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Connectedness in Single-Valued Neutrosophic Soft Grill Topological Spaces

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Abstract. The incentive of this article is to continue discovering more interesting results and concepts related to the single-valued neutrosophic soft topological spaces. The concept of the single-valued neutrosophic soft operator ϕ created from a single-valued neutrosophic soft grill $(\mathcal{K}^{\sigma}, \mathcal{K}^{\tau}, \mathcal{K}^{\delta})$ and a single-valued neutrosophic soft topological space $(\mathcal{B}, \tilde{\top}^{\tilde{\sigma}}, \tilde{\top}^{\tilde{\varsigma}}, \tilde{\top}^{\tilde{\delta}})$ is presented. Connectedness of single-valued neutrosophic soft topological spaces with single-valued neutrosophic soft grills is given. Moreover, the concept of γ-connectedness associated with a single-valued neutrosophic soft operator γ is extended on the set $\mathcal{B}$.

1. Introduction and Preliminaries

In real life, there are many mathematical tools that are precise, deterministic, and crisp-like for that of computing, reasoning, and formal modeling in character. On the other hand, others are not, such as the problems in engineering, social science, economics, environment and medical science, etc. The inadequacy of the classical parameterization tool in general may be considered to be the reason for these difficulties. For this and to avoid the above difficulties, Molodtsov (1999) [14] created the concept of soft set theory as a new mathematical tool for dealing with uncertainties and vagueness. The soft set theory was applied in several directions, such as game theory, theory of measurement, Riemann integration, smoothness of functions, and Perron integration by Molodtsov (2001) [15]. Practical application of soft sets in decision-making problems has been also given by Maji et al. (2002) [13].

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Maji et al. (2001) [12], have also introduced the concept of fuzzy soft set which is a more generalized concept and a combination of fuzzy set (Zadeh 1965) [30] and soft set (Molodtsov 1999) [14] and also studied some of its properties. Later, some researchers studied the concept of fuzzy soft sets (Acharjee and Tripathy [4]; Ahmad and Kharal (2009) [5]; Kharal and Ahmad (2009) [11], Tanay and Kandemir (2011) [26]; Aygünoglu et al. (2014) [8]; Çetkin et al. (2014) [9]; Abbas et al. (2016, 2018) [1, 2]; Gunduz and Bayramov (2013) [10]).

Smarandache [24] initiated the neutrosophic set as a generalization of an intuitionistic fuzzy set. Salama et al [23] set up the notion of neutrosophic crisp set. Correspondingly, Salama and Alblowi [22], introduced neutrosophic topology as they claimed a number of its characteristics. The single-valued neutrosophic set concept was given by Wang et al [27]. The concept of fuzzy ideal topological spaces, single-valued neutrosophic ideal, single-valued neutrosophic ideal open local function, connectedness in single-valued neutrosophic topological spaces $(\pounds, \tilde{\top}^{\sigma}, \tilde{\top}^{\varsigma}, \tilde{\top}^{\delta})$ and compactness in single-valued neutrosophic ideal topological spaces and studied the basic notions by following Šostak's [25] fuzzy topological spaces were obtained by Saber et al [3, 6, 7, 16–21, 31, 32].

This article aims to explore and define the properties and characterizations of the single-valued neutrosophic soft operator $\ominus$ in single-valued neutrosophic soft grill topological spaces. Also, an r-single-valued neutrosophic soft grill connectedness which has relations with an r-single-valued neutrosophic soft connectedness and some basic definitions and theorems about it have been given and investigated. Moreover, the r-single-valued neutrosophic soft $\aleph$-connectedness and r-fuzzy soft $\aleph$-disconnectedness related to a single-valued neutrosophic soft operator $\aleph$ on the set $\mathcal{B}$ is introduced.

Throughout this work, $\mathcal{B}$ denotes the initial universe, $\xi^{\mathcal{B}}$ is the collection of all single-valued neutrosophic sets (simply, svns) on $\mathcal{B}$ (where, $\xi = [0, 1], \xi_0 = (0, 1]$ and $\xi_1 = [0, 1)$) and $\mathcal{E}$ is the set of each parameters on $\mathcal{B}$.

All characterizations and concepts of svns are originate in Smarandache [24], Wang et al. [27], Yang et al. [28], Ye et al. [29].

$\hbar_z$ is a single-valued neutrosophic soft set [17] (simply, svnfs) on $\mathcal{B}$ where, $\hbar_z : \mathbf{E} \to \xi^{\mathcal{B}}$; i.e., $\hbar_e \cong \hbar(\tilde{e})$ is a svns on $\mathcal{B}$, for all $\tilde{e} \in z$ and $\hbar(\tilde{e}) = \langle 0, 1, 1\rangle$, if $\tilde{e} \not\in \ell$.

The svns $\hbar(\tilde{e})$ is termed as an element of the svnfs $\hbar_z$. Thus, a svnfs $\hbar_{\mathbf{E}}$ on $\mathcal{B}$ it can be defined as:

$$\begin{aligned}(\hbar, \mathbf{E}) &= \left\{(\tilde{e}, \hbar(\tilde{e})) \mid \tilde{e} \in \mathbf{E}, \hbar(\tilde{e}) \in \xi^{\mathcal{B}}\right\} \\ &= \left\{(\tilde{e}, \langle \sigma_{\hbar}(\tilde{e}), \tau_{\hbar}(\tilde{e}), \delta_{\hbar}(\tilde{e})\rangle) \mid \tilde{e} \in \mathbf{E}, \hbar(\tilde{e}) \in \xi^{\mathcal{B}}\right\},\end{aligned}$$

where $\sigma_{\hbar} : \mathbf{E} \to \xi$ ($\sigma_{\hbar}$ is termed as a membership function), $\tau_{\hbar} : \mathbf{E} \to \xi$ ($\tau_{\hbar}$ is termed as indeterminacy function), and $\delta_{\hbar} : \mathbf{E} \to \xi$ ($\delta_{\hbar}$ is termed as a non-membership function) of svnf set. $\widetilde{(\mathcal{B}, \mathbf{E})}$ refers to the collection of all svnfss on $\mathcal{B}$ and is termed svnfs-universe.

A svnfs $\hbar_z$ on $\mathcal{B}$ is termed as a null svnfs (simply, ϕ), if $\sigma_{\hbar}(\tilde{e}) = 0, \tau_{\hbar}(\tilde{e}) = 1$ and $\delta_{\hbar}(\tilde{e}) = 1$, for any $\tilde{e} \in \mathbf{E}$.

A svnf set $\hbar_{\mathbf{E}}$ on $\mathcal{B}$ is termed as an absolute svnf set (simply, $\tilde{\mathbf{E}}$), if $\sigma_{\hbar}(\tilde{e}) = 1, \tau_{\hbar}(\tilde{e}) = 0$ and $\delta_{\hbar}(\tilde{e}) = 0$, for any $\tilde{e} \in \mathbf{E}$.

A svnf set $\hbar_{\mathrm{E}}$ on $\mathcal{B}$ is termed as an t-absolute svnf set (simply, $\tilde{\mathbf{E}}^{t}$), if $\sigma_{\hbar}(\tilde{e}) = t, \tau_{\hbar}(\tilde{e}) = 0$ and $\delta_{\hbar}(\tilde{e}) = 0$, for any $\tilde{e} \in \mathbf{E}$ and $t \in \xi$.

For $\hbar_{z}, l_{y} \in (\widetilde{\mathcal{B}, \mathbf{E}})$, $\hbar_{z} \bar{\wedge} l_{y} = \phi$ if $\hbar_{z} \sqsubseteq l_{y}$ and $\hbar_{z} \bar{\wedge} l_{y} = \hbar_{z} \sqcap (l_{y})^{c}$ otherwise.

Definition 1.1. *[17] Let $\hbar_z$, l_y be svnf sets over $\mathcal{B}$. The union of svnf sets $\hbar_z$, l_y is a svnf set g_x, where $x = z \cup y$ and for any $\tilde{e} \in x$ and $\sigma_{\mathrm{g}} : \mathbf{E} \to \xi$ (σ_{g} called truth-membership) $\tau_{\mathrm{g}} : \mathbf{E} \to \xi$ (τ_{g} called indeterminacy), $\delta_{\mathrm{g}} : \mathbf{E} \to \xi$ (δ_{g} called falsity-membership) of g_x are as next:*

$$\sigma_{\mathrm{g}(\tilde{e})}(\varpi) = \begin{cases} \sigma_{\hbar(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{\hbar(\tilde{e})}(\varpi) \cup \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z \cup y. \end{cases}$$

$$\tau_{\mathrm{g}(\tilde{e})}(\varpi) = \begin{cases} \sigma_{\hbar(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{\hbar(\tilde{e})}(\varpi) \cap \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z \cap y. \end{cases}$$

$$\delta_{\mathrm{g}(\tilde{e})}(\varpi) = \begin{cases} \sigma_{\hbar(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{\hbar(\tilde{e})}(\varpi) \cap \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z \cap y. \end{cases}$$

Definition 1.2. *[17] The intersection of svnf sets $\hbar_z$, l_y is a svnf set g_x, where $x = z \cap y$ and for any $\tilde{e} \in C$, $\mathrm{g}_{\tilde{e}} = \hbar_{\tilde{e}} \widetilde{\cap} l_{\tilde{e}}$. We write as next:*

$$\sigma_{\mathrm{g}(\tilde{e})}(\varpi) = \begin{cases} \sigma_{\hbar(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{\hbar(\tilde{e})}(\varpi) \cap \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z \cap y. \end{cases}$$

$$\tau_{\mathrm{g}(\tilde{e})}(\varpi) = \begin{cases} \sigma_{\hbar(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{\hbar(\tilde{e})}(\varpi) \cup \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z \cup y. \end{cases}$$

$$\delta_{\mathrm{g}(\tilde{e})}(\varpi) = \begin{cases} \sigma_{\hbar(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z - y, \\ \sigma_{\hbar(\tilde{e})}(\varpi) \cup \sigma_{l(\tilde{e})}(\varpi), & \textit{if } \ \tilde{e} \in z \cup y. \end{cases}$$

Definition 1.3. *[17] Let $\hbar_z, l_y \in (\widetilde{\mathcal{B}, \mathbf{E}})$. Then,*

(1) *$\hbar_z$ is a svnf subset of l_y (simply, $\hbar_z \widetilde{\subseteq} l_y$) iff for every $\tilde{e} \in \mathbf{E}$,*

$$\sigma_{\hbar}(\tilde{e}) \leq \sigma_{l}(\tilde{e}), \quad \tau_{\hbar}(\tilde{e}) \geq \tau_{l}(\tilde{e}), \quad \delta_{\hbar}(\tilde{e}) \geq \delta_{l}(\tilde{e}).$$

(2) *The complement of* $\hbar_z$ *(simply,* $\hbar_z^c$*) [where* $\hbar^c : \mathbf{E} \to \xi^{\mathcal{B}}$*] is given by:*

$$\hbar_z^c = \{(\tilde{e}, \langle \delta_\hbar(\tilde{e}), \tau_{\hbar^c}(\tilde{e}), \sigma_\hbar(\tilde{e})\rangle) \mid \tilde{e} \in \mathcal{E}\}.$$

Theorem 1.1. *[17] Let* $\hbar_z, l_y, \mathrm{g}_x \in (\widetilde{\mathcal{B}, \mathbf{E}})$ *and* $(\hbar_z)_j = (\hbar_j)_z$, $(l_y)_j = (l_j)_y \in (\widetilde{\mathcal{B}, \mathbf{E}})$ $j \in \Gamma$*, where* Γ *is called the index set. Then*

(1) $\hbar_z \sqcap l_y = l_y \sqcap \hbar_z$ *and* $\hbar_z \sqcup l_y = l_y \sqcup \hbar_z$.
(2) $\hbar_z \sqcup (l_y \sqcup \mathrm{g}_x) = (\hbar_z \sqcup l_y) \sqcup \mathrm{g}_x$ *and* $\hbar_z \sqcap (l_y \sqcap \mathrm{g}_x) = (\hbar_z \sqcap l_y) \sqcap \mathrm{g}_x$.
(3) $\hbar_z \sqcup (\sqcap_{j\in\Gamma}[l_y]_j) = \sqcap_{j\in\Gamma}(\hbar_z \sqcup l_y)$.
(4) $\hbar_z \sqcap (\sqcup_{j\in\Gamma}[l_y]_j) = \sqcup_{j\in\Gamma}(\hbar_z \sqcap l_y)$.
(5) $[\hbar_z^c]^c = \hbar_z^c$.
(6) *If* $\hbar f_z \sqsubseteq l_y$*, then* $\hbar_z^c \sqsubseteq l_y^c$.
(7) $\hbar_z \sqcap \hbar_z = \hbar_z$ *and* $\hbar_z \sqcup \hbar_z = \hbar_z$.
(8) $\phi \leq \hbar_z \sqsubseteq \tilde{\mathbf{E}}$.
(9) $(\sqcup_{j\in\Gamma}[\hbar_z]_j)^c = \sqcap_{j\in\Gamma}[\hbar_z]_j^c$.

