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Some Statistical Estimations for Voting in Hough Transform

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ABSTRACT. Transform The Hough Transform (HT) is an essential method in detecting geometric shapes in images. In this work, we concentrate on enhancing the accuracy and efficiency of the HT through the statistical estimation for the voting process in lines detection. We propose a statistical pattern detection method, which aims to introduce a new estimation of the polar angle θ of a detected line in an image and its radial distance r after estimating the slope and intercept of line of detection.

1. Introduction

The Hough transform (HT) method, developed in [16], is a technique, which is typically used to extract edges or curves from an image. It is obvious that the input of the HT method is normally an image that has been edge detected by a suitable edge detector, and so the HT method is regarded as an edge linker that groups edge pixels together. The HT method detects curves in an image by interchanging roles between parameters of an analytic curve and points lying on that curve. The HT method has been generalised to detect arbitrary shapes [2], this includes both analytic [13] and nonanalytic curves [22]. These analytic curves are lines [23], parabolas [28], circles [19] and ellipses [27], which can be detected using some HT algorithms.

The idea behind HT is to extract curves in the image

$$f(x_o, y_o; a_1, \dots, a_k) = 0 \tag{1.1}$$

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where (x_o, y_o) is a point on the curve and $(a_1, \dots, a_k)$ is the vector of parameters. For example, a line $y_o - mx_o + c = 0$ has the vector of parameters (m, c). Another example is the circle $(x_o - a)^2 + (y_o - b)^2 - c^2 = 0$ with the vector of parameters (a, b, c). The HT method starts with the assumption that the coordinates of points (x_o, y_o) when substituted in Eq(1.1) are considered constants, and the parameters $a_1, \dots, a_k$ are variables. If a point is substituted in Eq(1.1), then the result represents a new equation in the new variables (the parameters) that has its own geometrical representation in the space of parameters. This means that the set of points to be fitted by the HT method is transformed to a family of new geometrical representations in the space of parameters. All these geometrical representations should cross at a value in the k-Euclidean space of parameters which represents the vector of the required parameters.

In our work, we represented the space of parameters by a matrix which collects particular voting regarding discrete values for the parameters in the space. The array of accumulators, $A(a_1, \dots, a_k)$, is originally set to zero for every parameter vector $(a_1, \dots, a_k)$. For each point (x_o, y_o), one then searches vectors $(a_1, \dots, a_k)$ in the parametric space and increase the accumulator corresponding to this vector by one whenever

$$f(x_o, y_o; a_1, \dots, a_k) < \in \tag{1.2}$$

for a suitable determined small number $\in$.

According to Eq(1.2),each value in a pixel of the image contributes votes to multiple parameter. To find the vector of parameters that characterises the curve, one detects maximum values in the parameter universe, which is equivalent to detecting the statistical relative frequency.

The problem in the execution of this method is that it is affected by the dimension of the space of parameters. If the HT technique is used to fit lines, then the parameter space is 2-dimentional. That is to say, this way is suitable for finding simple curves with a small number of parameters, a line as an example, which is the core of this work.

As mentioned in [3, 14, 18, 21], the HT stands as a good statistically robust estimator in the process of detecting lines in the image. In [5], authors managed to introduce an estimator for the probability density function (pdf) of variables enhanced by HT using mixtures of the kernel with a bandwidth of variables. The modelling of pdf in [5] takes advantage of all pixels in the image including edges to enhance variables. The usual HT is considered as a special event of our new statistical method. The HT can be applied in many life sectors, some examples are tennis broadcasts of snooker [6, 12], and sports trends [20] studied.

Relative descriptors of the intensity of images were defined in [25] to identify corners and match points. By using local descriptors, other distributions can also be modelled with multi-dimensional histograms, as shown in [24]. For a study of the uncertainty of the extraction of an edge by a corner detector, see [26]. Authors in [26] introduced a new rule for feature extraction uncertainty that supports special pixel covariance to derive a more accurate relation uncertainty estimate and for line detection in images using a regularized HT, see [1]. In [4], the mean shift clustering approach was utilized to detect fast line segments in the Hough domain. In [15], authors presented an active and more reliable scheme for a fast discovery of the optic disk. In [7], authors proposed a novel method which uses different procedures that answer the problem of skew discovery, taking into account having at least one variable standing as a compensation for the absence of more variables, and so each brand of interested text can be manipulated with better accuracy and more reliability in the output outcome. For more survey of literature related to the HT, see [11, 8, 17].

