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A Study of Fixed Point Results for Incompatible Mappings in Neutrosophic Double Controlled Metric Spaces with Application

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Abstract. We looked at and illustrated a few axioms of $\mathfrak{NDCMS}$ (Neutrosophic double controlled metric space) in this article. As a way to generalise the Banach contraction principle in the earlier mentioned spaces, we employed $\mathfrak{NDCMS}$. For the purpose of reviewing what we discovered, we graphically validated several examples and supported some findings. Furthermore, we provide evidence of usage and implemented it by proving their presence with a distinctive and integrative solution.

1. Introduction

Banach [3] reached some important discoveries that laid the framework for the abstraction of FP hypothesis. The idea of a fuzzy set was initially coined by Zadeh [16]. The fuzzy set term aided us in understanding the degree of ambiguity in items using a mathematical technique.The term fuzzy metric space ($\mathfrak{FMS}$) was invented by Kramosil and Michalek [11]. Schweizer and Sklar [15] invented the term "continuous (CTS) t- norms". The Banach contraction principle is widely acknowledged for playing a significant role in the conceptualization of $\mathfrak{FMS}$. Grabiec [7] talked about a hazy interpretation of the Banach contraction principle. Park [13] created a fantastic effort intuitionistic fuzzy metric space. In 1998, Smarandache developed the term "neuosophic set" and showed it with Sowndrarajan [17], they demonstrated some important findings from $\mathfrak{NMS}$. Kirisci and Simsek [9] then devised the topic of $\mathfrak{NMS}$ in 2019. Some $\mathfrak{FP}$ findings in $\mathfrak{NMS}$

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were confirmed by Sowdrarajan and Jeyaraman et al [10] in 2020. Although the Neutrosophic tackles naturalness, the intuitionistic and fuzzy do not. Milaiki [1] presented the abstract of controlled metric space ($\mathfrak{CMS}$), and Sezen [16] described the term controlled fuzzy metric space ($\mathfrak{CFMS}$), and both demonstrated various contraction mappings outcomes. Abdeljawad et al. [1] characterised Double controlled metric space ($\mathfrak{DCMS}$) as a space that comprises two irreconcilable transformations. Some further extensions of contraction mappings and metric spaces can be found in [4,8,14].

This research looked at the conviction of $\mathfrak{NDCMS}$. Graphics were utilised to show some outcomes and validate many examples. We also provide execution of the results. We utilised it to show the significance of an integral solutions as well as the occurrence of a unique solution.

2. Preliminaries

Authors provide an essential context for the lucidity of the content along with to make it simpler for people to work by comprehending the major portion in this section.

Definition 2.1. *[10] A 6-tuple $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ is called $\mathfrak{NMS}$ if $\mathfrak{K}$ is an arbitrary non empty set, $\star$ neutrosophic CTN, $\diamond$ neutrosophic CTC and $\mathfrak{A}, \mathfrak{I}, \mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{K} \times (0, \infty)$ satisfying the following condition: For all $\varsigma, \Theta, \eta \in \mathfrak{K}, \mathfrak{z} \in (0, \infty)$*
a) $0 \leq \mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) \leq 1; 0 \leq \mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) \leq 1; 0 \leq \mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) \leq 1;$
b) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) + \mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) + \mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) \leq 3;$
c) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = 1, \forall\, \mathfrak{z} > 0, \Leftrightarrow \varsigma = \Theta;$
d) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{A}(\Theta, \varsigma, \mathfrak{z});$ *for* $\mathfrak{z} > 0$
e) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) \star \mathfrak{A}(\Theta, \eta, \mathfrak{x}) \geq \mathfrak{A}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \forall\, \mathfrak{z}, \mathfrak{x} > 0$
f) $\mathfrak{A}(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is neutrosophic CTS and* $\lim_{\mathfrak{z} \to +\infty} \mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = 1$
g) $\mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) = 0, \forall\, \mathfrak{z} > 0, \Leftrightarrow \varsigma = \Theta;$
h) $\mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{I}(\Theta, \varsigma, \mathfrak{z});$ *for* $\mathfrak{z} > 0$
i) $\mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) \star \mathfrak{I}(\Theta, \eta, \mathfrak{x}) \leq \mathfrak{I}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \forall\, \mathfrak{z}, \mathfrak{x} > 0$
j) $\mathfrak{I}(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is neutrosophic CTS and* $\lim_{\mathfrak{z} \to +\infty} (\varsigma, \Theta, \mathfrak{z}) = 0$
k) $\mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) = 0, \forall\, \mathfrak{z} > 0, \Leftrightarrow \varsigma = \Theta;$
l) $\mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{D}(\Theta, \varsigma, \mathfrak{z});$ *for* $\mathfrak{z} > 0$
m) $\mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) \star \mathfrak{D}(\Theta, \eta, \mathfrak{x}) \leq \mathfrak{D}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \forall\, \mathfrak{z}, \mathfrak{x} > 0$
n) $\mathfrak{D}(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is neutrosophic CTS and* $\lim_{\mathfrak{z} \to +\infty} \mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) = 0$
Then, $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ *is called a* $\mathfrak{NMS}$.

Definition 2.2. *[17]Given* $\urcorner$*, let* $\mathfrak{K}$ *be a non empty set and* $\urcorner : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty)$ *are incompetent mapping, if* $\delta : \mathfrak{K} \times \mathfrak{K} \to (0, +\infty)$ *is called as a Controlled metric type (CMS) if*
a) $\delta(\varsigma, \Theta) = 0$ *iff* $\varsigma = \Theta;$
b) $\delta(\varsigma, \Theta) = \delta(\Theta, \varsigma);$
c) $\delta(\varsigma, \Theta) \leq \urcorner(\varsigma, \eta)\delta(\varsigma, \eta) + \urcorner(\eta, \Theta)\delta(\eta, \Theta);$ *for every* $\varsigma, \Theta, \eta \in \mathfrak{K}$.

Definition 2.3. *[5] Let $\mathfrak{K}$ be a non empty set and $\urcorner : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty), \star$ neutrosophic CTN, $\diamond$ neutrosophic CTC and $\mathfrak{A}, \mathfrak{I}, \mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{K} \times (0, \infty)$ satisfying the following condition: For all $\varsigma, \Theta, \eta \in \mathfrak{K}$,*

a) $0 \leq \mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) \leq 1; 0 \leq \mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) \leq 1; 0 \leq \mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) \leq 1;$

b) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) + \mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) + \mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) \leq 3;$

c) $\mathfrak{A}(\varsigma, \Theta, 0) = 0$

d) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = 1, \forall\, \mathfrak{z} > 0, \Leftrightarrow \varsigma = \Theta;$

e) $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{A}(\Theta, \varsigma, \mathfrak{z});$

f) $\mathfrak{A}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \geq \mathfrak{A}\left(\varsigma, \Theta, \frac{\mathfrak{z}}{\urcorner(\varsigma, \Theta)}\right) \star \mathfrak{A}\left(\Theta, \eta, \frac{\mathfrak{x}}{\urcorner(\Theta, \eta)}\right);$

g) $\mathfrak{A}(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS and* $\lim_{\mathfrak{z} \to +\infty} \mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = 1;$

h) $\mathfrak{I}(\varsigma, \Theta, 0) = 1$

i) $\mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) = 0, \forall\, \mathfrak{z} > 0 \Leftrightarrow \varsigma = \Theta;$

j) $\mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{I}(\Theta, \varsigma, \mathfrak{z});$

k) $\mathfrak{I}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \leq \mathfrak{I}\left(\varsigma, \Theta, \frac{\mathfrak{z}}{\urcorner(\varsigma, \Theta)}\right) \diamond \mathfrak{I}\left(\Theta, \eta, \frac{\mathfrak{x}}{(\urcorner(\Theta, \eta))}\right);$

l) $\mathfrak{I}(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS and* $\lim_{\mathfrak{z} \to +\infty} \mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) = 0;$

m) $\mathfrak{D}(\varsigma, \Theta, 0) = 1$

n) $\mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) = 0, \forall\, \mathfrak{z} > 0 \Leftrightarrow \varsigma = \Theta;$

o) $\mathfrak{D}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{I}(\Theta, \varsigma, \mathfrak{z});$

p) $\mathfrak{D}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \leq \mathfrak{I}\left(\varsigma, \Theta, \frac{\mathfrak{z}}{\urcorner(\varsigma, \Theta)}\right) \diamond \mathfrak{I}\left(\Theta, \eta, \frac{\mathfrak{x}}{\urcorner(\Theta, \eta)}\right);$

q) $(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS and* $\lim_{\mathfrak{z} \to +\infty} \mathfrak{I}(\varsigma, \Theta, \mathfrak{z}) = 0;$

Then, $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ *is called a* $\mathfrak{NCMS}$.

Definition 2.4. *[17] Given $\urcorner, \varphi : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty)$ are incompetent mapping, if $\delta : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty)$ fulfils the required prerequisite:*

a) $\delta(\varsigma, \Theta) = 0$ *iff* $\varsigma = \Theta;$

b) $\delta(\varsigma, \Theta) = \delta(\Theta, \varsigma);$

c) $\delta(\varsigma, \Theta) \leq \urcorner(\varsigma, \eta)\delta(\varsigma, \eta) + \varphi(\eta, \Theta)$ *for every* $\varsigma, \Theta, \eta \in \mathfrak{K}$, *then,* $(\mathfrak{K}, \delta)$ *is termed as DCMS.*

Definition 2.5. *[17] Suppose $\mathfrak{K} \neq \emptyset$ and $\urcorner, \varphi : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty)$ provided incomparable mappings, where $\star$ is a CTS t-norm and $\mathfrak{A}$ is a fuzzy set on $\mathfrak{K} \times \mathfrak{K} \times (0, \infty)$ is identified as $\mathfrak{FDCMS}$ on $\mathfrak{K}$, for every $\varsigma, \Theta, \eta \in \mathfrak{K}$ if:*

1. $\mathfrak{A}(\varsigma, \Theta, 0) = 0;$

2. $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = 1$ *for all* $\mathfrak{z} > 0$, *iff* $\varsigma = \Theta;$

3. $\mathfrak{A}(\varsigma, \Theta, \mathfrak{z}) = \mathfrak{A}(\Theta, \varsigma, \mathfrak{z});$

4. $\mathfrak{A}(\varsigma, \eta, \mathfrak{z} + \mathfrak{x}) \geq \mathfrak{A}\left(\varsigma, \Theta, \frac{\mathfrak{z}}{\urcorner(\varsigma, g)}\right) \star \mathfrak{A}\left(\varsigma, \Theta, \frac{\mathfrak{x}}{\varphi(\varsigma, \Theta)}\right)$

5. $\mathfrak{A}(\varsigma, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS. Then,* $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \star)$ *is named as a* $\mathfrak{FDCMS}$.

3. Main results

We have now clarified the meaning of the $\mathfrak{NDCMS}$ and included illustrations to support certain of the arguments.

