Int. J. Anal. Appl. (2024), 22:16

International Journal of Analysis and Applications

Sufficient Reduction Method for Bivariate Zero-Inflated Poisson Process

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Abstract. The sufficient reduction (SR) method was developed for detecting a mean shift in a bivariate zero-inflated Poisson process. The derived sequence of statistics from the reduction was monitored with the EWMA and EWMA-SN charts for monitoring a mean shift in a process. The detection performance was compared against other SR methods developed for a Poisson process and evaluated via the simulations under the different shift sizes and proportions of zero in the process. The results showed that the presence of zeros in the process influenced the performance of SR methods by delaying shift detection and reducing the detection accuracy, especially when shift size was small. The proposed method with the EWMA chart gave the shortest delay for detecting a small to moderate shift and gave the highest true alarm rate and the lowest non-detection rate for detecting a small shift compared to other methods.

1. Introduction

Various statistical techniques have been developed for multivariate data under the different purposes. One of the main purposes is to reduce the dimension of the multivariate data so that the reduced data or components can be used or easy to implement with other statistical methods. In a multivariate process, where the multivariate data are observed and monitored routinely, a shift detection in a process is also of interest as the sooner a shift is detected, the less damage caused. However, to be able to detect a shift in a multivariate process is rather complicated than those in a univariate process as it requires multivariate tools to handle this type of data. Although multivariate control charts have been used to monitor a change in a process, they face the problems of signal interpretation and threshold calculation which are more difficult than the univariate one.

With regard to the dimensionality reduction, the sufficient reduction (SR) methods use the principle of sufficiency to reduce the dimension of the multivariate series by summarizing all relevant information regarding to a parameter of interest to a single series so that the reduced series from the reduction can be simply used with other univariate statistical techniques. Several

Received: Oct. 12, 2023 .

2020 *Mathematics Subject Classification.* 62B05.

Key words and phrases. sufficent reduction; bivariate Poisson; zero-inflated.

https://doi.org/10.28924/2291-8639-22-2024-16

ISSN: 2291-8639

SR methods have been proposed to detect a mean shift in a multivariate process under different conditions regarding to the relations between/within series (correlation between series and/or autocorrelation within series) [11, 15] and types of change (simultaneous change and/or changes with time lags) [8, 11, 15]. All methods were proposed and illustrated under the assumption that the data in process are normally distributed [8, 11, 13, 15]. However, in real life there might be other types of data to be considered. Count data such as number of nonconforming products from manufacturing process or number of infected patients of diseases are considered as Poisson distributed data. Some SR methods were developed for detecting a shift in an exponential family distribution including Poisson distribution [8] and in bivariate Poisson distribution allowing for correlation between series and autocorrelation within series [14]. However, in some situations, for instance, number of nonconforming items in a well-managed process (or in a perfect state) or number of cases of a rare disease, might contain lots of zero and need to be handled properly. A large proportion of zeros in data causes the over-dispersion and might affect the detection performance of SR methods.

To overcome the over-dispersion, several studies have developed the zero-inflated Poisson (ZIP) distribution to model count data with excess zeros. Both univariate and multivariate ZIP distributions have been considered under different circumstances. In this study we aim to develop the sufficient reduction method for detecting a mean shift in a bivariate zero-inflated Poisson process, while the multivariate case, which is more complicated, might be considered as a further study. To evaluate the detection performance, a reduced sequence of statistics from the reduction will be monitored with univariate exponentially weighted moving average (EWMA) control charts. Details of the reduction, monitoring tools and performance evaluation are provided in the following sections.

2. Methods

2.1. Bivariate zero-inflated Poisson distribution.

A bivariate Poisson (BP) distribution can be constructed by the trivariate method. Let Z_1, Z_2 and Z_3 be mutually independent random variables from Poisson distributions with parameters λ_1, λ_2 and λ_3, respectively. Define

$$X_1 = Z_1 + Z_3 \quad \text{and} \quad X_2 = Z_2 + Z_3.$$

The random variables X_1 and X_2 follow a bivariate Poission distribution with parameters λ_1, λ_2 and λ_3, denoted by $X_1, X_2 \sim BP(\lambda_1, \lambda_2, \lambda_3)$. The joint probability function of X_1 and X_2 can be written as

$$f_{BP}(x_1, x_2; \lambda_1, \lambda_2, \lambda_3) = e^{-(\lambda_1+\lambda_2+\lambda_3)} \frac{\lambda_1^{x_1}}{x_1!} \frac{\lambda_2^{x_2}}{x_2!} \sum_{j=0}^{m} \binom{x_1}{j} \binom{x_2}{j} j! \left(\frac{\lambda_3}{\lambda_1 \lambda_2} \right)^j$$

when $m = \min(x_1, x_2)$. Parameter λ_3 represents the covariance between X_1 and X_2. If $\lambda_3 = 0$, X_1 and X_2 are independent random variables from Poisson distributions with parameters λ_1 and λ_2, respectively [4,6,10].

A bivariate zero-inflated Poission (BZIP) distribution has been defined in several studies as a mixture of a bivariate Poisson, two univariate Poisson distributions and a point mass at (0,0) [1] or a mixture of a bivariate Poisson distribution and a point mass at (0,0) [3,7,16].