In this article, we suggest a statistical design discovery scheme related to the HT. This approach introduces a novel estimation of the polar angle θ and the radial distance r of a detected line in an image, following the estimation of the line's slope m and intercept c.

2. Estimation of the statistics

Assuming that the noisy image is defined by $I(x_o, y_o) = c(x_o, y_o) + n(x_o, y_o)$ where $c(x_o)$ is the clean portion of the image and $n(x_o)$ is the noisy portion of the image, which is the compliment of $c(x_o)$. In our work, if $X = (x_o, y_o)$ is the position vector in the image, then X is modelled to be a random variable that is normally scattered with $x_o \sim N\left(\mu_{x_o}, \sigma_{x_o}^2\right)$ *and* $y_o \sim N\left(\mu_{y_o}, \sigma_{y_o}^2\right)$. In this case, the values of the partial derivatives of the image follow the normal distribution with $I_{x_o} \sim N\left(c_{x_o}, \sigma^2\right)$ and $I_{y_o} \sim N\left(c_{y_o}, \sigma^2\right)$. Moreover, if $W = \frac{I_{y_o}}{I_{x_o}}$, then we can assume that $\frac{n_{y_o}}{c_{x_o}}$ is very small, and so $W \approx \frac{c_{y_o} + n_{y_o}}{c_{x_o}}$.

Here the mean of W *is* $M(W) = \frac{c_{y_o}}{c_{x_o}}$ and the variance of W *is* $V(W) = \frac{\sigma^2}{c_{x_o}^2}$.

The modulus of the derivative of the intensity as a random variable is given by $\|\nabla I\| = \sqrt{I_{x_o}^2 + I_{y_o}^2}$ with Rayleigh distribution when $c_{x_o} = c_{y_o} = 0$. Assuming that our image has a large proportion of flat regions, the approximation of the standard deviation σ of the noise should be strongly estimated by calculating a peak in the distribution of the modulus $\|\nabla I\|$ with domain being the intensity itself [9, 10].

In [11], the author studied the statistical HT. The main work in [11] lies in estimation of the statistics related to the equation of a line written in terms of polar coordinates. Also, the author in [11] made estimation of the modes of such statistics.

In [29], the authors suggested a scheme that is built on minimum entropy analyse to find all values of parameters in an oriented line segment. Here the normal angle as well the length of the line segment were calculated by pasting a curve with a quadratic polynomial approximation towards entropies under intersect of voting.

The input of our work is a 2-dimentional image, on which we apply HT. We restrict our aim on extracting lines out of the image. A set of points with Cartesian coordinator (x, y) is available and we fit it to the polar equation of the line

$$y_o = mx_o + c \tag{2.1}$$

Let $\mathcal{A} = \{(x_1, y_1), \dots, (x_n, y_n)\}$ be the set of observations, which is aligned on a straight line with parameters $(\hat{m}, \hat{c})$ where $\hat{m}$ *and* $\hat{c}$ are both estimated using HT. Hence we can consider $\mathcal{B} = \{(m_1, x_1, y_1), \dots, (m_n, x_n, y_n)\}$ as the set of locations with an observation of the slope m. Thus, for an image, the slope (derivative) of the line be able to be nearby calculated and used as an observation of m. Then $c_1, \dots, c_n$ in the set $\mathcal{C} = \{(m_1, c_1), \dots, (m_n, c_n)\}$ can be computed using the set $\mathcal{A}$ and the well-known values $m_1, \dots, m_n$ in the set $\mathcal{B}$, all considered as observations. Two vectors of values are needed for both m and c to apply the HT, which enable us to build a two dimensional matrix of size $K_m \times K_c$ where K_m *and* K_c are the dimensions of the vectors of m and c respectively. This means that there are $K_m \times K_c$ different lines under voting. If

$$|y_o - mx_o - c| < \in \tag{2.2}$$

for some $\in > 0$, then there is a natural number $\mathcal{N}(\in)$, which counts the number of lines satisfying (2.2).