Definition 3.1. *[15] Suppose $\mathfrak{K} \neq \emptyset$ and $\urcorner, \varphi : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty)$ are considered as a incompetant mappings, $\star$ as t-norm, $\diamond$ as t-conorm, and $\mathfrak{A}, \mathfrak{I}, \mathfrak{D}$ are neutrosophic sets on $\mathfrak{K} \times \mathfrak{K} \times (0, +\infty)$ is characterised $\mathfrak{NDCMS}$ on $\mathfrak{K}$, if for each one $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ fulfills all $\Xi, \Theta, \eta \in \mathfrak{K}$ holds the following:*

(a) $0 \leq \mathfrak{A}(\Xi, \Theta, \zeta) \leq 1; 0 \leq \mathfrak{I}(\Xi, \Theta, \zeta) \leq 1; 0 \leq \mathfrak{D}(\Xi, \Theta, \zeta) \leq 1;$

(b) $\mathfrak{A}(\Xi, \Theta, \zeta) + \mathfrak{I}(\Xi, \Theta, \zeta) + \mathfrak{D}(\Xi, \Theta, \zeta) \leq 3;$

(c) $\mathfrak{A}(\Xi, \Theta, \zeta) = 1, \forall \zeta > 0, \Longleftrightarrow \Xi = \Theta;$

(d) $\mathfrak{A}(\Xi, \Theta, \zeta) = \mathfrak{A}(\Theta, \Xi, \zeta);$

(e) $\mathfrak{A}(\Xi, \eta, \zeta + \mathfrak{x}) \geq \mathfrak{A}\left(\Xi, \Theta, \frac{\zeta}{\urcorner(\Xi,\Theta)}\right) \star \mathfrak{A}\left(\Theta, \eta, \frac{\mathfrak{x}}{\varphi(\eta,\Theta)}\right);$

(f) $\mathfrak{A}(\Xi, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS and* $\lim_{\zeta \to +\infty} \mathfrak{A}(\Xi, \Theta, \zeta) = 1;$

(g) $\mathfrak{I}(\Xi, \Theta, \zeta) = 0, \forall \zeta > 0 \Longleftrightarrow \Xi = \Theta;$

(h) $\mathfrak{I}(\Xi, \Theta, \zeta) = \mathfrak{I}(\Theta, \Xi, \zeta);$

(i) $\mathfrak{I}(\Xi, \eta, \zeta + \mathfrak{x}) \leq \mathfrak{I}\left(\Xi, \Theta, \frac{\zeta}{\urcorner(\Xi,\Theta)}\right) \diamond \mathfrak{I}\left(\Theta, \eta, \frac{\mathfrak{x}}{\varphi(\eta,\Theta)}\right);$

(j) $\mathfrak{I}(\Xi, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS and* $\lim_{\zeta \to +\infty} \mathfrak{I}(\Xi, \Theta, \zeta) = 0;$

(k) $\mathfrak{D}(\Xi, \Theta, \zeta) = 0, \forall \zeta > 0 \Longleftrightarrow \Xi = \Theta;$

(l) $\mathfrak{D}(\Xi, \Theta, \zeta) = \mathfrak{D}(\Theta, \Xi, \zeta);$

(m) $\mathfrak{D}(\Xi, \eta, \zeta + \mathfrak{x}) \leq \mathfrak{D}\left(\Xi, \Theta, \frac{\zeta}{\urcorner(\Xi,\Theta)}\right) \diamond \mathfrak{D}\left(\Theta, \eta, \frac{\mathfrak{x}}{\varphi(\eta,\Theta)}\right);$

(n) $\mathfrak{D}(\Xi, \Theta, .) : (0, +\infty) \to [0, 1]$ *is CTS and* $\lim_{\zeta \to +\infty)} \mathfrak{I}(\Xi, \Theta, \zeta) = 0;$

(o) If $\zeta \leq 0$, *then* $\mathfrak{A}(\Xi, \Theta, \zeta) = 0, \mathfrak{I}(\Xi, \Theta, \zeta) = 1$ *and* $\mathfrak{D}(\Xi, \Theta, \zeta) = 1.$

Then, $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ *is called a* $\mathfrak{NDCMS}$.

Example 3.1. *Let $\mathfrak{K} \neq \emptyset$ and $\mathfrak{K} = \mathfrak{M} \cup \mathfrak{P}$, where $\mathfrak{M} = (0, 3)$ and $\mathfrak{P} = [3, \infty)$ and $\urcorner, \varphi : \mathfrak{K} \times \mathfrak{K} \to [1, +\infty)$ are given incomparable mappings defined as follows:* $\urcorner(\Xi, \Theta) = \begin{cases} 1 & \text{if } \Xi, \Theta \in \mathfrak{M} \\ \max\{\Xi, \Theta\} & \text{otherwise} \end{cases}$ *and*

$$\varphi(\Xi, \Delta) = \begin{cases} \frac{2}{\Xi} & \text{if } \Xi, \Theta \in \mathfrak{M} \\ 1 & \text{otherwise} \end{cases}$$

$\star$ is a CTS t-norm $\varpi \star \hbar = \varpi\hbar$, $\diamond$ be a CTS t-conorm as $\varpi \diamond \hbar = \varpi\hbar$. Define $\mathfrak{A}, \mathfrak{I}, \mathfrak{D} : \mathfrak{K} \times \mathfrak{K} \times (0, +\infty) \to [0, 1]$ as follows :

$$\mathfrak{A}(\Xi, \Delta, \zeta) = \begin{cases} 1 & \text{if } \Xi = \Theta \\ e^{\frac{-3}{\Theta\zeta}} & \text{if } \Xi \in \mathfrak{M} \text{ and } \Theta \in \mathfrak{P} \\ e^{\frac{-4}{\Xi\zeta}} & \text{if } \Xi \in \mathfrak{P} \text{ and } \Theta \in \mathfrak{M} \\ e^{\frac{-3}{\zeta}} & \text{otherwise} \end{cases}$$

$$\mathfrak{I}(\Xi, \Delta, \zeta) = \begin{cases} 0 & \text{if } \Xi = \Theta \\ 1 - e^{\frac{-3}{\Theta\zeta}} & \text{if } \Xi \in \mathfrak{M} \text{ and } \Theta \in \mathfrak{P} \\ 1 - e^{\frac{-4}{\Xi\zeta}} & \text{if } \Xi \in \mathfrak{P} \text{ and } \Theta \in \mathfrak{M} \\ 1 - e^{\frac{-3}{\zeta}} & \text{otherwise} \end{cases}$$

$$\mathfrak{D}(\Xi,\Delta,\zeta)=\begin{cases}0 & if\,\Xi=\Theta\\ e^{\frac{3}{\Theta\zeta}}-1 & if\,\Xi\in\mathfrak{M}\,and\,\Theta\in\mathfrak{P}\\ e^{\frac{4}{\Xi\zeta}}-1 & if\,\Xi\in\mathfrak{P}\,and\,\Theta\in\mathfrak{M}\\ e^{\frac{3}{\zeta}}-1 & otherwise\end{cases}$$

Then, $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ *is called a* $\mathfrak{NDCMS}$*.*

Here, the results except (e), (i) and (m) of above definition are trivially true. Hence, it is essential to prove the exceptional results for to say it as a $\mathfrak{NDCMS}$*.*

Now, we show that the results for the following cases to show that the condition (e), (i) and (m) holds.

Case I: *If* $\eta=\Xi$ *and* $\eta=\Theta$*, then the respective conditions satisfied.*

Case II: *Suppose* $\eta\neq\Xi$ *and* $\eta\neq\Theta$*, subsequently the conditions (e), (i),(m) holds when* $\Xi=\Theta$*.*
If $\Xi\neq\Theta$ *then we get* $\Xi\neq\Theta\neq\eta$*. Now, the conditions (e), (i), (m) satisfied in the following cases:*

	values	*t-norm & t-conorm*	$\mathfrak{A}(\Xi,\Theta,\zeta)$	$\mathfrak{I}(\Xi,\Theta,\zeta)$	$\mathfrak{D}(\Xi,\Theta,\zeta)$
$\Xi,\Theta,\eta\in\mathfrak{M}$ or $\Xi,\Theta,\eta\in\mathfrak{P}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{3}{2},\Theta=\frac{5}{2},\eta=\frac{1}{2}$	$\urcorner(\Xi,\Theta)=1,\varphi(\Theta,\eta)=\frac{2}{\Xi}$	$e^{-1.5}\geq e^{-7}$	$(1-e^{-1.5})\leq(1-e^{-7})$	$(e^{1.5}-1)\leq(e^{7}-1)$
$\Xi,\eta\in\mathfrak{M},\Theta\in\mathfrak{P}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{5}{2},\Theta=4,\eta=\frac{1}{2}$	$\urcorner(\Xi,\Theta)=4,\varphi(\Theta,\eta)=1$	$e^{-0.4}\geq e^{-3.8}$	$(1-e^{-0.4})\leq(1-e^{-3.8})$	$(e^{0.4}-1)\leq(e^{3.8}-1)$
$\Xi,\eta\in\mathfrak{P},\Theta\in\mathfrak{M}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{9}{2},\Theta=\frac{5}{2},\eta=\frac{7}{2}$	$\urcorner(\Xi,\Theta)=\frac{9}{2},\varphi(\Theta,\eta)=1$	$e^{-0.4}\geq e^{-3.6}$	$(1-e^{-0.4})\leq(1-e^{-3.6})$	$(e^{0.4}-1)\leq(e^{3.6}-1)$
$\Xi\in\mathfrak{P},\Theta,\eta\in\mathfrak{M}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{9}{2},\Theta=\frac{5}{2},\eta=\frac{3}{2}$	$\urcorner(\Xi,\Theta)=\frac{9}{2},\varphi(\Theta,\eta)=1$	$e^{-0.4}\geq e^{-4.9}$	$(1-e^{-0.4})\leq(1-e^{-4.9})$	$(e^{0.4}-1)\leq(e^{4.9}-1)$
$\Xi,\Theta\in\mathfrak{M},\eta\in\mathfrak{P}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{1}{2},\Theta=\frac{3}{2},\eta=3$	$\urcorner(\Xi,\Theta)=1,\varphi(\Theta,\eta)=\frac{2}{\Xi}$	$e^{-1.5}\geq e^{-15}$	$(1-e^{-1.5})\leq(1-e^{-15})$	$(e^{1.5}-1)\leq(e^{15}-1)$
$\Xi\in\mathfrak{P},\Theta,\eta\in\mathfrak{M}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{3}{2},\Theta=\frac{7}{2},\eta=\frac{9}{2}$	$\urcorner(\Xi,\Theta)=\frac{7}{2},\varphi(\Theta,\eta)=1$	$e^{-0.4}\geq e^{-3.9}$	$(1-e^{-0.4})\leq(1-e^{-3.9})$	$(e^{0.4}-1)\leq(e^{3.9}-1)$
$\Xi,\Theta\in\mathfrak{P},\eta\in\mathfrak{M}$	$\zeta=1,\mathfrak{x}=1,\Xi=\frac{7}{2},\Theta=\frac{9}{2},\eta=\frac{1}{2}$	$\urcorner(\Xi,\Theta)=\frac{9}{2},\varphi(\Theta,\eta)=1$	$e^{-1.5}\geq e^{-16.5}$	$(1-e^{-1.5})\leq(1-e^{-16.5})$	$(e^{1.5}-1)\leq(e^{16.5}-1)$

