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Some Results on Subspace Cesaro-Hypercyclic Operators

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Abstract. In this paper we characterize the notion of subspace Cesàro-hypercyclic. At the same time, we also provide a Subspace Cesàro-hypercyclic Criterion and offer an equivalent conditions of this criterion.

1. Introduction

Let $\mathcal{H}$ be a separable infinite dimensional Hilbert space over the scalar field $\mathbb{C}$. As usual, $\mathbb{N}$ is the set of all non-negative integers, $\mathbb{Z}$ is the set of all integers, and $B(\mathcal{H})$ is the space of all bounded linear operators on $\mathcal{H}$. A bounded linear operator $T : \mathcal{H} \to \mathcal{H}$ is called hypercyclic if there is some vector $x \in \mathcal{H}$ such that $Orb(T, x) = \{T^n x : n \in \mathbb{N}\}$ is dense in $\mathcal{H}$, where such a vector x is said hypercyclic for T.

The first example of hypercyclic operator was given by Rolewicz in [16]. He proved that if B is a backward shift on the Banach space l^p, then λB is hypercyclic if and only if $|\lambda| > 1$.

Let $\{e_n\}_{n\geq 0}$ be the canonical basis of $l^2(\mathbb{N})$. If $\{w_n\}_{n\in\geq 1}$ is a bounded sequence in $\mathbb{C}\backslash\{0\}$, then the unilateral backward weighted shift $T : l^2(\mathbb{N}) \longrightarrow l^2(\mathbb{N})$ is defined by $Te_n = w_n e_{n-1}, \quad n \geq 1, Te_0 = 0$, and let $\{e_n\}_{n\in\mathbb{Z}}$ be the canonical basis of $l^2(\mathbb{Z})$. If $\{w_n\}_{n\in\mathbb{Z}}$ is a bounded sequence in $\mathbb{C}\backslash\{0\}$, then the bilateral weighted shift $T : l^2(\mathbb{Z}) \longrightarrow l^2(\mathbb{Z})$ is defined by $Te_n = w_n e_{n-1}$.

The definition and the properties of supercyclicity operators were introduced by Hilden and Wallen [9]. They proved that all unilateral backward weighted shifts on a Hilbert space are supercyclic.

A bounded linear operator $T \in B(\mathcal{H})$ is called supercyclic if there is some vector $x \in \mathcal{H}$ such that the projective orbit $\mathbb{C}.Orb(T, x) = \{\lambda T^n x : \lambda \in \mathbb{C}, n \in \mathbb{N}\}$ is dense in X. Such a vector x is said supercyclic for T. Refer to ([1], [8], [4], [19]) for more informations about hypercyclicity and

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supercyclicity.
A nice criterion namely Hypercyclicity Criterion, was developed independently by Kitai [11] and, Gethner and Shapiro [7]. The Hypercyclicity Criterion has been widely used to show that many different types of operators are hypercyclic. For instance hypercyclic operators arise in the classes of composition operators [3], adjoints of multiplication operators [7], cohyponormal operators [6], and weighted shifts [17].
For the following theorem, see ([1], [8]).

Theorem 1.1. *(Hypercyclicity Criterion). Suppose that $T \in B(\mathcal{H})$. If there exist two dense subsets X_0 and Y_0 in $\mathcal{H}$ and an increasing sequence n_j of positive integer such that:*

(1) $T^{n_j}x \to 0$ *for each* $x \in X_0$, *and*
(2) *there exist mappings* $S_{n_j} : Y_0 \longrightarrow \mathcal{H}$ *such that* $S_{n_j}y \to 0$, *and* $T^{n_j}S_{n_j}y \to y$ *for each* $y \in Y_0$,

then T is hypercyclic.

In [17] and [18], Salas characterized the bilateral weighted shifts that are hypercyclic and those that are supercyclic in terms of their weight sequence. In [5], N. Feldman gave a characterization of the invertible bilateral weighted shifts that are hypercyclic or supercyclic.
Let $\mathcal{M}_n(T)$ denote the arithmetic mean of the powers of $T \in B(\mathcal{H})$, that is

$$\mathcal{M}_n(T) = \frac{I + T + T^2 + ... + T^{n-1}}{n}, n \in \mathbb{N}^*.$$

If the arithmetic means of the orbit of x are dense in $\mathcal{H}$ then the operator T is said to be Cesàro-hypercyclic. In [13], Fernando Leόn-Saavedra proved that an operator is Cesàro-hypercyclic if and only if there exists a vector $x \in \mathcal{H}$ such that the orbit $\{n^{-1}T^n x\}_{n\geq 1}$ is dense in $\mathcal{H}$ and characterized the bilateral weighted. The following examples give an operator which is Cesàro-hypercyclic but not hypercyclic and vice versa.