Definition 1.4. *[17] A single-valued neutrosophic soft topological space is ordered as* $(\mathcal{B}, \tilde{\top}^\sigma, \tilde{\top}^\tau, \tilde{\top}^\delta)$ *where* $\tilde{\top}^\sigma, \tilde{\top}^\tau, \tilde{\top}^\delta : \mathbf{E} \to \xi^{(\widehat{\mathcal{B},\mathbf{E}})}$ *is a mapping that satisfies the following axioms, for every* $\hbar_z, l_z \in (\widetilde{\mathcal{B}, \mathbf{E}})$ *and* $\tilde{e} \in \mathbf{E}$*:*

$(\top_1)$ $\tilde{\top}^\sigma_{\tilde{e}}(\phi) = \tilde{\top}^\sigma_{\tilde{e}}(\tilde{\mathbf{E}}) = 1$ *and* $\tilde{\top}^\tau_{\tilde{e}}(\phi) = \tilde{\top}^\tau_{\tilde{e}}(\tilde{\mathbf{E}}) = \tilde{\top}^\delta_{\tilde{e}}(\phi) = \tilde{\top}^\delta_{\tilde{e}}(\tilde{\mathbf{E}}) = 0$,
$(\top_2)$ $\tilde{\top}^\sigma_{\tilde{e}}(\hbar_z \sqcap l_y) \geq \tilde{\top}^\sigma_{\tilde{e}}(\hbar_z) \cap \tilde{\top}^\sigma_{\tilde{e}}(l_y)$, $\tilde{\top}^\tau_{\tilde{e}}(\hbar_z \sqcap l_y) \leq \tilde{\top}^\tau_{\tilde{e}}(\hbar_z) \cup \tilde{\top}^\tau_{\tilde{e}}(l_y)$,
$\tilde{\top}^\delta_{\tilde{e}}(\hbar_z \sqcap l_y) \leq \tilde{\top}^\delta_{\tilde{e}}(\hbar_z) \cup \tilde{\top}^\delta_{\tilde{e}}(l_y)$,
$(\top_3)$ $\tilde{\top}^\sigma_{\tilde{e}}(\bigsqcup_{j\in\Gamma}[\hbar_z]_j) \geq \bigcap_{j\in\Gamma} \tilde{\top}^{\tilde{\sigma}}_{\tilde{e}}([\hbar_z]_j)$, $\tilde{\top}^\tau_{\tilde{e}}(\bigsqcup_{j\in\Gamma}[\hbar_z]_j) \leq \bigcup_{i\in\triangle} \tilde{\top}^{\tilde{\tau}}_{\tilde{e}}([\hbar_z]_j)$,
$\tilde{\top}^\delta_{\tilde{e}}(\bigsqcup_{j\in\Gamma}[\hbar_z]_j) \leq \bigcup_{j\in\triangle} \tilde{\top}^\delta_{\tilde{e}}([\hbar_z]_j)$.

The svnft is termed to be stratified if it satisfies the following conditions:

$(\top_1^s)$ $\tilde{\top}^\sigma_{\tilde{e}}(\tilde{E}^t) = 1$, $\tilde{\top}^\tau_{\tilde{e}}(\tilde{E}^t) = 0$ *and* $\tilde{\top}^\delta_{\tilde{e}}(\tilde{E}^t) = 0$.

The Quadruple $(\mathcal{B}, \tilde{\top}^\sigma, \tilde{\top}^\varsigma, \tilde{\top}^\delta)$ *is known as a single-valued neutrosophic soft topological space (svnft-space), representing the degree of openness* $(\tilde{\top}^\sigma_{\tilde{e}}(\hbar_z))$*, the degree of indeterminacy* $(\tilde{\top}^\tau_{\tilde{e}}(\hbar_z))$*, and the degree of non-openness* $(\tilde{\top}^\delta_{\tilde{e}}(\hbar_z))$*; of a svnfs* $\hbar_A$ *with respect to the parameter* $\tilde{e} \in \mathbf{E}$ *respectively.*

Occasionally, $(\tilde{\top}^\sigma, \tilde{\top}^\tau, \tilde{\top}^\delta)$ *is written as* $\tilde{\top}^{\sigma\tau\delta}$ *here into avoid ambiguity.*

2. Single-Valued Neutrosophic Soft Grill

Definition 2.1. *A mapping* $\mathcal{K}^\sigma, \mathcal{K}^\tau, \mathcal{K}^\delta : \mathbf{E} \to \xi^{(\widetilde{\mathcal{B},\mathbf{E}})}$ *is called single-valued neutrosophic soft grill on* $\mathcal{B}$ *(abbreviated,* `svnf-grill`*) if it satisfies the following conditions* $\forall$ $\hbar_z, l_y \in (\widetilde{\mathcal{B}, \mathbf{E}})$ *and* $\tilde{e} \in \mathbf{E}$*:*

$(\mathcal{K}_1)$ $\mathcal{K}^{\sigma}_{\tilde{e}}(\phi)=0$, $\mathcal{K}^{\tau}_{\tilde{e}}(\phi)=1$, $\mathcal{K}^{\delta}_{\tilde{e}}(\phi)=1$ *and* $\mathcal{K}^{\sigma}_{\tilde{e}}(\tilde{\mathbf{E}})=1$, $\mathcal{K}^{\tau}_{\tilde{e}}(\tilde{\mathbf{E}})=0$, $\mathcal{K}^{\delta}_{\tilde{e}}(\tilde{\mathbf{E}})=0$,
$(\mathcal{K}_2)$ If $\hbar_z \sqsubseteq l_y$, *then* $\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z)\le \mathcal{K}^{\sigma}_{\tilde{e}}(l_y)$, $\mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z)\ge \mathcal{K}^{\tau}_{\tilde{e}}(l_y)$ *and* $\mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z)\ge \mathcal{K}^{\delta}_{\tilde{e}}(l_y)$,

$(\mathcal{K}_3)$ $\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z\sqcup l_y)\le \mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z)\vee\mathcal{K}^{\sigma}_{\tilde{e}}(l_y)$, $\mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z\sqcup l_y)\ge \mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z)\wedge\mathcal{K}^{\tau}_{\tilde{e}}(l_y)$ *and* $\mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z\sqcup l_y)\ge \mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z)\wedge\mathcal{K}^{\delta}_{\tilde{e}}(l_y)$.

Let $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$ *and* $\mathcal{K}^{\star\sigma\tau\delta}_{\mathbf{E}}$ *be* `svnf-grills` *on* $\mathcal{B}$, *we say* $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$ *is finer than* $\mathcal{K}^{\star\sigma\tau\delta}_{\mathbf{E}}$ *(*$\mathcal{K}^{\star\sigma\tau\delta}_{\mathbf{E}}$ *is coarser than* $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$ *) denoted by* $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}\sqsubseteq\mathcal{K}^{\star\sigma\tau\delta}_{\mathbf{E}}$ *if*

$$\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z)\le \mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z),\ \ \mathcal{K}^{\varsigma}_{\tilde{e}}(\hbar_z)\ge \mathcal{K}^{\varsigma}_{\tilde{e}}(\hbar_z),\ \ \mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z)\ge \mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z),\ \ \forall\ \hbar_z\in\widetilde{(\mathcal{B},\mathbf{E})},\ \tilde{e}\in\mathbf{E}.$$

The triple $(\mathcal{B},\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}},\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ *is termed the single-valued neutrosophic soft grill topological space (abbreviated, svnfgt-space).*

Definition 2.2. *Let* $(\mathcal{B},\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}},\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ *be svnfgt-space,* $\tilde{e}\in\mathbf{E}$, $r\in\xi_0$ *and* $\hbar_z\in\widetilde{(\mathcal{B},\mathbf{E})}$. *We define* $\varphi:\mathbf{E}\times\widetilde{(\mathcal{B},\mathbf{E})}\times\xi_0\to\widetilde{(\mathcal{B},\mathbf{E})}$, *indicated by* $\varphi(\tilde{e},\hbar_z,r)$ *or* $\varphi_{(\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}},\mathcal{K}^{\tilde{\sigma}\tilde{\varsigma}\tilde{\delta}}_{\mathbf{E}})}(\tilde{e},\hbar_z,r)$ *and called the svnf-operator related to* $(\mathcal{K}^{\sigma},\mathcal{K}^{\tau},\mathcal{K}^{\delta})$ *and* $(\top^{\sigma},\top^{\tau},\top^{\delta})$ *can be defined as follows:*

$$\begin{aligned}\varphi(\tilde{e},\hbar_z,r)=\ &\sqcap\{l_y\in\widetilde{(\mathcal{B},\mathbf{E})}\mid\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z\bar{\wedge}l_y)<r,\mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z\bar{\wedge}l_y)>1-r,\mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z\bar{\wedge}l_y)>1-r\\ &\text{and } \tilde{\top}^{\sigma}_{\tilde{e}}([l_y]^c)\ge r,\tilde{\top}^{\tau}_{\tilde{e}}([l_y]^c)\le 1-r,\tilde{\top}^{\delta}_{\tilde{e}}([l_y]^c)\le 1-r\}.\end{aligned}$$

Sometimes in this pape, we will write $\varphi_{\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}}(\tilde{e},\hbar_z,r)$ *or* $\varphi(\tilde{e},\hbar_z,r)$ *for* $\varphi_{(\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}},\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})}(\tilde{e},\hbar_z,r)$, *and also, sometimes, we will write* $\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},\hbar_z,r)$, $\varphi_{\mathcal{K}^{\tau}}(\tilde{e},\hbar_z,r)$, $\varphi_{\mathcal{K}^{\delta}}(\tilde{e},\hbar_z,r)$ *for* $\sigma_{[\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},\hbar_z,r)]}$, $\tau_{[\varphi_{\mathcal{K}^{\tau}}(\tilde{e},\hbar_z,r)]}$, $\delta_{[\varphi_{\mathcal{K}^{\delta}}(\tilde{e},\hbar_z,r)]}$ *respectively.*

If we take $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}=(\mathcal{K}^{\sigma\tau\delta}_{0})_{\mathbf{E}}$, then $\varphi(\tilde{e},\hbar_z,r)=\mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e},\hbar_z,r)$ for any $\tilde{e}\in\mathbf{E}$, $\hbar_z\in\widetilde{(\mathcal{B},\mathbf{E})}$, $r\in\zeta_0$.