For simplicity, in this study, we define a BZIP distribution as a mixture of a BP distribution and a point mass at (0,0) with parameters λ_1, λ_2, λ_3 and p, denoted by $X_1, X_2 \sim BZIP(\lambda_1, \lambda_2, \lambda_3, p)$, where p is the probability of getting a point mass at (0,0), $0 \leq p < 1$. The joint probability function of X_1 and X_2 can be written as

$$f(x_1, x_2; \lambda_1, \lambda_2, \lambda_3, p) = \begin{cases} p + (1-p) f_{BP}(0, 0; \lambda_1, \lambda_2, \lambda_3) & ; x_1, x_2 = 0 \\ (1-p) f_{BP}(x_1, x_2; \lambda_1, \lambda_2, \lambda_3) & ; x_1 \text{ } or \text{ } x_2 \neq 0 \end{cases}$$

The mean and variance of X_i, $i = 1, 2$,

$$\begin{aligned} E(X_i) &= (1-p)(\lambda_i + \lambda_3) \\ V(X_i) &= E(X_i)[1 + p(\lambda_i + \lambda_3)] \end{aligned}$$

The likelihood function is given by

$$L(\lambda_1, \lambda_2, \lambda_3, p; x_{1,t}, x_{2,t}) = \prod_{t=1}^{s} [p + (1-p) f_{BP}(0, 0; \lambda_1, \lambda_2, \lambda_3)]^{1-a_t} [(1-p) f_{BP}(x_{1,t}, x_{2,t}; \lambda_1, \lambda_2, \lambda_3)]^{a_t}$$

where $t = 1, 2, \ldots, s$, $a_t = 0$, if $x_{1,t}, x_{2,t} = 0$ and $a_t = 1$, if $x_{1,t}$ or $x_{2,t} \neq 0$ [3,4].

2.2. **Sufficient reduction method for BZIP process.**

The sufficient reduction (SR) methods were proposed for detecting a shift in a multivariate process in several studies under different circumstances [8,11,14,15]. The principle of sufficiency was used to reduce a p dimensional multivariate series to a single series of statistics summarizing all relevant information from the original series. Then the reduce series can be used to monitor with a univariate control chart for the purpose of shift detection. Wessman (1998) proposed the SR method to detect a simultaneous shift in a multivariate process. Data were assumed normally distributed with the correlation between the p series [11]. Frisén (2011) proposed the SR method to detect shifts with known time lags. The data were considered from the exponential family distributions including normal, exponential, gamma and Poisson, etc. Although lags between series were allowed, the p series were assumed independent [8]. Later, Siripanthana (2013) proposed the SR method for detecting a shift in a bivariate Poisson process, where correlations between the p series were cooperated [14]. Siripanthana and Stillman (2016) extended the SR methods of Wessman and Frisén by allowing autocorrelation in a multivariate normal data [15].

Let $\boldsymbol{x}_t = (x_{1,t}, x_{2,t})'$ be a vector of observations of random variables X_1 and X_2 observed at time t, $t = 1, 2, \ldots, s$. X_1 and X_2 follow a bivariate zero-inflated Poission (BZIP) distribution with parameters λ_1, λ_2, λ_3 and p. Let τ_i and τ be a change point of variable X_i, $i = 1, 2$, and a change point of a process, respectively. In this study, we focus on detecting a simultaneous change in the process, thus $\tau_1 = \tau_2 = \tau$. According to a change point τ, the process is divided into two stages: in control stage and out of control stage. The distribution parameters of these two stages are defined as follow.

$$\begin{array}{lll} X_1, X_2 \sim BZIP(\lambda_1, \lambda_2, \lambda_3^I, p) & \text{in control stage ,} & t = 1, 2, \ldots, \tau - 1 \\ X_1, X_2 \sim BZIP(\lambda_1, \lambda_2, \lambda_3^O, p) & \text{out of control stage,} & t = \tau, \tau + 1, \ldots, s \end{array}$$

where s is a decision time, $s = 1, 2, \ldots$. At each decision time s, the decision will be made whether the process is in control or out of control. λ_3^I and λ_3^O represent parameter λ_3 when the process is in control and out of control, respectively, where $\lambda_3^O = \lambda_3^I + c$ and c is a constant representing a shift size we desire to detect. Assuming that parameters λ_1, λ_2 and p are unchanged over time, in this study, we investigate a mean shift in the process via a change in parameter λ_3 since it reflects a mean (and also a variance) shift in both series if X_1 and X_2 are not independent.

Let $f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^I, p)$ and $f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^O, p)$ be the joint probability functions of X_1 and X_2 for the in control and out of control stages, respectively. As a shift in parameter λ_3 (i.e. $\lambda_3^O - \lambda_3^I$), is of interest, we use the principle of sufficiency to factorize the log-likelihood function regarding to parameters λ_3^O and λ_3^I. The log-likelihood function regarding to a change point τ can be define as follow.

$$\begin{aligned} l(\lambda_1, \lambda_2, \lambda_3, p|\tau; \boldsymbol{x}_t) &= \ln L(\lambda_1, \lambda_2, \lambda_3, p|\tau; \boldsymbol{x}_t) \\ &= \sum_{t=1}^{\tau-1} \ln f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^I, p) + \sum_{t=\tau}^{s} \ln f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^O, p) \\ &= \sum_{t=1}^{s} \ln f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^I, p) + \sum_{t=\tau}^{s} \ln \frac{f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^O, p)}{f(\boldsymbol{x}_t; \lambda_1, \lambda_2, \lambda_3^I, p)} \\ &= h(\mathbf{x}_s) + k(\mathbf{lr}_t(\mathbf{x}_s; \lambda_3^O - \lambda_3^I)) \end{aligned}$$

where h and k are the functions of $\mathbf{x}_s$ and $\mathbf{lr}_t(\mathbf{x}_s; \lambda_3^O - \lambda_3^I)$, respectively. As a decision will be made sequentially, data available at decision time s is $\mathbf{x}_s = (\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_s)$ and $\mathbf{lr}_t(\mathbf{x}_s; \lambda_3^O - \lambda_3^I)$ is a sequence of a statistic $T(\boldsymbol{x}_t)$ derived from the reduction according to the parameter of interest at time t, which can be written as

$$\mathbf{lr}_t(\mathbf{x}_s; \lambda_3^O - \lambda_3^I) = \{T_t(\boldsymbol{x}_t), \ldots, T_s(\boldsymbol{x}_s)\}$$