Example 3.2. *Let $\mathfrak{Y} = \{1,2,3\}$ and $\tilde{\mathfrak{k}}, \check{\mathfrak{f}} : \mathfrak{Y} \times \mathfrak{Y} \to [1,\infty)$ be duplet incomparable mappings stated as $\tilde{\mathfrak{k}}(\ddot{\gamma},\epsilon) = \ddot{\gamma} + \epsilon + 1$ and $\mathfrak{f}(\ddot{\gamma},\epsilon) = \ddot{\gamma}^2 + \epsilon^2 - 1$. Define $\mathfrak{A}, \mathfrak{I}, \mathfrak{D} : \mathfrak{Y} \times \mathfrak{Y} \times (0,+\infty) \to [0,1]$ as*

$$\mathfrak{A}(\ddot{\gamma},\epsilon,t) = \frac{\max\{\ddot{\gamma},\epsilon\} - \min\{\ddot{\gamma},\epsilon\} + t}{\max\{\ddot{\gamma},\epsilon\}}$$

$$\mathfrak{I}(\ddot{\gamma},\epsilon,t) = \frac{\min\{\ddot{\gamma},\epsilon\} - t}{\max\{\ddot{\gamma},\epsilon\}}$$

$$\mathfrak{D}(\ddot{\gamma},\epsilon,t) = \frac{\min\{\ddot{\gamma},\epsilon\} - t}{\max\{\ddot{\gamma},\epsilon\} - \min\{\ddot{\gamma},\epsilon\} + \dot{t}}$$

Now, the following conditions to be hold with product t-norm $\ddot{\gamma} \star \epsilon = \ddot{\gamma}\epsilon$ and t-conorm as $\ddot{\gamma} \diamond \epsilon = \ddot{\gamma}\epsilon$

$$\mathfrak{A}(\ddot{\gamma},\mathfrak{o},t+s) \geq \mathfrak{A}\left(\ddot{\gamma},\epsilon,\frac{t}{\tilde{\mathfrak{k}}(\ddot{\gamma},\epsilon)}\right) \star \mathfrak{A}\left(\epsilon,\mathfrak{o},\frac{\mathfrak{s}}{\mathfrak{f}(\epsilon,\mathfrak{o})}\right)$$

$$\mathfrak{I}(\ddot{\gamma},\mathfrak{o},t+s) \leq \mathfrak{I}\left(\ddot{\gamma},\epsilon,\frac{t}{\tilde{\mathfrak{k}}(\ddot{\gamma},\epsilon)}\right) \diamond \mathfrak{I}\left(\epsilon,\mathfrak{o},\frac{\mathfrak{s}}{\mathfrak{f}(\epsilon,\mathfrak{o})}\right)$$

$$\mathfrak{D}(\ddot{\gamma},\mathfrak{o},t+s) \leq \mathfrak{D}\left(\ddot{\gamma},\epsilon,\frac{t}{\tilde{\mathfrak{k}}(\ddot{\gamma},\epsilon)}\right) \diamond \mathfrak{D}\left(\epsilon,\mathfrak{o},\frac{\mathfrak{s}}{\mathfrak{f}(\epsilon,\mathfrak{o})}\right)$$

Then, $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ is described as a $\mathfrak{NDCMS}$.
If all Cauchy sequence convergent in $\mathfrak{K}$, then $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ is said to be a Complete $\mathfrak{NDCMS}$.

Theorem 3.1. *Consider $\psi, \xi : \mathfrak{K} \times \mathfrak{K} \to [1, 1/\mathcal{B}^n)$ where $\mathcal{B} \in (0,1)$ and $n \in \mathbb{N}$ are provided with non-comparison functions and $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ be a complete $\mathfrak{NFDCMS}$ and consider that*

$$\lim_{\zeta\to+\infty} \mathfrak{A}(\Xi,\mathfrak{r},\zeta) = 1, \lim_{\zeta\to+\infty} \mathfrak{I}(\Xi,\mathfrak{r},\zeta) = 0, \lim_{\zeta\to+\infty} \mathfrak{D}(\Xi,\mathfrak{r},\zeta) = 0 \ldots \tag{3.1}$$

in which $\Xi, \mathfrak{r} \in \mathfrak{K}$ for each and every $\zeta > 0$. Now, let $\mathfrak{q} : \mathfrak{K} \to \mathfrak{K}$ be a self mapping satisfying

$$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta) \geq \min\{\mathfrak{A}(\Xi,\mathfrak{r},\zeta),\mathfrak{A}(\Xi,\mathfrak{q}\Xi,\zeta),\mathfrak{A}(\mathfrak{r},\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{r},\zeta),\mathfrak{A}(\Xi,\mathfrak{q}\mathfrak{r},\zeta)\}$$

$$\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta) \leq \max\{\mathfrak{I}(\Xi,\mathfrak{r},\zeta),\mathfrak{I}(\Xi,\mathfrak{q}\Xi,\zeta),\mathfrak{I}(\mathfrak{r},\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{r},\zeta),\mathfrak{I}(\Xi,\mathfrak{q}\mathfrak{r},\zeta)\}$$

$$\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta) \leq \max\{\mathfrak{D}(\Xi,\mathfrak{r},\zeta),\mathfrak{D}(\Xi,\mathfrak{q}\Xi,\zeta),\mathfrak{D}(\mathfrak{r},\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{r},\zeta),\mathfrak{D}(\Xi,\mathfrak{q}\mathfrak{r},\zeta)\} \tag{3.2}$$

For each and every $\Xi_y \in \mathfrak{K}$. Let $\lim_{y\to\infty} \psi(\Xi_y,\mathfrak{r})$ and $\lim_{y\to\infty} \xi(\mathfrak{r},\Xi_y)$ exist. Then $\mathfrak{q}$ has a unique $\mathbb{FP}$ in $\mathfrak{K}$.

Proof. Let Ξ_0 be a point of $\mathfrak{K}$. If $\mathfrak{q}\Xi_0 = \Xi_0$, then Ξ_0 is the required $\mathbb{FP}$ and we define a sequence Ξ_y by $\Xi_y = \mathfrak{q}^n\Xi_0 = \mathfrak{q}\Xi_{y-1}, y \in \mathbb{N}$. By utilizing (3.2) for every $\zeta > 0$, we obtain

$$\mathfrak{A}(\Xi_y,\Xi_{y+1},\zeta) = \mathfrak{A}(\mathfrak{q}\Xi_{y-1},\mathfrak{q}\Xi_y,\zeta)$$
$$\geq \min\{\mathfrak{A}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_{y-1},\mathfrak{q}\Xi_{y-1},\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_y,\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\mathfrak{q}\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_{y-1},\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}})\}$$
$$= \min\{\mathfrak{A}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_y,\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_{y-1},\Xi_{y+1},\frac{\zeta}{\mathcal{B}})\}$$
$$= \min\{\mathfrak{A}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}),\mathfrak{A}(\Xi_{y-1},\Xi_{y+1},\frac{\zeta}{\mathcal{B}})\} \geq \mathfrak{A}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}) \tag{3.3}$$

$$\mathfrak{I}(\Xi_y,\Xi_{y+1},\zeta) = \mathfrak{I}(\mathfrak{q}\Xi_{y-1},\mathfrak{q}\Xi_y,\zeta)$$

$$\leq \max\{\mathfrak{I}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_{y-1},\mathfrak{q}\Xi_{y-1},\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_y,\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\mathfrak{q}\Xi_{y-1},\Xi_n,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_{y-1},\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}})\}$$
$$=\max\{\mathfrak{I}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_y,\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_{y-1},\Xi_{y+1},\frac{\zeta}{\mathcal{B}})\}$$
$$=\max\{\mathfrak{I}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}),\mathfrak{I}(\Xi_{y-1},\Xi_{y+1},\frac{\zeta}{\mathcal{B}})\}$$
$$\leq \mathfrak{I}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}) \tag{3.4}$$
$$\mathfrak{D}(\Xi_y,\Xi_{y+1},\zeta)=\mathfrak{D}(\mathfrak{q}\Xi_{y-1},\mathfrak{q}\Xi_y,\zeta)$$
$$\leq \max\{\mathfrak{D}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_{y-1},\mathfrak{q}\Xi_{y-1},\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_y,\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\mathfrak{q}\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_{y-1},\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}})\}$$
$$=\max\{\mathfrak{D}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_y,\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_{y-1},\Xi_{y+1},\frac{\zeta}{\mathcal{B}})\}$$
$$=\max\{\mathfrak{D}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}),\mathfrak{D}(\Xi_{y-1},\Xi_{y+1},\frac{\zeta}{\mathcal{B}})\}$$
$$\leq \mathfrak{D}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}) \tag{3.5}$$

Now, $\mathfrak{A}(\Xi_y,\Xi_{y+1},\zeta) \geq \mathfrak{A}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathfrak{B}})$, $\mathfrak{I}(\Xi_y,\Xi_{y+1},\zeta) \leq \mathfrak{I}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathfrak{B}})$, $\mathfrak{D}(\Xi_y,\Xi_{y+1},\zeta) \leq \mathfrak{D}(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathfrak{B}})$,
then, we conclude that $\Xi_y=\Xi_{y+1}$ for all $y\in\mathbb{N}$ and Ξ_y is a $\mathbb{FP}$ of $\mathfrak{q}$. It implies that,

$$\begin{aligned}\mathfrak{A}(\Xi_y,\Xi_{y+1},\zeta) &\geq \mathfrak{A}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathfrak{B}}) \geq \mathfrak{A}\left(\Xi_{y-2},\Xi_{y-1},\frac{\zeta}{\mathcal{B}}\right)\\ &\geq \mathfrak{A}\left(\Xi_{y-3},\Xi_{y-2},\frac{\zeta}{\mathcal{B}^2}\right)\ldots\cdots \geq \mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\zeta}{\mathcal{B}^y}\right), \end{aligned}\tag{3.6}$$
$$\begin{aligned}\mathfrak{I}(\Xi_y,\Xi_{y+1},\zeta) &\leq \mathfrak{I}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathfrak{B}}) \leq \mathfrak{I}\left(\Xi_{y-2},\Xi_{y-1},\frac{\zeta}{\mathcal{B}}\right)\\ &\leq \mathfrak{I}\left(\Xi_{y-3},\Xi_{y-2},\frac{\zeta}{\mathcal{B}^2}\right)\ldots\cdots \leq \mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\zeta}{\mathcal{B}^y}\right), \end{aligned}\tag{3.7}$$
$$\begin{aligned}\mathfrak{D}(\Xi_y,\Xi_{y+1},\zeta) &\leq \mathfrak{D}(\Xi_{y-1},\Xi_y,\frac{\zeta}{\mathfrak{B}}) \leq \mathfrak{D}\left(\Xi_{y-2},\Xi_{y-1},\frac{\zeta}{\mathcal{B}}\right)\\ &\leq \mathfrak{D}\left(\Xi_{y-3},\Xi_{y-2},\frac{\zeta}{\mathcal{B}^2}\right)\ldots\cdots \leq \mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\zeta}{\mathcal{B}^y}\right). \end{aligned}\tag{3.8}$$

