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A New Mood's Median Test for Imprecise Data

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ABSTRACT. The existing Mood's median test is a non-parametric test used to compare two or more sample data sets whether they are from the same population or not. There can be no application of this test to uncertain and indeterminate data. Therefore, it is necessary to find a generalization of this test that will enable us to apply it in uncertain environments. This study will present a new approach that utilizes neutrosophic statistics to apply Mood's median test. The approach involves defining hypotheses, determining a decision rule, and performing the test in an uncertain environment. An evaluation of the performance of the proposed test will be conducted using numerical examples. The results reveal that the proposed test under neutrosophic statistics is more informative, efficient, and flexible than the existing test under classical statistics in the presence of uncertain data.

1. INTRODUCTION

Nonparametric tests are techniques used in statistics for the analysis of data without specifying a particular distribution to meet the required assumptions to be analysed. For this reason, they are sometimes referred to as distribution-free tests. A wide variety of nonparametric tests have been used in the literature on statistical experimental design. One of the most useful non-parametric tests is Mood's median test, which is used to determine whether

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the medians of two or more independent samples are the same. In this context, Mood's median test was first introduced by Mood [1]. Brown and Mood [2] discussed median tests for linear hypotheses. Mood [3] studied the asymptotic efficiency of certain nonparametric two-sample tests. Fligner and Rust [4] described a modification of Mood's median test which allows one to test for differences between two medians without making any assumptions about the shape of the underlying populations. In a more recent article, Chen [5] proposed an extension of Mood's median test for survival data. Various articles and books discussing the Mood's median test can be found in ([6], [7], [8]). As mentioned above, the researchers applied this test to determinant datasets; however, experimenters are sometimes confronted with indeterminate data under uncertain conditions. Therefore, it is therefore necessary to look for an appropriate generalization of this test so that we may use indeterminate data in an uncertain environment.

Neutrosophic logic is an extension of fuzzy logic that provides information about the measure of indeterminacy, whereas fuzzy logic does not. Smarandache [9] claimed that the neutrosophic logic is more efficient than of fuzzy logic. Smarandache [10] demonstrated that neutrosophic statistics (NS) may be regarded as a generalization of classical statistics. A discussion on the differences between fuzzy statistics, NS, and classical statistics has been made by Aslam [11]. Aslam [12] in his publication highlighted the use of neutrosophic ANOVA. In a newer paper, AlAita and Aslam [13] discussed neutrosophic analysis of covariance with respect to completely randomized designs, randomized complete block designs, and split-plot designs. The neutrosophic sign test was applied to COVID-19 data by Sherwani, et al. [14]. The neutrosophic Kruskal Wallis H test was used to analyze COVID-19 data by Sherwani, et al. [15]. Aslam and Aldosari [16] presented a discussion about analyzing alloy melting points data using the Mann-Whitney test. Miari, et al. [17] suggested single valued neutrosophic Kruskal-Wallis and Mann Whitney tests. The median test for interval-valued data was highlighted by Grzegorzewski and Śpiewak [18]. Aslam and Saleem [19] introduced the F-test of testing linearity under neutrosophic statistics. Aslam, et al. [20] examined the monitoring of temperature using a moving average under uncertainty. Afzal and Aslam [21] presented a study of the use of neutrosophic statistics to analyze changes in blood pressure during pregnancy in women. An analysis of the temperature data of five different cities in Pakistan was conducted by Shahzadi [22] using the neutrosophic statistics. In the context of neutrosophic statistics, several statistical tests have been discussed ([23], [24], [25], [26]).

In complex systems, all observations are not determinable, thus the existing Mood's median test cannot be applied. It is the principal objective of this research to solve problems associated with studies that require the application of the neutrosophic mood's median test in instances of imprecise and uncertain data. For example, a study was conducted on women and men in Pakistan to analyze daily ICU occupancy data representing Corona-positive patients under NS. During an experiment, the experimenter often encounters numerous instances in which the results are uncertain. Therefore, it is not possible to apply classical tests to analyze this data. Accordingly, the neutrosophic mood's median test is necessary for this example.

