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Characterizations of Quasi $\theta(\tau_1, \tau_2)$-Continuous Multifunctions

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Abstract. This paper is concerned with the concepts of upper quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions and lower quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions. Furthermore, several characterizations of upper quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions and lower quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions are considered.

1. Introduction

The notion of continuity is an important concept in general topology as well as all branches of mathematics. Semi-open sets [44], preopen sets [47], α-open sets [49], β-open sets [1] and θ-open sets [76] play an important role in the research of generalizations of continuity. Using these notions many authors introduced and studied various types of generalizations of continuity for functions and multifunctions. Levine [44] introduced and studied the notion of semi-continuous functions. Arya and Bhamini [2] introduced the concept of θ-semi-continuity as a generalization of semi-continuity. Noiri [51] and Jafari and Noiri [35] have further investigated some characterizations of θ-semi-continuous functions. Marcus [46] introduced and investigated the notion of quasi continuous functions. The concepts of (Λ, sp)-open sets, $s(\Lambda, sp)$-open sets, $p(\Lambda, sp)$-open sets, $\alpha(\Lambda, sp)$-open sets, $\beta(\Lambda, sp)$-open sets and $b(\Lambda, sp)$-open sets were studied in [16]. Viriyapong and Boonpok [80] investigated some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets. Dungthaisong et al. [34] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [33] introduced and investigated the

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notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions and pairwise M-continuous functions were presented in [69], [73], [4], [63], [8], [9], [15], [23], [27] and [28], respectively. Popa [55] introduced and studied the notion of almost quasi continuous functions. Neubrunnovaá [48] showed that quasi continuity is equivalent to semi-continuity due to Levine [44]. Popa and Stan [57] introduced and investigated the notion of weakly quasi continuous functions. Weak quasi continuity is implied by quasi continuity and weak continuity [45] which are independent of each other. In [3], the present authors introduced and investigated the concept of (τ_1, τ_2)-continuous functions. Moreover, some characterizations of almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions, slightly $(\tau_1, \tau_2)s$-continuous functions, slightly (τ_1, τ_2)-continuous functions, $\delta(\tau_1, \tau_2)$-continuous functions, faintly (τ_1, τ_2)-continuous functions and almost weakly (τ_1, τ_2)-continuous functions were investigated in [5], [6], [61], [67], [58], [68] and [37], respectively. Kong-ied et al. [42] introduced and studied the notion of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [31] introduced and investigated the concept of weakly quasi (τ_1, τ_2)-continuous functions. Srisarakham et al. [66] introduced and studied the notion of quasi $\theta(\tau_1, \tau_2)$-continuous functions.

In 1975, Popa [56] extended the concept of quasicontinuous functions to the setting of multifunctions. In particular, Popa and Noiri [53] introduced the concept of almost quasi continuous multifunctions and investigated some characterizations of such multifunctions. Noiri and Popa [52] introduced and studied the notion of weakly quasi continuous multifunctions. Popa and Noiri [54] introduced the notion of θ-quasicontinuous multifunctions and investigated several further properties of such multifunctions. Some characterizations of upper θ-quasicontinuous multifunctions and lower θ-quasicontinuous multifunctions were investigated in [50]. Laprom et al. [43] introduced and investigated the concept of $\beta(\tau_1, \tau_2)$-continuous multifunctions. Furthermore, some characterizations of $(\tau_1, \tau_2)\alpha$-continuous multifunctions, $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\iota^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, s-$(\tau_1, \tau_2)p$-continuous multifunctions, weakly s-(τ_1, τ_2)-continuous multifunctions, almost nearly quasi (τ_1, τ_2)-continuous multifunctions, almost nearly (τ_1, τ_2)-continuous multifunctions, rarely s-$(\tau_1, \tau_2)p$-continuous multifunctions, s-(τ_1, τ_2)-continuous multifunctions and nearly (τ_1, τ_2)-continuous multifunctions were established in [81], [24], [20], [25], [19], [79], [7], [14], [21], [13], [11], [12], [18], [22], [10], [39], [17], [75], [62], [40], [72], [64], [78], [59], [36], [32], [41], [30] and [71], respectively. Khampakdee et al. [38] introduced and investigated the concept of c-(τ_1, τ_2)-continuous multifunctions. Pue-on et al. [65] introduced and studied the notion of almost quasi (τ_1, τ_2)-continuous multifunctions. Quite recently, Pue-on et al. [60] introduced the concepts of upper quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions and lower quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions. In this paper, we investigate several characterizations of upper quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions and lower quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called *$\tau_1\tau_2$-closed* [26] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called *$\tau_1\tau_2$-open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the *$\tau_1\tau_2$-closure* [26] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the *$\tau_1\tau_2$-interior* [26] of A and is denoted by $\tau_1\tau_2$-Int(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$\tau_1\tau_2$-clopen* [26] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$(\tau_1, \tau_2)r$-open* [81] (resp. *$(\tau_1, \tau_2)s$-open* [24], *$(\tau_1, \tau_2)p$-open* [24], *$(\tau_1, \tau_2)\beta$-open* [24]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called *$(\tau_1, \tau_2)r$-closed* (resp. *$(\tau_1, \tau_2)s$-closed*, *$(\tau_1, \tau_2)p$-closed*, *$(\tau_1, \tau_2)\beta$-closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$\alpha(\tau_1, \tau_2)$-open* [77] if $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be *$\alpha(\tau_1, \tau_2)$-closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a *$(\tau_1, \tau_2)\theta$-cluster point* [81] of A if $\tau_1\tau_2$-Cl$(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the *$(\tau_1, \tau_2)\theta$-closure* [81] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$(\tau_1, \tau_2)\theta$-closed* [81] if $(\tau_1, \tau_2)\theta$-Cl$(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be *$(\tau_1, \tau_2)\theta$-open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the *$(\tau_1, \tau_2)\theta$-interior* [81] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 2.1. [81] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2$-$Cl(A) = (\tau_1, \tau_2)\theta$-$Cl(A)$.*

(2) *$(\tau_1, \tau_2)\theta$-$Cl(A)$ is $\tau_1\tau_2$-closed in X.*

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a *$\theta(\tau_1, \tau_2)s$-cluster point* of A if (τ_1, τ_2)-sCl$(U) \cap A \neq \emptyset$ for every $(\tau_1, \tau_2)s$-open set U containing x. The set of all $\theta(\tau_1, \tau_2)s$-cluster points of A is called the *$\theta(\tau_1, \tau_2)s$-closure* of A and is denoted by $\theta(\tau_1, \tau_2)$-sCl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be *$\theta(\tau_1, \tau_2)s$-closed* if $\theta(\tau_1, \tau_2)$-sCl$(A) = A$. The complement of a $\theta(\tau_1, \tau_2)s$-closed set is said to be *$\theta(\tau_1, \tau_2)s$-open*. The union of all $\theta(\tau_1, \tau_2)s$-open sets of X contained in A is called the *$\theta(\tau_1, \tau_2)s$-interior* of A and is denoted by $\theta(\tau_1, \tau_2)$-sInt(A).

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$.

