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$(\in, \in \vee q_k)$-Intuitionistic Fuzzy Soft Boolean Near-Rings

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Abstract. This study proposes an enriched algebraic framework through the introduction of $(\in, \in \vee q_k)$-intuitionistic fuzzy soft Boolean near-rings (IFSBNs), a class of mathematical structures that generalize previous fuzzy and soft ideal systems within Boolean near-rings. Building upon established theories, we define the corresponding $(\in, \in \vee q_k)$-intuitionistic fuzzy soft ideals (IFSIs) and idealistic forms (IIFSBNs), and rigorously analyze their properties using formal definitions and examples. By expanding the capacity to model uncertainty and complex relationships, this work contributes to the theoretical backbone required for developing future intelligent systems. Importantly, the abstract nature of these algebraic tools makes them highly adaptable to curriculum designs in mathematics-focused educational environments, aligning with Sustainable Development Goal 4 (Quality Education). In particular, the framework can inspire high school and university students in research-intensive programs to engage in exploratory learning and abstract reasoning. Furthermore, this contribution exemplifies how collaborative academic efforts across institutions can produce foundational knowledge that transcends disciplinary boundaries, supporting SDG 17 (Partnerships for the Goals). The cross-institutional authorship and integration of interdisciplinary concepts promote educational equity and intellectual cooperation, fostering a culture of shared research innovation globally.

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1. Introduction

The concept of a fuzzy set was first put forward by Zadeh [20], and it presents a framework that enables many fundamental ideas in algebraic constructions to be generalized. Bhakat and Das [2] introduced the idea of an $(\in, \in \vee q)$-fuzzy subgroup. In [11], Narayanan and Manikantan stated the idea of $(\in, \in \vee q)$-fuzzy sub near-rings and $(\in, \in \vee q)$-fuzzy ideals of near-rings. The terms $(\in, \in \vee q_k)$-fuzzy sub near-rings and $(\in, \in \vee q_k)$-fuzzy ideals of near-rings, accordingly, have been defined by Dheena and Coumaressane [3]. A soft set is a mathematical concept that was first presented by Molodtsov [10], a Russian researcher, in 1999. It is useful for handling uncertainty. The terms fuzzy soft set (FSS) and FSS operations were defined by Maji et al. [7, 9]. Fuzzy soft rings and $(\in, \in \vee q)$-fuzzy soft rings over a ring were defined by Inan and Ozturk [6]. In [12], Ozturk and Inan elaborated on these concepts to near-rings. Rao et al. [17] defined fuzzy soft Boolean rings over a Boolean ring. Fuzzy soft Boolean near-rings and idealistic fuzzy soft Boolean near-rings were defined by Rao et al. [16], and also $(\in, \in \vee q_k)$-fuzzy soft Boolean near-rings over a Boolean near-ring were defined by Rao et al. [18]. Rao et al. [13–16] studied fuzzy soft Boolean rings (FSBRs) and fuzzy soft Boolean near-rings (FSBNRs), examining their algebraic properties, generalizations, and structural implications.

This article establishes the notions of $(\in, \in \vee q_k)$-IFSBNs and $(\in, \in \vee q_k)$-IFSIs over a BN. $(\in, \in \vee q_k)$-IFSBNs and $(\in, \in \vee q_k)$-IFSIs are generalizations of $(\in, \in \vee q)$-IFSBNs and $(\in, \in \vee q)$-IFSIs, respectively. Using examples, we also look at some of their properties. Furthermore, we define an $(\in, \in \vee q_k)$-IIFSBNs of an $(\in, \in \vee q_k)$-IFSBN and derive some associated results.

2. Preliminaries

This section shows some fundamental ideas that can be addressed in the sections that follow.

Definition 2.1. *[5] A nonempty set $\mathcal{R}$ with the binary operations $+$ and $\cdot$ that satisfies these axioms is referred to as a near-ring (NR):*

(i) the group $\mathcal{R}$ is operated by $+$,

(ii) the group $\mathcal{R}$ is operated by $\cdot$,

(iii) $(a+b)c = ac + bc, \forall a, b, c \in \mathcal{R}$.