From the factorization theorem, this sequence is proved to be a sufficient sequence of statistics regarding to a shift in parameter in the process [2, 12]. The statistic $T(\boldsymbol{x}_t)$ at time t can be derived from the log-likelihood ratio between in control and out of control stages as shown below.

$$
\begin{aligned}
&\sum_{t=\tau}^{s} \ln \frac{f(\boldsymbol{x}_t;\lambda_1,\lambda_2,\lambda_3^O,p)}{f(\boldsymbol{x}_t;\lambda_1,\lambda_2,\lambda_3^I,p)} \\
= &\sum_{t=\tau}^{s} \ln \frac{\left[p+(1-p)f_{BP}(0,0;\lambda_1,\lambda_2,\lambda_3^O)\right]^{1-a_t}\left[(1-p)f_{BP}(x_{1,t},x_{2,t};\lambda_1,\lambda_2,\lambda_3^O)\right]^{a_t}}{\left[p+(1-p)f_{BP}(0,0;\lambda_1,\lambda_2,\lambda_3^I)\right]^{1-a_t}\left[(1-p)f_{BP}(x_{1,t},x_{2,t};\lambda_1,\lambda_2,\lambda_3^I)\right]^{a_t}} \\
= &\sum_{t=\tau}^{s} \ln \frac{\left[p+(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^O)}\right]^{1-a_t}\left[(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^O)}\sum_{j=0}^{m}\frac{\lambda_1^{x_{1,t}-j}\lambda_2^{x_{2,t}-j}(\lambda_3^O)^j}{(x_{1,t}-j)!(x_{2,t}-j)!j!}\right]^{a_t}}{\left[p+(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^I)}\right]^{1-a_t}\left[(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^I)}\sum_{j=0}^{m}\frac{\lambda_1^{x_{1,t}-j}\lambda_2^{x_{2,t}-j}(\lambda_3^I)^j}{(x_{1,t}-j)!(x_{2,t}-j)!j!}\right]^{a_t}} \\
= &\sum_{t=\tau}^{s} \ln \frac{\left[p+(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^O)}\right]^{1-a_t}\left[e^{-\lambda_3^O}\sum_{j=0}^{m}\frac{\lambda_1^{x_{1,t}-j}\lambda_2^{x_{2,t}-j}(\lambda_3^O)^j}{(x_{1,t}-j)!(x_{2,t}-j)!j!}\right]^{a_t}}{\left[p+(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^I)}\right]^{1-a_t}\left[e^{-\lambda_3^I}\sum_{j=0}^{m}\frac{\lambda_1^{x_{1,t}-j}\lambda_2^{x_{2,t}-j}(\lambda_3^I)^j}{(x_{1,t}-j)!(x_{2,t}-j)!j!}\right]^{a_t}}
\end{aligned}
$$

where $m = \min(x_{1,t}, x_{2,t})$, $a_t = 0$, if $x_{1,t}, x_{2,t} = 0$, and $a_t = 1$, if $x_{1,t}$ or $x_{2,t} \neq 0$.
Thus, $T(\boldsymbol{x}_t)$ at time t is

$$
T_t(\boldsymbol{x}_t) = \ln \frac{\left[p+(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^O)}\right]^{1-a_t}\left[e^{-\lambda_3^O}\sum_{j=0}^{m}\frac{\lambda_1^{x_{1,t}-j}\lambda_2^{x_{2,t}-j}(\lambda_3^O)^j}{(x_{1,t}-j)!(x_{2,t}-j)!j!}\right]^{a_t}}{\left[p+(1-p)e^{-(\lambda_1+\lambda_2+\lambda_3^I)}\right]^{1-a_t}\left[e^{-\lambda_3^I}\sum_{j=0}^{m}\frac{\lambda_1^{x_{1,t}-j}\lambda_2^{x_{2,t}-j}(\lambda_3^I)^j}{(x_{1,t}-j)!(x_{2,t}-j)!j!}\right]^{a_t}}
$$

2.3. **Evaluation.**

To evaluate the performance of the proposed SR method, a simulation study was conducted by generating data from a BZIP distribution with parameters $\lambda_1 = \lambda_2 = \lambda_3^I = 4$ and $\lambda_3^O = \lambda_3^I + c$. c is defined as a size of shift to be detected, where $c = \delta\sigma_0$ (i.e. $\sigma_0 = \sqrt{V(X_i)}$ when process is in control). For a small shift $\delta = 0.1, 0.2, 0.3$, $\delta = 0.5, 0.7, 0.1$ for a moderate shift and $\delta = 2, 3$ for a large shift. In this study we investigate a point mass at (0,0) existing in a process with probability p = 0.05, 0.1, 0.2 and 0.3. Let $\tau = 100$ be a change point of the process, the distributions of in control and out of control stages can be defined as follow.

$$
\begin{aligned}
&X_1, X_2 \sim BZIP(4,4,4,p) && \text{in control stage,} && t = 1,2,\ldots,99 \\
&X_1, X_2 \sim BZIP(4,4,4+c,p) && \text{out of control stage,} && t = 100,101,\ldots,200
\end{aligned}
$$

After the reduction, the derived sequence of statistics $T(\boldsymbol{x}_t)$ will be monitored with a univariate control chart for the purpose of shift detection. In this study we use an exponentially weighted moving average (EWMA) control chart as a detection tools as it performs well for detecting a small shift and also quite robust to non-normal data compared to the Shewhart control charts [5]. Since the underlying distribution of the statistics $T(\boldsymbol{x}_t)$ is unknown, in this study we monitor a shift in the process by a parametric exponentially weighted moving average (EWMA) control chart along with a non-parametric exponentially weighted moving average sign (EWMA-SN) control chart defined as follows.