Example 1.1. *[13] Let T the bilateral backward shift with the weight sequence*

$$w_n = \begin{cases} 1 & \text{if } n \leq 0, \\ 2 & \text{if } n \geq 1. \end{cases}$$

Then T is not hypercyclic, but it is Cesàro-hypercyclic.

Example 1.2. *[20] Let T the bilateral backward shift with the weight sequence*

$$w_n = \begin{cases} 2 & \text{if } n < 0, \\ \frac{1}{2} & \text{if } n \geq 0. \end{cases}$$

Then T is not Cesàro-hypercyclic, but it is hypercyclic and supercyclic.

In 2011, B. F. Madore and R. A. Martnez-Avendano in [14] introduced and studied the concept of subspace-hypercyclicity for an operator. An operator T is subspace-hypercyclic or M-hypercyclic for a subspace M of X, if there exists $x \in X$ such that $Orb(T, x) \bigcap M$ is dense in M. Such a

vector x is called a M-hypercyclic vector for T, they showed that there are operators which are M-hypercyclic but not hypercyclic. They introduced analogously the concept of subspace-transitivity. Let $T \in \mathcal{B}(X)$ and M be a closed subspace of X, we say that T is M-transitive, if for any non-empty open sets U, V in M, there exists $n \geq 0$ such that $T^{-n}(U) \bigcap V$ contain a non-empty open subset of M.The authors showed that M-transitivity implies M-hypercyclicity. Note that the converse is not true, this is proven recently by C. M. Le in [12]; for more informations see ([10], [15]).
Similarly, for subspace-supercyclicity, Zhao, Y.L. Sun and Y.H. Zhou in [21] provided a Subspace-Supercyclicity Criterion and offered two necessary and sufficient conditions for a path of bounded linear operators to have a dense G_δ set of common subspace-hypercyclic vectors and common subspace-supercyclic vectors and they also constructed examples to show that subspace-supercyclic is not a strictly infinite dimensional phenomenon and that some subspace-supercyclic operators are not supercyclic.
In this present paper, we will partially characterize the notion of subspace Cesàro-hypercyclic. At the same time, we also provide a Subspace Cesàro-hypercyclic Criterion and offer an equivalent conditions of this criterion.

2. Main results

We will assume that the subspace $M \subset \mathcal{H}$ is topologically closed. We start with our main definitions.

Definition 2.1. *Let $T \in \mathcal{B}(\mathcal{H})$ and M be a closed subspace of $\mathcal{H}$. We say that T is M-cesàro-hypercyclic if there exists a vector $x \in \mathcal{H}$ such that $Orb(T,x) \bigcap M = \{n^{-1}T^n x : n \geq 1\} \bigcap M$ is dense in M. We call x a M-cesàro-hypercyclic vector.*

Remark 2.1. *The definition above reduces to the classical definition of cesàro-hypercyclic if $M = \mathcal{H}$ and we may assume that the subspace cesàro-hypercyclic vector $x \in M$, if needed.*

Example 2.1. *Let T be a cesàro-hypercyclic operator on $\mathcal{H}$ with cesàro-hypercyclic vector x and let I be the identity operator on $\mathcal{H}$. Then the operator $T \oplus I : \mathcal{H} \oplus \mathcal{H} \to \mathcal{H} \oplus \mathcal{H}$ is subspace cesàro-hypercyclic for the subspace $M := \mathcal{H} \oplus \{0\}$ with the subspace cesàro-hypercyclic vector $x \oplus \{0\}$, but $T \oplus I$ is not cesàro-hypercyclic on the space $\mathcal{H} \oplus \mathcal{H}$.*

Theorem 2.1. *Let T be a subspace cesàro-hypercyclic and M be a nonzero subspace of $\mathcal{H}$. Then*

$$CH(T,M) = \bigcap_{k\geq 1} \bigcup_{n \geq 1} \left(n^{-1}T^n\right)^{-1}(B_k),$$

where $(B_k)_{k\geq 1}$ is a countable open basis for the relative topology of M as a subspace of $\mathcal{H}$.