Theorem 2.1. *Let* $(\mathcal{B},\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}})$ *be svnft-space and* $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$, $\mathcal{K}^{\star\sigma\tau\delta}_{\mathbf{E}}$ *be two* `svnf-grills` *on* $\mathcal{B}$. *Therefore, for every* $\tilde{e}\in\mathbf{E}$, $\hbar_z,l_y\in\widetilde{(\mathcal{B},\mathbf{E})}$, $r\in\xi_0$*:*

(1) *If* $\hbar_z\sqsubseteq l_y$, *then* $\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},\hbar_z,r)\le\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},l_y,r)$, $\varphi_{\mathcal{K}^{\tau}}v(\tilde{e},\hbar_z,r)\ge\varphi_{\mathcal{K}^{\tau}}(\tilde{e},l_y,r)$ *and* $\varphi_{\mathcal{K}^{\delta}}(\tilde{e},\hbar_z,r)\ge\varphi_{\mathcal{K}^{\delta}}(\tilde{e},l_y,r)$.
(2) *If* $\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z)<r$, $\mathcal{K}^{\tau}_{\tilde{e}}(\hbar_{\tilde{e}})\ge 1-r$, $\mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z)\ge 1-r$, *then* $\varphi(\tilde{e},\hbar_z,r)=\phi$. *Furthermore,* $\varphi(\tilde{e},\phi,r)=\phi$.
(3) *If* $\mathcal{K}^{\sigma\tau\delta}_{\tilde{e}}\sqsubseteq\mathcal{K}^{\star\sigma\tau\delta}_{\tilde{e}}$, *then* $\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},\hbar_z,r)\le\varphi_{\mathcal{K}^{\star\sigma}}(\tilde{e},\hbar_z,r)$, $\varphi_{\mathcal{K}^{\tau}}(\tilde{e},\hbar_z,r)\ge\varphi_{\mathcal{K}^{\star\tau}}(\tilde{e},\hbar_z,r)$ *and* $\varphi_{\mathcal{K}^{\delta}}(\tilde{e},\hbar_z,r)\ge\varphi_{\mathcal{K}^{\star\delta}}(\tilde{e},\hbar_z,r)$.
(4) $\varphi(\tilde{e},\hbar_z\sqcap l_y,r)\sqsubseteq\varphi(\tilde{e},\hbar_z,r)\sqcap\varphi(\tilde{e},l_y,r)$.
(5) $\varphi(\tilde{e},\hbar_z\sqcup l_y,r)\sqsupseteq\varphi(\tilde{e},\hbar_z,r)\sqcup\varphi(\tilde{e},l_y,r)$.
(6) $\varphi(\tilde{e},\hbar_z,r)=\mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e},\varphi(\tilde{e},\hbar_z,r),r)=\mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e},\hbar_z,r)$.
(7) $\varphi(\tilde{e},\varphi(\tilde{e},\hbar_z,r),r)\sqsubseteq\varphi(\tilde{e},\hbar_z,r)$.

Proof. (1) Let

$$\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},\hbar_z,r)\not\le\varphi_{\mathcal{K}^{\sigma}}(\tilde{e},l_y,r),\quad\varphi_{\mathcal{K}^{\tau}}(\tilde{e},\hbar_z,r)\not\ge\varphi_{\mathcal{K}^{\tau}}(\tilde{e},l_y,r),\quad\varphi_{\mathcal{K}^{\delta}}(\tilde{e},\hbar_z,r)\not\ge\varphi_{\mathcal{K}^{\delta}}(\tilde{e},l_y,r)$$

Then, there is $g_x \in (\widetilde{\mathcal{B}, \mathbf{E}})$ with $\mathcal{K}^{\sigma}_{\tilde{e}}(l_y \bar{\wedge} g_x) < r, \mathcal{K}^{\tau}_{\tilde{e}}(l_y \bar{\wedge} g_x) > 1-r, \mathcal{K}^{\delta}_{\tilde{e}}(l_y \bar{\wedge} g_x) > 1-r$ and $\tilde{\top}^{\sigma}_{\tilde{e}}([g_x]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([g_x]^c) \leq 1-r$, $\tilde{\top}^{\delta}_{\tilde{e}}([g_x]^c) \leq 1-r$, such that

$$\varphi_{\mathcal{K}^\sigma}(\tilde{e}, \hbar_z, r) \geq g_x \geq \varphi_{\mathcal{K}^\sigma}(\tilde{e}, l_y, r), \quad \varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) \leq g_x \leq \varphi_{\mathcal{K}^\tau}(\tilde{e}, l_y, r),$$

$$\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \leq g_x \leq \varphi_{\mathcal{K}^\delta}(\tilde{e}, l_y, r).$$

On another side, since $\varphi_{\mathcal{K}^\sigma}(\tilde{e}, l_y, r) \geq g_x$, $\varphi_{\mathcal{K}^\tau} v(\tilde{e}, l_y, r) \leq g_x$, $\varphi_{\mathcal{K}^\delta}(\tilde{e}, l_y, r) \leq g_x$ and $\hbar_z \sqsubseteq l_y$ we obtain $\hbar_z \bar{\wedge} g_x \sqsubseteq l_y \bar{\wedge} g_x$. So,

$$\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) \leq \mathcal{K}^{\sigma}_{\tilde{e}}(l_y \bar{\wedge} g_x) < r, \ \mathcal{K}^{\tilde{\tau}}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) \geq \mathcal{K}^{\tau}_{\tilde{e}}(l_y \bar{\wedge} g_x) > 1-r, \ \mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) \geq \mathcal{K}^{\delta}_{\tilde{e}}(l_y \bar{\wedge} g_x) > 1-r.$$

Hence, $\varphi_{\mathcal{K}^\sigma}(\tilde{e}, \hbar_z, r) \leq g_x$, $\varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) \geq g_x$, and $\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \geq g_x$. A contradiction. Thus,

$$\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \leq \varphi_{\mathcal{K}^\delta}(\tilde{e}, l_y, r), \quad \varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) \geq \varphi_{\mathcal{K}^\tau}(\tilde{e}, l_y, r), \quad \varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \geq \varphi_{\mathcal{K}^\delta}(\tilde{e}, l_y, r).$$

(2) Since $\hbar_z \bar{\wedge} l_y \sqsubseteq \hbar_z$ we get

$$\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z \bar{\wedge} l_y) \leq \mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z) < r, \ \mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z \bar{\wedge} l_y) \geq \mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z) > 1-r, \ \mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z \bar{\wedge} l_y) \geq \mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z) > 1-r,$$

for each $l_y \in (\widetilde{\mathcal{B}, \mathbf{E}})$. Thus based on the concept of φ and if $\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z) < r$, $\mathcal{K}^{\tau}_{\tilde{e}}(\hbar_{\tilde{e}}) \geq 1-r$, $\mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z) \geq 1-r$, then $\varphi(\tilde{e}, \hbar_z, r) = \phi$.

(3) Assume that,

$$\varphi_{\mathcal{K}^\sigma}(\tilde{e}, \hbar_z, r) \not\leq \varphi_{\mathcal{K}^{\star\sigma}}(\tilde{e}, \hbar_z, r), \quad \varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) \not\geq \varphi_{\mathcal{K}^{\star\tau}}(\tilde{e}, \hbar_z, r),$$

$$\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \not\geq \varphi_{\mathcal{K}^{\star\delta}}(\tilde{e}, \hbar_z, r)$$

Then, there is $g_x \in (\widetilde{\mathcal{B}, \mathbf{E}})$ with $\mathcal{K}^{\star\sigma}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) < r, \mathcal{K}^{\star\tau}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) > 1-r, \mathcal{K}^{\star\delta}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) > 1-r$ and $\tilde{\top}^{\sigma}_{\tilde{e}}([g_x]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([g_x]^c) \leq 1-r$, $\tilde{\top}^{\delta}_{\tilde{e}}([g_x]^c) \leq 1-r$, such that

$$\varphi_{\mathcal{K}^\sigma}(\tilde{e}, \hbar_z, r) > g_x \geq \varphi_{\mathcal{K}^{\star\sigma}}(\tilde{e}, \hbar_z, r), \quad \varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) < g_x \leq \varphi_{\mathcal{K}^{\star\tau}}(\tilde{e}, \hbar_z, r),$$

$$\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) < g_x \leq \varphi_{\mathcal{K}^{\star\delta}}(\tilde{e}, \hbar_z, r).$$

Since $\varphi_{\mathcal{K}^{\star\sigma}}(\tilde{e}, \hbar_z, r) \leq g_x$, $\varphi_{\mathcal{K}^{\star\tau}}(\tilde{e}, \hbar_z, r) \geq g_x$, $\varphi_{\mathcal{K}^{\star\delta}}(\tilde{e}, \hbar_z, r) \geq g_x$ and $\mathcal{K}^{\sigma\tau\delta}_{\tilde{e}} \sqsubseteq \mathcal{K}^{\star\sigma\tau\delta}_{\tilde{e}}$, we get

$$\mathcal{K}^{\sigma}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) \leq \mathcal{K}^{\star\sigma}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) < r, \quad \mathcal{K}^{\tau}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) \geq \mathcal{K}^{\star\tau}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) > 1-r,$$

$$\mathcal{K}^{\delta}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) \geq \mathcal{K}^{\star\delta}_{\tilde{e}}(\hbar_z \bar{\wedge} g_x) > 1-r.$$

Hence, $\varphi_{\mathcal{K}^\sigma}(\tilde{e}, \hbar_z, r) \leq g_x$, $\varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) \geq g_x$, $\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \geq g_x$. A contradiction. Thus, $\varphi_{\mathcal{K}^\sigma}(\tilde{e}, \hbar_z, r) \leq \varphi_{\mathcal{K}^{\star\sigma}}(\tilde{e}, \hbar_z, r)$, $\varphi_{\mathcal{K}^\tau}(\tilde{e}, \hbar_z, r) \geq \varphi_{\mathcal{K}^{\star\tau}}(\tilde{e}, \hbar_z, r)$ and $\varphi_{\mathcal{K}^\delta}(\tilde{e}, \hbar_z, r) \geq \varphi_{\mathcal{K}^{\star\delta}}(\tilde{e}, \hbar_z, r)$.

(4) Since, $\hbar_z \sqcap l_y \sqsubseteq \hbar_z$ and $\hbar_z \sqcap l_y \sqsubseteq l_y$. So, from (1), we get $\varphi(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \varphi(\tilde{e}, \hbar_z, r)$ and $\varphi(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \varphi(\tilde{e}, l_{\mathcal{B}}, r)$. Therefore,

$$\varphi(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \varphi(\tilde{e}, \hbar_z, r) \sqcap \varphi(\tilde{e}, l_y, r).$$

(5) In a similar vein, we can demonstrate through a parallel line of reasoning that.