The statistic (Z_t) and control limits of EWMA chart can be calculated from

$$Z_t = \lambda T(\boldsymbol{x}_t) + (1-\lambda)Z_{t-1}\ , \quad Z_0 = \hat{\mu}_0$$

$$\begin{aligned} UCL &= \hat{\mu}_0 + L\hat{\sigma}_0\sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2t})} \\ CL &= \hat{\mu}_0 \\ LCL &= \hat{\mu}_0 - L\hat{\sigma}_0\sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2t})} \end{aligned}$$

where $T(\boldsymbol{x}_t)$ is a statistic derived at time t, $\hat{\mu}_0$ and $\hat{\sigma}_0$ are the estimated mean and variance of statistics $T(\boldsymbol{x}_t)$ when the process is in control ($t = 1, 2, \ldots, \tau - 1$). λ and L are a smoothing parameter and a width of the control limits, respectively. For a desired $\text{ARL}_0 = 370$, define $\lambda = 0.2$ and $L = 2.859$ [5]. An alarm will be flagged if the statistic (Z_t) goes outside the control limits.

For the EWMA-SN chart, define

$$S_t = \text{sign}(T(\boldsymbol{x}_t) - \hat{\theta}_0)$$

where sign() is a sign function and $\hat{\theta}_0$ is the median of statistics $T(\boldsymbol{x}_t)$ when the process is in control ($t = 1, 2, \ldots, \tau - 1$). $S_t = 0$, if $T(\boldsymbol{x}_t)$ equals $\hat{\theta}_0$. $S_t = 1$, if $T(\boldsymbol{x}_t) > \hat{\theta}_0$, otherwise $S_t = -1$. The statistic (W_t) and control limits of EWMA-SN chart can be calculated from

$$W_t = \lambda S_t + (1-\lambda)W_{t-1}, \quad W_0 = \hat{\theta}_0$$

$$\begin{aligned} UCL &= \hat{\theta}_0 + L\sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2t})} \\ CL &= \hat{\theta}_0 \\ LCL &= \hat{\theta}_0 - L\sqrt{\frac{\lambda}{2-\lambda}(1-(1-\lambda)^{2t})} \end{aligned}$$

For a desired $\text{ARL}_0 = 370$, define $\lambda = 0.2$ and $L = 2.471$ [9]. An alarm will be triggered if the statistic (W_t) goes beyond the control limits.

The performance of proposed method is compared against the two other SR methods of Frisén (2011) and Siripanthana (2013) developed for detecting a shift in a Poisson process [8, 14]. Let t_A be a time of an alarm. Four measures used to assess the detection performance include the conditional expected delay (CED), false alarm rate (FAR), true alarm rate (TAR) and non-detection rate (NDR) defined as follows.

$$\begin{aligned} CED &= E(t_A - \tau | t_A \geq \tau) \\ FAR &= P(t_A < \tau) \\ TAR &= E(t_A - \tau \leq 19 | t_A \geq \tau) \\ NDR &= E(t_A - \tau \geq 20 | t_A \geq \tau) = 1 - FAR - TAR \end{aligned}$$

where $TAR + FAR + NDR = 1$ [8, 13, 15]. The CED = 0, if an alarm is triggered at time τ. If the alarm flags before the change point τ, it is considered as a false alarm. To compare the detection performance, we expect the system to flag an alarm within 20 consecutive time points since the process has shifted. If it does, it is considered as a true alarm (or correct identification), otherwise it fails to detect a shift in the process.

3. Result

Let $\widehat{ced}$, $\widehat{far}$, $\widehat{tar}$ and $\widehat{ndr}$ be the estimates of CED, FAR, TAR and NDR from the simulations, respectively. The comparisons of the delay in detection are illustrated by line graphs (Figure 1), while bar charts (Figures 2 - 5) show the accuracy of the detection when p = 0.05, 0.1, 0.2 and 0.3, respectively. The notations of our proposed method (BZ), the Siripanthana method (SR) and the Frisén method (FR) monitored with the EWMA chart are denoted by BZ.EW, SR.EW and FR.EW, whereas BZ.SN, SR.SN and FR.SN denote those of three methods monitored with the EWMA-SN chart.

Unsurprisingly, large shift sizes are more likely to be detected than small shift sizes as $\widehat{ced}$ of all methods tend to decrease when shift sizes (δ) increase (Figure 1). Obviously, the presence of zeros in data affects the delay in detection, especially when shift size is small. All methods give longer $\widehat{ced}$ when p is large. It can be clearly seen that the methods with the EWMA chart give shorter delay than those with the EWMA-SN chart. Overall, the BZ.EW give shortest delay for detecting small to moderate shifts, while the SR.EW and FR.EW perform slightly better when shift size is large.