Definition 2.2. *[5] If $a^2 = a$, for all a in a near-ring $\mathcal{R}$, it is regarded as a Boolean near-ring (BN).*

Definition 2.3. *[20] A fuzzy subset (F-subset) μ in a nonempty set X is a function $\mu : X \to [0, 1]$. Then $F(X)$ represents the collection of all F-subsets in X.*

Definition 2.4. *[1] In an NR $\mathcal{R}$, an F-subset ρ is identified as a fuzzy sub-near-ring (F-sub-NR) of $\mathcal{R}$ if $\forall g, l, v \in \mathcal{R}$,*

(i) $\rho(g - l) \geq \min\{\rho(g), \rho(l)\}$,

(ii) $\rho(l + g - l) \geq \rho(g)$,

(iii) $\rho(gl) \geq \rho(g)$,

(iv) $\rho(g(l + v) - gl) \geq \rho(v)$.

Definition 2.5. *[3] In an NR $\mathcal{R}$, an F-subset ρ is identified as $(\in, \in \vee q_k)$-FN of $\mathcal{R}$ if*

(i) $g_u, l_w \in \rho \Rightarrow (g+l)_{U\{u,w\}} \in \vee q_k \rho$,

(ii) $g_u \in \rho \Rightarrow (-g)_u \in \vee q_k \rho$,

(iii) $g_u, l_w \in \rho \Rightarrow (gl)_{U\{u,w\}} \in \vee q_k \rho, \forall g, l \in \mathcal{R}, u, w \in (0,1]$.

An $(\in, \in \vee q_k)$-FN of $\mathcal{R}$ with $k = 0$ is an $(\in, \in \vee q)$-FN of $\mathcal{R}$ (see [19]).

Definition 2.6. *[1] In an NR $\mathcal{R}$, an F-subset ρ is identified as a fuzzy ideal (FI) of $\mathcal{R}$ if*

(i) $\rho(g-l) \geq U\{\rho(g), \rho(l)\}$,

(ii) $\rho(l+g-l) \geq \rho(g)$,

(iii) $\rho(gl) \geq \rho(g)$,

(iv) $\rho(g(l+v) - gl) \geq \rho(v), \forall g, l, v \in \mathcal{R}$.

Definition 2.7. *[3] In an NR $\mathcal{R}$, an F-subset ρ is identified as an $(\in, \in \vee q_k)$-FI of $\mathcal{R}$ if*

(i) $g_u, l_w \in \rho \Rightarrow (g-l)_{U\{u,w\}} \in \vee q_k \rho$,

(ii) $g_u \in \rho \Rightarrow (l+g-l)_u \in \vee q_k \rho$,

(iii) $g_u \in \rho \Rightarrow (gl)_u \in \vee q_k \rho$,

(iv) $g_u \in \rho \Rightarrow (g(l+v) - gv)_u \in \vee q_k \rho$, $\forall g, l, v \in \mathcal{R}, u, v \in (0,1]$.

An $(\in, \in \vee q_k)$-FI of $\mathcal{R}$ with $k = 0$ is an $(\in, \in \vee q)$-FI of $\mathcal{R}$. (see [19]).

Definition 2.8. *[7] Let K stand for the beginning of the universe, Z stand for the parameters, $B \subseteq Z$, and K's fuzzy power set is indicated by $P(K)$. An FSS over K is denoted by a pair (δ, M). In this instance, the mapping E is specified by $E : M \to P(K)$. A family of parameterized fuzzy subsets of K is known as an FSS.*

Definition 2.9. *[8] Let K stand for the beginning of the universe, Z stand for the parameters, $B \subseteq Z$, and K's IF power set is denoted by $IFS(K)$. An IFSS over K is denoted by a pair (δ, M). In this instance, the mapping E has been defined by $E : M \to IFS(K)$.*

An IFSS, or parameterized family of IF subsets of K, is a special case of an FSS. When all of K's IF subsets degenerate into F-subsets, an IFSS degenerates into an FSS.