3. Characterizations of upper and lower quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions

In this section, we investigate some characterizations of upper quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions and lower quasi $\theta(\tau_1, \tau_2)$-continuous multifunctions.

Definition 3.1. [60] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper quasi $\theta(\tau_1, \tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y containing $F(x)$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F((\tau_1, \tau_2)$-$sCl(U)) \subseteq \sigma_1\sigma_2$-$Cl(V)$.*

Lemma 3.1. [60] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper quasi $\theta(\tau_1, \tau_2)$-continuous;*
(2) *$\theta(\tau_1, \tau_2)$-$sCl(F^-(\sigma_1\sigma_2$-$Int((\sigma_1, \sigma_2)\theta$-$Cl(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta$-$Cl(B))$ for every subset B of Y;*
(3) *$\theta(\tau_1, \tau_2)$-$sCl(F^-(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(V)))) \subseteq F^-(\sigma_1\sigma_2$-$Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y;*
(4) *$\theta(\tau_1, \tau_2)$-$sCl(F^-(\sigma_1\sigma_2$-$Int(K))) \subseteq F^-(K)$ for every $(\sigma_1, \sigma_2)r$-closed set K of Y;*
(5) *$F^+(V) \subseteq \theta(\tau_1, \tau_2)$-$sInt(F^+(\sigma_1\sigma_2$-$Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;*
(6) *$\theta(\tau_1, \tau_2)$-$sCl(F^-(\sigma_1\sigma_2$-$Int(K))) \subseteq F^-(K)$ for every $\sigma_1\sigma_2$-closed set K of Y;*
(7) *$\theta(\tau_1, \tau_2)$-$sCl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2$-$Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y.*

For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, a multifunction $\mathrm{Cl}F_{\circledast} : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is defined in [26] as follows: $\mathrm{Cl}F_{\circledast}(x) = \sigma_1\sigma_2$-Cl$(F(x))$ for each $x \in X$.

Definition 3.2. [26] *A subset A of a bitopological space (X, τ_1, τ_2) is said to be:*

(1) *$\tau_1\tau_2$-paracompact if every cover of A by $\tau_1\tau_2$-open sets of X is refined by a cover of A which consists of $\tau_1\tau_2$-open sets of X and is $\tau_1\tau_2$-locally finite in X;*
(2) *$\tau_1\tau_2$-regular if for each $x \in A$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a $\tau_1\tau_2$-open set V of X such that $x \in V \subseteq \tau_1\tau_2$-$Cl(V) \subseteq U$.*

Lemma 3.2. [26] *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a multifunction such that $F(x)$ is $\tau_1\tau_2$-regular and $\tau_1\tau_2$-paracompact for each $x \in X$, then $ClF^+_{\circledast}(V) = F^+(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Lemma 3.3. [26] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, $ClF^-_{\circledast}(V) = F^-(V)$ for each $\sigma_1\sigma_2$-open set V of Y.*

Theorem 3.1. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-paracompact and $\sigma_1\sigma_2$-regular for each $x \in X$. Then, the following properties are equivalent:*

(1) *F is upper quasi $\theta(\tau_1, \tau_2)$-continuous;*
(2) *$ClF_{\circledast}$ is upper quasi $\theta(\tau_1, \tau_2)$-continuous.*

Proof. We put $G = \mathrm{Cl}F_{\circledast}$. Suppose that F is upper quasi $\theta(\tau_1, \tau_2)$-continuous. It follows from Lemmas 3.1, 3.2 and 3.3 that for every $\sigma_1\sigma_2$-open set V of Y,

$$\begin{aligned} G^+(V) = F^+(V) &\subseteq \theta(\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= \theta(\tau_1, \tau_2)\text{-sInt}(G^+(\sigma_1\sigma_2\text{-Cl}(V))). \end{aligned}$$

By Lemma 3.1, G is upper quasi $\theta(\tau_1, \tau_2)$-continuous.

Conversely, suppose that G is upper quasi $\theta(\tau_1, \tau_2)$-continuous. It follows from Lemmas 3.1, 3.2 and 3.3 that for every $\sigma_1\sigma_2$-open set V of Y,

$$\begin{aligned} F^+(V) = G^+(V) &\subseteq \theta(\tau_1, \tau_2)\text{-sInt}(G^+(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= \theta(\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V))). \end{aligned}$$

Thus by Lemma 3.1, F is upper quasi $\theta(\tau_1, \tau_2)$-continuous. □

Definition 3.3. [60] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower quasi $\theta(\tau_1, \tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $\sigma_1\sigma_2$-$Cl(V) \cap F(z) \neq \emptyset$ for every $z \in (\tau_1, \tau_2)$-$sCl(U)$.*

Theorem 3.2. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a multifunction such that $F(x)$ is $\sigma_1\sigma_2$-paracompact and $\sigma_1\sigma_2$-regular for each $x \in X$. Then, the following properties are equivalent:*

(1) *F is lower quasi $\theta(\tau_1, \tau_2)$-continuous;*
(2) *$ClF_{\circledast}$ is lower quasi $\theta(\tau_1, \tau_2)$-continuous.*

Proof. The proof is similar to that of Theorem 3.1 and is thus omitted. □

Definition 3.4. *A bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-Urysohn if for each pair of distinct points x and y in X, there exist $\tau_1\tau_2$-open sets U and V such that $x \in U$, $y \in V$ and $\tau_1\tau_2$-$Cl(U) \cap \tau_1\tau_2$-$Cl(V) = \emptyset$.*

Lemma 3.4. *If A and B are disjoint $\tau_1\tau_2$-compact subsets of a $\tau_1\tau_2$-Urysohn space (X, τ_1, τ_2), then there exist $\tau_1\tau_2$-open sets U and V of X such that $A \subseteq U$, $B \subseteq V$ and $\tau_1\tau_2$-$Cl(U) \cap \tau_1\tau_2$-$Cl(V) = \emptyset$.*

Definition 3.5. *A bitopological space (X, τ_1, τ_2) is called $(\tau_1, \tau_2)s$-Hausdorff if for each pair of distinct points x and y in X, there exist $\tau_1\tau_2$-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

A subset A of a bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)*s-regular* if A is $(\tau_1, \tau_2)s$-open and $(\tau_1, \tau_2)s$-closed.

Lemma 3.5. *Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$. If A is a (τ_1, τ_2)s-regular set of X, then (τ_1, τ_2)-sCl(A) is (τ_1, τ_2)s-regular.*

Lemma 3.6. *A bitopological space (X, τ_1, τ_2) is (τ_1, τ_2)s-Hausdorff if and only if for each pair of distinct points x and y in X, there exist (τ_1, τ_2)s-regular sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be *injective* if $x \neq y$ implies $F(x) \cap F(y) = \emptyset$.