The accuracy of detection also depends on shift size (Figures 2 - 5). All methods perform better by giving higher $\widehat{tar}$ and lower $\widehat{ndr}$ for detecting large shift sizes. The excess of zeros in the process also influences the detection performance especially when shift size is small. It can be clearly seen that the $\widehat{ndr}$s tend to increase as p increase. However, the performances improve when shift size is large. The methods with the EWMA chart outperform the methods with the EWMA-SN chart. The BZ.EW method give the highest $\widehat{tar}$ and lowest $\widehat{ndr}$ for detecting a small shift size. However, when shift size is moderate or large, the performance of FR.EW is slightly better than the others. Nevertheless, the SR.SN and FR.SN still fall behind the others even shift size is large. Having considered the false alarm rates, the $\widehat{far}$s of all methods are insignificantly different. Overall, the false alarm rates of the methods with EWMA chart are slightly lower than those with the EWMA-SN chart.

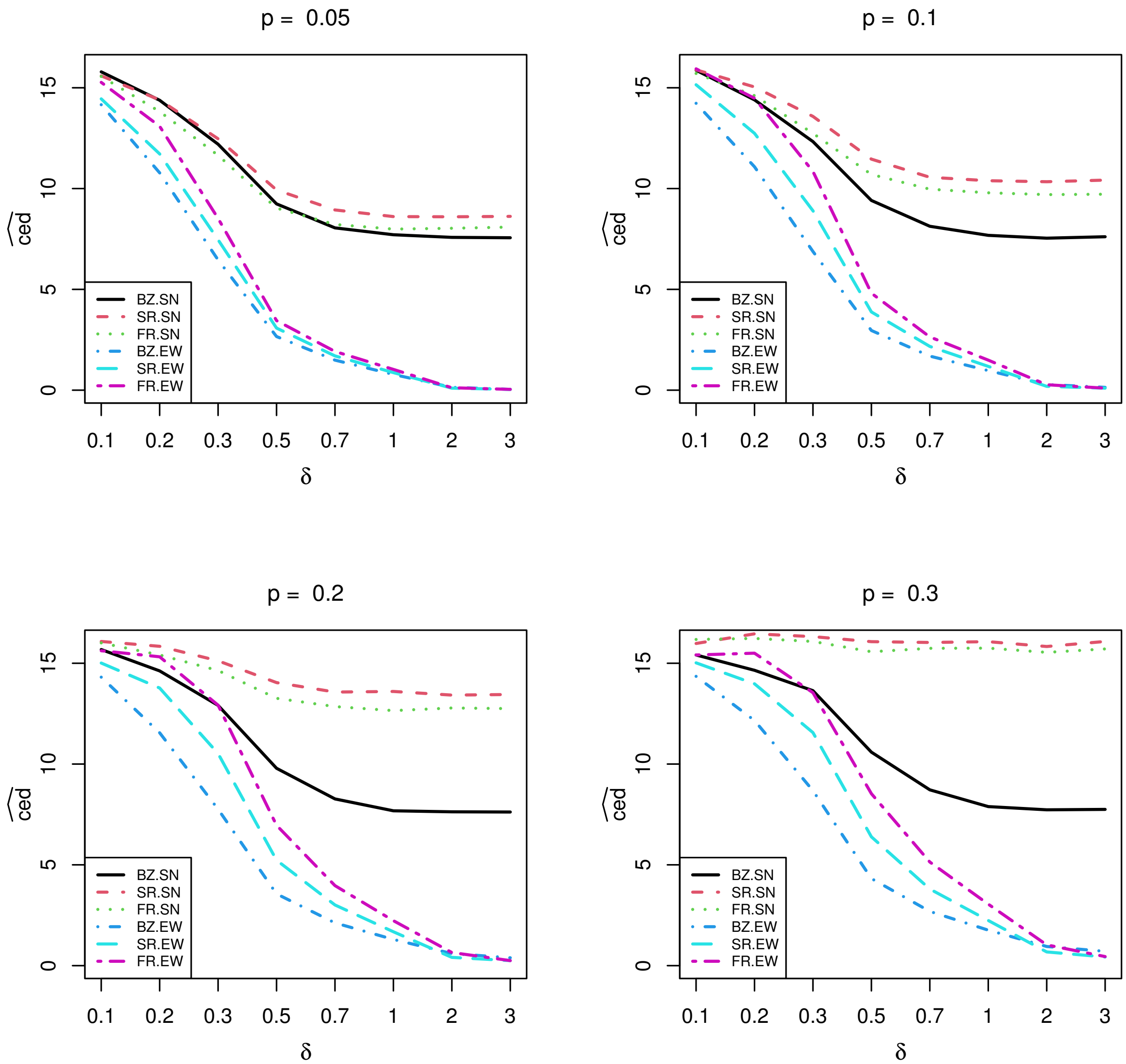

Figure 1. Line graphs comparing the conditional expected delay ($\widehat{ced}$).