For any $y,z\in\mathbb{N}$, then

$$\mathfrak{A}(\Xi_y,\Xi_{y+z},\zeta)$$
$$\geq \mathfrak{A}\left(\Xi_y,\Xi_{y+1},\frac{\frac{\zeta}{2}}{\psi(\Xi_y,\Xi_{y+1}}\right)\star\mathfrak{A}\left(\Xi_{y+1},\Xi_{y+z},\frac{\frac{\zeta}{2}}{\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$
$$\geq \mathfrak{A}\left(\Xi_y,\Xi_{y+1},\frac{\frac{\zeta}{2}}{\psi(\Xi_y,\Xi_{y+1})}\right)\star\mathfrak{A}\left(\Xi_{y+1},\Xi_{y+2},\frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$
$$\star\,\mathfrak{A}\left(\Xi_{y+2},\Xi_{y+z},\frac{\frac{\zeta}{2^2}}{\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\geq \mathfrak{A}\left(\Xi_y, \Xi_{y+1}, \frac{\frac{\zeta}{2}}{\psi(\Xi_y, \Xi_{y+1})}\right) \star \mathfrak{A}\left(\Xi_{y+1}, \Xi_{y+2}, \frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1}, \Xi_{y+2})\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\star \mathfrak{A}\left(\Xi_{y+2}, \Xi_{y+3}, \frac{\frac{\zeta}{2^3}}{\psi(\Xi_{y+2}, \Xi_{y+3})\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})}\right)$$

$$\star \mathfrak{A}\left(\Xi_{y+3}, \Xi_{y+z}, \frac{\frac{\zeta}{2^3}}{\xi(\Xi_{y+2}, \Xi_{y+z})\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+3}, \Xi_{y+z})}\right)$$

$$\geq \mathfrak{A}\left(\Xi_y, \Xi_{y+1}, \frac{\frac{\zeta}{2}}{\psi(\Xi_y, \Xi_{y+1})}\right) \star \mathfrak{A}\left(\Xi_{y+1}, \Xi_{y+2}, \frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1}, \Xi_{y+2})\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\star \mathfrak{A}\left(\Xi_{y+2}, \Xi_{y+3}, \frac{\frac{\zeta}{2^3}}{\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})\psi(\Xi_{y+2}, \Xi_{y+3})}\right)$$

$$\star \mathfrak{A}\left(\Xi_{y+3}, \Xi_{y+4}, \frac{\frac{\zeta}{2^4}}{\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})\xi(\Xi_{y+3}, \Xi_{y+z})\psi(\Xi_{y+3}, \Xi_{y+4})}\right)$$

$$\star \cdots \star \mathfrak{A}\left(\Xi_{y+z-2}, \Xi_{y+z-1}, \frac{\frac{\zeta}{2^{y+z-1}}}{\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})\ldots\xi(\Xi_{y+z-2}, \Xi_{y+z})\psi(\Xi_{y+z-2}, \Xi_{y+z-1})}\right)$$

$$\star \mathfrak{A}\left(\Xi_{y+z-1}, \Xi_{y+z}, \frac{\frac{\zeta}{2^{y+z+1}}}{\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})\ldots\xi(\Xi_{y+z-1}, \Xi_{y+z})}\right)$$

$$\mathfrak{I}(\Xi_y, \Xi_{y+z}, \zeta)$$

$$\leq \mathfrak{I}\left(\Xi_y, \Xi_{y+1}, \frac{\frac{\zeta}{2}}{\psi(\Xi_y, \Xi_{y+1}}\right) \diamond \mathfrak{I}\left(\Xi_{y+1}, \Xi_{y+z}, \frac{\frac{\zeta}{2}}{\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\leq \mathfrak{I}\left(\Xi_y, \Xi_{y+1}, \frac{\frac{\zeta}{2}}{\psi(\Xi_y, \Xi_{y+1})}\right) \diamond \mathfrak{I}\left(\Xi_{y+1}, \Xi_{y+2}, \frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1}, \Xi_{y+2})\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\diamond \mathfrak{I}\left(\Xi_{y+2}, \Xi_{y+z}, \frac{\frac{\zeta}{2^2}}{\xi(\Xi_{y+2}, \Xi_{y+z})\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\leq \mathfrak{I}\left(\Xi_y, \Xi_{y+1}, \frac{\frac{\zeta}{2}}{\psi(\Xi_y, \Xi_{y+1})}\right) \diamond \mathfrak{I}\left(\Xi_{y+1}, \Xi_{y+2}, \frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1}, \Xi_{y+2})\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\diamond \mathfrak{I}\left(\Xi_{y+2}, \Xi_{y+3}, \frac{\frac{\zeta}{2^3}}{\psi(\Xi_{y+2}, \Xi_{y+3})\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})}\right)$$

$$\diamond \mathfrak{I}\left(\Xi_{y+3}, \Xi_{y+z}, \frac{\frac{\zeta}{2^3}}{\xi(\Xi_{y+2}, \Xi_{y+z})\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+3}, \Xi_{y+z})}\right)$$

$$\leq \mathfrak{I}\left(\Xi_y, \Xi_{y+1}, \frac{\frac{\zeta}{2}}{\psi(\Xi_y, \Xi_{y+1})}\right) \diamond \mathfrak{I}\left(\Xi_{y+1}, \Xi_{y+2}, \frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1}, \Xi_{y+2})\xi(\Xi_{y+1}, \Xi_{y+z})}\right)$$

$$\diamond \mathfrak{I}\left(\Xi_{y+2}, \Xi_{y+3}, \frac{\frac{\zeta}{2^3}}{\xi(\Xi_{y+1}, \Xi_{y+z})\xi(\Xi_{y+2}, \Xi_{y+z})\psi(\Xi_{y+2}, \Xi_{y+3})}\right)$$

$$\diamond\mathfrak{I}\left(\Xi_{y+3},\Xi_{y+4},\frac{\frac{\zeta}{2^4}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+3},\Xi_{y+z})\psi(\Xi_{y+3},\Xi_{y+4})}\right)$$

$$\diamond\ldots\diamond\mathfrak{I}\left(\Xi_{y+z-2},\Xi_{y+z-1},\frac{\frac{\zeta}{2^{y+z-1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-2},\Xi_{y+z})\psi(\Xi_{y+z-2},\Xi_{y+z-1})}\right)$$

$$\diamond\mathfrak{I}\left(\Xi_{y+z-1},\Xi_{y+z},\frac{\frac{\zeta}{2^{y+z+1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-1},\Xi_{y+z})}\right)$$

$$\mathfrak{D}(\Xi_y,\Xi_{y+z},\zeta)$$

$$\leq\mathfrak{D}\left(\Xi_y,\Xi_{y+1},\frac{\frac{\zeta}{2}}{\psi(\Xi_y,\Xi_{y+1}}\right)\diamond\mathfrak{D}\left(\Xi_{y+1},\Xi_{y+z},\frac{\frac{\zeta}{2}}{\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\leq\mathfrak{D}\left(\Xi_y,\Xi_{y+1},\frac{\frac{\zeta}{2}}{\psi(\Xi_y,\Xi_{y+1})}\right)\diamond\mathfrak{D}\left(\Xi_{y+1},\Xi_{y+2},\frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\diamond\mathfrak{D}\left(\Xi_{y+2},\Xi_{y+z},\frac{\frac{\zeta}{2^2}}{\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\leq\mathfrak{D}\left(\Xi_y,\Xi_{y+1},\frac{\frac{\zeta}{2}}{\psi(\Xi_y,\Xi_{y+1})}\right)\diamond\mathfrak{D}\left(\Xi_{y+1},\Xi_{y+2},\frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\diamond\mathfrak{D}\left(\Xi_{y+2},\Xi_{y+3},\frac{\frac{\zeta}{2^3}}{\psi(\Xi_{y+2},\Xi_{y+3})\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})}\right)$$

$$\diamond\mathfrak{D}\left(\Xi_{y+3},\Xi_{y+z},\frac{\frac{\zeta}{2^3}}{\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+3},\Xi_{y+z})}\right)$$

$$\leq\mathfrak{D}\left(\Xi_y,\Xi_{y+1},\frac{\frac{\zeta}{2}}{\psi(\Xi_y,\Xi_{y+1})}\right)\diamond\mathfrak{D}\left(\Xi_{y+1},\Xi_{y+2},\frac{\frac{\zeta}{2^2}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\diamond\mathfrak{D}\left(\Xi_{y+2},\Xi_{y+3},\frac{\frac{\zeta}{2^3}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\psi(\Xi_{y+2},\Xi_{y+3})}\right)$$

$$\diamond\mathfrak{D}\left(\Xi_{y+3},\Xi_{y+4},\frac{\frac{\zeta}{2^4}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+3},\Xi_{y+z})\psi(\Xi_{y+3},\Xi_{y+4})}\right)$$

$$\diamond\ldots\diamond\mathfrak{D}\left(\Xi_{y+z-2},\Xi_{y+z-1},\frac{\frac{\zeta}{2^{y+z-1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-2},\Xi_{y+z})\psi(\Xi_{y+z-2},\Xi_{y+z-1})}\right)$$

$$\diamond\mathfrak{D}\left(\Xi_{y+z-1},\Xi_{y+z},\frac{\frac{\zeta}{2^{y+z+1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-1},\Xi_{y+z})}\right)$$

Using (3.6),(3.7),(3.8) in the above inequalities, we deduce

$$\mathfrak{A}(\Xi_y,\Xi_{y+z},\zeta)$$

$$\geq\mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2}\cdot\frac{1}{\mathcal{B}^y}}{\psi(\Xi_y,\Xi_{y+1})}\right)\star\mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^2}\cdot\frac{1}{\mathcal{B}^{y+1}}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$

$$\star\,\mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^3}\cdot\frac{1}{\mathcal{B}^{y+2}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\psi(\Xi_{y+2},\Xi_{y+3})}\right)$$
$$\star\,\mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^4}\cdot\frac{1}{\mathcal{B}^{y+3}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+3},\Xi_{y+z})\psi(\Xi_{y+3},\Xi_{y+4})}\right)$$
$$\star\cdots\star\,\mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^{y+z-1}}\cdot\frac{1}{\mathcal{B}^{2y+z-2}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-2},\Xi_{y+z})\psi(\Xi_{y+z-2},\Xi_{y+z-1})}\right)$$
$$\star\,\mathfrak{A}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^{y+z-1}}\cdot\frac{1}{\mathcal{B}^{2y+z-1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-1},\Xi_{y+z})}\right)$$

$$\mathfrak{I}(\Xi_y,\Xi_{y+z},\zeta)$$
$$\leq\mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2}\cdot\frac{1}{\mathcal{B}^{y}}}{\psi(\Xi_y,\Xi_{y+1})}\right)\diamond\mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^2}\cdot\frac{1}{\mathcal{B}^{y+1}}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$
$$\diamond\mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^3}\cdot\frac{1}{\mathcal{B}^{y+2}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\psi(\Xi_{y+2},\Xi_{y+3})}\right)$$
$$\diamond\mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^4}\cdot\frac{1}{\mathcal{B}^{y+3}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+3},\Xi_{y+z})\psi(\Xi_{y+3},\Xi_{y+4})}\right)$$
$$\diamond\ldots\diamond\mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^{y+z-1}}\cdot\frac{1}{\mathcal{B}^{2y+z-2}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-2},\Xi_{y+z})\psi(\Xi_{y+z-2},\Xi_{y+z-1})}\right)$$
$$\diamond\mathfrak{I}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^{y+z-1}}\cdot\frac{1}{\mathcal{B}^{2y+z-1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-1},\Xi_{y+z})}\right)$$