(6) From the concept of $\varphi(\tilde{e}, \hbar_z, r)$, $\mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \varphi(\tilde{e}, \hbar_z, r), r) = \varphi(\tilde{e}, \hbar_z, r)$. Now we will just verify $\varphi(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$. For each `svns-grill` $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$ we have $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}} \sqsubseteq \mathcal{K}^{0\sigma\tau\delta}_{\mathbf{E}}$, so by (3), we have

$$\varphi_{\mathcal{K}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) \sqsubseteq \varphi_{\mathcal{K}^{0\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) = \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r).$$

Therefore,

$$\varphi(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r).$$

(7) Likewise, we can establish through a similar line of reasoning that. □

Example 2.1. *Assume that, $\mathcal{B} = \{x_1, x_2\}$ be a universal set, $\mathbf{E} = \{\tilde{e}_1, \tilde{e}_2\}$ be a set of parameters. Define* `svnf-topology` *($\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}$) and* `svnf-grill` *($\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$) as follow, for every $\tilde{e} \in \mathbf{E}$*

$$\tilde{\top}^{\sigma}_{e}(\hbar_{\mathbf{E}}) = \begin{cases} 1, & if\ \ \hbar_{\boldsymbol{E}} = \phi \text{ or } \tilde{\boldsymbol{E}}, \\ \frac{1}{2}, & if\ \ \hbar_{\boldsymbol{E}} = \{(\tilde{e}_1, \langle 0.3, 0.3, 0.3\rangle), (\tilde{e}_2, \langle 0.6, 0.6, 0.6\rangle)\}, \\ 0, & if\ \ otherwise, \end{cases}$$

$$\tilde{\top}^{\tau}_{e}(\hbar_{\mathbf{E}}) = \begin{cases} 0, & if\ \ \hbar_{\boldsymbol{E}} = \phi \text{ or } \tilde{\boldsymbol{E}}, \\ \frac{1}{2}, & if\ \ \hbar_{\boldsymbol{E}} = \{(\tilde{e}_1, \langle 0.3, 0.3, 0.3\rangle), (\tilde{e}_2, \langle 0.6, 0.6, 0.6\rangle)\}, \\ 1, & if\ \ otherwise, \end{cases}$$

$$\tilde{\top}^{\delta}_{e}(\hbar_{\mathbf{E}}) = \begin{cases} 0, & if\ \ \hbar_{\boldsymbol{E}} = \phi \text{ or } \tilde{\boldsymbol{E}}, \\ \frac{1}{2}, & if\ \ \hbar_{\boldsymbol{E}} = \{(\tilde{e}_1, \langle 0.3, 0.3, 0.3\rangle), (\tilde{e}_2, \langle 0.6, 0.6, 0.6\rangle)\}, \\ 1, & if\ \ otherwise, \end{cases}$$

$$\mathcal{K}^{\sigma}_{e}(\hbar_{\mathbf{E}}) = \begin{cases} 1, & if\ \ \{(\tilde{e}_1, \langle 1, 0, 0\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0.7, & if\ \ \{(\tilde{e}_1, \langle 0.5, 0, 0\rangle), (\tilde{e}_2, \langle 0.5, 0, 0\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0, & if\ \ otherwise, \end{cases}$$

$$\mathcal{K}^{\tau}_{e}(\hbar_{\mathbf{E}}) = \begin{cases} 0, & if\ \ \{(\tilde{e}_1, \langle 1, 0, 0\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0.3, & if\ \ \{(\tilde{e}_1, \langle 0.5, 0, 0\rangle), (\tilde{e}_2, \langle 0.5, 0, 0\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 1, & if\ \ otherwise, \end{cases}$$

$$\mathcal{K}^{\delta}_{e}(\hbar_{\mathbf{E}}) = \begin{cases} 0, & if\ \ \{(\tilde{e}_1, \langle 1, 0, 0\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0.2, & if\ \ \{(\tilde{e}_1, \langle 0.5, 0, 0\rangle), (\tilde{e}_2, \langle 0.5, 0, 0\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 1, & if\ \ otherwise. \end{cases}$$

Then $\{(\tilde{e}_1, \langle 0.7, 0.7, 0.7\rangle), (\tilde{e}_2, \langle 0.4, 0.4, 0.4\rangle)\} = \varphi(\tilde{e}, \tilde{\mathbf{E}}^{0.6}, \frac{1}{2}) \neq \varphi(\tilde{e}, \varphi(\tilde{e}, \tilde{\mathbf{E}}^{0.6}, \frac{1}{2}), \frac{1}{2}) = \phi$

Theorem 2.2. *Let $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \Upsilon^{\sigma\tau\delta}_{\mathbf{E}})$ be svnfgt-space, $\{(\hbar_z)_i \in \widetilde{(\mathcal{B}, \mathbf{E})} : i \in \Gamma\}$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$. Then:*

(1) $(\sqcup(\varphi(\tilde{e}, (\hbar_z)_i, r)) : i \in \Gamma) \sqsubseteq (\varphi(\tilde{e}, \sqcup(\hbar_z)_i, r) : i \in \Gamma)$.

(2) $(\varphi(\tilde{e}, \sqcap(\hbar_z)_i, r) : i \in \Gamma) \sqsubseteq (\sqcap(\varphi(\tilde{e}, (\hbar_z)_i, r)) : i \in \Gamma)$.

Proof. (1) Since $((\hbar_z)_i \sqsubseteq \sqcup(\hbar_z)_i, \ \ \forall \ \ i \in \Gamma)$, so by theorem 2.1 (1), we have, $\varphi(\tilde{e}, (\hbar_z)_i, r) \sqsubseteq \varphi(\tilde{e}, \sqcup(\hbar_z)_i, r)$. Hence, $\sqcup(\varphi(\tilde{e}, (\hbar_z)_i, r)) \sqsubseteq \varphi(\tilde{e}, \sqcup(\hbar_z)_i, r), \ \forall \ i \in \Gamma$

(2) Since $(\sqcap(\hbar_z)_i \sqsubseteq (\hbar_z)_i, \ \forall \ i \in \Gamma)$, so by theorem 2.1 (1), we have, $\sqcap(\varphi(\tilde{e}, (\hbar_z)_i, r)) \sqsubseteq \varphi(\tilde{e}, \sqcap(\hbar_{\mathcal{A}})_i, r)$. Thus, $\varphi(\tilde{e}, \sqcap(\hbar_z)_i, r) \sqsubseteq \sqcap(\varphi(\tilde{e}, (\hbar_z)_i, r)), \ \forall \ i \in \Gamma$ □

Definition 2.3. *Let $(\mathcal{B}, \tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta}, \mathcal{K}_{\mathbf{E}}^{\sigma\tau\delta})$ be svnfgt-space, Then for all $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$ and $r \in \xi_0$ we define a mapping $\mathcal{C}^\star : \mathbf{E} \times \widetilde{(\mathcal{B}, \mathbf{E})} \times \xi_0 \longrightarrow \xi^{\widetilde{(\mathcal{B}, \mathbf{E})}}$ as next:*

$$\mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \hbar_z \sqcup \varphi(\tilde{e}, \hbar_z, r).$$

Clear that

$$(\tilde{\top}_{\mathcal{K}^{\star\sigma}})_{\tilde{e}}(\hbar_z) = \bigvee\{r \mid \mathcal{C}^\star(\tilde{e}, \hbar_z^c, r) = \hbar_z^c\}.$$
$$(\tilde{\top}_{\mathcal{K}^{\star\tau}})_{\tilde{e}}(\hbar_z) = \bigwedge\{1 - r \mid \mathcal{C}^\star(\tilde{e}, \hbar_z^c, 1 - r) = \hbar_z^c\}.$$
$$(\tilde{\top}_{\mathcal{K}^{\star\delta}})_{\tilde{e}}(\hbar_z) = \bigwedge\{1 - r \mid \mathcal{C}^\star(\tilde{e}, \hbar_z^c, 1 - r) = \hbar_z^c\}.$$

is a supra single-valued neutrosophic Soft topology generated by $\mathcal{C}^\star$ and $\tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta} \sqsubseteq (\tilde{\top}_{\mathcal{K}}^{\star\sigma\tau\delta})_{\mathbf{E}}$. If $\mathcal{K}_{\mathbf{E}}^{\sigma\tau\delta} = \mathcal{K}_{\mathbf{E}}^{0\sigma\tau\delta}$, therefor for any $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$ and $r \in \xi_0$, we have,

$$\mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \hbar_z \sqcup \varphi(\tilde{e}, \hbar_z, r) = \hbar_z \sqcup \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) = \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r).$$

Thus in this case, $\tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta} \sqsubseteq (\tilde{\top}_{\mathcal{K}^0}^{\star\sigma\tau\delta})_{\mathbf{E}}$.

Theorem 2.3. *For every $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$ and $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, the operator $\mathcal{C}^\star$ fulfills the next conditions:*

(1) $\mathcal{C}^\star(\tilde{e}, \phi, r) = \phi$.

(2) $\hbar_z \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$.

(3) *If $\hbar_z \sqsubseteq l_y$, then $\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, l_y, r)$.*

(4) $\mathcal{C}^\star(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap \mathcal{C}^\star(\tilde{e}, l_{\mathcal{B}}, r)$.

(5) $\mathcal{C}^\star(\tilde{e}, \hbar_z \sqcup l_y, r) \sqsupseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcup \mathcal{C}^\star(\tilde{e}, l_y, r)$.

(6) $\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, \mathcal{C}^\star(\tilde{e}, \hbar_z, r), r)$.

Proof. (1) $\mathcal{C}^\star(\tilde{e}, \phi, r) = \phi \sqcup \varphi(\tilde{e}, \phi, r) = \phi \sqcup \phi = \phi$.

(2) From the concept of $\mathcal{C}^\star$, we get than $\hbar_z \sqsubseteq \hbar_z \sqcup \varphi(\tilde{e}, \hbar_z, r) = \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$. Since $\hbar_z \sqsubseteq \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$ and by Theorem 2.1 (6), we obtain $\varphi(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$ implies that

$$\hbar_z \sqcup \varphi(\tilde{e}, \hbar_z, r) = \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r).$$

Therefore, $\hbar_z \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$.

(3) Because $\hbar_z \sqsubseteq l_y$ and by Theorem 2.1 (1), we obtain $\varphi(\tilde{e}, \hbar_z, r) \sqsubseteq \varphi(\tilde{e}, l_y, r)$. Therefore, $\hbar_z \sqcup \varphi(\tilde{e}, \hbar_z, r) \sqsubseteq l_y \sqcup \varphi(\tilde{e}, l_y, r)$. Thus, $\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, l_{\mathcal{B}}, r)$.

(4) From (3), we get that $\mathcal{C}^\star(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$ and $\mathcal{C}^\star(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, l_y, r)$ implies

$$\mathcal{C}^\star(\tilde{e}, \hbar_z \sqcap l_y, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap \mathcal{C}^\star(\tilde{e}, l_y, r).$$

(5) Similarly, we can affirm through a corresponding argument that.