Taking everything into account, we have an IF set on K, $E(a)$, for every $a \in B \subseteq Z$. This is known as the parameter a's IF set. The intuitionistic value

$$\langle \delta_a(u), \delta'_a(u) \rangle$$

indicates the degree to which object $u \in K$ has parameter a. δ_a able to be written in the following ways:

$$E(a) = \{\langle u, \delta_a(u), \delta'_a(u) \rangle \mid u \in K\}.$$

Fuzzy set $E(a)$ results from, for all $u \in K, a \in B \subseteq Z, \delta_a(u) + \delta'_a(u) = 1$, for all $u \in K, a \in B \subseteq Z$, the IFSS (δ, M) degenerates into an FSS.

Definition 2.10. *[8] Consider two IFSSs (δ, M) and (υ, O). Afterward, (δ, M) is an IFS subset of (υ, O) if*

(i) $M \subseteq O$,

(ii) $\forall a \in M, \upsilon(a)$ represents an IF subset of $\delta(a)$.

The notation $(\delta, M) \subseteq (\upsilon, O)$ indicates the preceding relationship of inclusion.

In the same way, (δ, M) is addressed as an IFS superset of (υ, O) if (υ, O) is an IFS subset of (δ, M). The relationship was denoted by $(\delta, M) \supseteq (\upsilon, O)$ above. If $(\delta, M) \subseteq (\upsilon, O)$ and $(\upsilon, O) \subseteq (\delta, M)$, then (δ, M) and (υ, O) are considered IFS equivalents.

Definition 2.11. *[8] Consider two IFSSs (δ, M) and (υ, O), following that, the set*

(i) (δ, M) AND (υ, O), so $(\delta, M) \wedge (\upsilon, O)$ can be explained as $(\delta, M) \wedge (\upsilon, O) = (\gamma, Q)$, where $Q = M \times O, \forall (r, v) \in M \times O, I(r, v) = E(r) \cap G(v)$,

(ii) (δ, M) OR (υ, O), so $(\delta, M) \vee (\upsilon, O)$ can be explained as $(\delta, M) \vee (\upsilon, O) = (\gamma, Q)$, where $Q = M \times O, \forall (r, v) \in M \times O, I(r, v) = E(r) \cup G(v)$.

Definition 2.12. *[8] An intersection of an IFSSs (δ, M) and (υ, O) is addressed as an IFSS, and its represented by the symbol (γ, Q), where $Q = M \cup O$ and $\forall a \in Q$,*

$$\gamma_a = \begin{cases} \delta_a, & \text{if} \quad a \in M - O \\ \upsilon_a, & \text{if} \quad a \in O - M \\ \delta_a \cap \upsilon_a, & \text{if} \quad a \in M \cap O \end{cases}$$

Then, it looks like this: $(\gamma, Q) = (\delta, M) \cap (\upsilon, O)$.

Definition 2.13. *[8] A union of an IFSSs (δ, M) and (υ, O) is addressed as an IFSS, and it is represented by the symbol (γ, Q), where $Q = M \cup O$ and $\forall\ a \in Q$,*

$$\gamma_a = \begin{cases} \delta_a, & \text{if} \quad a \in M - O \\ \upsilon_a, & \text{if} \quad a \in O - M \\ \delta_a \cup \upsilon_a, & \text{if} \quad a \in M \cap O \end{cases}$$

Then, it looks like this: $(\gamma, Q) = (\delta, M) \cup (\upsilon, O)$.

Occasionally, we might define intersection and union differently than the IFSS definitions that were previously given. These definitions are as follows:

Definition 2.14. *[8] Let (δ, M) and (υ, O) be two IFSSs such that $M \cap O \neq \emptyset$.*

(i) The IFSS (γ, Q), where $Q = M \cap O$ and $I(u) = E(u) \cup G(u), \forall u \in Q$, is the bi-union of (δ, M) and (υ, O). $(\gamma, Q) = (\delta, M) \sqcup (\upsilon, O)$ indicates this.

