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Strictly Wider Class of Soft Sets via Supra Soft δ-Closure Operator

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Abstract. In this work, we use the supra soft δ-closure operator to present a new notion of generalized closed sets in supra soft topological spaces (or SSTSs), named supra soft δ-generalized closed sets. We show that, this notion is more general than many of previous notions, which presented before in famous papers. We illustrate many of its essential properties in detail. Specifically, we illustrate that the new collection neither forms soft topology nor supra soft topology. Moreover, we study the behavior of the soft image and soft pre-images of supra soft δ-generalized closed sets under new types of soft mappings, named supra soft irresolute and supra soft δ-irresolute closed. In addition, we define the concept of supra soft δ-generalized open sets, as a complement of supra soft δ-generalized closed sets. Finally, the relationships with other forms of generalized open sets in SSTSs are explored, supported by concrete examples and counterexamples. Therefore, I think the development of the notions presented in this paper are sufficiently general relevance to allow for future extensions.

1. Introduction

In 1983, Mashhour et al. [1] generalized notions the topological spaces by presenting the concept of supra topological spaces. A. Alpers [2], in 2002, used these new notions to present applications to digital topologies. Kozae et al. [3], in 2002, presented new applications for theses notions in digital plane. Recently, in 2023 [4] Al-shami and Alshammari applied the supra topological spaces to information systems. Many applications based on closure operators [5, 6] and novel types of open sets, named C-open sets [7], F-open [8] have been introduced.

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In 2011, Ahmad and Kharal [9] applied the continuity notions to soft set theory [10, 11]. Several classes of generalized soft open sets and soft functions defined in [12–14]. Zorlutuna [15] investigated more interesting properties of soft continuity. Wardowski [16] presented the notions of soft functions based on its fixed points. Alqahtani and Ameen [17, 18] introduced Baire category soft sets in soft nodec spaces.

The definitions of soft ideal presented by Kandil et al. [19], in 2014. Theses approves have been investigated by using the concepts of soft semi-open sets [20, 21]. In 2018, applications in medical [22] via certain soft ideal rough topological spaces have been presented by Abd El-latif. Several weaker classes of soft open sets have been provided by using the soft ideal notions [23–25]. Also, new versions of separation axioms [26, 27] in STS have been studied.

In 2014, El-Sheikh and Abd El-latif [28] presented the notions of SSTSs by ignoring the condition of finite soft intersection in the concept of soft topological space (or STS) [29]. So, the new collection became wider than the old one. It leading to investigate and generalize many topological properties to such spaces like different types of separation axioms [30, 31] and irresolute functions bases on supra soft b-open sets [32, 33].

Later, several generalizations and topological properties were provided; To name a few: Supra soft regular generalized closed sets [34,35], soft connected spaces and soft paracompact spaces [36], supra soft separated sets and supra soft connectedness based on supra soft b-open sets [37,38], supra soft compactness [39, 40]. Several types of weaker forms of supra soft separation axioms [41, 42] based on supra semi open soft sets [43], supra β-open soft sets [44] ave been introduced and studied. Recently, Abd El-latif introduced the approach of supra soft somewhere dense sets [45]. He and his co-author [46] presented new categories of supra soft continuous functions based on this new notion.

Recently, [47], Abd El-latif defined the concept of supra soft δ-open sets in SSTSs. He introduced new soft operators named, supra soft δ-closure (interior, boundary, cluster) operator. Moreover, he applied these operators to introduce new weaker classes of supra soft continuity.

The concepts of soft generalized closed (or soft g-closed) sets in STS have been defined, in 2012, by Kannan [48]. He and his co-authors [49] investigated this notion and presented the notion of soft strongly g-closed sets. Kandil et al. [50] presented the concepts of supra soft g-closed sets (based on soft ideals) [51]. More application on weaker forms of generalized open sets were recently introduced in [52, 53], to improve the measurement accuracy for information systems.

The purpose of this manuscript, is to present the concepts of supra soft δ-generalized closed sets in SSTSs by using the notion of supra soft δ-closure operator, in Section 3. With supporting by examples, we prove that our new notions are more general than many previous notions. More interesting properties such soft union (respectively, soft intersection) of finite numbers of supra soft δ-generalized closed sets are discussed. Moreover, the soft pre-images and soft images of supra soft δ-generalized closed sets are explored and studied under the supra soft irresolute and supra soft δ-irresolute closed functions. In Section 4, the concepts of supra soft δ-generalized open sets by using the supra soft δ-interior operator in SSTSs are presented. Furthermore, many of its basic properties are presented.

2. Preliminaries

Definition 2.1. *[10] A pair (Z, Δ) defined on an initial universe set Z and set of parameters Δ, is called a soft set over Z, defined by $K_\Delta = \{K(\vartriangle) : \vartriangle \in \Delta,\ K : \Delta \to P(Z)\}$. If $K(\vartriangle) = \Phi$ (respectively, $K(\vartriangle) = Z$) for all $\vartriangle \in \Delta$, then (K, Δ) is called a null (respectively, an absolute) soft set and will denoted by $\tilde{\Phi}$ (respectively, $\tilde{Z}$). Henceforth, the family of all soft sets will dented by by $S(Z)_\Delta$.*

Definition 2.2. *[29] The collection $\tau \subseteq S(Z)_\Delta$ is called STS on Z if:*

- **(1):** $\tilde{Z}, \tilde{\Phi} \in \tau$,
- **(2):** *The soft union of arbitrary numbers of soft sets in τ belongs to τ,*
- **(3):** *The soft intersection of finite numbers of soft sets in τ belongs to τ.*

The triplet (Z, τ, Δ) is called an STS over Z. Also, the elements of τ are called soft open sets, and their soft complements are called soft closed sets.