(6) From (2) and (5) we obtain $\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, \mathcal{C}^\star(\tilde{e}, \hbar_z, r), r)$. □

Theorem 2.4. *Let $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \Upsilon^{\sigma\tau\delta}_{\mathbf{E}})$ be svnfgt-space, $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$. Then:*

(1) *If $\hbar_z \sqsubseteq \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$, then*

$$\mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) = \mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \varphi(\tilde{e}, \hbar_Z, r), r) = \varphi(\tilde{e}, \hbar_Z, r).$$

(2) *If $\tilde{\top}^{\sigma}_{\tilde{e}}([\hbar_Z]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([\hbar_Z]^c) \leq 1 - r$, $\tilde{\top}^{\delta}_{\tilde{e}}([\hbar_Z]^c) \leq 1 - r$, then $\varphi(\tilde{e}, \hbar_Z, r) \sqsubseteq \hbar_Z$.*

Proof. (1) Because $\hbar_z \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$ and $\varphi(\tilde{e}, \hbar_Z, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$, so we obtain,

$$\hbar_z \sqcup \varphi(\tilde{e}, \hbar_Z, r) = \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r).$$

In view of Theorem 2.1 (6), we get,

$$\varphi(\tilde{e}, \hbar_Z, r) = \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \varphi(\tilde{e}, \hbar_Z, r), r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r).$$

Because, $\hbar_z \sqsubseteq \varphi(\tilde{e}, \hbar_z, r)$ we have $\mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}\mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \varphi(\tilde{e}, \hbar_Z, r), r)$ and since $\varphi(\tilde{e}, \hbar_z, r) \sqsubseteq$ $\mathrm{cl}^\star(\tilde{e}, \hbar_z, r)$. Hence,

$$\mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_{\mathcal{A}}, r) = \mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \varphi(\tilde{e}, \hbar_Z, r), r) = \varphi(\tilde{e}, \hbar_Z, r).$$

(2) Form Theorem 2.3 (2), we have

$$\varphi(\tilde{e}, \hbar_Z, r) = \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \varphi(\tilde{e}, \hbar_Z, r), r) \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) = \hbar_z.$$

□

3. Connectedness in Single-Valued Neutrosophic Soft Grill Topological Spaces

In this unit, we familiarize the r-single-valued neutrosophic grill connectedness (for short, r-svnfg-connectedness) of a svnfgt-space $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$. Recall that, the svnfs $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$ are called r-single-valued neutrosophic separated (for short, r-svnf-separated) if $\hbar_z$ and l_y satisfy the following condition

$$\mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) \sqcap l_y = \phi = \hbar_z \sqcap \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, l_y, r), \quad \tilde{e} \in \mathbf{E}, \quad r \in \xi_0.$$

Definition 3.1. *Let $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ be r-svnfgt-space. Then,*

(1) *the svnfs $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$ are called r-single-valued neutrosophic grill separated (r-svnfg-separated) if $\hbar_z$ and l_y satisfy the following condition*

$$\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap l_y = \phi = \hbar_z \sqcap \mathrm{cl}^\star(\tilde{e}, l_y, r), \quad \tilde{e} \in \mathbf{E}, \quad r \in i_0.$$

(2) *$(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ r-single-valued neutrosophic grill connected (abbreviated r-svnfg-connected space) if it could not be found two r-svnfg-separated sets $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\hbar_z \neq \phi$, $l_y \neq \phi$ such that $\hbar_z \sqcup l_y = \tilde{E}$. That is, there do not exist r-svnfg-separated sets $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\hbar_z \neq \phi$ except $\hbar_z = \phi$, $l_y = \phi$.*

Remark 3.1. *Any two r-svnf-separated sets are r-svnfg-separated sets. That is from*

$$\mathcal{C}^{\star}(\tilde{e}, \mathrm{g}_{x}, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \mathrm{g}_{x}, r), \ \ \forall \ \mathrm{g}_{x} \in \widetilde{(\mathcal{B}, \mathbf{E})}, \ \tilde{e} \in \mathbf{E}, \ r \in \xi_0.$$

However, the converse is not true in general, as shown in the following example.

Example 3.1. *Assume that, $\mathcal{B} = \{x_1, x_2\}$ be a universal set, $\mathbf{E} = \{\tilde{e}_1, \tilde{e}_2\}$ be a set of parameters. Define `svnf-topology` $\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}$ and `svnf-grill` $\mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}}$ as follow, for every $\tilde{e} \in \mathbf{E}$*

$$\tilde{\top}^{\sigma}_{e}(\hbar_{\mathrm{E}}) = \begin{cases} 1, & if \ \ \hbar_{\boldsymbol{E}} = \phi \text{ or } \tilde{\boldsymbol{E}}, \\ \frac{1}{2}, & if \ \ \hbar_{\boldsymbol{E}} = \{(\tilde{e}_1, \langle 1, 0.4, 0.4\rangle), (\tilde{e}_2, \langle 0.5, 1, 1\rangle)\}, \\ 0, & if \ \ otherwise, \end{cases}$$

$$\tilde{\top}^{\tau}_{e}(\hbar_{\mathrm{E}}) = \begin{cases} 0, & if \ \ \hbar_{\boldsymbol{E}} = \phi \text{ or } \tilde{\boldsymbol{E}}, \\ \frac{1}{2}, & if \ \ \hbar_{\boldsymbol{E}} = \{(\tilde{e}_1, \langle 1, 0.4, 0.4\rangle), (\tilde{e}_2, \langle 0.5, 1, 1\rangle)\}, \\ 1, & if \ \ otherwise, \end{cases}$$

$$\tilde{\top}^{\delta}_{e}(\hbar_{\mathrm{E}}) = \begin{cases} 0, & if \ \ \hbar_{\boldsymbol{E}} = \phi \text{ or } \tilde{\boldsymbol{E}}, \\ \frac{1}{2}, & if \ \ \hbar_{\boldsymbol{E}} = \{(\tilde{e}_1, \langle 1, 0.4, 0.4\rangle), (\tilde{e}_2, \langle 0.5, 1, 1\rangle)\}, \\ 1, & if \ \ otherwise, \end{cases}$$

$$\tilde{\mathcal{K}}^{\sigma}_{e}(\hbar_{\mathrm{E}}) = \begin{cases} 1, & if \ \ \{(\tilde{e}_1, \langle 1, 0, 0\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0.5, & if \ \ \{(\tilde{e}_1, \langle 0, 0.3, 0.3\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0, & if \ \ otherwise, \end{cases}$$

$$\tilde{\mathcal{K}}^{\tau}_{e}(\hbar_{\mathrm{E}}) = \begin{cases} 0, & if \ \ \{(\tilde{e}_1, \langle 1, 0, 0\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0.5, & if \ \ \{(\tilde{e}_1, \langle 0, 0.3, 0.3\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 1, & if \ \ otherwise, \end{cases}$$

$$\tilde{\mathcal{K}}^{\delta}_{e}(\hbar_{\mathrm{E}}) = \begin{cases} 0, & if \ \ \{(\tilde{e}_1, \langle 1, 0, 0\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 0.25, & if \ \ \{(\tilde{e}_1, \langle 0, 0.3, 0.3\rangle), (\tilde{e}_2, \langle 0, 1, 1\rangle)\} \sqsubseteq \hbar_{\boldsymbol{E}} \sqsubseteq \tilde{\boldsymbol{E}}, \\ 1, & if \ \ otherwise. \end{cases}$$

Let $l_{\mathrm{E}} = \{(\tilde{e}_1, \langle 0.8, 0, 0\rangle), (\tilde{e}_2, \langle 0, 0.5, 0.5\rangle)\}$ *and* $\mathrm{g}_{\mathrm{E}} = \{(\tilde{e}_1, \langle 0, 0, 0.2\rangle), \ (\tilde{e}_2, \langle 0.5, 0.5, 0\rangle)\}$. *Since* $\mathcal{K}^{\sigma}_{\tilde{e}}(l_{\mathrm{E}}) < \frac{1}{2}$, $\mathcal{K}^{\tau}_{\tilde{e}}(l_{\mathrm{E}}) \geq 1 - \frac{1}{2}$, $\mathcal{K}^{\delta}_{\tilde{e}}(l_{\mathrm{E}}) \geq 1 - \frac{1}{2}$ *and* $\mathcal{K}^{\sigma}_{\tilde{e}}(\mathrm{g}_{\mathrm{E}}) < \frac{1}{2}$, $\mathcal{K}^{\tau}_{\tilde{e}}(\mathrm{g}_{\mathrm{E}}) \geq 1 - \frac{1}{2}$, $\mathcal{K}^{\delta}_{\tilde{e}}(\mathrm{g}_{\mathrm{E}}) \geq 1 - \frac{1}{2}$, *we have* $\varphi(\tilde{e}, l_{\mathrm{E}}, \frac{1}{2}) = \varphi(\tilde{e}, \mathrm{g}_{\mathrm{E}}, \frac{1}{2}) = \phi$. *So,* $\mathrm{cl}^{\star}(\tilde{e}, l_{\mathrm{E}}, \frac{1}{2}) = l_{\mathrm{E}}$ *and* $\mathrm{cl}^{\star}(\tilde{e}, \mathrm{g}_{\mathrm{E}}, \frac{1}{2}) = \mathrm{g}_{\mathrm{E}}$. *Thus,*

$$\mathrm{cl}^{\star}(\tilde{e}, l_{\mathrm{E}}, \frac{1}{2}) \sqcap \mathrm{g}_{\mathrm{E}} = l_{\mathrm{E}} \sqcap \mathrm{g}_{\mathrm{E}} = l_{\mathrm{E}} \sqcap \mathrm{cl}^{\star}(\tilde{e}, \mathrm{g}_{\mathrm{E}}, \frac{1}{2}) = \phi.$$

Hence, l_{E} *and* g_{E} *are r-svnfg-separated sets. However,* l_{E} *and* g_{E} *are not r-svnf-separated sets where* $\mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, l_{\mathrm{E}}, \frac{1}{2}) = \tilde{\mathbf{E}}$ *and thus* $\mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, l_{\mathrm{E}}, \frac{1}{2}) \sqcap \mathrm{g}_{\mathrm{E}} \neq \phi$.

Definition 3.2. *Let* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \Upsilon^{\sigma\tau\delta}_{\mathbf{E}})$ *be r-svnfgt-space, and let* $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$ *be nonempty svnf sets, such that*

(1) $\hbar_z, l_y$ *are r-svnfg-separated with* $\hbar_z \sqcup l_y = \tilde{E}$. *Therefore,* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ *is termed r-single-valued neutrosophic grill disconnected (abbreviated r-svnfg-disconnected space).*

(2) $\hbar_z, l_y$ *are r-svnfg-separated with* $\hbar_z \sqcup l_y = \mathrm{g}_x$. *Therefore,* g_x *is termed r-svnfg-disconnected on* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$.

Theorem 3.1. *Let* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ *be r-svnfgt-space. Therefore, the following statements are equivalent.*

(1) $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ *is r-svnfg-connected.*

(2) *If* $\hbar_z \sqcup l_y = \tilde{E}$ *and* $\hbar_z \sqcap l_y = \phi$ *with* $\tilde{\top}^{\sigma}_{\tilde{e}}(\hbar_z) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}(\hbar_z) \leq 1-r$ $\tilde{\top}^{\delta}_{\tilde{e}}(\hbar_z) \leq 1-r$ *and* $\tilde{\top}^{\sigma}_{\tilde{e}}(l_y) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}(l_y) \leq 1-r$ $\tilde{\top}^{\delta}_{\tilde{e}}(l_y) \leq 1-r$, $\tilde{e} \in \mathbf{E}$, $r \in \zeta_0$, *then* $\hbar_z = \phi$ *or* $l_y = \phi$.

(3) *If* $\hbar_z \sqcup l_y = \tilde{E}$ *and* $\hbar_z \sqcap l_y = \phi$ *with* $\tilde{\top}^{\sigma}_{\tilde{e}}([\hbar_z]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([\hbar_z]^c) \leq 1-r$, $\tilde{\top}^{\delta}_{\tilde{e}}([\hbar_z]^c) \leq 1-r$ *and* $\tilde{\top}^{\sigma}_{\tilde{e}}([l_y]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([l_y]^c) \leq 1-r$, $\tilde{\top}^{\delta}_{\tilde{e}}([l_y]^c) \leq 1-r$, $\tilde{e} \in \mathbf{E}$, $r \in \zeta_0$, *then* $\hbar_z = \phi$ *or* $l_y = \phi$.