In our literature review, and to the best of our knowledge, there are no studies on Mood's median test under NS. In this paper, we will introduce neutrosophic Mood's median test for the first time. We will determine the neutrosophic hypotheses and decision rule. Moreover, we will compute χ_N^2-test for the proposed test under uncertain conditions. Based on real data, we will evaluate the performance of the proposed test. Considering neutrosophic Mood's median test as an alternative to the existing test in the presence of uncertainty, we can conclude that the proposed test is more informative and efficient than those under CS.

The remainder of this paper is organized as follows: In the following section, we discuss some basic concepts under NS. Section 3 provides Mood's median test under NS. Section 4 presents and explains numerical examples. Section 5 introduces comparison analysis. Section 6 presents conclusions.

2. PRELIMINARIES

Suppose that a neutrosophic random variable (NRV) $X_N \in [X_L, X_U]$ hasn't particular neutrosophic distribution, where X_L and X_U are smaller and larger values of indeterminacy interval. Let $X_N = X_L + X_U I_N$ is the neutrosophic form of NRV having determinate part X_L and indeterminate part $X_U I_N$; $I_N \in [I_L, I_U]$, where $I_N \in [I_L, I_U]$ is measure of uncertainty.

Suppose n is a neutrosophic random sample selected from a population of size N having indeterminate observations. The neutrosophic population median θ_N is expressed as follows;

If n is odd, $\theta_N(x) = x_{N(\frac{n+1}{2})}$.

If n is even, $\theta_N(x) = \frac{x_{N(\frac{n}{2})} + x_{N(\frac{n}{2}+1)}}{2}$.

The neutrosophic form of $\theta_N(x)$ is $\theta_N(x) = \theta_L(x) + \theta_U(x) I_N;\ I_N \in [I_L, I_U]$.

3. MOOD'S MEDIAN TEST UNDER NEUTROSOPHIC STATISTICS

In neutrosophic statistics, neutrosophic nonparametric tests refer to methods of statistical analysis that require no particular distribution to be specified (especially when data are not normally distributed). The neutrosophic nonparametric tests can be used as an alternative to neutrosophic parametric tests, such as the neutrosophic T-test or NANOVA, if the underlying neutrosophic data fits certain criteria and assumptions. According to the literature on neutrosophic nonparametric tests, many statistical neutrosophic tests have been studied (e.g., neutrosophic Kruskal Wallis H test, neutrosophic sign test, etc.). Our proposed test is similar to the sign test. The main objective of the proposed neutrosophic Mood's median test is to compare two or more terms of population medians when the data contain neutrosophic numbers.

Under neutrosophic statistics, the neutrosophic Mood's median test is used to test the null hypothesis that all two or more samples have equal medians against the alternative hypothesis that the medians population are not equal. In other words; Suppose θ_{Ni} represents the population median for the ith population, the neutrosophic null and alternative hypotheses are formulated as follows;

$H_{N0}: \theta_{N1} = \theta_{N2} = \cdots = \theta_{Nk}$ vs H_{N1}: At least one median differs.

This test can be performed by following the steps below:

Step 1: Compute the median θ_N of the samples.

Step 2: Establish a $2 \times k$ contingency table consisting of the number of elements that are greater than θ_N in each sample and of the number of elements that are less than or equal to θ_{Ni} in each sample. Both samples are represented by columns. The frequencies of neutrosophic data is shown in Table 1.

TABLE 1. Frequencies of neutrosophic data

	Sample 1	Sample 2	…	Sample k	Total
>Median	O_{N11}	O_{N12}	…	O_{N1k}	$n_{N1.}$
≤Median	O_{N23}	O_{N24}	…	O_{N2k}	$n_{N2.}$
Total	n_{N1}	n_{N2}	…	n_{Nk}	n_N

$$E_{Nij} = \frac{n_{Ni.} \times n_{Nj}}{n_N}, i = 1,2; j = 1,2, \dots, k.$$

Step 3: Conduct a chi-square test of independence.

$$\chi_N^2 = \sum \frac{\left(O_{Nij} - E_{Nij}\right)^2}{E_{Nij}} = [\chi_L^2, \chi_U^2], \quad (1)$$

where O_{Nij} represents the observed neutrosophic frequencies, E_{Nij} represents the expected neutrosophic frequencies, where $i = 1,2,\ j = 1,2,\dots,k$. The degrees of freedom are $k-1$.

Step 4: Compute the $p_N - value$.

Step 5: According to Smarandache [10], the neutrosophic decision rule can be summarized as follows:

If $min\{p_N - value\} > \alpha$, then we accept the null hypothesis H_{N0} at the level α.