Theorem 3.3. *If $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper quasi $\theta(\tau_1, \tau_2)$-continuous injective multifunction into a $\sigma_1\sigma_2$-Urysohn space (Y, σ_1, σ_2) and $F(x)$ is $\sigma_1\sigma_2$-compact for each $x \in X$, then (X, τ_1, τ_2) is (τ_1, τ_2)s-Hausdorff.*

Proof. For any distinct points x and y of X, we have $F(x) \cap F(y) = \emptyset$, since F is injective. Since $F(x)$ is $\sigma_1\sigma_2$-compact for each $x \in X$ and (Y, σ_1, σ_2) is $\sigma_1\sigma_2$-Urysohn, by Lemma 3.4, there exist $\sigma_1\sigma_2$-open sets V and W such that $F(x) \subseteq V$, $F(y) \subseteq W$ and $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(W) = \emptyset$. Since F is upper quasi $\theta(\tau_1, \tau_2)$-continuous, there exist $(\tau_1, \tau_2)s$-open sets U and G of X containing x and y, respectively, such that $F((\tau_1, \tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$ and $F((\tau_1, \tau_2)\text{-sCl}(G)) \subseteq \sigma_1\sigma_2\text{-Cl}(W)$. Thus,

$$(\tau_1, \tau_2)\text{-sCl}(G) \cap (\tau_1, \tau_2)\text{-sCl}(U) = \emptyset$$

because $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(W) = \emptyset$. By Lemmas 3.5 and 3.6, (X, τ_1, τ_2) is (τ_1, τ_2)s-Hausdorff. □

Corollary 3.1. *If (Y, σ_1, σ_2) is a $\sigma_1\sigma_2$-Urysohn space and $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a quasi $\theta(\tau_1, \tau_2)$-continuous injection, then (X, τ_1, τ_2) is (τ_1, τ_2)s-Hausdorff.*

Definition 3.6. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the graph $G(F) = \{(x, F(x)) \mid x \in X\}$ is called $\theta(\sigma_1, \sigma_2)$s-closed if for each $(x, y) \in (X \times Y) - G(F)$, there exist a (τ_1, τ_2)s-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $[(\tau_1, \tau_2)\text{-sCl}(U) \times \sigma_1\sigma_2\text{-Cl}(V)] \cap G(F) = \emptyset$.*

Lemma 3.7. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ has a $\theta(\sigma_1, \sigma_2)$s-closed graph if and only if for each $(x, y) \in (X \times Y) - G(F)$, there exist a (τ_1, τ_2)s-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $F((\tau_1, \tau_2)\text{-sCl}(U)) \cap \sigma_1\sigma_2\text{-Cl}(V) = \emptyset$.*

Theorem 3.4. *If (Y, σ_1, σ_2) is a $\sigma_1\sigma_2$-Urysohn space and $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is an upper quasi $\theta(\tau_1, \tau_2)$-continuous multifunction such that $F(x)$ is $\sigma_1\sigma_2$-compact for each $x \in X$, then $G(F)$ is $\theta(\sigma_1, \sigma_2)$s-closed.*

Proof. Let $(x_0, y_0) \in (X \times Y) - G(F)$. Then, $y_0 \in Y - F(x_0)$. Since (Y, σ_1, σ_2) is a $\sigma_1\sigma_2$-Urysohn, for each $y \in F(x_0)$, there exist $\sigma_1\sigma_2$-open sets $V(y)$ and $W(y)$ such that $y \in V(y)$, $y_0 \in W(y)$ and $\sigma_1\sigma_2$-Cl$(V(y)) \cap \sigma_1\sigma_2$-Cl$(W(y)) = \emptyset$. The family $\{V(y) \mid y \in F(x_0)\}$ is a $\sigma_1\sigma_2$-open cover of $F(x_0)$ and there exist a finite number of points, say, $y_1, y_2, ..., y_n$ in $F(x_0)$ such that $F(x_0) \subseteq \bigcup_{i=1}^{n} V(y_i)$. Put $V = \bigcup_{i=1}^{n} V(y_i)$ and $W = \bigcap_{i=1}^{n} W(y_i)$. Then, V and W are $\sigma_1\sigma_2$-open sets, $y_0 \in W$, $F(x_0) \subseteq V$ and $\sigma_1\sigma_2$-Cl$(V) \cap \sigma_1\sigma_2$-Cl$(W) = \emptyset$. Since F is upper quasi $\theta(\tau_1, \tau_2)$-continuous, there exists a

$(\tau_1, \tau_2)s$-open set U of X containing x_0 such that $F((\tau_1, \tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. Thus, we have $F((\tau_1, \tau_2)\text{-sCl}(U)) \cap \sigma_1\sigma_2\text{-Cl}(W) = \emptyset$ and by Lemma 3.7, $G(F)$ is $\theta(\sigma_1, \sigma_2)s$-closed. □

Definition 3.7. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the graph $G(f) = \{(x, f(x)) \mid x \in X\}$ is called $\theta(\sigma_1, \sigma_2)s$-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $(\tau_1, \tau_2)s$-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $[(\tau_1, \tau_2)\text{-sCl}(U) \times \sigma_1\sigma_2\text{-Cl}(V)] \cap G(f) = \emptyset$.*

Lemma 3.8. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ has a $\theta(\sigma_1, \sigma_2)s$-closed graph if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist a $(\tau_1, \tau_2)s$-open set U of X containing x and a $\sigma_1\sigma_2$-open set V of Y containing y such that $f((\tau_1, \tau_2)\text{-sCl}(U)) \cap \sigma_1\sigma_2\text{-Cl}(V) = \emptyset$.*

Corollary 3.2. *If (Y, σ_1, σ_2) is a $\sigma_1\sigma_2$-Urysohn space and $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is a quasi $\theta(\tau_1, \tau_2)$-continuous function, then $G(f)$ is $\theta(\sigma_1, \sigma_2)s$-closed in X.*

Definition 3.8. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper $\theta(\tau_1, \tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(\tau_1\tau_2\text{-Cl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$.*

Definition 3.9. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower $\theta(\tau_1, \tau_2)$-continuous if for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $\sigma_1\sigma_2\text{-Cl}(V) \cap F(z) \neq \emptyset$ for every $z \in \tau_1\tau_2\text{-Cl}(U)$.*

Theorem 3.5. *Let $G, H : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be multifunctions. Assume that the following four conditions:*

1. *(Y, σ_1, σ_2) is a $\sigma_1\sigma_2$-Urysohn space,*
2. *G is upper $\theta(\tau_1, \tau_2)$-continuous and H is upper quasi $\theta(\tau_1, \tau_2)$-continuous,*
3. *$G(x)$ and $H(x)$ are $\sigma_1\sigma_2$-compact for each $x \in X$, and*
4. *$G(x) \cap H(x) \neq \emptyset$ for each $x \in X$*

are satisfied. Then a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, defined by $F(x) = G(x) \cap H(x)$ for each $x \in X$, is upper quasi $\theta(\tau_1, \tau_2)$-continuous.