$$\mathfrak{D}(\Xi_y,\Xi_{y+z},\zeta)$$
$$\leq\mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2}\cdot\frac{1}{\mathcal{B}^{y}}}{\psi(\Xi_y,\Xi_{y+1})}\right)\diamond\mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^2}\cdot\frac{1}{\mathcal{B}^{y+1}}}{\psi(\Xi_{y+1},\Xi_{y+2})\xi(\Xi_{y+1},\Xi_{y+z})}\right)$$
$$\diamond\mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^3}\cdot\frac{1}{\mathcal{B}^{y+2}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\psi(\Xi_{y+2},\Xi_{y+3})}\right)$$
$$\diamond\mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^4}\cdot\frac{1}{\mathcal{B}^{y+3}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\xi(\Xi_{y+3},\Xi_{y+z})\psi(\Xi_{y+3},\Xi_{y+4})}\right)$$
$$\diamond\ldots\diamond\mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^{y+z-1}}\cdot\frac{1}{\mathcal{B}^{2y+z-2}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-2},\Xi_{y+z})\psi(\Xi_{y+z-2},\Xi_{y+z-1})}\right)$$
$$\diamond\mathfrak{D}\left(\Xi_0,\Xi_1,\frac{\frac{\zeta}{2^{y+z-1}}\cdot\frac{1}{\mathcal{B}^{2y+z-1}}}{\xi(\Xi_{y+1},\Xi_{y+z})\xi(\Xi_{y+2},\Xi_{y+z})\ldots\xi(\Xi_{y+z-1},\Xi_{y+z})}\right)$$

Applying (3.1) for $y\to+\infty$, we suggest that

$$\lim_{y\to\infty}\mathfrak{A}(\Xi_y,\Xi_{y+z},\zeta)=1\star 1\star\cdots\star 1=1$$

$$\lim_{y\to\infty}\mathfrak{I}(\Xi_y,\Xi_{y+z},\zeta)=0\diamond 0\diamond\ldots\diamond 0=0$$

$$\lim_{y\to\infty}\mathfrak{D}(\Xi_y,\Xi_{y+z},\zeta)=0\diamond 0\diamond\ldots\diamond 0=0$$

Which recommend that the sequence $\{\Xi_y\}$ is a Cauchy sequence in $\mathfrak{K}$. Already, $\mathfrak{K}$ is complete $\mathfrak{FDCMS}$, accordingly others do exist.
$\Xi\in\mathfrak{K}$ like that $\lim_{y\to\infty}\mathfrak{A}(\Xi_y,\Xi,\zeta)=1$, $\lim_{y\to\infty}\mathfrak{I}(\Xi_y,\Xi,\zeta)=0$, $\lim_{y\to\infty}\mathfrak{D}(\Xi_y,\Xi,\zeta)=0$.

From (3.2) we have

$$\begin{aligned}
&\mathfrak{A}(\Xi_{y+1},\mathfrak{q}\Xi,\zeta)=\mathfrak{A}(\mathfrak{q}\Xi_y,\mathfrak{q}\Xi,\zeta)\\
&\geq\min\left\{\mathfrak{A}\left(\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi_y,\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\mathfrak{q}\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi_y,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&=\min\left\{\mathfrak{A}\left(\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi_{y+1},\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi_y,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&\mathfrak{I}(\Xi_{y+1},\mathfrak{q}\Xi,\zeta)=\mathfrak{I}(\mathfrak{q}\Xi_y,\mathfrak{q}\Xi,\zeta)\\
&\leq\max\left\{\mathfrak{I}\left(\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi_y,\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\mathfrak{q}\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi_y,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&=\max\left\{\mathfrak{I}\left(\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi_{y+1},\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi_y,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&\mathfrak{D}(\Xi_{y+1},\mathfrak{q}\Xi,\zeta)=\mathfrak{D}(\mathfrak{q}\Xi_y,\mathfrak{q}\Xi,\zeta)\\
&\leq\max\left\{\mathfrak{D}\left(\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi_y,\mathfrak{q}\Xi_y,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\mathfrak{q}\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi_y,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&=\max\left\{\mathfrak{D}\left(\Xi_y,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi_y,\Xi_{y+1},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi_{y+1},\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi_y,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}
\end{aligned}$$

Since, $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ is $\mathfrak{NDCMS}$ and taking limit $y\to\infty$ there exist $\lim_{y\to+\infty}\Xi_y=\Xi$. Then, we rewrite the above equation as

$$\begin{aligned}
\mathfrak{A}(\Xi,\mathfrak{q}\Xi,\zeta)&\geq\min\left\{\mathfrak{A}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&=\min\left\{\mathfrak{A}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\geq\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\\
\mathfrak{I}(\Xi,\mathfrak{q}\Xi,\zeta)&\leq\max\left\{\mathfrak{I}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&=\max\left\{\mathfrak{I}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\leq\mathfrak{I}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\\
\mathfrak{D}(\Xi,\mathfrak{q}\Xi,\zeta)&\leq\max\left\{\mathfrak{D}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\\
&=\max\left\{\mathfrak{D}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right)\right\}\leq\mathfrak{D}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right).
\end{aligned}$$

Which concludes Ξ is the $\mathbb{FP}$ of $\mathfrak{q}$.
We must now demonstrate that Ξ is unique. Assume for the moment that $\check{z}$ is a $\mathbb{FP}$ of $\mathfrak{q}$ as well. Then,
$\mathfrak{A}(\Xi,\check{z},\zeta)=\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\check{z},\zeta)\geq\min\left\{\mathfrak{A}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\check{z},\mathfrak{q}\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\mathfrak{q}\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\mathfrak{q}\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right)\right\}$.

Since $\mathfrak{q}$ is $\mathfrak{FP}$ of Ξ, the above equation becomes
$\geq \min\left\{\mathfrak{A}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\Xi,\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\check{z},\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{A}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right)\right\}\geq \mathfrak{A}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right)$
$\mathfrak{A}\left(\Xi,\check{z},\zeta\right)\geq \mathfrak{A}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{I}\left(\Xi,\check{z},\zeta\right)\leq \mathfrak{I}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right),\mathfrak{D}\left(\Xi,\check{z},\zeta\right)\leq \mathfrak{D}\left(\Xi,\check{z},\frac{\zeta}{\mathcal{B}}\right)$
Implies that $\Xi=\check{z}$. This proves the uniqueness. Now, we prove the main result by taking $\Xi,\mathfrak{r}$ and ζ in equations we get,

$$\mathfrak{A}(\Xi,\mathfrak{r},\mathcal{B}\zeta)\geq \min\{\mathfrak{A}(\Xi,\mathfrak{r},\zeta),\mathfrak{A}(\Xi,\mathfrak{q}\Xi,\zeta),\mathfrak{A}(\mathfrak{r},\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{A}(\Xi,\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{r},\zeta)\}$$
$$\mathfrak{I}(\Xi,\mathfrak{r},\mathcal{B}\zeta)\leq \max\{(\Xi,\mathfrak{r},\zeta),\mathfrak{I}(\Xi,\mathfrak{q}\Xi,\zeta),\mathfrak{I}(\mathfrak{r},\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{I}(\Xi,\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{r},\zeta)\}$$
$$\mathfrak{D}(\Xi,\mathfrak{r},\mathcal{B}\zeta)\leq \max\{(\Xi,\mathfrak{r},\zeta),\mathfrak{D}(\Xi,\mathfrak{q}\Xi,\zeta),\mathfrak{D}(\mathfrak{r},\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{D}(\Xi,\mathfrak{q}\mathfrak{r},\zeta),\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{r},\zeta)\}$$

□

Corollary 3.1. *Consider $\psi,\xi:\mathfrak{K}\times\mathfrak{K}\to[1,1/\mathcal{B}^n)$ where $\mathcal{B}\in(0,1)$ and $n\in\mathbb{N}$ are designated incomparable functions and $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ be a complete $\mathfrak{NFDCMS}$ and assume that $\lim_{\zeta\to+\infty}\mathfrak{A}(\Xi,\mathfrak{r},\zeta)=1$, $\lim_{\zeta\to+\infty}\mathfrak{I}(\Xi,\mathfrak{r},\zeta)=0$, $\lim_{\zeta\to+\infty}\mathfrak{D}(\Xi,\mathfrak{r},\zeta)=0$ in which $\Xi,\mathfrak{r}\in\mathfrak{K}$ for all $\zeta>0$. Now, let $:\mathfrak{K}\to\mathfrak{K}$ be a self mapping satisfying*

$$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)\geq\mathfrak{A}(\Xi,\mathfrak{r},\zeta),\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)\leq\mathfrak{I}(\Xi,\mathfrak{r},\zeta),\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)\leq\mathfrak{D}(\Xi,\mathfrak{r},\zeta),\zeta>0,$$

for each and every $\varsigma,\mathfrak{r}\in\mathfrak{K}$. Then $\mathfrak{q}$ has a unique $\mathbb{FP}$.

Example 3.3. *Let $\mathfrak{K}=[0,1]$ and $\psi,\xi:\mathfrak{K}\times\mathfrak{K}\to[1,1/\mathcal{B}^n)$ be interpret as $\psi(g,h)=2(g+h)$ and $\xi(g,h)=2(g^2+h^2+1)$. Define $\mathfrak{A}(g,h,\zeta)=e^{\frac{-(g-h)^2}{\zeta}},\mathfrak{I}(g,h,\zeta)=1-e^{\frac{-(g-h)^2}{\zeta}}$, $\mathfrak{D}(g,h,\zeta)=e^{\frac{(g-h)^2}{\zeta}}-1$, for all $g,h\in\mathfrak{K},\zeta>0$.*
Then $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ is a complete $\mathfrak{FDCMS}$ with product t-norm and t-conorm.
Define $\mathfrak{q}:\mathfrak{K}\to\mathfrak{K}$ by $\mathfrak{q}(g)=\frac{1-2^{-g}}{3}$. Then,