Definition 2.3. *[29] Let (Z, τ, Δ) be an STS and $(T, \Delta) \in S(Z)_\Delta$. The soft closure of (T, Δ), denoted by $cl(T, \Delta)$ is the soft intersection of all soft closed supersets of (T, Δ).*

Definition 2.4. *[54] Let $(Z, \tau\Delta)$ be an STS and $(V, \Delta) \in S(Z)_\Delta$. The soft interior of (V, Δ), denoted by $int(V, \Delta)$ is the soft union of all soft open subsets of (V, Δ).*

Theorem 2.1. *[9] For the soft function $f_{pu} : (Z_1, \tau_1, \Delta_1) \to (Z_2, \tau_2, \Delta_2)$, the following statements hold,*

- **(a):** $f_{pu}^{-1}((N^{\tilde{c}}, \Delta_2)) = (f_{pu}^{-1}(N, \Delta_2))^{\tilde{c}}\ \forall\ (N, \Delta_2) \in S(Z_2)_{\Delta_2}$.
- **(b):** $f_{pu}(f_{pu}^{-1}((N, \Delta_2)))\tilde{\subseteq}(N, \Delta_2)\ \forall\ (N, \Delta_2) \in S(Z_2)_{\Delta_2}$. *If f_{pu} is surjective, then the equality holds.*
- **(c):** $(M, \Delta_1)\tilde{\subseteq} f_{pu}^{-1}(f_{pu}((M, \Delta_1)))\ \forall\ (M, \Delta_1) \in S(Z_1)_{\Delta_1}$. *If f_{pu} is injective, then the equality holds.*
- **(d):** $f_{pu}(\tilde{Z}_1)\tilde{\subseteq}\tilde{Z}_2$. *If f_{pu} is surjective, then the equality holds.*

Definition 2.5. *[28] The collection $\Theta \subseteq S(Z)_\Delta$ is called supra soft topology (or SSTS) on Z if:*

- **(1):** $\tilde{Z}, \tilde{\Phi} \in \Theta$,
- **(2):** *The soft union of arbitrary numbers of soft sets in Θ belongs to Θ.*

The elements of Θ are called supra soft open sets, and their soft complements are called supra soft closed sets.

Definition 2.6. *[28] Let (Z, τ, Δ) be an STS and (Z, Θ, Δ) be an SSTS. We say that, Θ is an SSTS associated with τ if $\tau \subset \Theta$.*

Definition 2.7. *[28] Let (Z, Θ, Δ) be an SSTS over Z and $(T, \Delta) \in S(Z)_\Delta$. Then, the supra soft interior of (V, Δ), denoted by $int^s(V, \Delta)$ is the soft union of all supra soft open subsets of (V, Δ). Also, the supra soft closure of (T, Δ), denoted by $cl^s(T, \Delta)$ is the soft intersection of all supra soft closed supersets of (T, Δ).*

Definition 2.8. *[55] A soft set (F, Δ) is called supra soft regular closed (respectively, open) set in an SSTS (Z, Θ, Δ) if $cl^s(int^s(F, \Delta)) = (F, \Delta)$ (respectively, $int^s(cl^s(F, \Delta)) = (F, \Delta)$).*

Definition 2.9. *[47] Let (Z, Θ, Δ) be an SSTS and $(R, \Delta) \in S(Z)_\Delta$. Then, (R, Δ) is called a supra soft δ-open set if there is $\tilde{Z} \neq (V, \Delta) \in \Theta$ such that $(R, \Delta) \tilde{\subseteq} cl^s((R, \Delta) \cap (V, \Delta))$. The complement of a supra soft δ-open set is a supra soft δ-closed. The collection of all supra soft δ-open sets will denoted by $SOS_\delta(Z)$ and the collection of all supra soft δ-closed sets will denoted by $SCS_\delta(Z)$.*

Proposition 2.1. *[47] Let (Z, Θ, Δ) be an SSTS and $(L, \Delta) \in S(Z)_\Delta$.*

- **(1):** *If $(L, \Delta) \in SOS_\delta(X)$, then there is $\tilde{Z} \neq (V, \Delta) \in \Theta$ such that $(L, \Delta) \tilde{\subseteq} cl^s(V, \Delta)$.*
- **(2):** *If $(L, \Delta) \in SCS_\delta(X)$, then there is $\tilde{\Phi} \neq (H, \Delta) \in \Theta^c$ such that $int^s(H, E) \tilde{\subseteq} (L, \Delta)$.*

Proposition 2.2. *[47] Let (Z, Θ, Δ) be an SSTS and $(K, \Delta) \in S(Z)_\Delta$. Then,*

- **(1):** $int^s_\delta(K, \Delta)) = \tilde{\bigcup}\{(O, \Delta) : (O, \Delta) \tilde{\subseteq} (K, \Delta),\ (O, \Delta) \in SOS_\delta(Z)\}$.
- **(2):** $cl^s_\delta(K, \Delta)) = \tilde{\bigcap}\{(C, \Delta) : (K, \Delta) \tilde{\subseteq} (C, \Delta),\ (C, \Delta) \in SCS_\delta(Z)\}$.

Definition 2.10. *[47] Let (Z, Θ, Δ) be an SSTS and $(S, \Delta) \in S(Z)_\Delta$. Then, supra soft δ-boundary of (S, Δ), is denoted by $b^s_\delta(S, \Delta)$, defined by $b^s_\delta(S, \Delta) = cl^s_\delta(S, \Delta) - int^s_\delta(S, \Delta)$.*

3. Weaker class of supra soft closed sets via supra soft δ-closure operator

This section aims to define the concepts of supra soft δ-generalized closed sets (briefly, g^s_δ-closed sets) in SSTSs by using the supra soft δ-closure operator. We show that, this notion is wider than the notion of supra soft generalized closed sets [50], supra soft strongly generalized closed sets [34] and supra soft regular generalized closed sets [35]. Moreover, we fond out that this notion does not success to form an STS or SSTS, since the soft intersection (respectively, soft union) of any two g^s_δ-closed sets need not to be an g^s_δ-closed, in general. In addition, we discuss the soft pre-images and soft images of g^s_δ-closed sets under supra soft irresolute and supra soft δ-irresolute closed functions. Furthermore, many examples and counterexamples are provided.

Definition 3.1. *A soft subset (A, Δ) of an SSTS (Z, Θ, Δ) is called g^s_δ-closed set if $cl^s_\delta(A, \Delta) \tilde{\subseteq} (G, \Delta)$ whenever $(A, \Delta) \tilde{\subseteq} (G, \Delta)$ and $(G, \Delta) \in \Theta$. The family of all g^s_δ-closed sets will denoted by $G^s_\delta C(Z)$.*

Example 3.1. *Suppose that $Z = \{z_1, z_2, z_3, z_4\}$ and $\Delta = \{\vartriangle_1, \vartriangle_2\}$. Let $(G_1, \Delta), (G_2, \Delta), (G_3, \Delta)$ be soft sets over Z, defined as follows:*

$G_1(\vartriangle_1) = \{z_1, z_2\}, \quad G_1(\vartriangle_2) = \{z_2, z_3\}$,
$G_2(\vartriangle_1) = \{z_2, z_3\}, \quad G_2(\vartriangle_2) = \{z_1, z_3\}$,
$G_3(\vartriangle_1) = \{z_1, z_2, z_3\}, \quad G_3(\vartriangle_2) = \{z_1, z_2, z_3\}$.