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y such that $F(x) \subseteq V$. Then, $G(x) - V$ and $H(x) - V$ are disjoint $\sigma_1\sigma_2$-compact sets. By Lemma 3.4, there exist $\sigma_1\sigma_2$-open sets W and W' such that $G(x) - V \subseteq W$, $H(x) - V \subseteq W'$ and $\sigma_1\sigma_2\text{-Cl}(W) \cap \sigma_1\sigma_2\text{-Cl}(W') = \emptyset$. Since G is upper $\theta(\tau_1, \tau_2)$-continuous, there exists a $\tau_1\tau_2$-open set U' of X containing x such that $F(\tau_1\tau_2\text{-Cl}(U')) \subseteq \sigma_1\sigma_2\text{-Cl}(W \cup V)$. Since H is upper quasi $\theta(\tau_1, \tau_2)$-continuous, there exists a $(\tau_1, \tau_2)s$-open set U'' of X containing x such that $F((\tau_1, \tau_2)\text{-sCl}(U'')) \subseteq \sigma_1\sigma_2\text{-Cl}(W' \cup V)$. Let $U = U' \cap U''$. Then, U is a $(\tau_1, \tau_2)s$-open set containing x. If $x_0 \in (\tau_1, \tau_2)\text{-sCl}(U)$, then $x_0 \in \tau_1\tau_2\text{-Cl}(U') \cap (\tau_1, \tau_2)\text{-sCl}(U'')$. If $y \in F(x_0)$ for each $x_0 \in (\tau_1, \tau_2)\text{-sCl}(U)$, then

$$y \in \sigma_1\sigma_2\text{-Cl}(W \cup V) \cap \sigma_1\sigma_2\text{-Cl}(W'' \cup V) = [\sigma_1\sigma_2\text{-Cl}(W') \cap \sigma_1\sigma_2\text{-Cl}(W'')] \cup \sigma_1\sigma_2\text{-Cl}(V).$$

Since $\sigma_1\sigma_2\text{-Cl}(W') \cap \sigma_1\sigma_2\text{-Cl}(W'') = \emptyset$, we have $y \in \sigma_1\sigma_2\text{-Cl}(V)$ and hence $F((\tau_1, \tau_2)\text{-sCl}(U)) \subseteq \sigma_1\sigma_2\text{-Cl}(V)$. Thus, F is quasi $\theta(\tau_1, \tau_2)$-continuous. □

Definition 3.10. [70] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$, $F(G) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be upper weakly quasi (τ_1, τ_2)-continuous if F is upper weakly quasi (τ_1, τ_2)-continuous at each point of X.*

Lemma 3.9. [70] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper weakly quasi (τ_1, τ_2)-continuous;*
(2) *for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2\text{-}Cl(V)$;*
(3) *$\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(F^-(\sigma_1\sigma_2\text{-}Int(K)))) \subseteq F^-(K)$ for every $\sigma_1\sigma_2$-closed set K of Y;*
(4) *$F^+(V) \subseteq (\tau_1, \tau_2)\text{-}sInt(F^+(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;*
(5) *$(\tau_1, \tau_2)\text{-}sCl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y.*

Definition 3.11. [70] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$, $\sigma_1\sigma_2\text{-}Cl(V) \cap F(z) \neq \emptyset$ for every $z \in G$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be lower weakly quasi (τ_1, τ_2)-continuous if F is lower weakly quasi (τ_1, τ_2)-continuous at each point of X.*

Lemma 3.10. [70] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower weakly quasi (τ_1, τ_2)-continuous;*
(2) *for each $x \in X$ and each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $U \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$;*
(3) *$\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(F^+(\sigma_1\sigma_2\text{-}Int(K)))) \subseteq F^+(K)$ for every $\sigma_1\sigma_2$-closed set K of Y;*
(4) *$F^-(V) \subseteq (\tau_1, \tau_2)\text{-}sInt(F^-(\sigma_1\sigma_2\text{-}Cl(V)))$ for every $\sigma_1\sigma_2$-open set V of Y;*
(5) *$(\tau_1, \tau_2)\text{-}sCl(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ for every $\sigma_1\sigma_2$-open set V of Y.*

Definition 3.12. [29] *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper almost quasi (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$, $F(G) \subseteq (\sigma_1, \sigma_2)\text{-}sCl(V)$. A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called upper weakly quasi (τ_1, τ_2)-continuous if F is upper almost quasi (τ_1, τ_2)-continuous at each point of X.*

Lemma 3.11. [29] *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper almost quasi (τ_1, τ_2)-continuous;*
(2) *for each $x \in X$ and every $\sigma_1\sigma_2$-open set V of Y such that $F(x) \subseteq V$, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq (\sigma_1, \sigma_2)\text{-}sCl(V)$;*
(3) *$F^+(V)$ is $(\tau_1, \tau_2)s$-open in X for every $(\sigma_1, \sigma_2)r$-open set V of Y;*

(4) $F^+(V) \subseteq (\tau_1, \tau_2)$-$sInt(F^+((\sigma_1, \sigma_2)$-$sCl(V)))$ *for every* $\sigma_1\sigma_2$-*open set* V *of* Y;
(5) (τ_1, τ_2)-$sCl(F^-(\sigma_1\sigma_2$-$Cl(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(B))))) \subseteq F^-(\sigma_1\sigma_2$-$Cl(B))$ *for every subset* B *of* Y;
(6) $F^+(V) \subseteq \tau_1\tau_2$-$Cl(\tau_1\tau_2$-$Int(F^+((\sigma_1, \sigma_2)$-$sCl(V))))$ *for every* $\sigma_1\sigma_2$-*open set* V *of* Y.

Definition 3.13. [29] *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called lower almost quasi* (τ_1, τ_2)-*continuous at a point* $x \in X$ *if for each* $\sigma_1\sigma_2$-*open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$ *and each* $\tau_1\tau_2$-*open set* U *of* X *containing* x, *there exists a nonempty* $\tau_1\tau_2$-*open set* G *such that* $G \subseteq U$, (σ_1, σ_2)-$sCl(V) \cap F(z) \neq \emptyset$ *for every* $z \in G$. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is called lower almost quasi* (τ_1, τ_2)-*continuous if* F *is lower weakly quasi* (τ_1, τ_2)-*continuous at each point of* X.

Lemma 3.12. [29] *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, *the following properties are equivalent:*

(1) F *is lower almost quasi* (τ_1, τ_2)-*continuous;*
(2) *for each* $x \in X$ *and every* $\sigma_1\sigma_2$-*open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$, *there exists a* $(\tau_1, \tau_2)s$-*open set* U *of* X *containing* x *such that* $U \subseteq F^-((\sigma_1, \sigma_2)$-$sCl(V))$;
(3) $F^-(V)$ *is* $(\tau_1, \tau_2)s$-*open in* X *for every* $(\sigma_1, \sigma_2)r$-*open set* V *of* Y;
(4) $F^-(V) \subseteq (\tau_1, \tau_2)$-$sInt(F^-((\sigma_1, \sigma_2)$-$sCl(V)))$ *for every* $\sigma_1\sigma_2$-*open set* V *of* Y;
(5) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2$-$Cl(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(B))))) \subseteq F^+(\sigma_1\sigma_2$-$Cl(B))$ *for every subset* B *of* Y;
(6) $F^-(V) \subseteq \tau_1\tau_2$-$Cl(\tau_1\tau_2$-$Int(F^-((\sigma_1, \sigma_2)$-$sCl(V))))$ *for every* $\sigma_1\sigma_2$-*open set* V *of* Y.