$$\begin{aligned}
\mathfrak{A}(\mathfrak{q}g,\mathfrak{q}h,\mathcal{B}\zeta)&=\mathfrak{A}\left(\frac{1-2^{-g}}{3},\frac{1-2^{-h}}{3},\mathcal{B}\zeta\right)\\
&=e^{-\frac{\left(\frac{1-2^{-g}}{3}-\frac{1-2^{-h}}{3}\right)^2}{\mathcal{B}\zeta}}\\
&=e^{\frac{-(2^{-g}-2^{-h})^2}{9\mathcal{B}\zeta}}\geq e^{\frac{-(g-h)^2}{9\mathcal{B}\zeta}}\geq e^{\frac{-(g-h)^2}{\zeta}}=\mathfrak{A}(g,h,\zeta)\\
&\geq\min\{\mathfrak{A}(g,h,\zeta),\mathfrak{A}(g,\mathfrak{q}h,\zeta),\mathfrak{A}(h,\mathfrak{q}h,\zeta),\mathfrak{A}(g,\mathfrak{q}h,\zeta),\mathfrak{A}(\mathfrak{q}g,h,\zeta)\}\\
\mathfrak{I}(\mathfrak{q}g,\mathfrak{q}h,\mathcal{B}\zeta)&=\mathfrak{I}\left(\frac{1-2^{-g}}{3},\frac{1-2^{-h}}{3},\mathcal{B}\zeta\right)\\
&=e^{-\frac{\left(\frac{1-2^{-g}}{3}-\frac{1-2^{-h}}{3}\right)^2}{\mathcal{B}\zeta}}\\
&=e^{\frac{-(2^{-g}-2^{-h})^2}{9\mathcal{B}\zeta}}\leq e^{\frac{-(g-h)^2}{9\mathcal{B}\zeta}}\leq e^{\frac{-(g-h)^2}{\zeta}}=\mathfrak{I}(g,h,\zeta)\\
&\leq\max\{\mathfrak{I}(g,h,\zeta),\mathfrak{I}(g,\mathfrak{q}h,\zeta),\mathfrak{I}(h,\mathfrak{q}h,\zeta),\mathfrak{I}(g,\mathfrak{q}h,\zeta),\mathfrak{I}(\mathfrak{q}g,h,\zeta)\}
\end{aligned}$$

$$\begin{aligned}\mathfrak{D}(\mathfrak{q}g,\mathfrak{q}h,\mathcal{B}\zeta) &= \mathfrak{D}\left(\frac{1-2^{-g}}{3},\frac{1-2^{-h}}{3},\mathcal{B}\zeta\right)\\ &= e^{\frac{\left(\frac{1-2^{-g}}{3}-\frac{1-2^{-h}}{3}\right)^2}{\mathcal{B}\zeta}}-1\\ &= e^{\frac{(2^{-g}-2^{-h})^2}{9\mathcal{B}\zeta}}-1 \le e^{\frac{(g-h)^2}{9\mathcal{B}\zeta}}-1 \le e^{\frac{(g-h)^2}{\zeta}}-1 = \mathfrak{D}(g,h,\zeta)\\ &\le \max\{\mathfrak{D}(g,h,\zeta),\mathfrak{D}(g,\mathfrak{q}h,\zeta),\mathfrak{D}(h,\mathfrak{q}h,\zeta),\mathfrak{D}(g,\mathfrak{q}h,\zeta),\mathfrak{D}(\mathfrak{q}g,h,\zeta)\}\end{aligned}$$

For every $g,h\in\mathfrak{K}$, where $\mathcal{B}\in[1/9,1)$. Thus, all of the theorem's parameters are verified. Consequently, $\mathfrak{q}$ *has a unique* $\mathbb{FP}$.

Corollary 3.2. *Consider $\psi,\xi:\mathfrak{K}\times\mathfrak{K}\to[1,1/\mathcal{B}^n)$ where $\mathcal{B}\in(0,1)$ and $n\in\mathbb{N}$ are specified functions which are impossible to compare and $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ be a complete $\mathfrak{NDCFMS}$ and consider that*

$$\lim_{\zeta\to+\infty}\mathfrak{A}(\Xi,\mathfrak{r},\zeta)=1,\ \lim_{\zeta\to+\infty}\mathfrak{I}(\Xi,\mathfrak{r},\zeta)=0,\ \lim_{\zeta\to+\infty}\mathfrak{D}(\Xi,\mathfrak{r},\zeta)=0$$

in which $\Xi,\mathfrak{r}\in\mathfrak{K}$ for every $\zeta>0$. Now, let $:\mathfrak{K}\to\mathfrak{K}$ is a self mapping satisfying

$$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)\ge\mathfrak{A}(\Xi,\mathfrak{r},\zeta),\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)\le\mathfrak{I}(\Xi,\mathfrak{r},\zeta),\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)\le\mathfrak{D}(\Xi,\mathfrak{r},\zeta),\zeta>0.$$

For each and every $\Xi,\mathfrak{r}\in\mathfrak{K}$. Finally proves $\mathfrak{q}$ *has a unique* $\mathbb{FP}$.

Example 3.4. *Let $\mathfrak{K}=\mathfrak{M}\cup\mathfrak{W}$ where $\mathfrak{M}=(0,2)$ and $\mathfrak{W}=(2,\infty)$. Define $\mathfrak{A},\mathfrak{I},\mathfrak{D}:\mathfrak{K}\times\mathfrak{K}\times(0,+\infty)\to[0,1]$ as*

$$\mathfrak{A}(\Xi,\mathfrak{r},\zeta)=\begin{cases}1 & if\,\Xi=\mathfrak{r}\\ \frac{\zeta}{\zeta+\frac{2}{\mathfrak{r}}} & if\ \Xi\in\mathfrak{M}\ and\,\mathfrak{r}\in\mathfrak{W}\\ \frac{\zeta}{\zeta+\frac{2}{\Xi}} & if\,\Xi\in\mathfrak{W}\ and\,\mathfrak{r}\in\mathfrak{M}\\ \frac{1}{\zeta+1} & otherwise\end{cases}$$

$$\mathfrak{I}(\Xi,\mathfrak{r},\zeta)=\begin{cases}0 & if\,\Xi=\mathfrak{r}\\ \frac{\frac{2}{\mathfrak{r}}}{\zeta+\frac{2}{\mathfrak{r}}} & if\ \Xi\in\mathfrak{M}\ and\,\mathfrak{r}\in\mathfrak{W}\\ \frac{\frac{2}{\Xi}}{\zeta+\frac{2}{\Xi}} & if\,\Xi\in\mathfrak{W}\ and\,\mathfrak{r}\in\mathfrak{M}\\ \frac{\zeta}{\zeta+1} & otherwise\end{cases}$$

$$\mathfrak{D}(\Xi,\mathfrak{r},\zeta)=\begin{cases}0 & if\,\Xi=\mathfrak{r}\\ \frac{2}{\mathfrak{r}\zeta} & if\ \Xi\in\mathfrak{M}\ and\,\mathfrak{r}\in\mathfrak{W}\\ \frac{2}{\Xi\zeta} & if\,\Xi\in\mathfrak{W}\ and\,\mathfrak{r}\in\mathfrak{M}\\ \zeta & otherwise\end{cases}$$

For CTS product t-norm and t-conorm. Consider $\psi,\xi:\mathfrak{K}\times\mathfrak{K}\to[1,1/\mathcal{B}^n)$ where $\mathcal{B}\in(0,1)$ and $n\in\mathbb{N}$ as

$$\psi(\Xi,\mathfrak{r})=\begin{cases}1 & if\,\Xi,\mathfrak{r}\in\mathfrak{M}\\ \max\{\Xi,\mathfrak{r}\} & otherwise\end{cases}\quad and\ \xi(\Xi,\mathfrak{r})=\begin{cases}1+\frac{1}{\Xi} & if\,\Xi,\mathfrak{r}\in\mathfrak{M}\\ 1 & otherwise\end{cases}$$

Clearly, $(\mathfrak{K}, \mathfrak{A}, \mathfrak{I}, \mathfrak{D}, \star, \diamond)$ *be a complete* $\mathfrak{NDCFMS}$*. Consider* $\mathfrak{q} : \mathfrak{K} \times \mathfrak{K}$ *by*

$$\mathfrak{q}(v) = \begin{cases} v & if v \in \mathfrak{M} \\ v^2 + 1 & if v \in \mathfrak{W} \end{cases}$$

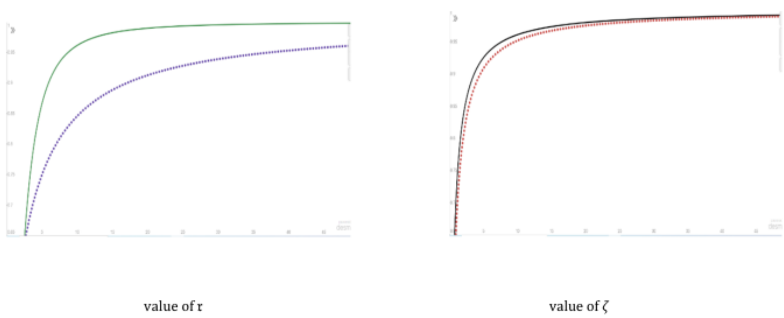
For all $v \in \mathfrak{K}$ *and* $\mathcal{B} = 0.5$*. For each of the four scenarios listed below, the disparity needs to be confirmed.*
Case I: *If* $\Xi = \mathfrak{r}$ *then there's* $\mathfrak{q}\Xi = \mathfrak{q}\mathfrak{r}$*. In the present case:*

$$\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) = 1 = \mathfrak{A}(\Xi, \mathfrak{r}, \zeta), \mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) = 0 = \mathfrak{I}(\Xi, \mathfrak{r}, \zeta), \mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) = 0 = \mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$$

Case II: *If* $\Xi \in \mathfrak{M}, \mathfrak{r} \in \mathfrak{W}$*, we have* $\mathfrak{q}\Xi \in \mathfrak{M}, \mathfrak{q}\mathfrak{r} \in \mathfrak{W}$*.*

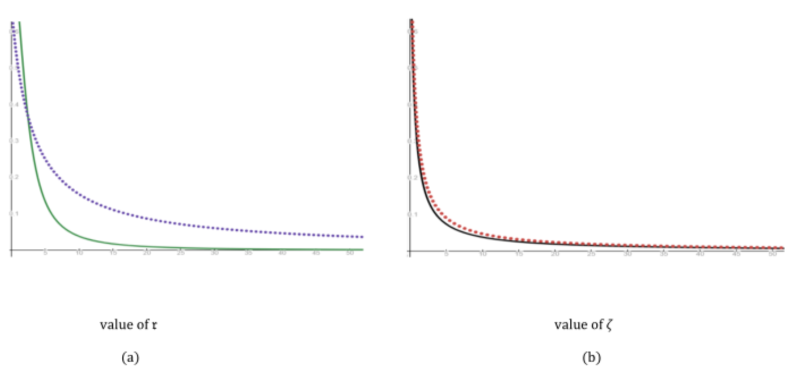
$$\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) = \frac{\mathcal{B}\zeta}{\mathcal{B}\zeta + \frac{2}{\mathfrak{q}\mathfrak{r}}} = \frac{0.5\zeta}{0.5\zeta + \frac{2}{\mathfrak{r}^2+1}} \geq \frac{\zeta}{\zeta + \frac{2}{\mathfrak{r}+1}} = \mathfrak{A}(\Xi, \mathfrak{r}, \zeta)$$