Proof. Let $(B_k)_{k\geq 1}$ is a countable open basis for the relative topology of M as a subspace of $\mathcal{H}$. We have $x \in CH(T,M)$ if and only if $\{n^{-1}T^n x : n \geq 1\} \bigcap M$ is dense in M if and only if for each $k \geq 1$, there exist $n \geq 1$ such that $n^{-1}T^n x \in B_k$ if and only if $x \in \bigcap_{k\geq 1} \bigcup_{n \geq 1} \left(n^{-1}T^n\right)(B_k)$.

□

Theorem 2.2. *Let T be a subspace cesàro-hypercyclic and M be a nonzero subspace of $\mathcal{H}$. Then the following conditions are equivalent:*

(1) *For every non-empty open U and V of M, there exist $n \geq 1$ such that $\left(n^{-1}T^n\right)^{-1}(U) \bigcap V$ contains a non-empty open subset of M.*
(2) *For every non-empty open U and V of M, there exist $n \geq 1$ such that $\left(n^{-1}T^n\right)^{-1}(U) \bigcap V$ is non-empty and $\left(n^{-1}T^n\right)(M) \subset M$.*
(3) *For every non-empty open U and V of M, there exist $n \geq 1$ such that $\left(n^{-1}T^n\right)^{-1}(U) \bigcap V$ is non-empty open subset of M.*

Proof. $(1) \Rightarrow (2)$. Let U and V be tow nonempty open subsets of M. By (1) there exist $n \geq 1$ such that $\left(n^{-1}T^n\right)^{-1}(U) \bigcap V$ contains a non-empty open W of M, it follows that $W \subset \left(n^{-1}T^n\right)^{-1}(U) \cap V$ and $\left(n^{-1}T^n\right)^{-1}(U) \cap V \neq \emptyset$.
Next, We prove that $\left(n^{-1}T^n\right)(M) \subset M$.
Let $x \in M$, we have $W \subset \left(n^{-1}T^n\right)^{-1}(U) \cap V$, this implies that $n^{-1}T^n(W) \subset U \subset M$. Let $x_0 \in W$, since W is open of M then for all r enough small we have $x_0 + rx \in W$, therefore $\left(n^{-1}T^n\right)^{-1}(x_0 + rx) = \left(\left(n^{-1}T^n\right)x_0 + r\left(n^{-1}T^n\right)x\right) \in M$. Since $n^{-1}T^n x_0 \in M$, it follows that $r\left(n^{-1}T^n\right)x \in M$. We then conclude that $\left(n^{-1}T^n\right)(M) \subset M$.
$(2) \Rightarrow (3)$. Let U and V be nonempty open subsets of M, by (2) there exist $n \geq 1$ such that $\left(n^{-1}T^n\right)^{-1}(U) \bigcap V$ is non-empty and $\left(n^{-1}T^n\right)(M) \subset M$.
Since $\left(n^{-1}T^n\right)|M : M \longrightarrow M$ is continuous, then $\left(n^{-1}T^n\right)^{-1}(U)$ is open in M, therefore $\left(n^{-1}T^n\right)^{-1}(U) \cap V$ is nonempty open of M.
At last, we see that the implication $(3) \Rightarrow (1)$ is obvious and this completes the whole proof of the theorem. □

Corollary 2.1. *Let T be a subspace cesàro-hypercyclic and M be a nonzero subspace of $\mathcal{H}$. If any of the conditions in theorem 2.2 is satisfied, then $CH(T, M)$ is a dense subset of M.*

Proof. We may assume the condition (3) in theorem 2.2 is satisfied, then for every non-empty open U of M and for all $k \geq 1$, there exist $n \geq 1$ such that the set $\left(n^{-1}T^n\right)^{-1}(U) \bigcap B_k$ is nonempty and open. Hence the set

$$A_k = \bigcup_n \left(n^{-1}T^n\right)^{-1}(B_k)$$

is nonempty and open. Furthermore, $U \cap A_k \neq \emptyset$ for all $k \geq 1$. Thus each A_k is dense in M and so by the Baire category theorem and theorem 2.1 $CH(T, M)$ is also dense in M. □

Next we get the following theorem, which is the Subspace Cesàro-Hypercyclic Criterion, which is similar to the Supercyclictiy Criterion that was stated in [2]; see also [8].