Theorem 3.6. *If a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is upper weakly quasi* (τ_1, τ_2)-*continuous and lower almost quasi* (τ_1, τ_2)-*continuous, then* F *is upper quasi* $\theta(\tau_1, \tau_2)$-*continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$. Since F is upper weakly quasi (τ_1, τ_2)-continuous, by Lemma 3.9 there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$ and hence $U \subseteq F^+(\sigma_1\sigma_2$-Cl$(V))$. Since F is lower almost quasi (τ_1, τ_2)-continuous and $\sigma_1\sigma_2$-Cl(V) is a $(\sigma_1, \sigma_2)r$-closed set of Y, by Lemma 3.12 we have $F^+(\sigma_1\sigma_2$-Cl$(V))$ is $(\tau_1, \tau_2)s$-closed in X. Thus,

$$(\tau_1, \tau_2)\text{-sCl}(U) \subseteq F^+(\sigma_1\sigma_2\text{-Cl}(V))$$

and hence $F((\tau_1, \tau_2)$-sCl$(U)) \subseteq \sigma_1\sigma_2$-Cl$(V)$. This shows that F is upper quasi $\theta(\tau_1, \tau_2)$-continuous. □

Theorem 3.7. *If a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is lower weakly quasi* (τ_1, τ_2)-*continuous and upper almost quasi* (τ_1, τ_2)-*continuous, then* F *is lower quasi* $\theta(\tau_1, \tau_2)$-*continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y such that $F(x) \cap V \neq \emptyset$. Since F is lower weakly quasi (τ_1, τ_2)-continuous, by Lemma 3.10 there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $U \subseteq F^-(\sigma_1\sigma_2$-Cl$(V))$. Since F is upper almost quasi (τ_1, τ_2)-continuous and $\sigma_1\sigma_2$-Cl(V) is a $(\sigma_1, \sigma_2)r$-closed set of Y, by Lemma 3.11 $F^-(\sigma_1\sigma_2$-Cl$(V))$ is $(\tau_1, \tau_2)s$-closed in X and hence (τ_1, τ_2)-sCl$(U) \subseteq F^-(\sigma_1\sigma_2$-Cl$(V))$. This implies that $\sigma_1\sigma_2$-Cl$(V) \cap F(z) \neq \emptyset$ for every $z \in (\tau_1, \tau_2)$-sCl(U). Thus, F is lower quasi $\theta(\tau_1, \tau_2)$-continuous. □

Definition 3.14. [74] *A bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)s$-regular if for each $(\tau_1, \tau_2)s$-closed set F of X and each $x \notin F$, there exist disjoint $(\tau_1, \tau_2)s$-open sets V and V such that $x \in U$ and $F \subseteq V$.*

Lemma 3.13. [74] *A bitopological space (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-regular if and only if for each $x \in X$ and each $(\tau_1, \tau_2)s$-open set U containing x, there exists a $(\tau_1, \tau_2)s$-open set V such that $x \in V \subseteq (\tau_1, \tau_2)$-$sCl(V) \subseteq U$.*

Theorem 3.8. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper weakly quasi (τ_1, τ_2)-continuous and (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-regular, then F is upper quasi $\theta(\tau_1, \tau_2)$-continuous.*

Proof. Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y containing $F(x)$. Since F is upper weakly quasi (τ_1, τ_2)-continuous, by Lemma 3.9, there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. By Lemma 3.13, there exists a $(\tau_1, \tau_2)s$-open set W such that $x \in W \subseteq (\tau_1, \tau_2)$-sCl$(W) \subseteq U$. Thus, $F((\tau_1, \tau_2)$-sCl$(W) \subseteq \sigma_1\sigma_2$-Cl$(V)$ and hence F is upper quasi $\theta(\tau_1, \tau_2)$-continuous. □

Theorem 3.9. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is lower weakly quasi (τ_1, τ_2)-continuous and (X, τ_1, τ_2) is $(\tau_1, \tau_2)s$-regular, then F is lower quasi $\theta(\tau_1, \tau_2)$-continuous.*

Proof. The proof is similar to that of Theorem 3.8. □

Recall that a collection $\mathscr{U}$ of subsets of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2$-*locally finite* [26] if every $x \in X$ has a $\tau_1\tau_2$-neighbourhood which intersects only finitely many elements of $\mathscr{U}$.

Definition 3.15. *A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-almost regular if for each $x \in A$ and each $(\tau_1, \tau_2)r$-open set U of X containing x, there exists a $\tau_1\tau_2$-open set V such that $x \in V \subseteq \tau_1\tau_2$-$Cl(V) \subseteq U$.*

Definition 3.16. *A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-nearly paracompact if every cover of A by $(\tau_1, \tau_2)r$-open sets of X has a $\tau_1\tau_2$-open $\tau_1\tau_2$-locally finite refinement which covers A.*

Lemma 3.14. *Let (X, τ_1, τ_2) be a bitopological space. If A is a $\tau_1\tau_2$-almost regular $\tau_1\tau_2$-nearly paracompact set of X and U is a $(\tau_1, \tau_2)r$-open set such that $A \subseteq U$, then there exists a $\tau_1\tau_2$-open set V such that $A \subseteq V \subseteq \tau_1\tau_2$-$Cl(V) \subseteq U$.*

Theorem 3.10. *If a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is upper weakly quasi (τ_1, τ_2)-continuous and $F(x)$ is $\tau_1\tau_2$-almost regular $\tau_1\tau_2$-nearly paracompact in Y for each $x \in X$, then F is upper almost quasi (τ_1, τ_2)-continuous.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y and $F(x) \subseteq V$. Since $F(x)$ is $\tau_1\tau_2$-almost regular $\tau_1\tau_2$-nearly paracompact, by Lemma 3.14 there exists a $\sigma_1\sigma_2$-open set W of Y such that $F(x) \subseteq W \subseteq \sigma_1\sigma_2$-Cl$(W) \subseteq V$. Thus, $x \in U \subseteq F^+(V)$ and hence $F^+(V)$ is $(\tau_1\tau_2)s$-open in X. It follows from Lemma 3.11 that F is upper almost quasi (τ_1, τ_2)-continuous. □

Definition 3.17. *A bitopological space (X, τ_1, τ_2) is said to be $\mathscr{S}$-(τ_1, τ_2)-closed if every $(\tau_1, \tau_2)s$-open cover $\{U_\gamma \mid \gamma \in \Gamma\}$, there exists a finite subset Γ_0 of Γ such that $X = \cup\{\tau_1\tau_2\text{-}Cl(U_\gamma) \mid \gamma \in \Gamma_0\}$.*

Theorem 3.11. *Let $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a surjective multifunction and $F(x)$ is $\sigma_1\sigma_2$-compact for each $x \in X$. If F is upper weakly quasi (τ_1, τ_2)-continuous and lower almost quasi (τ_1, τ_2)-continuous and (X, τ_1, τ_2) is $\mathscr{S}$-(τ_1, τ_2)-closed, then (Y, σ_1, σ_2) is quasi (σ_1, σ_2)-$\mathscr{H}$-closed.*

