U be any (Λ, p)-open set of X. By (3), we have

$$\begin{aligned}Y - [f(U^{(\Lambda,p)})]_{p(\Lambda,p)} &= [Y - f(U^{(\Lambda,p)})]^{p(\Lambda,p)} \\ &\subseteq f(X - U) \\ &= Y - f(U)\end{aligned}$$

and hence $f(U) \subseteq [f(U^{(\Lambda,p)})]_{p(\Lambda,p)}$. Thus, f is weakly $p(\Lambda, p)$-open. □

4. On Weakly $p(\Lambda, p)$-Closed Functions

We begin this section by introducing the concept of weakly $p(\Lambda, p)$-closed functions.

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly $p(\Lambda, p)$-closed if $[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$ for each (Λ, p)-closed set K of X.*

Theorem 4.1. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is weakly $p(\Lambda, p)$-closed;*
(2) *$[f(U)]^{p(\Lambda,p)}) \subseteq f(U^{(\Lambda,p)})$ for every (Λ, p)-open set U of X.*

Proof. (1) ⇒ (2): Let U be any (Λ, p)-open set of X. Then by (1),

$$\begin{aligned}[f(U)]^{p(\Lambda,p)} &= [f(U_{(\Lambda,p)})]^{p(\Lambda,p)} \\ &\subseteq [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)} \\ &\subseteq f(U^{(\Lambda,p)}).\end{aligned}$$

(2) ⇒ (1): Let K be any (Λ, p)-closed set of X. Using (2), we have

$$\begin{aligned}[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} &\subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)}) \\ &\subseteq f(K^{(\Lambda,p)}) \\ &= f(K).\end{aligned}$$

This shows that f is weakly $p(\Lambda, p)$-closed. □

Theorem 4.2. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is weakly $p(\Lambda, p)$-closed;*
(2) *$[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$ for every $p(\Lambda, p)$-closed set K of X;*
(3) *$[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$ for every $\alpha(\Lambda, p)$-closed set K of X.*

Proof. (1) ⇒ (2): Let K be any $p(\Lambda,p)$-closed set of X. Then, $[K_{(\Lambda,p)}]^{(\Lambda,p)} \subseteq K$. Thus by (1),

$$\begin{aligned}[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} &\subseteq [f([[K_{(\Lambda,p)}]^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)}\\ &\subseteq f([K_{(\Lambda,p)}]^{(\Lambda,p)})\\ &\subseteq f(K).\end{aligned}$$

(2) ⇒ (3): Let K be any $\alpha(\Lambda,p)$-closed set of X. Then, K is $p(\Lambda,p)$-closed in X. Using (2), we have $[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$.

(3) ⇒ (1): Let K be any (Λ,p)-closed set of X. Then, we have K is $\alpha(\Lambda,p)$-closed in X. By (3), $[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$. Thus, f is weakly $p(\Lambda,p)$-closed. □

Theorem 4.3. *For a function $f:(X,\tau)\to(Y,\sigma)$, the following properties are equivalent:*

(1) *f is weakly $p(\Lambda,p)$-closed;*
(2) *$[f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every (Λ,p)-open set U of X;*
(3) *$[f([U^{\theta(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(U^{\theta(\Lambda,p)})$ for every (Λ,p)-open set U of X;*
(4) *$[f(U)]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $p(\Lambda,p)$-open set U of X;*
(5) *$[f(U)]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $r(\Lambda,p)$-open set U of X;*
(6) *for each subset B of Y and each (Λ,p)-open set U of X with $f^{-}(B) \subseteq U$, there exists a $p(\Lambda,p)$-open set V of Y such that $B \subseteq V$ and $f^{-1}(V) \subseteq U^{(\Lambda,p)}$;*
(7) *for each point $y \in Y$ and each (Λ,p)-open set U of X with $f^{-}(y) \subseteq U$, there exists a $p(\Lambda,p)$-open set V of Y containing y and $f^{-1}(V) \subseteq U^{(\Lambda,p)}$.*

Proof. (1) ⇒ (2): Let U be any (Λ,p)-open set of X. Then, $U^{(\Lambda,p)}$ is (Λ,p)-closed in X. Since f is weakly $p(\Lambda,p)$-closed, $[f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)}) \subseteq f(U^{(\Lambda,p)})$.