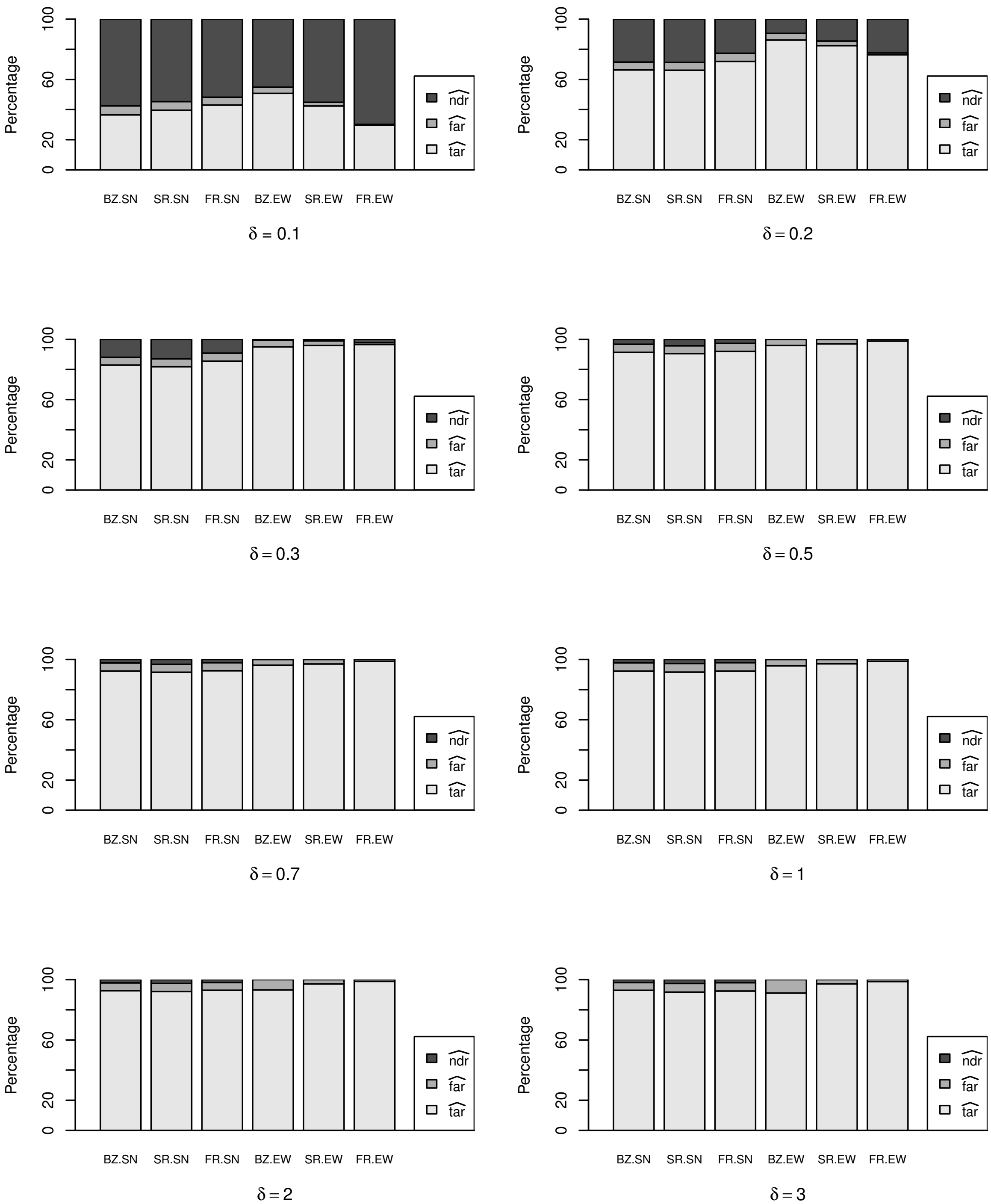

Figure 2. Bar charts comparing the detection performance when $p = 0.05$

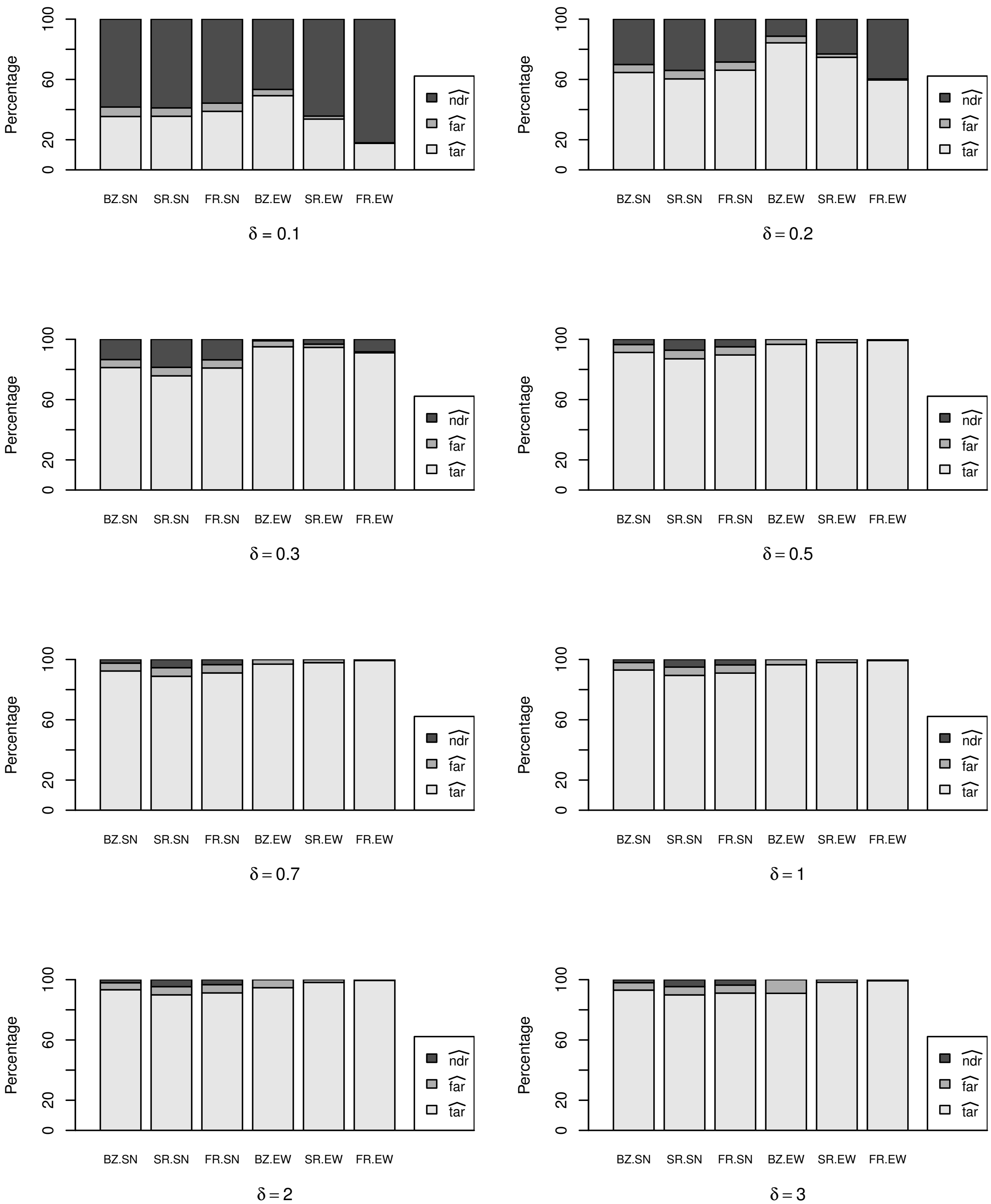

FIGURE 3. Bar charts comparing the detection performance when $p = 0.1$

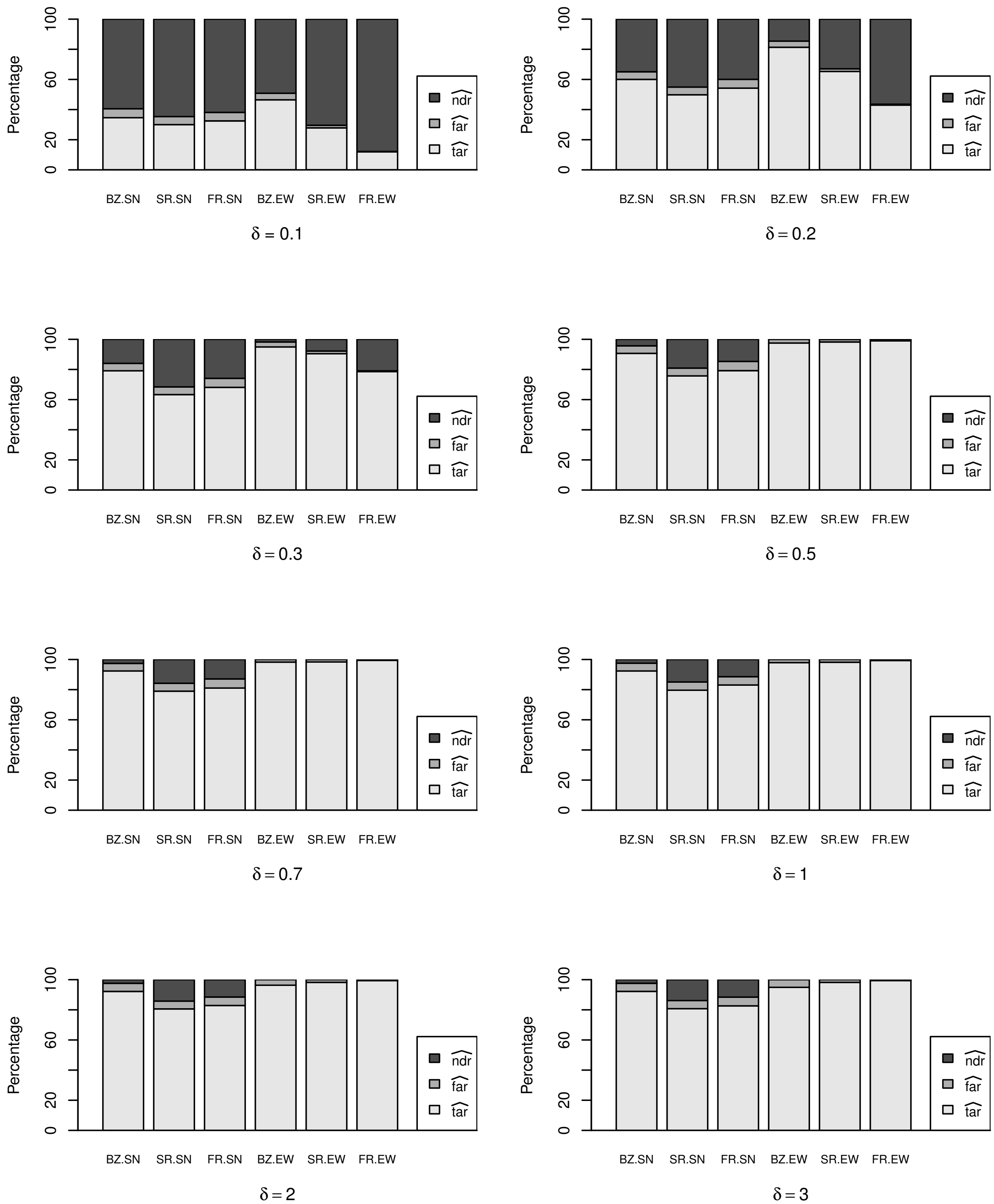

FIGURE 4. Bar charts comparing the detection performance when $p = 0.2$

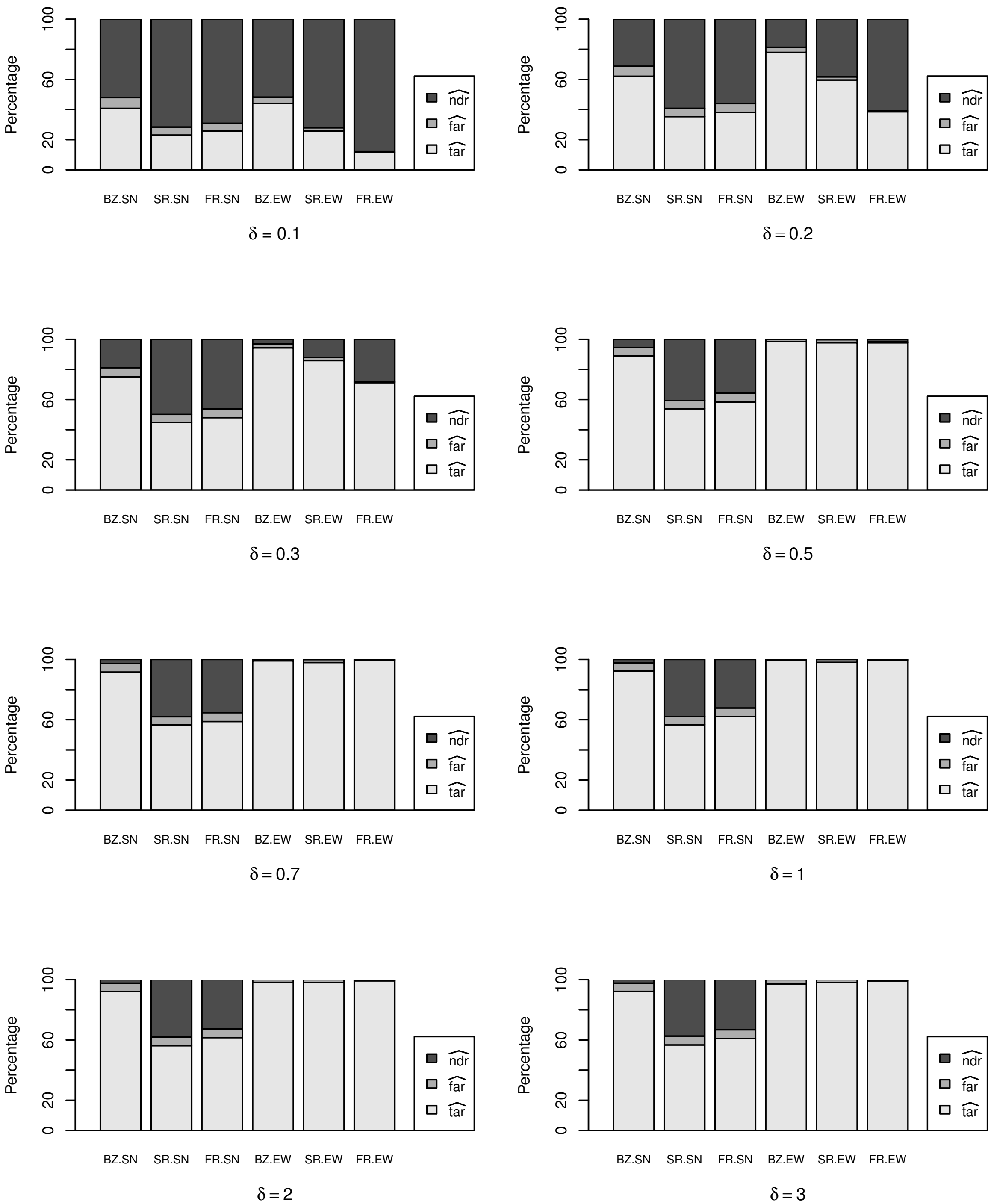

FIGURE 5. Bar charts comparing the detection performance when $p = 0.3$

4. Conclusion

In this study, the sufficient reduction method was developed for detecting a change in a bivariate inflated-Poisson process. The derived sequence of statistics was monitored with the EWMA and EWMA-SN chart. The simulation study was conducted for evaluating the detection performance of the proposed method against the Siripanthana and the Frisén methods by considering the delay and the accuracy of shift detection. As expected, large shift sizes are detected rapidly than small shift sizes. All methods give shorter delay when shift size is large. The excess of zeros in the process causes the over-dispersion and affects the detection performance. The proposed method, which takes the excess of zero into account in the reduction, gives the shortest delay for detecting a small to moderate shift and gives the highest true alarm rate and the lowest non-detection rate for detecting a small shift compared to other methods.

Since the underlying distribution of the derived statistics is unknown, in this study we use the EWMA and the EWMA-SN as detection tools. The EWMA-SN chart uses only a sign function to compare a statistic whether it is lower or higher than the median of in control process and does not take any differences into account, while the EWMA chart does. Therefore, it is obviously seen that the methods monitored with the EMMA chart give shorter delay and higher true alarm rate compared to those with the EWMA-SN chart. In addition to a Poisson process, the mean and variance of the process are inseparable. To improve the detection performance, one might consider the use of a control chart monitoring a mean shift along with a variance shift in a process as a further study.

Acknowledgments: The author would like to thank the National Science and Technology Development Agency, Thailand for supporting this research.

Conflicts of Interest: The author declares that there are no conflicts of interest regarding the publication of this paper.

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