Then, $\Theta = \{\tilde{Z}, \tilde{\Phi}, (G_1, \Delta), (G_2, \Delta), (G_3, \Delta)\}$ is an SSTS over Z. Hence, the soft set (G_2, Δ) is a g^s_δ-closed whereas the soft set (G_3, Δ) is not g^s_δ-closed.

Remark 3.1. *The soft intersection (respectively, soft union) of any two g^s_δ-closed sets need not to be a g^s_δ-closed in general as shall shown in the following examples.*

Examples 3.1. **(1):** *In Example 3.1, the soft sets $(H_1, \Delta), (H_2, \Delta)$, where:*

$H_1(\vartriangle_1) = \{z_1, z_2\}, \quad H_1(\vartriangle_2) = \{z_1, z_3\}$,

$H_2(\triangle_1) = \{z_3\}$, $H_2(\triangle_2) = \{z_2\}$,
are g^s_δ*-closed, but their soft union* $(H_1, \Delta)\tilde{\cup}(H_2, \Delta) = \{(\triangle_1, \{z_1, z_2, z_3\}), (\triangle_2, \{z_1, z_2, z_3\})\}$ *is not* g^s_δ*-closed.*

(2): *In Example 3.1, the soft sets* $(I_1, \Delta), (I_2, \Delta)$*, where:*
$I_1(\triangle_1) = \{z_1, z_2, z_3\}$, $I_1(\triangle_2) = Z$,
$I_2(\triangle_1) = Z$, $H_2(\triangle_2) = \{z_1, z_2, z_3\}$,
are g^s_δ*-closed, but their soft intersection* $(I_1, \Delta)\tilde{\cap}(I_2, \Delta) = \{(\triangle_1, \{z_1, z_2, z_3\}), (\triangle_2, \{z_1, z_2, z_3\})\}$ *is not* g^s_δ*-closed.*

Definition 3.2. *A soft subset* (T, Δ) *of an SSTS* (Z, Θ, Δ) *is called:*

(1): *[50] Supra soft g-closed, if* $cl^s(T, \Delta)\tilde{\subseteq}(V, \Delta)$ *whenever* $(T, \Delta)\tilde{\subseteq}(V, \Delta)$ *and* $(V, \Delta) \in \Theta$*. The soft complement of supra soft g-closed set is called supra soft g-open.*

(2): *[35] Supra soft regular generalized closed (supra soft rg-closed), if* $cl^s(T, \Delta)\tilde{\subseteq}(V, \Delta)$ *whenever* $(T, \Delta)\tilde{\subseteq}(V, \Delta)$ *and* (V, Δ) *is supra soft regular open. The soft complement of supra soft rg-closed set is called supra soft rg-open.*

Theorem 3.1. *Every supra soft g-closed subset* (A, Δ) *of an SSTS* (X, Θ, Δ) *is a* g^s_δ*-closed.*

Proof. Assume that (A, Δ) be a supra soft g-closed subset of an SSTS (X, Θ, Δ), then $cl^s(A, \Delta)\tilde{\subseteq}(G, \Delta)$ whenever $(A, \Delta)\tilde{\subseteq}(G, \Delta)$ and $(G, \Delta) \in \Theta$. Since $cl^s_\delta(A, \Delta)\tilde{\subseteq}cl^s(A, \Delta)$, $cl^s_\delta(A, \Delta)\tilde{\subseteq}(G, \Delta)$, $(G, \Delta) \in \Theta$. Thus, (A, Δ) is g^s_δ-closed set.

Remark 3.2. *The next example shall show that, the converse of the above theorem is not satisfied in general.*

Example 3.2. *Suppose that* $Z = \{z_1, z_2, z_3\}$ *and* $\Delta = \{\triangle_1, \triangle_2\}$ *be the set of parameters. Let* $(G_1, \Delta), (G_2, \Delta), (G_3, \Delta), (G_4, \Delta), (G_5, \Delta)$ *be soft sets over Z, defined as follows:*
$G_1(\triangle_1) = \{z_1\}$, $G_1(\triangle_2) = \{z_1\}$,
$G_2(\triangle_1) = \{z_1, z_3\}$, $G_2(\triangle_2) = \{z_2, z_3\}$,
$G_3(\triangle_1) = \{z_2, z_3\}$, $G_3(\triangle_2) = \{z_1, z_3\}$,
$G_4(\triangle_1) = Z$, $G_4(\triangle_2) = \{z_1, z_3\}$,
$G_5(\triangle_1) = \{z_1, z_3\}$, $G_5(\triangle_2) = Z$.
Then, $\Theta = \{\tilde{Z}, \tilde{\Phi}, (G_1, \Delta), (G_2, \Delta), (G_3, \Delta), (G_4, \Delta), (G_5, \Delta)\}$ *is an SSTS over Z. Hence, the soft sets* (F, Δ)*, where:*
$F(\triangle_1) = \{z_3\}$, $F(\triangle_2) = \{z_1\}$*, is a* g^s_δ*-closed, but it is not supra soft g-closed.*

Proposition 3.1. *Let* (Q, Δ) *be a soft subset of an SSTS* (X, Θ, Δ)*, which is both supra soft open and* g^s_δ*-closed. Then,* $cl^s_\delta(Q, \Delta) = (Q, \Delta)$

Proof. Since (Q, Δ) is g^s_δ-closed, $cl^s_\delta(Q, \Delta)\tilde{\subseteq}(Q, \Delta)\tilde{\subseteq}cl^s_\delta(Q, \Delta)$, $(Q, \Delta) \in \Theta$. Therefore,

$$cl^s_\delta(Q, \Delta) = (Q, \Delta).$$

Theorem 3.2. *Every supra soft rg-closed subset* (W, Δ) *of an SSTS* (X, Θ, Δ) *is a* g^s_δ*-closed.*

Proof. Assume that (W, Δ) is supra soft rg-closed subset of an SSTS (Z, Θ, Δ), then

$$cl^s(W, \Delta)\tilde{\subseteq}(G, \Delta) \text{ whenever } (W, \Delta)\tilde{\subseteq}(G, \Delta) \text{ and } (G, \Delta) \text{ is supra soft regular open.}$$

So,

$$(G, \Delta) \in \Theta \text{ from [[55], Theorem 3.3].}$$

It follows,

$$cl^s_\delta(W, \Delta)\tilde{\subseteq}cl^s(W, \Delta)\tilde{\subseteq}(G, \Delta), \ (G, \Delta) \in \Theta.$$