FIGURE 1: *Fluctuation of* $\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ *with* $\mathfrak{A}(\Xi, \mathfrak{r}, \zeta)$ *of case II on 2D view, for:*
(a) $\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ *(green curve) vs* $\mathfrak{A}(\Xi, \mathfrak{r}, \zeta)$*(violet dotted curve) at* $\zeta = 1$ *and* $\mathfrak{r} \in (3, 50)$*.*
(b) $\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ *(black curve) vs* $\mathfrak{A}(\Xi, \mathfrak{r}, \zeta)$ *(red dotted curve) at* $\zeta \in (1, 50)$ *and* $\mathfrak{r} = 3$



$$\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) = \frac{\frac{2}{\mathfrak{q}\mathfrak{r}}}{\mathcal{B}\zeta + \frac{2}{\mathfrak{q}\mathfrak{r}}} = \frac{\frac{2}{\mathfrak{r}^2+1}}{0.5\zeta + \frac{2}{\mathfrak{r}^2+1}} \leq \frac{\frac{2}{\mathfrak{r}+1}}{\zeta + \frac{2}{\mathfrak{r}+1}} = \mathfrak{I}(\Xi, \mathfrak{r}, \zeta)$$

FIGURE 2: *Fluctuation of* $\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ *with* $\mathfrak{I}(\Xi, \mathfrak{r}, \zeta)$ *of case II on 2D view, for:*
(a) $\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ *(green curve) vs* $\mathfrak{I}(\Xi, \mathfrak{r}, \zeta)$ *(violet dotted curve) at* $\zeta = 1$ *and* $\mathfrak{r} \in (3, 50)$*.*
(b) $\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ *(black curve) vs* $\mathfrak{I}(\Xi, \mathfrak{r}, \zeta)$ *(red dotted curve)at* $\zeta \in (1, 50)$ *and* $\mathfrak{r} = 3$



$$\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) = \frac{2}{\mathfrak{q}\mathfrak{r}\mathcal{B}\zeta} = \frac{2}{0.5\zeta(\mathfrak{r}^2+1)} \leq \frac{2}{(\mathfrak{r}+1)\zeta} = \mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$$

FIGURE 3: *Fluctuation of $\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ with $\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$ of case II on 2D view, for:*
(a) $\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ (blue curve) vs $\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$ (green dotted curve) at $\zeta = 1$ and $\mathfrak{r} \in (3, 50)$.
(b) $\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ (violet curve) vs $\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$ (black dotted curve) at $\zeta \in (1, 50)$ and $\mathfrak{r} = 3$

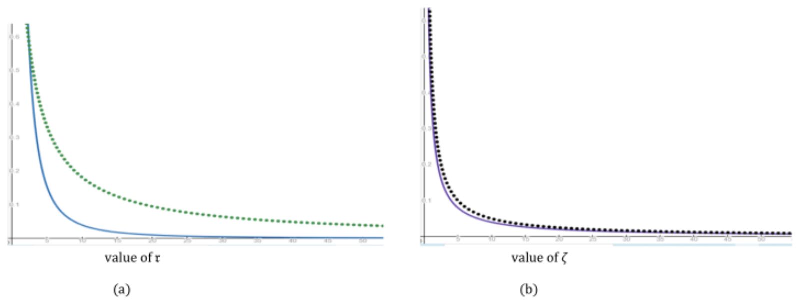


Table 1 and 2 shows the fluctuation between $\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ and $\mathfrak{A}(\Xi, \mathfrak{r}, \zeta), \mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ and $\mathfrak{I}(\Xi, \mathfrak{r}, \zeta), \mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ and $\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$ as a mapping of $\mathfrak{r}$ with relative to ζ. The contour for the estimation of ζ is towering to 50 as a mapping of $\mathfrak{r}$.
At $\zeta = 70, \mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ changed to 1, and after higher values of ζ, it changeless.
$(\zeta = 1).\mathfrak{A}(\Xi, \mathfrak{r}, \zeta)$ doesn't change till $\zeta = 100$, but it arrived nearer to 1.
$\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ changed 0, and after higher values of ζ, it changeless. $(\zeta = 0)$. $\mathfrak{I}(\Xi, \mathfrak{r}, \zeta)$ doesn't change till $\zeta = 100$, but it arrived nearer to 0.
$\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ changed 0, and after higher values of z , it changeless. $(\zeta = 0)$. $\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$ doesn't change till $\zeta = 100$, but it arrived nearer to 0.

Value of ζ	*Value of $\mathfrak{r}$*	$\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$	$\mathfrak{A}(\Xi, \mathfrak{r}, \zeta)$	$\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$	$\mathfrak{I}(\Xi, \mathfrak{r}, \zeta)$	$\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$	$\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$
1	*3*	*0.7143*	*0.6667*	*0.2857*	*0.4170*	*0.4000*	*0.5000*
	20	*0.9901*	*0.9130*	*0.0099*	*0.0868*	*0.0100*	*0.0952*
	50	*0.9984*	*0.9623*	*0.0016*	*0.0348*	*0.0040*	*0.0392*
50	*3*	*0.9921*	*0.9901*	*0.0079*	*0.0099*	*0.0080*	*0.0133*
	20	*0.9998*	*0.9981*	*0.0002*	*0.0019*	*0.0002*	*0.0019*
	50	*1.0000*	*0.9992*	*0.0003*	*0.0008*	*0.00003*	*0.0008*

TABLE 1 : *Fluctuation between*
$\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ and $\mathfrak{A}(\Xi, \mathfrak{r}, \zeta), \mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ and $\mathfrak{I}(\Xi, \mathfrak{r}, \zeta), \mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta)$ and $\mathfrak{D}(\Xi, \mathfrak{r}, \zeta)$ as a mapping of r with unchanged estimation of $\zeta = 1$ and $\zeta = 50$.

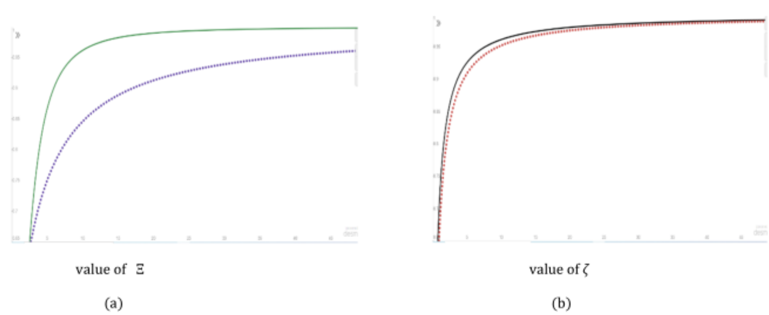
Value of $\mathfrak{r}$	*Value of* ζ	$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$	$\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$	$\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$	$\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$	$\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$	$\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$
3	1	0.7143	0.6667	0.2857	0.3333	0.4000	0.5000
	20	0.9901	0.9130	0.0196	0.0244	0.0200	0.0250
	50	0.9984	0.9623	0.0079	0.0099	0.0080	0.0100
50	1	0.9921	0.9901	0.0016	0.0377	0.0016	0.0392
	20	0.9998	0.9981	0.0001	0.0020	0.0001	0.0020
	50	1.0000	0.9992	0.0000	0.0008	0.0000	0.0008

TABLE 2: *Fluctuation between*
$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *and* $\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$, $\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *and* $\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$, $\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *and* $\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$ *as a mapping of* ζ *with unchanged value of* $\mathfrak{r}=3$ *and* $\mathfrak{r}=50$.

Case III: *If* $\Xi\in\mathfrak{W},\mathfrak{r}\in\mathfrak{M}$, *we have* $\mathfrak{q}\Xi\in\mathfrak{W},\mathfrak{q}\mathfrak{r}\in\mathfrak{M}$.

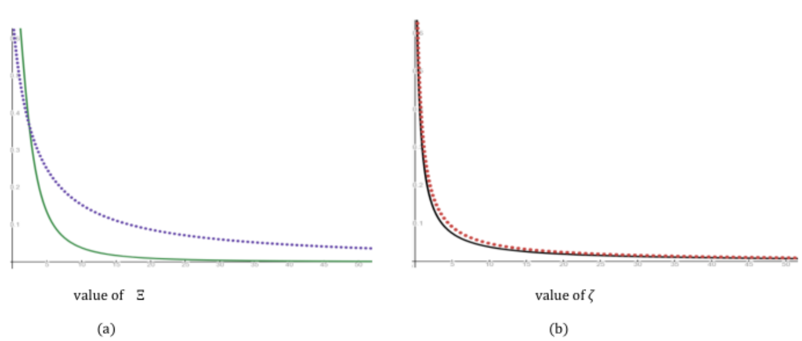
$$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)=\frac{\mathfrak{B}\zeta}{\mathcal{B}\zeta+\frac{2}{\mathfrak{q}\Xi}}=\frac{0.5\zeta}{0.5\zeta+\frac{2}{\Xi^2+1}}\geq\frac{\zeta}{\zeta+\frac{2}{\Xi+1}}=\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$$

FIGURE 4: *Fluctuation of* $\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *with* $\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$ *of case III on 2D view, for:*
(a) $\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *(green curve) vs* $\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$ *(violet dotted curve) at* $\zeta=1$ *and* $\Xi\in(3,50)$.
(b) $\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *(black curve) vs* $\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$ *(red dotted curve) at* $\zeta\in(1,50)$ *and* $\Xi=3$



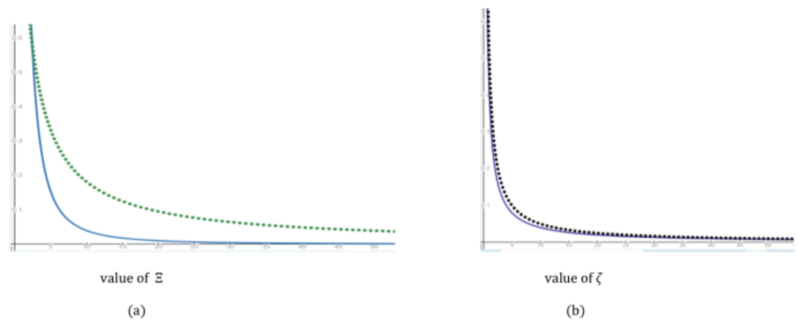
$$\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)=\frac{\frac{2}{\mathfrak{q}\Xi}}{\mathcal{B}\zeta+\frac{2}{\mathfrak{q}\Xi}}=\frac{\frac{2}{\Xi^2+1}}{0.5\zeta+\frac{2}{\Xi^2+1}}\leq\frac{\frac{2}{\Xi+1}}{\zeta+\frac{2}{\Xi+1}}=\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$$

FIGURE 5: *Fluctuation of* $\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *with* $\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$ *of case III on 2D view, for:*
(a) $\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *(green) vs* $\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$ *(violet dotted) at* $\zeta=1$ *and* $\Xi\in(3,50)$.
(b) $\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ *(black) vs* $\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$ *(red dotted) at* $\zeta\in(1,50)$ *and* $\Xi=3$



$$\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)=\frac{2}{\mathfrak{q}\Xi\mathcal{B}\zeta}=\frac{2}{0.5\zeta(\Xi^2+1)}\leq\frac{2}{(\Xi+1)\zeta}=\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$$

FIGURE 6: *Fluctuation of $\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ with $\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$ of case III on 2D view, for:*
(a) $\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ (blue) vs $\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$ (green dotted)at $\zeta=1$ and $\Xi\in(3,50)$.
(b) $\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ (violet) vs $\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$ (black dotted) at $\zeta\in(1,50)$ and $\Xi=3$



Case IV: *If $\Xi,\mathfrak{r}$ does not belongs to above any cases, then $\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ and $\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$, $\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ and $\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$, $\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)$ and $\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$ depends on only ζ, we have*

$$\mathfrak{A}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)=\frac{1}{\mathcal{B}\zeta+1}=\frac{1}{0.5\zeta+1}\geq\frac{1}{\zeta+1}=\mathfrak{A}(\Xi,\mathfrak{r},\zeta)$$
$$\mathfrak{I}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)=\frac{\zeta}{\mathcal{B}\zeta+1}=\frac{\zeta}{0.5\zeta+1}\leq\frac{\zeta}{\zeta+1}=\mathfrak{I}(\Xi,\mathfrak{r},\zeta)$$
$$\mathfrak{D}(\mathfrak{q}\Xi,\mathfrak{q}\mathfrak{r},\mathcal{B}\zeta)=\mathcal{B}\zeta=0.5\zeta\leq\zeta=\mathfrak{D}(\Xi,\mathfrak{r},\zeta)$$

Which implies that all the condition of corollary (3.2) holds and $\mathcal{B}$ has a unique $\mathbb{FP}$.