If $max\{p_N - value\} \leq \alpha$, then we reject the null hypothesis H_{N0} at the level α.

If $min\{p_N - value\} < \alpha < max\{p_N - value\}$, then there is indeterminacy. Thus $\frac{\alpha - min\{p_N - value\}}{max\{p_N - value\} - min\{p_N - value\}}$ represents the chance to reject H_{N0} at the level α, and $\frac{max\{p_N - value\} - \alpha}{max\{p_N - value\} - min\{p_N - value\}}$ represents the chance to accept H_{N0} at the level α.

4. NUMERICAL EXAMPLES

In this section, we numerically assess the performance of our proposed test by examples using uncertain observations. For assessing the efficiency of the proposed test, the proposed test is calculated and compared with the existing test under classical statistics in terms of a measure of uncertainty.

Example 4.1. A study was conducted on the melting points of two alloys based upon not precise observations. Assume that each observation within and between samples is independent and symmetrical distribution. The purpose of this study is to determine whether the melting point of the first alloy (x) is much higher than that of the second alloy (y). Table 2 summarizes the data. For more information see [18]. Data frequencies of melting points of two alloys are shown in Table 3.

TABLE 2. Measurement of melting point of two alloys

	$\boldsymbol{x}$	$\boldsymbol{y}$
1	[545.5, 563.3]	[426.1, 444.1]
2	[511.6, 529.4]	[406.7, 430.5]
3	[503.5, 523.1]	[387.3, 407.2]
4	[449.2, 470.1]	[450.9, 475.8]
5	[489.0, 506.7]	[440.2, 458.5]
6	[479.1, 495.6]	[490.6, 507.7]
7	[467.9, 495.3]	[480.2, 496.8]
8	[495.6, 520.9]	[503.8, 520.9]
9	[472.8, 496.9]	[482.8, 503.6]
10	[519.1, 542.9]	[432.8, 458.2]
11	[484.0, 505.4]	[453.3, 480.5]
12	[525.9, 550.7]	[446.9, 473.8]
13	[500.9, 517.7]	[451.2, 468.4]
14	[483.0, 499.2]	[466.1, 482.2]
15	[485.0, 500.6]	[459.4, 477.0]
16	[499.6, 516.8]	[479.4, 496.3]
17	[515.1, 535.0]	
18	[464.4, 489.3]	
$Median$	[492.30, 511.75]	[452.25, 476.40]

The neutrosophic null and alternative hypotheses are

$H_{N0}: \theta_{N\mathrm{x}} = \theta_{N\mathrm{y}}$ vs $H_{N1}: \theta_{N\mathrm{x}} > \theta_{N\mathrm{y}}$

$Median = [479.80, 496.85]$.

$\chi_N^2 = [7.556, 11.806]$.

TABLE 3. Data frequencies of melting points of two alloys

	x	y	Total
$> Median$	[13, 14]	[4, 3]	[17, 17]
$\leq Median$	[5, 4]	[12, 13]	[17, 17]
Total	[18, 18]	[16, 16]	[34, 34]

The neutrosophic form of the proposed test-χ_N^2 can be expressed as $\chi_N^2 = 7.556 + 11.806 I_N$; $I_N \in [0,0.360]$.

$p_N - value = [0.006, 0.001] < 0.05$. Then we reject the null hypothesis H_{N0} and accept H_{N1} at the level $\alpha = 0.05$. That is, the melting point of the first alloy (x) is much higher than that of the second alloy (y).

Example 4.2. To apply the proposed neutrosophic Mood's median test, daily ICU occupancy data representing Corona-positive patients from Pakistan have been analyzed. In this study, the hypothesis being tested is whether there is a statistically significant difference in ICU occupancy of Covid-19 patients according to their age group. The data shown in Table 4 is uncertainty data. This study uses the neutrosophic Mood's median test to test the null hypothesis that there are no differences in the daily occupancy of Covid-19 patients from different age groups in Pakistan for the month of December 2020. There are three categories of age groups in Covid-19 for the daily occupancy of ICUs (35 years and below, 35 to 55 years, and 55 years and above). For more information see [15]. The data frequency of Covid-19 patients is shown in Table 5.