Therefore,

$$(W, \Delta) \text{ is a } g^s_\delta\text{-closed set.}$$

Remark 3.3. *The converse of Theorem 3.2 is not to true in general. In Example 3.2, the soft sets (F, Δ) is g^s_δ-closed, but it is not supra soft rg-closed.*

Theorem 3.3. *Every supra soft (respectively, δ-) closed subset (A, Δ) of an SSTS (Z, Θ, Δ) is a g^s_δ-closed.*

Proof. We prove the case of supra soft δ-closed set. Let $(A, \Delta)\tilde{\subseteq}(G, \Delta)$ s.t $(G, \Delta) \in \Theta$ and $(A, \Delta) \in SCS_\delta(Z)$. Then,

$$cl^s_\delta(A, \Delta) = (A, \Delta)\tilde{\subseteq}(G, \Delta) \text{ from [[47], Proposition 4.7 (1)].}$$

Hence,

$$(A, \Delta) \text{ is a } g^s_\delta\text{-closed.}$$

The other case follows directly from the fact that, every supra soft closed set is a supra soft δ-closed [[47], Theorem 3.6 (1)].

Remark 3.4. *The converse of Theorem 3.3 is not true in general, as shown in the following example.*

Example 3.3. *Assume that $Z = \{z_1, z_2, z_3\}$ and $\Delta = \{\vartriangle_1, \vartriangle_2\}$ be the set of parameters. Let $(O, \Delta), (P, \Delta)$ be soft sets over the universe Z, where:*
$O(\vartriangle_1) = \{z_1, z_3\}$, $\quad O(\vartriangle_2) = \{z_2, z_3\}$,
$P(\vartriangle_1) = \{z_2, z_3\}$, $\quad P(\vartriangle_2) = \{z_1, z_3\}$.
Then, $\Theta = \{\tilde{Z}, \tilde{\Phi}, (O, \Delta), (P, \Delta)\}$ defines an SSTS on Z. Hence, the soft set (C, Δ), where:
$C(\vartriangle_1) = \{z_2, z_3\}$, $\quad C(\vartriangle_2) = \{z_2, z_3\}$,
is a g^s_δ-closed, but it is neither supra soft δ-closed nor supra soft closed.

Theorem 3.4. *Let (V, Δ) be a g^s_δ-closed subset of an SSTS (Z, Θ, Δ). If there is $(U, \Delta) \in S(Z)_\Delta$ s.t $(V, \Delta)\tilde{\subseteq}(U, \Delta)\tilde{\subseteq}cl^s_\delta(V, \Delta)$, then (U, Δ) is a g^s_δ-closed set.*

Proof. Suppose that $(U, \Delta)\tilde{\subseteq}(G, \Delta)$ and $(G, \Delta) \in \Theta$. Since $(V, \Delta)\tilde{\subseteq}(U, \Delta)$ and (V, Δ) is g^s_δ-closed, $cl^s_\delta(V, \Delta)\tilde{\subseteq}(G, \Delta)$. It follows,

$$cl^s_\delta(U, \Delta)\tilde{\subseteq}cl^s_\delta(V, \Delta)\tilde{\subseteq}(G, \Delta).$$

Hence,

$$cl^s_\delta(U, \Delta)\tilde{\subseteq}(G, \Delta).$$

Therefore,

$$(U,\Delta) \text{ is a } g^s_\delta\text{-closed.}$$

Theorem 3.5. *A soft subset (S,Δ) of an SSTS (Z,Θ,Δ) is g^s_δ-closed set if $cl^s_\delta(S,\Delta)\setminus(S,\Delta)$ contains empty supra soft closed set only.*

Proof. Assume that (S,Δ) be a g^s_δ-closed set and (T,Δ) be a non empty supra soft closed subset of $\tilde{Z}$ s.t $(T,\Delta)\tilde{\subseteq}cl^s_\delta(S,\Delta)\setminus(S,\Delta)$. So,

$$(T,\Delta)\tilde{\subseteq}cl^s_\delta(S,\Delta) \tag{3.1}$$

Also, $(S,\Delta)\tilde{\subseteq}(T^{\tilde{c}},\Delta)$, $(T^{\tilde{c}},\Delta)\in\Theta$. Since (S,Δ) is g^s_δ-closed, $cl^s_\delta(S,\Delta)\tilde{\subseteq}(T^{\tilde{c}},\Delta)$. It follows,

$$(T,\Delta)\tilde{\subseteq}[cl^s_\delta(S,\Delta)]^{\tilde{c}} \tag{3.2}$$

From Eqs (3.1) and (3.2), we have $(T,\Delta)\tilde{\subseteq}cl^s_\delta(S,\Delta)\tilde{\cap}[cl^s_\delta(S,\Delta)]^{\tilde{c}}=\tilde{\Phi}$. Therefore, $(T,\Delta)=\tilde{\Phi}$, which is a contradiction.

Definition 3.3. *[47] Let $f_{pu}:(Z_1,\tau_1,\Delta_1)\to(Z_2,\tau_2,\Delta_2)$ be a soft function with Θ_1 as an associated SSTS with τ_1 is said to be a supra soft δ-continuous if $f^{-1}_{pu}(G,\Delta_2)\in SOS_\delta(Z_1)\ \forall\ (G,\Delta_2)\in\tau_2$.*

Definition 3.4. *A soft function $f_{pu}:(Z_1,\tau_1,\Delta_1)\to(Z_2,\tau_2,\Delta_2)$ with Θ_1,Θ_2 associated SSTSs with τ_1 and τ_2, respectively, is said to be:*

- **(1):** *[56] A supra soft irresolute if $f^{-1}_{pu}(G,\Delta_2)\in\Theta_1\ \forall\ (G,\Delta_2)\in\Theta_2$*
- **(2):** *A supra soft δ-irresolute closed if $f_{pu}(G,\Delta_1)\in SCS_\delta(Z_2)\ \forall\ (G,\Delta_1)\in SCS_\delta(Z_1)$.*
- **(3):** *A supra soft δ-irresolute open if $f_{pu}(G,\Delta_1)\in SOS_\delta(Z_2)\ \forall\ (G,\Delta_1)\in SOS_\delta(Z_1)$.*

Proposition 3.2. *Let $f_{pu}:(Z_1,\tau_1,\Delta_1)\to(Z_2,\tau_2,\Delta_2)$ be a soft function with Θ_1,Θ_2 associated SSTSs with τ_1 and τ_2, respectively and $(G,\Delta_1)\tilde{\subseteq}\tilde{Z}_1$. Then,*

- **(1):** *If f_{pu} is supra soft δ-irresolute open, then $f_{pu}(int^s_\delta(G,\Delta_1))\tilde{\subseteq}int^s_\delta[f_{pu}(G,\Delta_1)]$.*
- **(2):** *If f_{pu} is supra soft δ-irresolute closed, then $cl^s_\delta(f_{pu}(G,\Delta_1))\tilde{\subseteq}f_{pu}(cl^s_\delta(G,\Delta_1))$.*

Proof.