Proof. Let $\{V_\gamma \mid \gamma \in \Gamma\}$ be any $\sigma_1\sigma_2$-open cover of Y. For each $x \in X$, $F(x)$ is $\sigma_1\sigma_2$-compact and there exists a finite subset $\Gamma(x)$ of Γ such that $F(x) \subseteq \cup\{V_\gamma \mid \gamma \in \Gamma(x)\}$. Now, put

$$V(x) = \cup\{V_\gamma \mid \gamma \in \Gamma(x)\}.$$

Then, $V(x)$ is $\sigma_1\sigma_2$-open in Y and $F(x) \subseteq V(x)$. It follows from Theorem 3.6 that F is upper quasi $\theta(\tau_1, \tau_2)$-continuous. By Lemma 3.1, $x \in F^+(V(x)) \subseteq \theta(\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V(x))))$. Since

$$\theta(\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V(x))))$$

is $(\tau_1, \tau_2)s$-open in X, we have $\{\theta(\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V(x))) \mid x \in X\}$ is a $(\tau_1, \tau_2)s$-open cover of X. Since (X, τ_1, τ_2) is $\mathscr{S}$-(τ_1, τ_2)-closed, there exists a finite number of points, say, $x_1, x_2, ..., x_n$ in X such that

$$\begin{aligned} X &= \bigcup_{i=1}^{n}(\theta(\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V(x_i))))) \\ &= \bigcup_{i=1}^{n}\tau_1\tau_2\text{-Cl}(F^+(\sigma_1\sigma_2\text{-Cl}(V(x_i)))) \\ &= \tau_1\tau_2\text{-Cl}(\bigcup_{i=1}^{n}F^+(\sigma_1\sigma_2\text{-Cl}(V(x_i)))). \end{aligned}$$

Thus,

$$\begin{aligned} X = (\tau_1, \tau_2)\text{-sCl}(\bigcup_{i=1}^{n}F^+(\sigma_1\sigma_2\text{-Cl}(V(x_i)))) &\subseteq (\tau_1, \tau_2)\text{-sCl}(F^+(\bigcup_{i=1}^{n}\sigma_1\sigma_2\text{-Cl}(V(x_i)))) \\ &= (\tau_1, \tau_2)\text{-sCl}(F^+(\bigcup_{i=1}^{n}V(x_i))). \end{aligned}$$

Since $\sigma_1\sigma_2\text{-Cl}(\bigcup_{i=1}^{n}V(x_i))$ is $(\sigma_1, \sigma_2)r$-closed in Y, by Lemma 3.12 $F^+(\sigma_1\sigma_2\text{-Cl}(\bigcup_{i=1}^{n}V(x_i)))$ is $(\tau_1, \tau_2)s$-closed in X. Therefore, we have

$$\begin{aligned} Y = F(X) &= F(F^+(\sigma_1\sigma_2\text{-Cl}(\bigcup_{i=1}^{n}V(x_i)))) \\ &\subseteq \sigma_1\sigma_2\text{-Cl}(\bigcup_{i=1}^{n}V(x_i)) \\ &= \bigcup_{i=1}^{n}\sigma_1\sigma_2\text{-Cl}(V(x_i)) \\ &= \bigcup_{i=1}^{n}\cup_{\gamma\in\Gamma(x_i)}\sigma_1\sigma_2\text{-Cl}(V_\gamma). \end{aligned}$$

This shows that (Y, σ_1, σ_2) is quasi (σ_1, σ_2)-$\mathscr{H}$-closed. □

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References

[1] M.E. Abd El-Monsef, S.N. El-Deeb, R.A. Mahmoud, β-Open Sets and β-Continuous Mappings, Bull. Fac. Sci. Assiut Univ. 12 (1983), 77–90.

[2] S.P. Arya, M.P. Bhamini, Some Weaker Forms of Semi-Continuous Functions, Ganita, 33 (1982), 124–134.

[3] C. Boonpok, N. Srisarakham, (τ_1, τ_2)-Continuity for Functions, Asia Pac. J. Math. 11 (2024), 21. https://doi.org/10.28924/APJM/11-21.

[4] C. Boonpok, J. Khampakdee, Almost Strong $\theta(\Lambda, p)$-Continuity for Functions, Eur. J. Pure Appl. Math. 17 (2024), 300–309. https://doi.org/10.29020/nybg.ejpam.v17i1.4975.

[5] C. Boonpok, P. Pue-On, Characterizations of Almost (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 22 (2024), 33. https://doi.org/10.28924/2291-8639-22-2024-33.

[6] C. Boonpok, C. Klanarong, On Weakly (τ_1, τ_2)-Continuous Functions, Eur. J. Pure Appl. Math. 17 (2024), 416–425. https://doi.org/10.29020/nybg.ejpam.v17i1.4976.

[7] C. Boonpok, J. Khampakdee, Upper and Lower α-$\star$-Continuity, Eur. J. Pure Appl. Math. 17 (2024), 201–211. https://doi.org/10.29020/nybg.ejpam.v17i1.4858.

[8] C. Boonpok, N. Srisarakham, Weak Forms of (Λ, b)-Open Sets and Weak (Λ, b)-Continuity, Eur. J. Pure Appl. Math. 16 (2023), 29–43. https://doi.org/10.29020/nybg.ejpam.v16i1.4571.

[9] C. Boonpok, $\theta(\star)$-Precontinuity, Mathematica, 65 (2023), 31–42. https://doi.org/10.24193/mathcluj.2023.1.04.

[10] C. Boonpok, P. Pue-on, Upper and Lower Weakly (Λ, sp)-Continuous Multifunctions, Eur. J. Pure Appl. Math. 16 (2023), 1047–1058. https://doi.org/10.29020/nybg.ejpam.v16i2.4573.

[11] C. Boonpok, P. Pue-on, Upper and Lower $s\beta(\star)$-Continuous Multifunctions, Eur. J. Pure Appl. Math. 16 (2023), 1634–1646. https://doi.org/10.29020/nybg.ejpam.v16i3.4732.

[12] C. Boonpok, J. Khampakdee, Upper and Lower Weak $s\beta(\star)$-Continuity, Eur. J. Pure Appl. Math. 16 (2023), 2544–2556. https://doi.org/10.29020/nybg.ejpam.v16i4.4734.

[13] C. Boonpok, P. Pue-on, Upper and Lower Weakly α-$\star$-Continuous Multifunctions, Int. J. Anal. Appl. 21 (2023), 90. https://doi.org/10.28924/2291-8639-21-2023-90.

[14] C. Boonpok, N. Srisarakham, Almost α-$\star$-Continuity for Multifunctions, Int. J. Anal. Appl. 21 (2023), 107. https://doi.org/10.28924/2291-8639-21-2023-107.

[15] C. Boonpok, on Some Spaces via Topological Ideals, Open Math. 21 (2023), 20230118. https://doi.org/10.1515/math-2023-0118.

[16] C. Boonpok, J. Khampakdee, (Λ, sp)-Open Sets in Topological Spaces, Eur. J. Pure Appl. Math. 15 (2022), 572–588. https://doi.org/10.29020/nybg.ejpam.v15i2.4276.

[17] C. Boonpok, J. Khampakdee, On Almost $\alpha(\Lambda, sp)$-Continuous Multifunctions, Eur. J. Pure Appl. Math. 15 (2022), 626–634. https://doi.org/10.29020/nybg.ejpam.v15i2.4277.