The neutrosophic null and alternative hypotheses are

$H_{N0}: \theta_{N1} = \theta_{N2} = \theta_{N3}$ vs H_{N1}: At least one median differs.

$Median = [405, 410]$.

$\chi_N^2 = [14.800, 17.753]$.

The neutrosophic form of the proposed test-χ_N^2 can be expressed as $\chi_N^2 = 14.800 + 17.753 I_N$; $I_N \in [0,0.166]$.

$p_N - value = [0.001, 0.000] < 0.05$. Then we reject the null hypothesis H_{N0} and accept H_{N1} at the level $\alpha = 0.05$. That is, there is a difference in the medians of the samples.

5. COMPAISON ANALYSIS USING THE NEUTROSOPHIC FORM OF MOOD'S MEDIAN TEST

In this section, we numerically assess the performance of our proposed test data analysis through the previous examples. In order to determine the efficiency of the proposed test, we calculate and compare the proposed Mood's median test in terms of a measure of uncertainty with the existing Mood's median test under classic statistics.

TABLE 4. Daily ICU occupancy data representing Corona-positive patients from Pakistan

Day	**Age**		
	35 and below	**35-55**	**55 and above**
1	[460, 465]	[359, 361]	[443, 450]
2	[427, 429]	[352, 365]	[421, 426]
3	[407, 410]	[445, 455]	[436, 450]
4	[378, 380]	410	[376, 385]
5	[364, 368]	[458, 464]	458
6	[345, 349]	[410, 415]	[408, 420]
7	[342, 346]	[463, 470]	[422, 425]
8	345	[580, 584]	[431, 440]
9	[313, 318]	[432, 440]	[459, 462]
10	[277, 280]	379	369
11	[268, 271]	370	360
12	[259, 262]	[584, 589]	[431, 445]
13	[256, 260]	[410, 416]	[403, 415]
14	251	[587, 590]	[436, 445]
15	249	415	376
16	[233, 227]	[419, 422]	370
17	[209, 211]	357	443
15	[187, 191]	[467, 472]	445
19	[173, 175]	[415, 418]	[355, 365]
20	168	358	450
Median	[272.5, 275.5]	[415, 417]	[426.5, 433]

TABLE 5. Data frequencies of Covid-19 patients

	35 and below	35-55	55 and above	Total
>Median	[13, 14]	[14, 13]	[3, 2]	[30, 29]
≤Median	[7, 6]	[6, 7]	[17, 18]	[30, 31]
Total	[20, 20]	[20, 20]	[20, 20]	[60, 60]

According to what was mentioned earlier, the proposed test represents a generalization of Mood's median test. The neutrosophic logic literature has indicated that a method based on data in an indeterminate interval is more effective and suitable for use in uncertainty than determined values under classical statistics. The neutrosophic form of χ_N^2-test is $\chi_N^2 = \chi_L^2 + \chi_U^2 I_N;\ I_N \in [I_L, I_U]$, where the first part χ_L^2 is known as the determined part and presents the value of χ^2-test under CS. The second part $\chi_U^2 I_N;\ I_N \in [I_L, I_U]$ is known as the indeterminate part. Note that the χ_N^2-test reduces to χ^2-test under classical statistics if $I_{F_N} = 0$. This means that the NS approach provides the χ_N^2-test values in an interval with the measure of indeterminacy which is more general and includes the determinate part of the CS. For example, in the Example 1, the neutrosophic form of χ_N^2-test is $\chi_N^2 = 7.556 + 11.806 I_N;\ I_N \in [0, 0.360]$. It means that the proposed χ_N^2-test indicate that χ_N^2 ranges between 7.556 and 11.806 with the degree of indeterminacy 0.360. Therefore, in light of the comparisons, it is concluded that the proposed test under NS is more informative than the existing test under CS.

6. CONCLUSIONS

The use of classical statistics in analysing indeterminate data due to uncertain experimental measurements may result in incorrect conclusions regarding Mood's median test. The Mood's median test with indeterminate observations was considered in light of this study, and a neutrosophic statistical approach was suggested for analysing the data. We formulated neutrosophic hypotheses, a neutrosophic decision rule, and a test statistic. An illustration of the proposed test application was provided through real-life examples. This study indicates that the proposed approach outperforms the classical approach in terms of accuracy, flexibility, and generality of analysis under uncertain circumstances. We hope the proposed test will be used in numerous future scientific experiments.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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