- **(1):** Let f_{pu} be a supra soft δ-irresolute open, then $f_{pu}(int^s_\delta(G,\Delta_1))\in SOS_\delta(Z_2)$. Follows,
$$f_{pu}(int^s_\delta(G,\Delta_1))=int^s_\delta[f_{pu}(int^s_\delta(G,\Delta_1))]\tilde{\subseteq}int^s_\delta[f_{pu}(G,\Delta_1)].$$
- **(2):** It is similar to (1).

Theorem 3.6. *Let $f_{pu}:(Z_1,\tau_1,\Delta_1)\to(Z_2,\tau_2,\Delta_2)$ be a soft function with Θ_1,Θ_2 associated SSTSs with τ_1 and τ_2, respectively, which is supra soft irresolute and supra soft δ-irresolute closed. If $(S,\Delta_1)\in G^s_\delta C(Z_1)$, then $f_{pu}(S,\Delta_1)\in G^s_\delta C(Z_2)$.*

Proof. Assume that, $(S,\Delta_1)\in G^s_\delta C(Z_1)$ and $(W,\Delta_2)\in\Theta_2$ s.t $f_{pu}(S,\Delta_1)\tilde{\subseteq}(W,\Delta_2)$. So,

$$(S,\Delta_1)\tilde{\subseteq}f^{-1}_{pu}(W,\Delta_2) \text{ and } f^{-1}_{pu}(W,\Delta_2)\in\Theta_1, \text{ where } f_{pu} \text{ is supra soft irresolute.}$$

Since $(S,\Delta_1)\in G^s_\delta C(Z_1)$, $cl^s_\delta(S,\Delta_1)\tilde{\subseteq}f^{-1}_{pu}(W,\Delta_2)$, and hence

$$f_{pu}[cl^s_\delta(S,\Delta_1)]\tilde{\subseteq}f_{pu}(f^{-1}_{pu}(W,\Delta_2))\tilde{\subseteq}(W,\Delta_2).$$

Since f_{pu} is supra soft δ-irresolute closed and $cl^s_\delta(S,\Delta_1) \in SCS_\delta(Z_1)$, so

$$f_{pu}[cl^s_\delta(S,\Delta_1)] \in SCS_\delta(Z_2).$$

Therefore,

$$cl^s_\delta[f_{pu}(S,\Delta_1)] \tilde{\subseteq} cl^s_\delta[f_{pu}[cl^s_\delta(S,\Delta_1)]] = f_{pu}[cl^s_\delta(S,\Delta_1)] \tilde{\subseteq} (W,\Delta_2).$$

Hence,

$$f_{pu}(S,\Delta_1) \in G^s_\delta C(Z_2).$$

4. Supra soft δ-generalized open sets

In this section, we provide the concept of g^s_δ-open sets in SSTSs as a complement to the notion of g^s_δ-closed sets. An equivalent condition to the definition is provided. Also, several interested properties supported by examples are explored.

Definition 4.1. *A soft subset (A,Δ) of an SSTS (Z,Θ,Δ) is called supra soft δ-generalized open set (briefly, g^s_δ-open) if its relative complement $(A^{\tilde{c}},\Delta)$ is g^s_δ-closed. The category of all g^s_δ-open sets will denoted by $G^s_\delta O(Z)$.*

Example 4.1. *In Example 3.1, the soft set $(G^{\tilde{c}}_2,\Delta)$ is g^s_δ-open and the soft set $(G^{\tilde{c}}_3,\Delta)$ is not g^s_δ-open.*

Theorem 4.1. *A soft subset (A,Δ) of an SSTS (Z,Θ,Δ) is g^s_δ-open set if, and only if, $(H,\Delta) \tilde{\subseteq} int^s_\delta(A,\Delta)$ whenever $(H,\Delta) \tilde{\subseteq} (A,\Delta)$ and $(H,\Delta) \in \Theta^c$.*

Proof. Necessity: Assume that (A,Δ) is a g^s_δ-open subset of $\tilde{Z}$ and $(A,\Delta) \tilde{\subseteq} (H,\Delta)$ s.t $(H,\Delta) \in \Theta^c$. It follows,

$$(H^{\tilde{c}},\Delta) \tilde{\subseteq} (A^{\tilde{c}},\Delta),\ (H^{\tilde{c}},\Delta) \in \Theta.$$

Since $(A^{\tilde{c}},\Delta)$ is g^s_δ-closed,

$$cl^s_\delta(H^{\tilde{c}},\Delta) \tilde{\subseteq} (A^{\tilde{c}},\Delta).$$

Hence,

$$(A,\Delta) \tilde{\subseteq} [cl^s_\delta(H^{\tilde{c}},\Delta)]^{\tilde{c}} = int^s_\delta(H,\Delta) \text{ from [[47], Theorem 4.12 (1)].}$$

Sufficient: Suppose that $(A^{\tilde{c}},\Delta) \tilde{\subseteq} (G,\Delta)$ s.t $(G,\Delta) \in \Theta$. So,

$$(G^{\tilde{c}},\Delta) \tilde{\subseteq} (A,\Delta) \text{ and } (G^{\tilde{c}},\Delta) \in \Theta^c.$$

From the necessary condition,

$$(G^{\tilde{c}},\Delta) \tilde{\subseteq} int^s_\delta(A,\Delta).$$

Hence,

$$[int^s_\delta(A,\Delta)]^{\tilde{c}} = cl^s_\delta[(A^{\tilde{c}},\Delta)] \tilde{\subseteq} (G,\Delta) \text{ and } (G,\Delta) \in \Theta.$$

Therefore,

$$(A^{\tilde{c}},\Delta) \text{ is a } g^s_\delta\text{-closed set, and hence } (A,\Delta) \text{ is a } g^s_\delta\text{-open set.}$$

Theorem 4.2. *Every supra soft g-open subset of an SSTS (X,Θ,Δ) is a g^s_δ-open.*