Proof. (1)$\Longrightarrow$(2) Suppose there exist $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$ with $\tilde{\top}^{\sigma}_{\tilde{e}}(\hbar_z) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}(\hbar_z) \leq 1-r$ $\tilde{\top}^{\delta}_{\tilde{e}}(\hbar_z) \leq 1-r$, $\tilde{\top}^{\sigma}_{\tilde{e}}(l_y) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}(l_y) \leq 1-r$ $\tilde{\top}^{\delta}_{\tilde{e}}(l_y) \leq 1-r$, such that $\hbar_z \sqcup l_y = \tilde{E}$ and $\hbar_z \sqcup l_y = \phi$, which implies $\hbar_z = [l_y]^c$ and $l_y = [\hbar_{\mathcal{A}}]^c$. Then, by Theorem 2.3 (2) and Theorem 2.4 (2) we have;

$$\mathcal{C}^{\star}(\tilde{e}, [l_y]^c, r) \sqcap [\hbar_z]^c \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, [l_y]^c, r) \sqcap [\hbar_z]^c = [l_y]^c \sqcap [\hbar_z]^c = \hbar_z \sqcap l_y = \phi,$$

and

$$\mathcal{C}^{\star}(\tilde{e}, [\hbar_z]^c, r) \sqcap [l_y]^c \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}} v(\tilde{e}, [\hbar_z]^c, r) \sqcap [l_y]^c = [\hbar_z]^c \sqcap [l_y]^c = l_y \sqcap \hbar_z = \phi.$$

Therefore, $[l_y]^c$ and $[\hbar_z]^c$ are r-svnfg-separated sets with $[l_y]^c \sqcup [\hbar_z]^c = \hbar_z \sqcup l_y = \tilde{\mathbf{E}}$. But $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ is r-svnfg-connected implies $[l_y]^c = \phi$ or $[\hbar_z]^c = \phi$ and hence, $l_y = \phi$ or $\hbar_z = \phi$.

(2)$\Longrightarrow$(3) Clear.

(3)$\Longrightarrow$(1) Let $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\hbar_z \neq \phi$, $l_y \neq \phi$ such that $\hbar_z \sqcup l_y = \tilde{E}$. Assume that $\mathrm{g}_x = \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r)$ and $\mathrm{w}_D = \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, l_y, r)$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$, then $\mathrm{g}_x \sqcup \mathrm{w}_D = \tilde{E}$ with $\tilde{\top}^{\sigma}_{\tilde{e}}([\mathrm{g}_x]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([\mathrm{g}_x]^c) \leq 1-r$, $\tilde{\top}^{\delta}_{\tilde{e}}([\mathrm{g}_x]^c) \leq 1-r$, $\tilde{\top}^{\sigma}_{\tilde{e}}([\mathrm{w}_D]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([\mathrm{w}_D]^c) \leq 1-r$, $\tilde{\top}^{\delta}_{\tilde{e}}([\mathrm{w}_D]^c) \leq 1-r$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$. Now, suppose that (3) is not satisfied. That is, $\mathrm{g}_x \neq \phi$, $\mathrm{w}_D \neq \phi$, $\mathrm{g}_x \sqcup \mathrm{w}_D = \phi$. Thus, by Theorem 2.3 (2), we obtain,

$$\mathcal{C}^{\star}(\tilde{e}, \hbar_z, r) \sqcap l_y \sqsubseteq \mathcal{C}_{\top_{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r), r) \sqcap \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, l_y, r) = \mathrm{g}_x \sqcap \mathrm{w}_D = \phi.$$

and

$$\hbar_z \sqcap \mathcal{C}^{\star}(\tilde{e}, l_y, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r), r) \sqcap \mathcal{C}_{\top_{\sigma\tau\delta}}(\tilde{e}, l_y, r) = \mathrm{g}_c \sqcap \mathrm{w}_D = \phi.$$

Therefore, l_y and $\hbar_z$ are r-svnfg-separated sets, $l_y = \phi$, $\hbar_z = \phi$ with $\hbar_z \sqcup l_y = \tilde{\mathbf{E}}$. Hence, $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ is not r-svnfg-connected. □

Theorem 3.2. *Let* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ *be r-svnfgt-space and* $\hbar_z, l_y, \mathrm{g}_c \in \widetilde{(\mathcal{B}, \mathbf{E})}$. *If* l_y *and* g_c *are r-svnfg-separated sets, then* $\hbar_z \sqcap l_y$, $\hbar_z \sqcap \mathrm{g}_x$ *are r-svnfg-separated sets.*

Proof. Let l_y and g_x be r-svnfg-separated sets, that is,

$$\mathcal{C}^\star(\tilde{e}, l_y, r) \sqcap g_x = \phi = \text{cl}^\star(\tilde{e}, g_x, r) \sqcap l_y, \forall,\ \tilde{e} \in \mathbf{E},\ \ r \in \xi_0.$$

Then, from Theorem 2.3 (4) we get that

$$\begin{aligned}\mathcal{C}^\star(\tilde{e}, \sqcap[\hbar_z \sqcap l_y], r) \sqcap [\hbar_z \sqcap g_x] &\sqsubseteq [\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap \mathcal{C}^\star(\tilde{e}, l_y, r)] \sqcap [\hbar_z \sqcap g_x] \\ &\sqsubseteq [\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap \hbar_z] \sqcap [\mathcal{C}^\star(\tilde{e}, l_y, r) \sqcap g_x] \\ &= \hbar_z \sqcap \phi = \phi\end{aligned}$$

and

$$\begin{aligned}\mathcal{C}^\star(\tilde{e}, \sqcap[\hbar_z \sqcap g_x], r) \sqcap [\hbar_z \sqcap l_y] &\sqsubseteq [\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap \mathcal{C}^\star(\tilde{e}, g_x, r)] \sqcap [\hbar_z \sqcap l_y] \\ &\sqsubseteq [\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap \hbar_z] \sqcap [\mathcal{C}^\star(\tilde{e}, g_c, r) \sqcap l_y] \\ &= \hbar_z \sqcap \phi = \phi\end{aligned}$$

Therefore, $\hbar_z \sqcap l_y$, $\hbar_z \sqcap g_x$ are r-svnfg-separated sets. □

Theorem 3.3. *Let $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ be r-svnfgt-space and $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$. Therefore, the following statements are equivalent.*

(1) *$\hbar_z$ is r-svnfg-connected.*

(2) *If l_y and g_x are r-svnfg-separated with $\hbar_z \sqsubseteq l_y \sqcup g_c$, then $\hbar_z \sqcap l_y = \phi$ or $\hbar_z \sqcap g_x = \phi$*

(3) *If l_y and g_x are r-svnfg-separated with $\hbar_z \sqsubseteq l_y \sqcup g_x$, then $\hbar_z \sqsubseteq l_y$ or $\hbar_z \sqsubseteq g_x$.*

Proof. (1)$\Longrightarrow$(2) l_y and g_x are r-svnfg-separated such that $\hbar_z \sqsubseteq l_y \sqcup g_x$. Form Theorem 3.2, $\hbar_z \sqcap l_y$ and $\hbar_z \sqcap g_x$ are r-svnfg-separated. So, $\hbar_z = \hbar_z \sqcap [l_y \sqcup g_x] = (\hbar_z \sqcap l_y) \sqcup (\hbar_z \sqcap g_x)$. But $\hbar_z$ is r-svnfg-connected. Therefore, $\hbar_z \sqcap l_y = \phi$ or $\hbar_z \sqcap g_x = \phi$.

(2)$\Longrightarrow$(3) If $\hbar_z \sqcap l_y = \phi$, then $\hbar_z = \hbar_z \sqcap [l_{\mathcal{B}} \sqcup g_c] = (\hbar_z \sqcap l_y) \sqcup (\hbar_z \sqcap g_x) = \hbar_z \sqcap g_x$, and hence, $\hbar_z \sqsubseteq g_x$. Similarly, if $\hbar_z \sqcap g_x$ then $\hbar_z \sqsubseteq l_y$.

(3)$\Longrightarrow$(1) Let l_y and g_x be r-svnfg-separated such that $\hbar_z = l_y \sqcup g_x$, by (3), we have $\hbar_z \sqsubseteq l_y$ or $\hbar_z \sqsubseteq g_x$.

If $\hbar_z \sqsubseteq l_y$ and l_y, g_x are r-svnfg-separated sets, then $g_x = g_x \sqcap \hbar_z \sqsubseteq g_x \sqcap l_y \sqsubseteq g_x \sqcap \mathcal{C}^\star(\tilde{e}, l_y, r) = \phi$. Thus, $g_c = \phi$.

If $\hbar_z \sqsubseteq g_x$, similarly, we have $l_y = \phi$. Therefore, $\hbar_z$ is r-svnfg-connected. □

Theorem 3.4. *Let $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \Upsilon^{\sigma\tau\delta}_{\mathbf{E}})$ be svnfgt-space, $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$ and $r \in \xi_0$. If $\hbar_z \neq \phi$ is r-svnfg-connected and $l_y \sqsubseteq \hbar_z \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$, then l_y is r-svnfg-separated.*

Proof. Assume that, l_y is not r-svnfg-separated. So, there exist non-empty r-svnfg-separated g_x, $w_c \in \widetilde{(\mathcal{B}, \mathbf{E})}$ such that $l_y = g_x \sqcup w_D$. that is,

$$\mathcal{C}^\star(\tilde{e}, g_x, r) \sqcap w_D = \phi = \mathcal{C}^\star(\tilde{e}, w_D, r) \sqcap g_x, \forall,\ \tilde{e} \in \mathbf{E},\ \ r \in \xi_0.$$

Because, $\hbar_z \sqsubseteq l_y = g_x \sqcup w_D$ and $\hbar_z$ is r-svnfg-connected, and by Theorem 3.3 (3), we obtain either $\hbar_z \sqsubseteq g_x$ or $\hbar_z \sqsubseteq w_D$. Form $l_y \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$, we have

if $\hbar_z \sqsubseteq g_x$, then

$$w_D = (g_x \sqcap w_D) \sqcap w_D = l_y \sqcap w_D \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap w_D \sqsubseteq \mathcal{C}^\star(\tilde{e}, g_x, r) \sqcap w_D = \phi$$

which contradicts to $w_D \neq \phi$.

If $\hbar_z \sqsubseteq w_D$, then

$$g_x = (w_D \sqcap g_x) \sqcap g_x = l_y \sqcap g_x \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap g_x \sqsubseteq \mathcal{C}^\star(\tilde{e}, w_D, r) \sqcap g_x = \phi$$

which contradicts to $g_x \neq \phi$. Hence, l_y is r-svnfg-separated. □

Theorem 3.5. *Let $(\mathcal{B}, \tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta}, \mathcal{K}_{\mathbf{E}}^{\sigma\tau\delta})$ be svnfgt-space, $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$ and $r \in \xi_0$. If $\hbar_z$, l_y are r-svnfg-connected which are not r-svnfg-separated, therefore, $\hbar_z \sqcup l_y$ is r-svnfg-connected.*

Proof. Let w_D and g_x be r-svnfg-connected with $\hbar_z \sqcup l_y = w_D \sqcup g_c$. Because $\hbar_z$ is r-svnfg-connected and by theorem 3.3 (3), $\hbar_{\mathcal{A}} \sqsubseteq g_c$ or $\hbar_z \sqsubseteq w_D$. Say $\hbar_z \sqsubseteq w_D$. Assume that $l_y \sqsubseteq g_x$. Because

$$(\hbar_z \sqcup l_y) \sqcap w_D = (\hbar_{\mathcal{A}} \sqcup w_D) \sqcup (l_y \sqcap w_D) = \hbar_z \sqcup \phi = \hbar_z$$

and

$$(\hbar_z \sqcup l_{\mathcal{B}}) \sqcap g_c = (\hbar_z \sqcup g_c) \sqcup (l_y \sqcap g_x) = g_x \sqcup \phi = g_x.$$