4. Application

In this section, we'll look at how following Integral Equation may be solved using $\mathfrak{NDCMS}$.
Now, take the Integral Equation

$$\Xi(e)=\mathfrak{k}(e)+\int_0^e\mathfrak{H}(e,j,\Xi(j))dj. \tag{4.1}$$

For every $e\in[0,I]$ there $I>0$.
Let $\mathfrak{K}=C([0,I],\mathbb{R})$ be the collection of all CTS real-valued operations generated on $[0,I]$. Keep in mind that $\mathfrak{K}$ is a complete metric space in terms of the sup-metric.

$$d(\Xi,\mathfrak{r})=\sup_{e\in[0,I]}|\Xi(e)-\mathfrak{r}(e)|$$

Also the space $(\mathfrak{K},\mathfrak{A},\mathfrak{I},\mathfrak{D},\star,\diamond)$ with

$$\mathfrak{A}(\Xi,\mathfrak{r},\zeta)=e^{-\frac{\sup_{e\in[0,I]}|\Xi(e)-\mathfrak{r}(e)|}{\zeta}},$$
$$\mathfrak{I}(\Xi,\mathfrak{r},\zeta)=1-e^{-\frac{\sup_{e\in[0,I]}|\Xi(e)-\mathfrak{r}(e)|}{\zeta}},$$

$$\mathfrak{D}(\Xi, \mathfrak{r}, \zeta) = e^{\frac{\sup_{e\in[0,D]} |\Xi(e)-\mathfrak{r}(e)|}{\zeta}} - 1.$$

For each and every $\Xi, \mathfrak{r} \in \mathfrak{K}$ and $\zeta > 0$, with product t-norm and t-conorm ,is a complete $\mathfrak{N}\mathfrak{D}\mathfrak{C}\mathfrak{M}\mathfrak{S}$.

Theorem 4.1. *Let $\mathfrak{q} : \mathfrak{R} \to \mathfrak{R}$ be an integral operator defined by*

$$\mathfrak{q}\Xi(e) = \mathfrak{k}(e) + \int_0^e \mathfrak{H}(e, j, \Xi(j))dj.$$

where $\mathfrak{k} \in C([0, I], \mathbb{R})$ and $\mathfrak{H} \in C([0, I] \times [0, I] \times \mathbb{R}, \mathbb{R})$. If there exist $h : [0, I] \times [0, I] \to [0, \infty)$ such that for all $e, j \in [0, I], h(e, j) \in L^1([0, I], \mathbb{R})$, and for all $\Xi, \mathfrak{r} \in \mathfrak{R}$, we get

$$|\mathfrak{H}(e, j, \Xi(j)) - \mathfrak{H}(e, j, y(j))| \le h(e, j)|\Xi(j) - \mathfrak{r}(j)|,$$

where $\int_0^e h(e, j,)dj$ is bounded on $[0, I]$ and $0 < \sup_{e\in[0,I]} \int_0^e h(e, j,)dj \le \mathcal{B} < 1$. The subsequent Integral Equation (5) has a unique solution in $\mathfrak{K}$.

Proof. Take $\Xi, \mathfrak{r} \in \mathfrak{R}$ and assume

$$\begin{aligned}
\mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) &= e^{-\frac{\sup_{e\in[0,I]} |\mathfrak{q}\Xi(e)-\mathfrak{q}\mathfrak{r}(e)|}{\mathcal{B}\zeta}} \ge e^{-\frac{\sup_{e\in[0,I]} \int_0^e |\mathfrak{H}(e,j,\Xi(j))-\mathfrak{H}(e,j,\mathfrak{r}(j))|dj}{\mathcal{B}\zeta}} \\
&\ge e^{-\frac{\sup_{e\in[0,I]} \int_0^e h(e,j)|\Xi(j)-\mathfrak{r}(j)|dj}{\mathcal{B}\zeta}} \ge e^{-\frac{|\Xi(j)-\mathfrak{r}(j)| \sup_{e\in[0,I]} \int_0^e h(e,j)dj}{\mathcal{B}\zeta}} \\
&\ge e^{-\frac{\mathcal{B}|\Xi(j)-\mathfrak{r}(j)|}{\mathcal{B}\zeta}} = e^{-\frac{|\Xi(j)-\mathfrak{r}(j)|}{\zeta}} \\
&\ge e^{-\frac{\sup_{j\in[0,I]} |\Xi(j)-\mathfrak{r}(j)|}{\zeta}} \\
\mathfrak{A}(\Xi, \mathfrak{r}, \zeta) &\ge \min\{\mathfrak{A}(\Xi, \mathfrak{r}, \zeta), \mathfrak{A}(\Xi, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{A}(\mathfrak{r}, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{A}(\Xi, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{A}(\mathfrak{q}\Xi, \mathfrak{r}, \zeta)\} \\
\mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) &= 1 - e^{-\frac{\sup_{e\in[0,I]} |\mathfrak{q}\Xi(e)-\mathfrak{q}\mathfrak{r}(e)|}{\mathcal{B}\zeta}} \le 1 - e^{-\frac{\sup_{e\in[0,I]} \int_0^e |\mathfrak{H}(e,j,\Xi(j))-\mathfrak{H}(e,j,\mathfrak{r}(j))|dj}{\mathcal{B}\zeta}} \\
&\le 1 - e^{-\frac{\sup_{e\in[0,I]} \int_0^e h(e,j)|\Xi(j)-\mathfrak{r}(j)|dj}{\mathcal{B}\zeta}} \le 1 - e^{-\frac{|\Xi(j)-\mathfrak{r}(j)| \sup_{e\in[0,I]} \int_0^e h(e,j)dj}{\mathcal{B}\zeta}} \\
&\le 1 - e^{-\frac{\mathcal{B}|\Xi(j)-\mathfrak{r}(j)|}{\mathcal{B}\zeta}} = 1 - e^{-\frac{|\Xi(j)-\mathfrak{r}(j)|}{\zeta}} \\
&\le 1 - e^{-\frac{\sup_{j\in[0,I]} |\Xi(j)-\mathfrak{r}(j)|}{\zeta}} \\
\mathfrak{I}(\Xi, \mathfrak{r}, \zeta) &\le \max\{\mathfrak{I}(\Xi, \mathfrak{r}, \zeta), \mathfrak{I}(\Xi, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{I}(\mathfrak{r}, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{I}(\Xi, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{I}(\mathfrak{q}\Xi, \mathfrak{r}, \zeta)\} \\
\mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{q}\mathfrak{r}, \mathcal{B}\zeta) &= e^{\frac{\sup_{e\in[0,I]} |\mathfrak{q}\Xi(e)-\mathfrak{q}\mathfrak{r}(e)|}{\mathcal{B}\zeta}} - 1 \le e^{\frac{\sup_{e\in[0,I]} \int_0^e |\mathfrak{H}(e,j,\Xi(j))-\mathfrak{H}(e,j,\mathfrak{r}(j))|dj}{\mathcal{B}\zeta}} - 1 \\
&\le e^{\frac{\sup_{e\in[0,I]} \int_0^e h(e,j)|\Xi(j)-\mathfrak{r}(j)|dj}{\mathcal{B}\zeta}} - 1 \le e^{\frac{|\Xi(j)-\mathfrak{r}(j)| \sup_{e\in[0,I]} \int_0^e h(e,j)dj}{\mathcal{B}\zeta}} - 1 \\
&\le e^{\frac{\mathcal{B}|\Xi(j)-\mathfrak{r}(j)|}{\mathcal{B}\zeta}} - 1 = e^{\frac{|\Xi(j)-\mathfrak{r}(j)|}{\zeta}} - 1 \\
&\le e^{\frac{\sup_{j\in[0,I]} |\Xi(j)-\mathfrak{r}(j)|}{\zeta}} - 1 \\
\mathfrak{D}(\Xi, \mathfrak{r}, \zeta) &\le \max\{\mathfrak{D}(\Xi, \mathfrak{r}, \zeta), \mathfrak{D}(\Xi, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{D}(\mathfrak{r}, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{D}(\Xi, \mathfrak{q}\mathfrak{r}, \zeta), \mathfrak{D}(\mathfrak{q}\Xi, \mathfrak{r}, \zeta)\}
\end{aligned}$$

Theorem (3.1)'s criteria are all met, accordingly $\mathfrak{q}$ has a singular solution in $\mathfrak{K}$. □

Example 4.1. *Let the Integral Equation be* $\mathfrak{K} = C([0,1],\mathbb{R})$,

$$\Xi(e) = \sin(e) + \frac{2}{3}\cos(e) - \frac{1}{3}\cos^2(e) + \int_0^j \frac{\Xi(j)}{3}dj, \tag{4.2}$$

Here $\mathfrak{k}(e) = \sin(e) + \frac{2}{3}\cos(e) - \frac{1}{3}\cos^2(e)$ *and* $\mathfrak{H}(e,j,\Xi(j)) = \frac{\Xi(j)}{3}$ *which are continuous.*
Note that,

$$|\mathfrak{H}(e,j,\Xi(j)) - \mathfrak{H}(e,j,\mathfrak{r}(j))| = \left|\frac{\Xi(j)}{3} - \frac{\mathfrak{r}(j)}{3}\right| = \frac{1}{3}\left|\Xi(j) - \mathfrak{r}(j)\right| \leq h(e,j)|\Xi(j) - \mathfrak{r}(j)|$$

where $h(e,j) = \frac{1}{2e}$ *so that* $\sup_{e\in[0,1]} \int_0^e h(e,j)dj = \frac{1}{2} < 1$.
As every requirement of Theorem (4.1) has been achieved, there is a unique solution to the Integral Equation (4.2).

5. Conclusions

By introducing $\mathfrak{NDCFMS}$ and a number of new verifiable $\mathbb{FP}$ theorems, we enhance Sezen's $\mathfrak{CFMS}$ in the current investigation. We additionally addressed a few complex cases. Due to the fact that our structure is more generic than a category of fuzzy and $\mathfrak{DCFMS}$, our verdict and notions add to a number of previously reported findings that have been more specifically applied.

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