Proof. Assume that (H,Δ) is supra soft g-open set and $(H,\Delta)\tilde{\subseteq}(A,\Delta)$ s.t $(H,\Delta)\in\Theta^c$. It follows,

$$(H,\Delta)\tilde{\subseteq}int^s(A,\Delta).$$

Since $int^s(A,\Delta)\tilde{\subseteq}int^s_\delta(A,\Delta)$, $(H,\Delta)\tilde{\subseteq}int^s_\delta(A,\Delta)$. Therefore,

$$(A,\Delta) \text{ is a } g^s_\delta\text{-open.}$$

Remark 4.1. *The converse of Theorem 4.2 need not to be true in general. In Example 3.2, the soft sets $(F^{\tilde{c}},\Delta)$ is a g^s_δ-open , but it is not supra soft g-open.*

Theorem 4.3. *Every supra soft rg-open subset (Q,Δ) of an SSTS (X,Θ,Δ) is a g^s_δ-open.*

Proof. Suppose that, (Q,Δ) be a supra soft rg-open subset of an SSTS (Z,Θ,Δ). Then,

$$(H,\Delta)\tilde{\subseteq}int^s(Q,\Delta), \text{ whenever}$$

$(H,\Delta)\tilde{\subseteq}(Q,\Delta)$ and $(H,\Delta)\in RCS^s(Z)$. So,

$$(H,\Delta)\in\Theta^c \text{ from [[55], Theorem 3.4].}$$

It follows,

$$(H,\Delta)\tilde{\subseteq}int^s(Q,\Delta)\tilde{\subseteq}int^s_\delta(Q,\Delta),\ (H,\Delta)\in\Theta^c.$$

Therefore,

$$(Q,\Delta) \text{ is a } g^s_\delta\text{-open set.}$$

Remark 4.2. *The converse of Theorem 4.3 need not to be true in general. In Example 3.2, the soft sets $(F^{\tilde{c}},\Delta)$ is a g^s_δ-open, but it is not supra soft rg-closed.*

Proposition 4.1. *Let (Q,Δ) be a soft subset of an SSTS (X,Θ,Δ), which is both supra soft closed and g^s_δ-open. Then, $int^s_\delta(Q,\Delta)=(Q,\Delta)$*

Proof. Obvious from Proposition 3.1.

Note 4.1. *The soft union (respectively, soft intersection) of any two g^s_δ-open sets need not to be a g^s_δ-open in general. Examples 3.1 confirm our claim.*

Theorem 4.4. *Every supra soft (respectively, δ-) open subset (G,Δ) of an SSTS (Z,Θ,Δ) is a g^s_δ-open, but not conversely see Example 4.2.*

Proof. We prove the case of supra soft δ-open set. Let $(H,\Delta)\tilde{\subseteq}(G,\Delta)$ s.t $(H,\Delta)\in\Theta^c$ and $(G,\Delta)\in SOS_\delta(Z)$. Then,

$$(H,\Delta)\tilde{\subseteq}int^s_\delta(G,\Delta)=(G,\Delta)) \text{ from [[47], Proposition 4.4 (1)].}$$

Hence,

$$(G,\Delta) \text{ is a } g^s_\delta\text{-open set.}$$

The other case follows directly from the fact that, each supra soft open set is a supra soft δ-open [[47], Theorem 3.6 (1)].

Example 4.2. *In Example 3.3, the soft set $(C^{\tilde{c}}, \Delta)$ is a g^s_δ-open , but it is neither supra soft δ-open nor supra soft open.*

Theorem 4.5. *Let (J, Δ) is g^s_δ-open subset of an SSTS (Z, Θ, Δ). If $\exists$ $(K, \Delta) \in S(Z)_\Delta$ s.t $int^s_\delta(J, \Delta)\tilde{\subseteq}(K, \Delta)\tilde{\subseteq}(J, \Delta)$, then (K, Δ) is a g^s_δ-open set.*

Proof. Suppose that, $(H, \Delta)\tilde{\subseteq}(K, \Delta)$ and $(H, \Delta) \in \Theta^c$. Since $(K, \Delta)\tilde{\subseteq}(J, \Delta)$ and (J, Δ) is a g^s_δ-open,

$$(H, \Delta)\tilde{\subseteq}int^s_\delta(J, \Delta).$$

It follows,

$$(H, \Delta)\tilde{\subseteq}int^s_\delta(J, \Delta)\tilde{\subseteq}int^s_\delta(K, \Delta).$$

Hence,

$$(H, \Delta)\tilde{\subseteq}int^s_\delta(K, \Delta).$$

Therefore,

$$(J, \Delta) \text{ is a } g^s_\delta\text{-open set.}$$

Theorem 4.6. *Let $f_{pu} : (Z_1, \tau_1, \Delta_1) \to (Z_2, \tau_2, \Delta_2)$ be a soft function with Θ_1, Θ_2 associated SSTSs with τ_1 and τ_2, respectively, which is supra soft irresolute and supra soft δ-irresolute open. If $(S, \Delta_1) \in G^s_\delta O(Z_1)$, then $f_{pu}(S, \Delta_1) \in G^s_\delta O(Z_2)$.*

Proof. Assume that $(S, \Delta_1) \in G^s_\delta O(Z_1)$ and $(H, \Delta_2) \in \Theta_2^c$ s.t $(H, \Delta_2)\tilde{\subseteq}f_{pu}(S, \Delta_1)$. So,

$$f_{pu}(S^{\tilde{c}}, \Delta_1)\tilde{\subseteq}(H^{\tilde{c}}, \Delta_2) \ , (H^{\tilde{c}}, \Delta_2) \in \Theta_2.$$

It follows,

$$(S^{\tilde{c}}, \Delta_1)\tilde{\subseteq}f^{-1}_{pu}[f_{pu}(S^{\tilde{c}}, \Delta_1)]\tilde{\subseteq}f^{-1}_{pu}(H^{\tilde{c}}, \Delta_2),$$

Since f_{pu} is supra soft irresolute, $f^{-1}_{pu}(H^{\tilde{c}}, \Delta_2) \in \Theta_1$. Since $(S, \Delta_1) \in G^s_\delta O(Z_1)$, $(S^{\tilde{c}}, \Delta_1) \in G^s_\delta C(Z_1)$. Hence,

$$[int^s_\delta(S, \Delta_1)]^{\tilde{c}} = cl^s_\delta(S^{\tilde{c}}, \Delta_1)\tilde{\subseteq}f^{-1}_{pu}(H^{\tilde{c}}, \Delta_2).$$

It follows,

$$f^{-1}_{pu}(H, \Delta_2)\tilde{\subseteq}int^s_\delta(S, \Delta_1).$$

Therefore,

$$(H, \Delta_2)\tilde{\subseteq}f_{pu}[f^{-1}_{pu}(H, \Delta_2)]\tilde{\subseteq}f_{pu}[int^s_\delta(S, \Delta_1)]\tilde{\subseteq}int^s_\delta[f_{pu}(S, \Delta_1)],$$

f_{pu} is supra soft δ-irresolute open. Thus,

$$f_{pu}(S, \Delta_1) \in G^s_\delta O(Z_2).$$

5. Conclusion and upcoming work

Our purpose of this project, is to generalize many kinds of supra soft generalizes closed sets in SSTSs by using the of supra soft δ-closure operator. We showed that the new class contains strictly the class of supra soft generalized closed sets [50], supra soft strongly generalized closed sets [34] and supra soft regular generalized closed sets [35]. Also, the notions of g^s_δ-open sets were defined and several of its interested properties were studied. The presented results have strong depth that can herald future applications. So, our future work, is to generalize these notions by using the soft ideal notions [19] an soft semi open sets [57,58] and introduce many types of supra soft separation axioms, connectedness and compactness via the above-mentioned notions.