One method for the estimation of the sample size $\mathcal{N}(\in)$ is using the inequality

$$Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{\mathcal{N}(\in)}} \leq \in \tag{2.3}$$

where $\in$ is statistically considered as the margin of error, S is the known standard deviation of the sample and $Z_{1-\frac{\alpha}{2}}$ is the $1-\frac{\alpha}{2}$ quantile of the $N(0,1)$. The good news behind $\mathcal{N}(\in)$ lies in the point that the number of lines under voting is now short listed, which saves time and memory. Also, Ineq(2.3) provides us with a lower bound for $\in$, namely, $Z_{1-\frac{\alpha}{2}} \frac{S}{\sqrt{K_m \times K_c}}$ where $K_m \times K_c$ in this case is an upper bound for $\mathcal{N}(\in)$.

Considering the pairs $(m_1, c_1), \dots, (m_{\mathcal{N}(\in)}, c_{\mathcal{N}(\in)})$, we conclude the following:

$$\hat{m} = \frac{\sum_{i=1}^{\mathcal{N}(\in)} m_i}{\mathcal{N}(\in)} \tag{2.4}$$

$$\hat{\sigma}_m = \sqrt{\frac{\sum_{i=1}^{\mathcal{N}(\in)} (m_i - \hat{m})^2}{\mathcal{N}(\in) - 1}} \tag{2.5}$$

where $\mathcal{N}(\in) = \left\lceil \frac{Z_{1-\frac{\alpha}{2}} S}{\in} \right\rceil^2$. Also one can estimate both the mode and median of the values $m_1, \dots, m_{\mathcal{N}(\in)}$. Similarly, we can calculate

$$\hat{c} = \frac{\sum_{i=1}^{\mathcal{N}(\in)} c_i}{\mathcal{N}(\in)} \tag{2.6}$$

$$\hat{\sigma}_c = \sqrt{\frac{\sum_{i=1}^{\mathcal{N}(\in)} (c_i - \hat{c})^2}{\mathcal{N}(\in) - 1}} \tag{2.7}$$

Also one can estimate both the mode and median of the values $c_1, \dots, c_{\mathcal{N}(\epsilon)}$.

If $\sum_{i=1}^{\mathcal{N}(\epsilon)} (m_i - \widehat{m})^2 = u, then\ \hat{\sigma}_m = \sqrt{\dfrac{u}{\left\lfloor \frac{Z_{1-\frac{\alpha}{2}} S}{\epsilon} \right\rfloor^2 - 1}}$.

Now assume the $\hat{\sigma}_m$ is required to be very small, that is, $\hat{\sigma}_m \ll \sqrt{\delta}$ where $\delta > 0$.

Then $u \ll \delta \left(\left\lfloor \frac{Z_{1-\frac{\alpha}{2}} S}{\epsilon} \right\rfloor^2 - 1 \right)$.

Now, we go further by taking into account that $tan\,\widehat{\theta}_s = \widehat{m}$, , where $\hat{\theta}_s$ is the estimated slope angle of the line of estimation. Hence, if $(\hat{r}, \hat{\theta})$ are the estimated polar coordinates, then it is natural that $\tan \hat{\theta} = \frac{-1}{\widehat{m}}$, or equivalently, $\hat{\theta} = tan^{-1}\left(\frac{-1}{\widehat{m}}\right)$. Moreover, $\hat{r} = x_o \cos \hat{\theta} + y_o \sin \hat{\theta}$ where (x_o, y_o) is the point on the line of estimation being the nearest to the origin. The last statement ensures that the simulation should minimize the distance measured from data points to the origin. That is, (x_o, y_o) is chosen by minimizing $\left\{ \sqrt{x_{oi}^2 + y_{oi}^2} \right\}_{i=1}^{n}$. To avoid the idea of minimizing of the above distance, one can follow the following conclusion of $\hat{r}$ in term of $\hat{c}\ and\ \hat{\theta}$. It is well known that another form of representing a line in the plane is

$y_o = \frac{-1}{\tan \theta} x_o + c$, equivalently,

$$y_o \tan \theta + x_o = c\ \tan \theta \tag{2.8}$$

But $r = x_o \cos \theta + y_o \sin \theta$ implies that

$$\frac{r}{\cos \theta} = x_o + y_o \tan \theta \tag{2.9}$$

Eq(2.8) and Eq(2.9) imply that $\frac{r}{\cos \theta} = c \tan \theta$.