Theorem 2.3. *(Subspace Cesàro-Hypercyclic Criterion) Let T be a subspace cesàro-hypercyclic and M be a nonzero subspace of $\mathcal{H}$. Assume that there exist M_0 and M_1, dense subsets of M, an increasing sequence (n_k) of positive integer and a sequence of mappings $S_{n_k} : M_1 \to M$ such that*

(1) *For each* $x \in M_0$, $n_k^{-1}T^{n_k}x \rightarrow 0$,
(2) *For each* $y \in M_1$, $S_{n_k}y \rightarrow 0$,
(3) *For each* $y \in M_1$, $\left(n_k^{-1}T^{n_k} \circ S_{n_k}\right)y \rightarrow y$,
(4) M *is an invariant subspace for* $n_k^{-1}T^{n_k}$ *for all* $n \geq 1$.

Then T *is subspace cesàro-hypercyclic for* M.

Proof. Let U and V be non-empty open subsets of M. By Theorem 2.2, it is enough to prove that there exist $n \geq 1$ such that

$$(n_k^{-1}T^{n_k})^{-1}(U) \bigcap V \text{ is non-empty and } n_k^{-1}T^{n_k}(M) \subset M.$$

Since M_0 and M_1 are dense in M, there exist $x \in M_0 \cap V, y \in M_1 \cap U$. And since U and V are nonempty open subsets, there exists $\varepsilon > 0$ such that $B_M(x, \varepsilon) \subseteq V$ and $B_M(y, \varepsilon) \subseteq U$. By assumption, there exist (n_k) such that

$$\|n_k^{-1}T^{n_k}x\| \leq \tfrac{\varepsilon}{2}, \|S_{n_k}y\| \leq \tfrac{\varepsilon}{2} \text{ and } \|n_k^{-1}T^{n_k}S_{n_k}y - y\| \leq \tfrac{\varepsilon}{2}.$$

Define $u = x + S_{n_k}y$. We know that $u \in M$ and $u \in V$, since $\|u - x\| = \|S_{n_k}y\| \leq \frac{\varepsilon}{2}$. Observe that $n_k^{-1}T^{n_k}u = n_k^{-1}T^{n_k}x + n_k^{-1}T^{n_k}S_{n_k}y$, so $n_k^{-1}T^{n_k}u \in M$. Since

$$\|n_k^{-1}T^{n_k}u - y\| = \|n_k^{-1}T^{n_k}x\| + \|n_k^{-1}T^{n_k}y - y\| < \varepsilon,$$

we have that $n_k^{-1}T^{n_k}u \in U$. Then $(n_k^{-1}T^{n_k})^{-1}(U) \bigcap V \neq \emptyset$ and T is subspace cesàro-hypercyclic for M. □

Theorem 2.4. *Let* $T \in B(\mathcal{H})$ *and* M *be a nonzero subspace of* $\mathcal{H}$. *Then the following* (1) *and* (2) *are equivalent:*

(1) $\mathcal{T}$ *satisfies Subspace Cesàro-Hypercyclic Criterion.*
(2) *(Outer Subspace Cesàro-Hypercyclic Criterion) There exist an increasing sequence* (n_k) *of positive integer , a dense linear subspace* $Y_0 \subseteq M$ *and, for each* $y \in Y_0$, *a dense linear subspace* X_0 *of* M *such that:*
   (a) *There exists a sequence of mappings* $S_{n_k} : Y_0 \rightarrow M$, $k \in \mathbb{N}^*$ *such that* $(n_k^{-1}T^{n_k} \circ S_{n_k})y \rightarrow y$, *For each* $y \in Y_0$ *and*
   (b) $\|n_k^{-1}T^{n_k}x\| \, \|S_{n_k}y\| \rightarrow 0$ *For each* $y \in Y_0$ *and* $x \in X_0$,
   (c) M *is an invariant subspace for* $n_k^{-1}T^{n_k}$ *for all* $k \geq 1$.