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References

[1] A.S. Mashhour, A.A. Allam, F.S. Mahmoud, F.H. Khedr, On Supra Topological Spaces, Indian J. Pure Appl. Math. 4 (1983), 502–510.

[2] A. Alpers, Digital Topology: Regular Sets and Root Images of the Cross-Median Filter, J. Math. Imaging. Vis. 17 (2002), 7–14.

[3] A.M. Kozae, M. Shokry, M. Zidan, Supra Topologies for Digital Plane, AASCIT Commun. 3 (2016), 1–10.

[4] T.M. Al-shami, I. Alshammari, Rough Sets Models Inspired by Supra-Topology Structures, Artif. Intell. Rev. 56 (2022), 6855–6883. https://doi.org/10.1007/s10462-022-10346-7.

[5] A. Al-Omari, M.H. Alqahtani, Primal Structure with Closure Operators and Their Applications, Mathematics 11 (2023), 4946. https://doi.org/10.3390/math11244946.

[6] M.H. Alqahtani, New Topologies on a Graph, Adv. Appl. Discr. Math. 37 (2023), 77–93. https://doi.org/10.17654/0974165823013.

[7] M.H. Alqahtani, H.Y. Saleh, A Novel Class of Separation Axioms, Compactness, and Continuity via C-Open Sets, Mathematics. 11 (2023), 4729. https://doi.org/10.3390/math11234729.

[8] M.H. Alqahtani, F-open and F-closed sets in Topological Spaces, Eur. J. Pure Appl. Math. 16 (2023), 819–832. https://doi.org/10.29020/nybg.ejpam.v16i2.4583.

[9] A. Kharal, B. Ahmad, Mappings on Soft Classes, New Math. Nat. Comput. 07 (2011), 471–481. https://doi.org/10.1142/s1793005711002025.

[10] D. Molodtsov, Soft Set Theory-First Results, Comput. Math. Appl. 37 (1999), 19–31. https://doi.org/10.1016/s0898-1221(99)00056-5.

[11] P.K. Maji, R. Biswas, A.R. Roy, Soft Set Theory, Comput. Math. Appl. 45 (2003), 555–562. https://doi.org/10.1016/s0898-1221(03)00016-6.

[12] Z.A. Ameen, M.H. Alqahtani, Some Classes of Soft Functions Defined by Soft Open Sets Modulo Soft Sets of the First Category, Mathematics. 11 (2023), 4368. https://doi.org/10.3390/math11204368.

[13] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, γ-Operation and Decompositions of Some Forms of Soft Continuity in Soft Topological Spaces, Ann. Fuzzy Math. Inform. 7 (2014), 181–196.

[14] Z.A. Ameen, A Non-Continuous Soft Mapping That Preserves Some Structural Soft Sets, J. Intell. Fuzzy Syst. 42 (2022), 5839–5845. https://doi.org/10.3233/jifs-212410.

[15] I. Zorlutuna, H. akir, On Continuity of Soft Mappings, Appl. Math. Inf. Sci. 9 (2015), 403–409. https://doi.org/10.12785/amis/090147.

[16] D. Wardowski, On a Soft Mapping and Its Fixed Points, Fixed Point Theory Appl. 2013 (2013), 182. https://doi.org/10.1186/1687-1812-2013-182.

[17] Z.A. Ameen, M.H. Alqahtani, Baire Category Soft Sets and Their Symmetric Local Properties, Symmetry. 15 (2023), 1810. https://doi.org/10.3390/sym15101810.

[18] M.H. Alqahtani, Z.A. Ameen, Soft Nodec Spaces, AIMS Math. 9 (2024), 3289–3302. https://doi.org/10.3934/math.2024160.

[19] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, Soft Ideal Theory Soft Local Function and Generated Soft Topological Spaces, Appl. Math. Inf. Sci. 8 (2014), 1595–1603. https://doi.org/10.12785/amis/080413.

[20] A.H. Hussain, S.A. Abbas, A.M. Salman, N.A. Hussein, Semi Soft Local Function Which Generated a New Topology in Soft Ideal Spaces, J. Interdiscip. Math. 22 (2019), 1509–1517. https://doi.org/10.1080/09720502.2019.1706848.

[21] F. Gharib, A.M. Abd El-latif, Soft Semi Local Functions in Soft Ideal Topological Spaces, Eur. J. Pure Appl. Math. 12 (2019), 857–869.

[22] A.M. Abd El-Latif, Generalized Soft Rough Sets and Generated Soft Ideal Rough Topological Spaces, J. Intell. Fuzzy Syst. 34 (2018), 517–524. https://doi.org/10.3233/jifs-17610.

[23] M. Akdag, F. Erol, Soft I-Sets and Soft I-Continuity of Functions, Gazi Univ. J. Sci. 27 (2014), 923–932.

[24] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-Latif, γ-Operation and Decompositions of Some Forms of Soft Continuity of Soft Topological Spaces via Soft Ideal, Ann. Fuzzy Math. Inform. 9 (2015), 385–402.

[25] A. Nasef, M. Parimala, R. Jeevitha, M. ElSayed, Soft Ideal Theory and Applications, Int. J. Nonlinear Anal. Appl. 13 (2022), 1335–1342. https://doi.org/10.22075/ijnaa.2022.6266.

[26] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-Latif, Soft Regularity and Normality Based on Semi Open Soft Sets and Soft Ideals, Appl. Math. Inf. Sci. Lett. 3 (2015), 47–55.

[27] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, Soft Semi (Quasi) Hausdorff Spaces via Soft Ideals, South Asian J. Math. 4 (2014, 265–284.