[18] C. Boonpok, $\theta(\star)$-Quasi Continuity for Multifunctions, WSEAS Trans. Math. 21 (2022), 245–251. https://doi.org/10.37394/23206.2022.21.29.

[19] C. Boonpok, Upper and Lower $\beta(\star)$-Continuity, Heliyon, 7 (2021), e05986. https://doi.org/10.1016/j.heliyon.2021.e05986.

[20] C. Boonpok, C. Viriyapong, Upper and Lower Almost Weak (τ_1, τ_2)-Continuity, Eur. J. Pure Appl. Math. 14 (2021), 1212–1225. https://doi.org/10.29020/nybg.ejpam.v14i4.4072.

[21] C. Boonpok, Weak Quasi Continuity for Multifunctions in Ideal Topological Spaces, Adv. Math.: Sci. J. 9 (2020), 339–355.

[22] C. Boonpok, P. Pue-On, Continuity for Multifunctions in Ideal Topological Spaces, WSEAS Trans. Math. 19 (2021), 624–631. https://doi.org/10.37394/23206.2020.19.69.

[23] C. Boonpok, on Characterizations of ⋆-Hyperconnected Ideal Topological Spaces, J. Math. 2020 (2020), 9387601. https://doi.org/10.1155/2020/9387601.

[24] C. Boonpok, $(\tau_1, \tau_2)\delta$-Semicontinuous Multifunctions, Heliyon, 6 (2020), e05367. https://doi.org/10.1016/j.heliyon.2020.e05367.

[25] C. Boonpok, on Continuous Multifunctions in Ideal Topological Spaces, Lobachevskii J. Math. 40 (2019), 24–35. https://doi.org/10.1134/s1995080219010049.

[26] C. Boonpok, C. Viriyapong, M. Thongmoon, on Upper and Lower (τ_1, τ_2)-Precontinuous Multifunctions, J. Math. Computer Sci. 18 (2018), 282–293. https://doi.org/10.22436/jmcs.018.03.04.

[27] C. Boonpok, Almost (G, m)-Continuous Functions, Int. J. Math. Anal. 4 (2010), 1957–1964.

[28] C. Boonpok, M-Continuous Functions in Biminimal Structure Spaces, Far East J. Math. Sci. 43 (2010), 41–58.

[29] M. Chiangpradit, S. Sompong, C. Boonpok, Upper and Lower Almost Quasi (τ_1, τ_2)-Continuity, Asia Pac. J. Math. 12 (2025), 12. https://doi.org/10.28924/APJM/12-12.

[30] M. Chiangpradit, A. Sama-Ae, C. Boonpok, s-(τ_1, τ_2)-Continuity for Multifunctions, Eur. J. Pure Appl. Math. 18 (2025), 5634. https://doi.org/10.29020/nybg.ejpam.v18i1.5634.

[31] M. Chiangpradit, S. Sompong, C. Boonpok, Weakly Quasi (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 22 (2024), 125. https://doi.org/10.28924/2291-8639-22-2024-125.

[32] N. Chutiman, A. Sama-Ae, C. Boonpok, Almost Near (τ_1, τ_2)-Continuity for Multifunctions, Eur. J. Pure Appl. Math. 18 (2025), 5650. https://doi.org/10.29020/nybg.ejpam.v18i1.5650.

[33] T. Duangphui, C. Boonpok and C. Viriyapong, Continuous Functions on Bigeneralized Topological Spaces, Int. J. Math. Anal. 5 (2011), 1165–1174.

[34] W. Dungthaisong, C. Boonpok, C. Viriyapong, Generalized Closed Sets in Bigeneralized Topological Spaces, Int. J. Math. Anal. 5 (2011), 1175–1184.

[35] S. Jafari, T. Noiri, Properties of θ-Continuous Functions, J. Inst. Math. Comput. Sci. Math. Ser. 13 (2000), 123–128.

[36] J. Khampakdee, A. Sama-Ae, C. Boonpok, Almost Nearly Quasi (τ_1, τ_2)-Continuous Multifunctions, Eur. J. Pure Appl. Math. 18 (2025), 5720. https://doi.org/10.29020/nybg.ejpam.v18i1.5720.

[37] C. Boonpok, S. Sompong, J. Khampakdee, Almost Weakly (τ_1, τ_2)-Continuous Functions, Eur. J. Pure Appl. Math. 18 (2025), 5721. https://doi.org/10.29020/nybg.ejpam.v18i1.5721.

[38] J. Khampakdee, S. Sompong, C. Boonpok, c-(τ_1, τ_2)-Continuity for Multifunctions, Eur. J. Pure Appl. Math. 17 (2024), 2288–2298. https://doi.org/10.29020/nybg.ejpam.v17i3.5320.

[39] J. Khampakdee, C. Boonpok, Upper and Lower $\alpha(\Lambda, sp)$-Continuous Multifunctions, WSEAS Trans. Math. 21 (2022), 684–690. https://doi.org/10.37394/23206.2022.21.80.

[40] C. Klanarong, S. Sompong, C. Boonpok, Upper and Lower Almost (τ_1, τ_2)-Continuous Multifunctions, Eur. J. Pure Appl. Math. 17 (2024), 1244–1253. https://doi.org/10.29020/nybg.ejpam.v17i2.5192.

[41] B. Kong-ied, S. Sompong, C. Boonpok, Rarely s-$(\tau_1, \tau_2)P$-Continuous Multifunctions, Eur. J. Pure Appl. Math. 18 (2025), 5649. https://doi.org/10.29020/nybg.ejpam.v18i1.5649.

[42] B. Kong-ied, S. Sompong, C. Boonpok, Almost Quasi (τ_1, τ_2)-Continuous Functions, Asia Pac. J. Math. 23 (2025), 14. https://doi.org/10.28924/APJM/12-14.

[43] K. Laprom, C. Boonpok, C. Viriyapong, $\beta(\tau_1, \tau_2)$-Continuous Multifunctions on Bitopological Spaces, J. Math. 2020 (2020), 4020971. https://doi.org/10.1155/2020/4020971.

[44] N. Levine, Semi-Open Sets and Semi-Continuity in Topological Spaces, Amer. Math. Mon. 70 (1963), 36–41. https://doi.org/10.1080/00029890.1963.11990039.

[45] N. Levine, A Decomposition of Continuity in Topological Spaces, Amer. Math. Mon. 68 (1961), 44–46. https://doi.org/10.2307/2311363.

[46] S. Marcus, Sur les Fonctions Quasicontinues au Sense de S. Kempisty, Colloq. Math. 8 (1961), 47–53. http://eudml.org/doc/210887.

[47] A.S. Mashhour, M.E. Abd El-Monsef, S.N. El-Deeb, On Precontinuous and Weak Precontinuous Mappings, Proc. Math. Phys. Soc. Egypt. 53 (1982), 47–53. https://cir.nii.ac.jp/crid/1573950400472990592.

[48] A. Neubrunnová, on Certain Generalizations of the Notion of Continuity, Mat. časopis, 23 (1973), 374–380. http://dml.cz/dmlcz/126571.

[49] O. Njastad, On Some Classes of Nearly Open Sets, Pac. J. Math. 15 (1965), 961–970. https://doi.org/10.2140/pjm.1965.15.961.