That is,

$$\hat{r} = \hat{c} \sin \hat{\theta} \tag{2.10}$$

3. Matlab Programs

Program (1): Fitting through m and c

% A test for using the Hough transform in fitting data to a line.
% The Eq. of a line used here is of the formula 'y=mx+c' …..(*).
% The Eq. 'c=y-mx' represents the rewritten form of (*).
% The constants m and c now play the role of variables.
% The (m,c) space is called the parameter space.
% The (x,y) pairs represent the observed values.
% X is the vector of 1st coordinates of the observed pairs.

```
% Y is the vector of 2nd coordinates of the observed pairs.
Input X
Input Y
lxy=length (X);
m=im:fm;
c=ic:fc;
% m and c quantize the parameter space.
lm=length(m); lc=length(c);
acc=zeros(lm,lc);
% acc is the array of accumulators.
% The idea of the Hough transform is incrementing by 1 each accumulator
% that corresponds to the pair (m,c) which satisfies the relation in
% the If statement.
Input ∈
for ii=1:lxy
    for jj=1:lm
        for kk=1:lc
       if abs(y(ii)-m(jj)*x(ii)-c(kk))< ∈
           acc(jj,kk)=acc(jj,kk)+1;
                  end
         end
   end
end
% The method suggests that m and c solve the problem.
ac=acc(:);
[value, ind]=max(ac);
siz=size(acc);
% siz represents N(∈).
[a,b]=ind2sub(siz,ind);
m=m(a), c=c(b)
theta=invtan ((-1)/m)
r=c*sin (theta)
```

Program (2): Fitting direct by r *and* θ

```
% A test for using the Hough transform in fitting data to a line.
% The Eq. of a line used here is of the form 'xcos z +ysin z =r'.
% The constants z  and r now play the role of variables.
```

```
% The (z, r) space is called the parameter space.
% The (x,y) pairs represent the observed values.
% X is the vector of 1st  coordinates of the observed pairs.
% Y is the vector of 2nd  coordinates of the observed pairs.
Input X
Input Y
lxy=length(X);
lm=length(m); lc=length(c);
z=0:pi/72:2*pi;
r=0:0.5:lm;
% z and r quantize the parameter space.
cz=cos(z);
sz=sin(z);
acc=zeros(lm,lc);
% acc is the array of accumulators.
% The idea of the Hough transform is incrementing by 1 each accumulator
% that corresponds to the pair (z,r) which satisfies the relation in
% the If statement.
for ii=1:lxy
   for jj=1:lm
      for kk=1:lc
         if abs(x(ii)*cz(jj)+y(ii)*sz(jj)-r(kk))< ∈
              acc(jj,kk)=acc(jj,kk)+1;
                  end
          end
   end
end
ac=acc(:);
[value, ind]=max(ac);
siz=size(acc);
% siz represent N(∈)
[a,b]=ind2sub(siz,ind);
cz=cz(a), sz=szc(a),r=r(b)
```

4. Conclusion

In conclusion, this study has investigated the application of statistical estimations to enhance the voting process in the Hough Transform (HT) algorithm for line recognition. We have focused on improving the precision and efficiency of detecting lines, particularly in the presence of margin value $\in$. Firstly, we have demonstrated some statistical concepts that are embedded in using the HT estimator of the HT estimator, especially in the context of detecting one line whose equation is written using polar coordinates. Additionally, we have derived on enhancing method which gives more accuracy and efficiency while voting towards the best line of detection. Finally, the estimation of the slope of the line and its intercepts had been used in our work to introduce the new contribution in estimating the polar angle and radial distance of line of detection.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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