[28] S.A. El-Sheikh, A.M. Abd El-latif, Decompositions of Some Types of Supra Soft Sets and Soft Continuity, Int. J. Math. Trends Technol. 9 (2014), 37–56. https://doi.org/10.14445/22315373/ijmtt-v9p504.

[29] M. Shabir, M. Naz, On Soft Topological Spaces, Comput. Math. Appl. 61 (2011), 1786–1799. https://doi.org/10.1016/j.camwa.2011.02.006.

[30] T.M. Al-shami, M.E. El-Shafei, Two Types of Separation Axioms on Supra Soft Topological Spaces, Demonstr. Math. 52 (2019), 147–165. https://doi.org/10.1515/dema-2019-0016.

[31] C.G. Aras, S. Bayramov, Results of Some Separation Axioms in Supra Soft Topological Spaces, TWMS J. Appl. Eng. Math. 9 (2019), 58–63.

[32] A.M. Abd El-Latif, R.A. Hosny, Supra Soft Separation Axioms and Supra Irresoluteness Based on Supra b-Open Soft Sets, Gazi Univ. J. Sci. 29 (2016), 845–854.

[33] A.M. Abd El-latif, S. Karataş, Supra b-Open Soft Sets and Supra b-Soft Continuity on Soft Topological Spaces, J. Math. Comput. Appl. Res. 5 (2015), 1–18.

[34] A.M. Abd El-latif, Soft Supra Strongly Generalized Closed Sets, J. Intell. Fuzzy Syst. 31 (2016), 1311–1317. https://doi.org/10.3233/ifs-162197.

[35] Z.G. Ergul, S. Yuksel, Supra Regular Generalized Closed Sets in Supra Soft Topological Spaces, Ann. Fuzzy Math. Inform. 11 (2016), 349–360.

[36] W. Rong, F. Lin, Soft Connected Spaces and Soft Paracompact Spaces, Int. J. Appl. Math. Stat. 51 (2013), 667–681.

[37] A.M. Abd El-latif, Supra Soft b-Connectedness I: Supra Soft b-Irresoluteness and Separateness, Creat. Math. Inform. 25 (2016), 127–134.

[38] A.M. Abd El-latif, Supra Soft b-Connectedness II: Some Types of Supra Soft b-Connectedness, Creat. Math. Inform. 26 (2017), 1–8.

[39] T.M. Al-shami, A. Mhemdi, R. Abu-Gdairi, M.E. El-Shafei, Compactness and Connectedness via the Class of Soft Somewhat Open Sets, AIMS Math. 8 (2023), 815–840. https://doi.org/10.3934/math.2023040.

[40] A.M. Abd El-latif, On Soft Supra Compactness in Supra Soft Topological Spaces, Tbilisi Math. J. 11 (2018), 169–178. https://doi.org/10.32513/tbilisi/1524276038.

[41] T. Al-shami, J. Alcantud, A. Azzam, Two New Families of Supra-Soft Topological Spaces Defined by Separation Axioms, Mathematics. 10 (2022), 4488. https://doi.org/10.3390/math10234488.

[42] C.G. Aras, S. Bayramov, Separation Axioms in Supra Soft Bitopological Spaces, Filomat. 32 (2018), 3479–3486. https://doi.org/10.2298/fil1810479g.

[43] A.M. Abd El-latif, R.A. Hosny, Supra Semi Open Soft Sets and Associated Soft Separation Axioms, Appl. Math. Inf. Sci. 10 (2016), 2207–2215. https://doi.org/10.18576/amis/100623.

[44] A.M. Abd El-latif, Supra Soft Separation Axioms Based on Supra β-Open Soft Sets, Math. Sci. Lett. 5 (2016), 121–129. https://doi.org/10.18576/msl/050202.

[45] A.M. Abd El-latif, Novel Types of Supra Soft Operators via Supra Soft sd-Sets and Applications, AIMS Math. 9 (2024), 6586–6602. https://doi.org/10.3934/math.2024321.

[46] A.M. Abd El-latif, M.H. Alqahtani, Novel Categories of Supra Soft Continuous Maps via New Soft Operators, AIMS Math. 9 (2024), 7449–7470. https://doi.org/10.3934/math.2024361.

[47] A.M. Abd El-latif, M.H. Alqahtani, New Soft Operators Related to Supra Soft δ_i-Open Sets and Applications, AIMS Math. 9 (2024), 3076–3096. https://doi.org/10.3934/math.2024150.

[48] K. Kannan, Soft Generalized Closed Sets in Soft Topological Spaces, J. Theor. Appl. Inf. Technol. 37 (2012), 17–21.

[49] K. Kannan, D. Rajalakshmi, R. Srikanth, Soft Strongly g-Closed Sets, Indian J. Sci. Technol. 8 (2015), 1–6. https://doi.org/10.17485/ijst/2015/v8i18/65394.

[50] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, Supra Generalized Closed Soft Sets with Respect to an Soft Ideal in Supra Soft Topological Spaces, Appl. Math. Inf. Sci. 8 (2014), 1731–1740. https://doi.org/10.12785/amis/080430.

[51] Z.A. Ameen, M.H. Alqahtani, Congruence Representations via Soft Ideals in Soft Topological Spaces, Axioms 12 (2023), 1015. https://doi.org/10.3390/axioms12111015.

[52] T.M. Al-shami, Improvement of the Approximations and Accuracy Measure of a Rough Set Using Somewhere Dense Sets, Soft Comput. 25 (2021), 14449–14460. https://doi.org/10.1007/s00500-021-06358-0.

[53] T.M. Al-shami, A. Mhemdi, Approximation Operators and Accuracy Measures of Rough Sets From an Infra-Topology View, Soft Comput. 27 (2022), 1317–1330. https://doi.org/10.1007/s00500-022-07627-2.

[54] I. Zorlutuna, M. Akdag, W.K. Min, S. Atmaca, Remarks on Soft Topological Spaces, Ann. Fuzzy Math. Inform. 3 (2012), 171–185.

[55] L. Lincy, A. Kalaichelvi, Supra Soft Regular Open Sets, Supra Soft Regular Closed Sets and Supra Soft Regular Continuity, Int. J. Pure Appl. Math. 119 (2018), 1075–1079.

[56] A.M. Abd El-latif, R.A. Hosny, Supra Open Soft Sets and Associated Soft Separation Axioms, Int. J. Adv. Math. 6 (2017), 68–81.

[57] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, Soft Semi Compactness via Soft Ideals, Appl. Math. Inf. Sci. 8 (2014), 2297–2306.

[58] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. Abd El-latif, Soft Semi Separation Axioms and Irresolute Soft Functions, Ann. Fuzzy Math. Inform. 8 (2014), 305–318.