(2) ⇒ (3): It is suffices see that $U^{\theta(\Lambda,p)} = U^{(\Lambda,p)}$ for every (Λ,p)-open set U of X.

(3) ⇒ (4): It is suffices see that $U^{\theta(\Lambda,p)} = U^{(\Lambda,p)}$ for every $p(\Lambda,p)$-open set U of X.

(4) ⇒ (5): Let U be any $r(\Lambda,p)$-open set of X. Then, U is $p(\Lambda,p)$-open in X. Using (4), we have $[f(U)]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$.

(5) ⇒ (6): Let B be any subset of Y and U be any (Λ,p)-open set of X with $f^{-1}(B) \subseteq U$. Then, $f^{-1}(B) \cap [X - U^{(\Lambda,p)}]^{(\Lambda,p)} = \emptyset$ and hence $B \cap f([X - U^{(\Lambda,p)}]^{(\Lambda,p)}) = \emptyset$. Since $X - U^{(\Lambda,p)}$ is $r(\Lambda,p)$-open, $B \cap [f(X - U^{(\Lambda,p)})]^{p(\Lambda,p)} = \emptyset$ by (5). Put $V = Y - [f(X - U^{(\Lambda,p)})]^{p(\Lambda,p)}$. Then, we have V is $p(\Lambda,p)$-open such that $B \subseteq V$ and

$$\begin{aligned}f^{-1}(V) &\subseteq X - f^{-1}([f(X - U^{(\Lambda,p)})]^{p(\Lambda,p)})\\ &\subseteq X - f^{-1}(f(X - U^{(\Lambda,p)}))\\ &\subseteq U^{(\Lambda,p)}.\end{aligned}$$

(6) ⇒ (7): This is obvious.

(7) ⇒ (1): Let K be any (Λ,p)-closed set of X and $y \in Y - f(K)$. Since $f^{-1}(y) \subseteq X - K$, there exists a $p(\Lambda,p)$-open set V of Y such that $y \in V$ and $f^{-1}(V) \subseteq [X - K]^{(\Lambda,p)} = X - K_{(\Lambda,p)}$ by (7).

Thus, $V \cap f(K_{(\Lambda,p)}) = \emptyset$ and hence $y \in Y - [f(K_{(\Lambda,p)})]^{p(\Lambda,p)}$. It implies that $[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(K)$. Thus, f is weakly $p(\Lambda, p)$-closed. □

Theorem 4.4. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are equivalent:*

(1) *f is weakly $p(\Lambda, p)$-closed;*
(2) *$[f(U)]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)})$ for every $r(\Lambda, p)$-open set U of X.*

Proof. (1) ⇒ (2): Let U be any $r(\Lambda, p)$-open set of X. Then, U is (Λ, p)-open in X. Thus, by Theorem 4.3,

$$[f(U)]^{p(\Lambda,p)} = [f([U^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)} \subseteq f(U^{(\Lambda,p)}).$$

(2) ⇒ (1): Let K be any (Λ, p)-closed set of X. Then, $[K^{(\Lambda,p)}]_{(\Lambda,p)}$ is $r(\Lambda, p)$-open in X. By (2), we have

$$\begin{aligned}[f(K_{(\Lambda,p)})]^{p(\Lambda,p)} &\subseteq [f([K^{(\Lambda,p)}]_{(\Lambda,p)})]^{p(\Lambda,p)} \\ &\subseteq f([[K^{(\Lambda,p)}]_{(\Lambda,p)}]^{(\Lambda,p)}) \\ &\subseteq f(K^{(\Lambda,p)}) \\ &= f(K).\end{aligned}$$

This shows that f is weakly $p(\Lambda, p)$-closed. □

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