Proof. It is obvious that any operator satisfying the Subspace Cesàro-Hypercyclic Criterion also satisfie the criteria of (2). It suffices to show that (2) implies (1). Let $U_i, V_i \subseteq M$ non-empty open sets with $i = 1, 2$. The same argument as in the proof of Theorem 3.2 in [2] can be used to show that there exist (n_k) of positive integer such that

$$\left(n_k^{-1}T^{n_k}\right)^{-1}(U_i) \bigcap V_i \neq \emptyset, \text{ for } i = 1, 2.$$

Then we can know that $(T \oplus T)$ is subspace cesàro-hypercyclic for $M \oplus M$ and (x, y) is subspace cesàro-hypercyclic vector for $(T \oplus T)$. In particular, x be subspace cesàro-hypercyclic vector for T and $CH(T, M) \bigcap M$ is a dense G_δ subset of M. Let (U_k) be a base of 0-neighborhoods in M. Then there exist (n_k) of positive integer such that

$$n_k^{-1} T^{n_k} x \in U_k \text{ and } n_k^{-1} T^{n_k} y \in x + U_k \text{ for all } k \geq 1.$$

This implies that $n_k^{-1} T^{n_k} x \to 0$ and $n_k^{-1} T^{n_k} y \to x$. Let $M_0 = M_1 = Orb(T, x) \bigcap M$, which is dense G_δ in M. Also for all $k \geq 1$ define

$$S_{n_k}(n^{-1} T^n x) = n^{-1} T^n y.$$

Note that

$$n_k^{-1} T^{n_k} S_{n_k}(n^{-1} T^n x) = n_k^{-1} T^{n_k}(n^{-1} T^n y) = n^{-1} T^n \Big(n_k^{-1} T^{n_k} y \Big) \to n^{-1} T^n x.$$

Hence (1) holds. We complete the proof. □

Proposition 2.1. *Let $T \in B(\mathcal{H})$ satisfy the subspace cesàro-hypercyclicity criterion with respect to a sequence (n_k). Then T is subspace cesàro-mixing.*

Proof. We show that T is subspace cesàro-mixing. Let M_0 and M_1 be dense sets in M, that are given in the subspace cesàro-hypercyclicity criterion. Let U and V are two nonempty open sets in M, then choose $x \in M_0 \cap V$ and $y \in U \cap M_1$ and $\varepsilon > 0$ such that $B(x, \varepsilon) \subset V$ and $B(y, \varepsilon) \subset U$. By Theorem 2.3, there exist $k_0 \in \mathbb{N}^*$ so that for all $k \geq k_0$, $\|\frac{T^{n_k}}{n_k} x\| \leq \varepsilon, \|S_{n_k}(y)\| \leq \varepsilon$, and $\|\frac{T^{n_k}}{n_k} S_{n_k}(y) - y\| \leq \varepsilon$. Then for each $k \geq k_0$ we have $z_k = x + S_{n_k} y \in B(x, \varepsilon) \subset V$ and $\frac{T^{n_k}}{n_k} z_k \in B(y, \varepsilon) \subset U$. That is, $\frac{T^{n_k}}{n_k}(V) \cap U \neq \emptyset$, $\forall k \geq k_0$. Hence T is subspace cesàro-mixing. □

Theorem 2.5. *Let T and S in $B(\mathcal{H})$ and M_1, M_2 be a nonzero closed subspaces of $\mathcal{H}$ and $(T \oplus S)$ is $(M_1 \oplus M_2)$-cesàro-mixing operator, then T and S are M_1-cesàro-mixing and M_2-cesàro-mixing operators, respectively.*

Proof. let U_1 and U_2 be open sets in M_1, and V_1 and V_2 be open sets in M_2, then $U_1 \oplus V_1$ and $U_2 \oplus V_2$ are open in $M_1 \oplus M_2$. So there exists an $n_0 \geq 1$ such that

$$\Big(n^{-1} \, (T \oplus S) \Big)^{-1} (U_1 \oplus V_1) \cap (U_2 \oplus V_2) \neq \emptyset,$$

and

$$\Big(n^{-1} \, (T \oplus S) \Big) (M_1 \oplus M_2) \subseteq (M_1 \oplus M_2)$$

for all $n \geq n_0$. Then

$$\Big(n^{-1} T^n \Big)^{-1} (U_1) \bigcap U_2 \neq \emptyset, \; \Big(n^{-1} S^n \Big)^{-1} (V_1) \bigcap V_2 \neq \emptyset, \; \Big(n^{-1} T^n \Big) (M_1) \subset M_1$$

and

$$\Big(n^{-1} S^n \Big) (M_2) \subset M_2.$$

Therefore, T and S are M_1-cesàro-mixing and M_2-cesàro-mixing operators, respectively. □

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