Form Theorem 7, we obtain, $\hbar_z$ and l_y are r-svnfg-connected. Which is a contradiction. Therefore, $l_y \sqsubseteq w_D$. Thus, $\hbar_z \sqcup l_{\mathcal{B}} \sqsubseteq w_D$. In the same way, if $\hbar_z \sqsubseteq g_x$, we obtaian that $\hbar_z \sqcup l_y \sqsubseteq g_x$. Therefore by Theorem 8, we have, $\hbar_z \sqcup l_y$ is r-svnfg-connected. □

Theorem 3.6. *Let $(\mathcal{B}, \tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta}, \mathcal{K}_{\mathbf{E}}^{\sigma\tau\delta})$ be svnfgt-space and let $\pounds = \{(\hbar_z)_i \in \widetilde{(\mathcal{B}, \mathbf{E})},\ i \in \Gamma\}$ be a collection of r-svnfg-connected sets in $\mathcal{B}$, such that no two members of $\pounds$ are r-svnfg-separated. Then, $\bigsqcup_{i\in\Gamma}(\hbar_z)_i$ is r-svnfg-connected.*

Proof. Put $\hbar_z = \bigsqcup_{i\in\Gamma}(\hbar_z)_i$ and let $l_{\mathcal{B}}, g_x \in \widetilde{(\mathcal{B}, \mathbf{E})}$ be r-svnfg-separated sets such that $\hbar_z = l_y \sqcup g_c$. Because every two members $(\hbar_z)_i$, $(\hbar_z)_j \in \pounds$ are not r-svnfg-separated, by Theorem 3.5, $(\hbar_z)_i \sqcup (\hbar_z)_j$ is r-svnfg-connected. Form Theorem 3.3 (3), we have $(\hbar_z)_i \sqcup (\hbar_z)_j \sqsubseteq l_y$ or $(\hbar_z)_i \sqcup (\hbar_z)_j \sqsubseteq g_x$, say $(\hbar_{\mathcal{A}})_i \sqcup (\hbar_z)_j \sqsubseteq l_y$. Thus $\hbar_z$ is r-svnfg-connected. □

Theorem 3.7. *Let $(\mathcal{B}, \tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta}, \mathcal{K}_{\mathbf{E}}^{\sigma\tau\delta})$ be svnfgt-space and $\{(\hbar_z)_i \in \widetilde{(\mathcal{B}, \mathbf{E})},\ i \in \Gamma\}$ be a collection of r-svnfg-connected sets and $\sqcap_{i\in\gamma}(\hbar_z)_i \neq \phi$. Then, $\bigsqcup_{i\in\Gamma}(\hbar_z)_i$ is r-svnfg-connected.*

Proof. Clear. □

Definition 3.3. *Let $(\mathcal{B}, \tilde{\top}_{\mathbf{E}}^{\sigma\tau\delta}, \mathcal{K}_{\mathbf{E}}^{\sigma\tau\delta})$ be svnfgt-space. A non empty set $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$ is r-svnfg-component if $\hbar_z$ is a maximal r-svnfg-connected set in $\mathcal{B}$, that is if $\hbar_z \sqsubseteq l_{\mathcal{B}}$ and l_y is r-svnfg-connected set, then $\hbar_z = l_y$.*

Theorem 3.8. *Let $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \mathcal{K}^{\sigma\tau\delta}_{\mathbf{E}})$ be r-svnfgt-space and $\hbar_z$, $l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$. Therefore,*

(1) *if $\hbar_z$ is r-svnfg-component, then $\mathcal{C}^\star(\tilde{e}, \hbar_z, r) = \hbar_z$.*

(2) *If l_y and $\hbar_z$ are r-svnfg-components in $\mathcal{B}$ with $l_y \sqcap \hbar_z = \phi$, then l_y and $\hbar_z$ are r-svnfg-separated sets.*

Proof. (1) Because $\hbar_z$ is r-svnfg-connected set and $\hbar_z \sqsubseteq \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$, from Theorem 3.4, we obtain $\mathcal{C}^\star(\tilde{e}, \hbar_z, r)$ is r-svnfg-connected. On the other hand $\hbar_z$ is r-svnfg-component, it implies $\hbar_z = \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$.

(2) Because l_y and $\hbar_z$ are r-svnfg-components in $\mathcal{B}$ such that $l_y \sqcap \hbar_z = \phi$. So, Form (1), we obtain $l_y = \mathcal{C}^\star(\tilde{e}, l_{\mathcal{B}}, r)$ and $\hbar_z = \mathcal{C}^\star(\tilde{e}, \hbar_z, r)$. Hence

$$\mathcal{C}^\star(\tilde{e}, \hbar_z, r) \sqcap l_y = \phi = \hbar_z \sqcap \mathcal{C}^\star(\tilde{e}, l_y, r).$$

Therefore, l_y and $\hbar_z$ are r-svnfg-separated sets. □

4. Single-Valued Neutrosophic Soft γ-Connected Spaces

Here, we present the single-valued neutrosophic soft γ-connected Spaces r-svnf-connected of space $\mathcal{B}$ relative to a r-svnf operator γ. Suppose [with respect to any r-svnft $\tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}$ defined on $\mathcal{B}$ and $\text{cl}_{\sigma\tau\delta}$ is the single-valued neutrosophic soft closure operator on $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}})$] that:

$$\hbar_z \sqsubseteq \gamma(\tilde{e}, \hbar_z, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \hbar_z, r) \ \ \forall \ \ \hbar_z, \ \ \in \widetilde{(\mathcal{B}, \mathbf{E})}, \ \ \tilde{e} \in \mathbf{E}, \ \ r \in \xi_0.$$

Also, suppose that γ is a monotone operator, that is, $l_y \sqsubseteq \text{g}_x$ implies $\gamma(\tilde{e}, l_y, r) \sqsubseteq \mathcal{C}_{\top^{\sigma\tau\delta}}(\tilde{e}, \text{g}_x, r)$, $l_y, \text{g}_x \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$

Definition 4.1. *Let $\mathcal{B}$ be a non-nall set and $\mathbf{E}$ be a set of parameters. Therefore,*

(1) *the svnf-sets $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$ are called r-single-valued neutrosophic $\gamma-$ separated (abbreviated r-svnfγ-separated) if $\hbar_{\mathcal{A}}$ and l_y satisfy the following condition*

$$\gamma(\tilde{e}, \hbar_z, r) \sqcap l_y = \phi = \hbar_z \sqcap \gamma(\tilde{e}, l_y, r), \ for \ every \ \tilde{e} \in \mathbf{E}, \ \ r \in \xi_0.$$

(2) *$\mathcal{B}$ is termed r-single-valued neutrosophic $\gamma-$connected (abbreviated r-svnfγ-connected space) if one cannot find two svnf-sets $\hbar_{\mathcal{A}}, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$ $\hbar_z \neq \phi$, $l_y \neq \phi$ and $\hbar_{\mathcal{A}} \sqcup l_y = \tilde{E}$. That is, there do not exist r-svnfγ-separated sets $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, except $\hbar_z = \phi$, $l_y = \phi$.*

Definition 4.2. *Let $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $\hbar_z \neq \phi$, $l_y \neq \phi$, such that:*

(1) *$\hbar_z, l_y$ are r-svnfγ-separated with $\hbar_z \sqcup l_y = \tilde{E}$. Therefore, $\mathcal{B}$ is termed r-single-valued neutrosophic $\gamma-$disconnected (abbreviated r-svnfγ-disconnected space).*

(2) *$\hbar_z, l_y$ are r-svnfγ-separated with $\hbar_z \sqcup l_y = \text{g}_c$. Therefore, g_x is termed r-svnfγ-disconnected space in $\widetilde{(\mathcal{B}, \mathbf{E})}$.*

For a r-svnfgt-space $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \Upsilon^{\sigma\tau\delta}_{\mathbf{E}})$.

If $\gamma = \mathcal{C}_{\top^{\sigma\tau\delta}}$, then we obtain the r-svnf- connectedness.

If $\gamma = \mathcal{C}^\star_{\tilde{\top}^{\sigma\tau\delta}}$, then we obtain the r-svnfg- connectedness

Example 4.1. *Assume that,* $\mathcal{B} = \{a, b\}$, $\mathbf{E} = \{\tilde{e}_1, \tilde{e}_2\}$ *and* $(\hbar_{\mathbf{E}})_1, (\hbar_{\mathbf{E}})_2 \in \widetilde{(\mathcal{B}, \mathbf{E})}$ *where* $(\hbar_{\mathbf{E}})_1 = \{(\tilde{e}_1, \langle 1, 1, 0\rangle), (\tilde{e}_2, \langle 0, 0, 1\rangle)\}$ *and* $(\hbar_{\mathbf{E}})_2 = \{(\tilde{e}_1, \langle 0, 0, 1\rangle), (\tilde{e}_2, \langle 1, 1, 0\rangle)\}$ *for* $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$, *we define the single valued soft operator* γ *as follows:*

$$\gamma(\tilde{e}, \hbar_{\mathbf{E}}, r) = \begin{cases} \phi, & if\ \ \hbar_{\boldsymbol{E}} = \phi\ \forall r \in \xi_0, \\ (\hbar_{\mathbf{E}})_1, & if\ \ \phi \neq \hbar_{\boldsymbol{E}} \sqsubseteq (\hbar_{\boldsymbol{E}})_1, r \leq \frac{1}{2}, \\ (\hbar_{\mathbf{E}})_2, & if\ \ \phi \neq \hbar_{\boldsymbol{E}} \sqsubseteq (\hbar_{\boldsymbol{E}})_2, r \leq \frac{3}{5}, \\ \tilde{\mathbf{E}}, & if\ \ otherwise, \end{cases}$$

Now, let $\phi \neq \hbar_{\mathbf{E}} = (\hbar_{\mathbf{E}})_1$, $\phi \neq g_{\mathbf{E}} = (\hbar_{\mathbf{E}})_2$ *and* $r \leq \frac{1}{3}$ *then we have*

$$\gamma(\tilde{e}, \hbar_{\mathbf{E}}, r) \sqcap g_{\mathbf{E}} = \phi = \hbar_{\mathbf{E}} \sqcap \gamma(\tilde{e}, g_{\mathbf{E}}, r).$$

Thus, $\hbar_{\mathbf{E}}$ *and* $g_{\mathbf{E}}$ *are r-svnfγ-separated sets. At* $\hbar_{\mathbf{E}} = (\hbar_{\mathbf{E}})_1$, $g_{\mathbf{E}} = (\hbar_{\mathbf{E}})_2$ *and* $r \leq \frac{1}{3}$ *we obtain that* $\hbar_{\mathbf{E}}$ *and* $g_{\mathbf{E}}$ *are r-svnfγ-separated with* $\tilde{\mathbf{E}} = \hbar_{\mathbf{E}} \sqcap g_{\mathbf{E}}$. *Therefore,* $\mathcal{B}$ *is r-svnfγ-disconnected.*

If $r \geq \frac{1}{2}$, *then* $\mathcal{B}$ *is r-svnfγ-disconnected.*

The following theorem is similarly proved, as in Theorem 3.1.

Theorem 4.1. *Let* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}})$ *be r-svnft-space. Therefore, the following statements are equivalent.*

(1) $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}})$ *is r-svnfγ-connected.*

(2) *If* $\hbar_z \sqcup\ l_y = \tilde{E}$ *and* $\hbar_z \sqcap\ l_y = \phi$ *with* $\tilde{\top}^{\sigma}_{\tilde{e}}(\hbar_z) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}(\hbar_z) \leq 1 - r$ $\tilde{\top}^{\delta}_{\tilde{e}}(\hbar_z) \leq 1 - r$, $\tilde{\top}^{\sigma}_{\tilde{e}}(l_y) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}(l_y) \leq 1 - r$ $\tilde{\top}^{\delta}_{\tilde{e}}(l_y) \leq 1 - r$, $\tilde{e} \in \mathbf{E}$, $r \in \xi_0$, *then* $\hbar_z = \phi$ *or* $l_y = \phi$.