[50] T. Noiri, V. Popa, Some Properties of Upper and Lower θ-Quasicontinuous Multifunctions, Demonstr. Math. 38 (2005), 223–234. https://doi.org/10.1515/dema-2005-0124.

[51] T. Noiri, On θ-Continuous Functions, Indian J. Pure Appl. Math. 21 (1990), 410–415.

[52] T. Noiri, v. Popa, Weakly Quasi Continuous Multifunctions, Anal. Univ. Timişoara, Ser. St. Mat. 26 (1988), 33–38.

[53] V. Popa, T. Noiri, Almost Quasi Continuous Multifunctions, Tatra Mt. Math. Publ. 14 (1998), 81–90.

[54] V. Popa, T. Noiri, on θ-Quasi Continuous Multifunctions, Demonstr. Math. 28 (1995), 111–122.

[55] V. Popa, on a Decomposition of Quasicontinuity in Topological Spaces, Stud. Cerc. Mat. 30 (1978), 31–35.

[56] V. Popa, On Some Decomposition of Quasicontinuity of Multifunctions, Stud. Cerc. Mat. 27 (1975), 322–328.

[57] V. Popa, C. Stan, On a Decomposition of Quasicontinuity in Topological Spaces, Stud. Cerc. Mat. 25 (1973), 41–43.

[58] C. Prachanpol, C. Boonpok, C. Viriyapong, $\delta(\tau_1, \tau_2)$-Continuous Functions, Eur. J. Pure Appl. Math. 17 (2024), 3730–3742. https://doi.org/10.29020/nybg.ejpam.v17i4.5335.

[59] P. Pue-on, A. Sama-Ae, C. Boonpok, Upper and Lower Weakly s-(τ_1, τ_2)-Continuous Multifunctions, Eur. J. Pure Appl. Math. 18 (2025), 5718. https://doi.org/10.29020/nybg.ejpam.v18i1.5718.

[60] P. Pue-on, A. Sama-Ae, C. Boonpok, Quasi $\theta(\tau_1, \tau_2)$-Continuity for Multifunctions, Eur. J. Pure Appl. Math. 18 (2025), 5717. https://doi.org/10.29020/nybg.ejpam.v18i1.5717.

[61] P. Pue-on, S. Sompong, C. Boonpok, Slightly $(\tau_1, \tau_2)s$-Continuous Functions, Int. J. Math. Comput. Sci. 20 (2025), 217–221.

[62] P. Pue-on, S. Sompong, C. Boonpok, Upper and Lower (τ_1, τ_2)-Continuous Mulfunctions, Int. J. Math. Comput. Sci. 19 (2024), 1305–1310.

[63] P. Pue-on, C. Boonpok, $\theta(\Lambda, p)$-Continuity for Functions, Int. J. Math. Comput. Sci. 19 (2024), 491–495.

[64] P. Pue-on, S. Sompong, C. Boonpok, Weakly Quasi (τ_1, τ_2)-Continuous Multifunctions, Eur. J. Pure Appl. Math. 17 (2024), 1553–1564. https://doi.org/10.29020/nybg.ejpam.v17i3.5191.

[65] P. Pue-on, S. Sompong, C. Boonpok, Almost Quasi (τ_1, τ_2)-Continuity for Multifunctions, Int. J. Anal. Appl. 22 (2024), 97. https://doi.org/10.28924/2291-8639-22-2024-97.

[66] N. Srisarakham, S. Sompong, C. Boonpok, Quasi $\theta(\tau_1, \tau_2)$-Continuous Functions, Eur. J. Pure Appl. Math. 18 (2025), 5722. https://doi.org/10.29020/nybg.ejpam.v18i1.5722.

[67] N. Srisarakham, S. Sompong, C. Boonpok, Slight (τ_1, τ_2)-Continuity for Functions, Int. J. Math. Comput. Sci. 20 (2025), 211–215.

[68] N. Srisarakham, A. Sama-Ae, C. Boonpok, Characterizations of Faintly (τ_1, τ_2)-continuous functions, Eur. J. Pure Appl. Math. 17 (2024), 2753–2762.

[69] N. Srisarakham, C. Boonpok, Almost (Λ, p)-Continuous Functions, Int. J. Math. Comput. Sci. 18 (2023), 255–259.

[70] M. Thongmoon, A. Sama-Ae, C. Boonpok, Upper and Lower Weakly Quasi (τ_1, τ_2)-Continuous Multifunctions, Int. J. Anal. Appl. 23 (2025), 45. https://doi.org/10.28924/2291-8639-23-2025-45.

[71] M. Thongmoon, A. Sama-Ae, C. Boonpok, Upper and Lower Near (τ_1, τ_2)-Continuity, Eur. J. Pure Appl. Math. 18 (2025), 5633. https://doi.org/10.29020/nybg.ejpam.v18i1.5633.

[72] M. Thongmoon, S. Sompong, C. Boonpok, Upper and Lower Weak (τ_1, τ_2)-Continuity, Eur. J. Pure Appl. Math. 17 (2024), 1705–1716. https://doi.org/10.29020/nybg.ejpam.v17i3.5238.

[73] M. Thongmoon, C. Boonpok, Strongly $\theta(\Lambda, p)$-Continuous Functions, Int. J. Math. Comput. Sci. 19 (2024), 475–479.

[74] M. Thongmoon, S. Sompong, C. Boonpok, (τ_1, τ_2)-Continuous Multifunctions and $\tau_1\tau_2$-δ-Open Sets, Int. J. Math. Comput. Sci. 19 (2024), 1369–1375.

[75] M. Thongmoon and C. Boonpok, Upper and Lower Almost $\beta(\Lambda, sp)$-Continuous Multifunctions, WSEAS Trans. Math. 21 (2022), 844–853. https://doi.org/10.37394/23206.2022.21.96.

[76] N.V. Veličko, H-Closed Topological Spaces, Amer. Math. Soc. Transl. 78 (1967), 103-118. https://cir.nii.ac.jp/crid/1572543024587207808.

[77] N. Viriyapong, S. Sompong, C. Boonpok, (τ_1, τ_2)-Extremal Disconnectedness in Bitopological Spaces, Int. J. Math. Comput. Sci. 19 (2024), 855–860.

[78] N. Viriyapong, S. Sompong, C. Boonpok, Upper and Lower s-$(\tau_1, \tau_2)P$-Continuous Multifunctions, Eur. J. Pure Appl. Math. 17 (2024), 2210–2220. https://doi.org/10.29020/nybg.ejpam.v17i3.5322.

[79] C. Viriyapong and C. Boonpok, Weak Quasi (Λ, sp)-Continuity for Multifunctions, Int. J. Math. Comput. Sci. 17 (2022), 1201–1209.

[80] C. Viriyapong, C. Boonpok, (Λ, sp)-Continuous Functions, WSEAS Trans. Math. 21 (2022), 380–385. https://doi.org/10.37394/23206.2022.21.45.

[81] C. Viriyapong, C. Boonpok, $(\tau_1, \tau_2)\alpha$-Continuity for Multifunctions, J. Math. 2020 (2020), 6285763. https://doi.org/10.1155/2020/6285763.