(3) *If* $\hbar_z \sqcup l_y = \tilde{E}$ *and* $\hbar_z \sqcap l_y = \phi$ *with* $\tilde{\top}^{\sigma}_{\tilde{e}}([\hbar_z]^c) \geq r, \tilde{\top}^{\tau}_{\tilde{e}}([\hbar_z]^c) \leq 1 - r$, $\tilde{\top}^{\delta}_{\tilde{e}}([\hbar_z]^c) \leq 1 - r$ *and* $\tilde{\top}^{\sigma}_{\tilde{e}}([l_y]^c) \geq r$, $\tilde{\top}^{\tau}_{\tilde{e}}([l_y]^c) \leq 1 - r$, $\tilde{\top}^{\delta}_{\tilde{e}}([l_y]^c) \leq 1 - r$, $\tilde{e} \in \mathbf{E}$, $r \in \zeta_0$, *then* $\hbar_z = \phi$ *or* $l_y = \phi$.

The following theorem is similarly proved, as in Theorem 3.2.

Theorem 4.2. *Let* $\mathcal{B}$ *be a non-empty set,* $\mathbf{E}$ *be a set of parameters and* $\hbar_z, l_y, g_x \in \widetilde{(\mathcal{B}, \mathbf{E})}$. *If* l_y *and* g_x *are r-svnfγ-separated sets, then* $\hbar_z \sqcap l_y$, $\hbar_z \sqcap g_x$ *are r-svnfγ-separated sets.*

The following theorem is similarly proved, as in Theorem 3.3.

Theorem 4.3. *Let* $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$. *Then, the following statements are equivalent.*

(1) $\hbar_z$ *is r-svnfγ-connected.*

(2) *If* l_y *and* g_c *are r-svnfγ-separated with* $\hbar_z \sqsubseteq l_{\mathcal{B}} \sqcup g_x$, *then* $\hbar_z \sqcap l_y = \phi$ *or* $\hbar_z \sqcap g_x = \phi$

(3) *If* l_y *and* g_c *are r-svnfγ-separated with* $\hbar_z \sqsubseteq l_y \sqcup g_x$, *then* $\hbar_z \sqsubseteq l_y$ *or* $\hbar_z \sqsubseteq g_x$.

The following theorem is similarly proved, as in Theorem 3.4.

Theorem 4.4. *Let* $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $r \in \xi_0$. *If* $\hbar_z \neq \phi$ *is r-svnfγ-connected and* $\hbar_z \sqsubseteq l_y \sqsubseteq \gamma(\tilde{e}, \hbar_z, r)$, $\tilde{e} \in \mathbf{E}$, *then* l_y *is r-svnfγ-connected.*

Theorem 4.5. *Let* $\hbar_z, l_y \in \widetilde{(\mathcal{B}, \mathbf{E})}$, $r \in \xi_0$. *If* $\hbar_{\mathcal{A}}$ *and* $l_{\mathcal{B}}$ *are r-svnfγ-connected which are not r-svnfγ-separated, then* $\hbar_z \sqcup l_y$ *is r-svnfγ-connected.*

Proof. Let g_x and w_D be r-svnfγ-separated, such that, $\hbar_z \sqcup l_y = g_C \sqcup w_D$. Since, $\hbar_{\mathcal{A}}$ is r-svnfγ-connected, by Theorem 4.3 (3), $\hbar_z \sqsubseteq g_x$ or $\hbar_z \sqsubseteq w_D$. Let $\hbar_z \sqsubseteq w_D$. Suppose $l_y \sqsubseteq g_x$. Since $(\hbar_z \sqcup l_y) \sqcap w_D = (\hbar_z \sqcap w_D) \sqcup (l_y \sqcap w_D) = \hbar_z \sqcup \phi = \hbar_z$, by Theorem 4.2, $\hbar_{\mathcal{A}}$ and $l_{\mathcal{B}}$ are r-svnfγ-separated. Which is a contradiction. Hence we have $l_y \sqsubseteq w_D$. Therefore $\hbar_z \sqcup l_y \sqsubseteq w_D$. By the same way, if $\hbar_z \sqsubseteq g_x$, we have $\hbar_z \sqcup l_y \sqsubseteq g_x$. Then by Theorem 4.3 (3),r-svnfγ-separated, then $\hbar_z \sqcup l_y$ is r-svnfγ-connected. □

The following theorem is similarly proved, as in Theorem 3.6.

Theorem 4.6. *Let $\zeta = \{(\hbar_z)_i \in \widetilde{(\mathcal{B}, \mathbf{E})},\ i \in \Gamma\}$ be a collection of r-svnfγ-connected sets in $\mathcal{B}$ such that no two members of ζ are r-svnfγ-separated. Then, $\bigsqcup_{i\in\Gamma}(\hbar_z)_i$ is r-svnfγ-connected.*

The following corollary follows from Theorem 4.6.

Corollary 4.1. *Let $\{(\hbar_z)_i \in \widetilde{(\mathcal{B}, \mathbf{E})},\ i \in \Gamma\}$ be a family of r-svnfγ-connected sets and $\sqcap_{i\in\gamma}(\hbar_z)_i \neq \phi$. Then, $\bigsqcup_{i\in\Gamma}(\hbar_z)_i$ is r-svnfγ-connected.*

Theorem 4.7. *Let $\vartheta_\psi : \widetilde{(\mathcal{B}, \mathbf{E})} \to \widetilde{(\pounds, \mathbf{F})}$ be a mapping such that,*

$$\gamma(\tilde{e}, \vartheta_\psi^{-1}(l_y), r) \sqsubseteq \vartheta_\psi^{-1}(\theta(\psi(\tilde{e})), l_y, r), \forall\ l_y \in \widetilde{(\pounds, \mathbf{F})}, r \in \xi_0, \tilde{e} \in \mathbf{E},$$

where γ is a svnfγ-operator on $\mathcal{B}$ and θ is a r-svnfγ-operator on $\pounds$. Then, the set $\vartheta_\psi(\hbar_z) \in \widetilde{(\pounds, \mathbf{F})}$ is r-svnfθ-connected if the set $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$ is r-svnfγ-connected.

Proof. Let $l_y \neq \phi$ and $g_x \neq \phi$ be a r-svnfθ-separated sets in $\widetilde{(\pounds, \mathbf{F})}$ with $\vartheta_\psi(\hbar_z) = l_y \sqcup g_x$. That is $\theta(\psi(\tilde{e}), g_x, r) \sqcap l_y \sqsubseteq \theta(\psi(\tilde{e}), l_y, r) \sqcap g_x = \phi$, for all $r \in \xi_0, \tilde{e} \in \mathbf{E}$, then we have $\hbar_z \sqsubseteq \vartheta_\psi^{-1}(\vartheta_\psi(\hbar_z)) = \vartheta_\psi^{-1}(l_y \sqcup g_x) = \vartheta_\psi^{-1}(l_y) \sqcup \vartheta_\psi^{-1}(g_x)$,

$$\begin{aligned} \gamma(\tilde{e}, \vartheta_\psi^{-1}(l_y), r) \sqcap \vartheta_\psi^{-1}(g_x) &\sqsubseteq \vartheta_\psi^{-1}(\theta(\psi(\tilde{e}), l_y, r)) \sqcap \vartheta_\psi^{-1}(g_x) \\ &= \vartheta_\psi^{-1}(\theta(\psi(\tilde{e}), l_y, r) \sqcap g_x) \\ &= \vartheta_\psi^{-1}(\phi) = \phi. \end{aligned}$$

Also

$$\begin{aligned} \gamma(\tilde{e}, \vartheta_\psi^{-1}(g_x), r) \sqcap \vartheta_\psi^{-1}(l_y) &\sqsubseteq \vartheta_\psi^{-1}(\theta(\psi(\tilde{e}), g_x, r)) \sqcap \vartheta_\psi^{-1}(l_y) \\ &= \vartheta_\psi^{-1}(\theta(\psi(\tilde{e}), g_x, r) \sqcap l_y) \\ &= \vartheta_\psi^{-1}(\phi) = \phi. \end{aligned}$$

Hence $\vartheta_\psi^{-1}(l_y)$ and $\vartheta_\psi^{-1}(g_x)$ r-svnfγ-separated sets in $\mathcal{B}$. So that, $\hbar_z \sqsubseteq \vartheta_\psi^{-1}(l_y) \sqcup \vartheta_\psi^{-1}(g_x)$. But $\hbar_z$ is r-svnfγ-connected. by Theorem 3.3 (3), $\hbar_z \sqsubseteq \vartheta_\psi^{-1}(l_y)$ or $\hbar_z \sqsubseteq \vartheta_\psi^{-1}(g_x)$, which means, $\vartheta_\psi(\hbar_z) \sqsubseteq l_y$ or $\vartheta_\psi^{-1}(\hbar_z) \sqsubseteq g_x$. Hence, by using Theorem 3.3 (3), we have $\vartheta_\psi(\hbar_z)$. is r-svnfθ-connected □

Corollary 4.2. *Let* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}})$ *and* $(\pounds, \tilde{\top}^{\star\sigma\tau\delta}_{\mathbf{F}})$ *be two svnft-spaces. If* $\vartheta_\psi : \widetilde{(\mathcal{B}, \mathbf{E})} \to \widetilde{(\pounds, \mathbf{F})}$ *is a svnf-continuous mapping and* $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$ *is r-svnf-connected in* $\mathcal{B}$*, then* $\vartheta_\psi(\hbar_z)$ *is r-svnfθ-connected in* $\pounds$*.*

Note, if $\gamma = \mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}$ *and* $\theta = \mathcal{C}_{\tilde{\top}^{\star\sigma\tau\delta}}$*. Then, the result follows from Theorem 4.7.*

Corollary 4.3. *Let* $(\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}, \tilde{\mathcal{K}}^{\sigma\tau\delta}_{\mathbf{E}})$ *and* $(\pounds, \tilde{\top}^{\star\sigma\tau\delta}_{\mathbf{F}}, \tilde{\mathcal{K}}^{\star\sigma\tau\delta}_{\mathbf{F}})$ *be two svnfgt-spaces and* $\vartheta_\psi : (\mathcal{B}, \tilde{\top}^{\sigma\tau\delta}_{\mathbf{E}}) \to (\pounds, \tilde{\top}^{\star\sigma\tau\delta}_{\mathbf{F}}, \tilde{\mathcal{K}}^{\star\sigma\tau\delta}_{\mathbf{F}})$ *be a mapping satisfying the condition,*

(1) $\mathcal{C}_{\tilde{\top}^{\sigma\tau\delta}}(\tilde{e}, \vartheta_\psi^{-1}(l_y), r) \sqsubseteq \vartheta_\psi^{-1}(\mathcal{C}^{\star}_{\tilde{\top}^{\star\sigma\tau\delta}}(\psi(\tilde{e})), l_y, r)\ \forall\ l_y \in \widetilde{(\pounds, \mathbf{F})}, r \in \xi_0, \tilde{e} \in \mathbf{E}$.

Then, the set $\vartheta_\psi(\hbar_z) \in \widetilde{(\pounds, \mathbf{F})}$ *is r-svnfg-connected if the set* $\hbar_z \in \widetilde{(\mathcal{B}, \mathbf{E})}$ *is r-svnfg-connected.*

Proof. Note, if $\gamma = \mathcal{C}^{\star}_{\tilde{\top}^{\sigma\tau\delta}}$ and $\theta = \mathcal{C}^{\star}_{\tilde{\top}^{\star\sigma\tau\delta}}$. Then, the result follows from Theorem 4.7. □

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