(ii) The IFSS (γ, Q), where $Q = M \cap O$ and $I(u) = E(u) \cap G(u), \forall u \in Q$, is the bi-intersection of (δ, M) and (υ, O). $(\gamma, Q) = (\delta, M) \sqcap (\upsilon, O)$ indicates this.

Definition 2.15. *[4] Let there be two IFSSs (δ, M) and (υ, O). The IFSS $(\delta \circ \upsilon, Q)$, where $Q = M \cup O$ and $\forall a \in Q, r \in \mathcal{R}$, is defined as the product of (δ, M) and (υ, O),*

$$(\delta \circ \upsilon)_a(r) = \begin{cases} \delta_a(r), & \text{if} \quad a \in M - O \\ \upsilon_a(r), & \text{if} \quad a \in O - M \\ \sup_{c=xy} \delta_a(x) \cup \upsilon_a(y), & \text{if} \quad a \in M \cap O \end{cases}$$

and

$$(\delta \circ \upsilon)'_a(r) = \begin{cases} \delta'_a(r), & \text{if} \quad a \in M - O \\ \upsilon'_a(r), & \text{if} \quad a \in O - M \\ \inf_{c=xy} \delta'_a(x) \cup \upsilon'_a(y), & \text{if} \quad a \in M \cap O \end{cases}$$

The way this is represented is $(\delta \circ \upsilon, Q) = (\delta, M) \circ (\upsilon, O)$.

3. $(\in, \in \vee q_k)$-Intuitionistic Fuzzy Soft Boolean Near-Rings

This part introduces the ideas of $(\in, \in \vee q_k)$-IFSBNs and $(\in, \in \vee q_k)$-IFSIs of $\mathcal{R}$. Further definitions and an analysis of some of its properties are provided for the notion of an $(\in, \in \vee q_k)$-IFSBN of an $(\in, \in \vee q_k)$-IFSBN.

Definition 3.1. *Let* (δ, M) *be an IFSS of* $\mathcal{R}$. *Afterward,* (δ, M) *is addressed as an* $(\in, \in \vee q_k)$*-IFSBN of* $\mathcal{R}$ *if for each* $a \in M$ *and* $r, v \in \mathcal{R}$,

(i) $\delta_a(r+v) \geq U\{\delta_a(r), \delta_a(v), (1-k)/2\}$ *and* $\delta_a^{'}(r+v) \leq V\{\delta_a^{'}(r), \delta_a^{'}(v), (1-k)/2\}$,

(ii) $\delta_a(rv) \geq U\{\delta_a(r), \delta_a(v), (1-k)/2\}$ *and* $\delta_a^{'}(rv) \leq V\{\delta_a^{'}(r), \delta_a^{'}(v), (1-k)/2\}$,

(iii) $\delta_a(-r) \geq U\{\delta_a(r), (1-k)/2\}$ *and* $\delta_a^{'}(-r) \leq V\{\delta_a^{'}(r), (1-k)/2\}$.

An $(\in, \in \vee q_k)$*-IFSBN of* $\mathcal{R}$ *with* $k = 0$ *is an* $(\in, \in \vee q)$*-IFSBN (*U *represents the minimum and* V *represents the maximum).*

Example 3.1. *Let the binary operations* $+$ *and* $\cdot$ *be present on the nonempty set* $\mathcal{R} = \{0, g, l, v\}$ *in the following terms:*

$+$	0	g	l	v
0	0	g	l	v
g	g	0	v	l
l	l	v	0	a
v	v	0	g	0

$\cdot$	0	g	l	v
0	0	0	0	0
g	0	g	0	g
l	0	0	l	l
v	0	g	l	v

Then $(\mathcal{R}, +, \cdot)$ *is a BN. Set the parameters to* $M = \{e_1, e_2, e_3\}$, *and define an IFSS* (δ, M) *over* $\mathcal{R}$ *as follows:*

δ	e_1	e_2	e_3
0	0.2	0.4	0.3
g	0.2	0.4	0.3
l	0.1	0.3	0.2
v	0.1	0.3	0.2

$\delta^{'}$	e_1	e_2	e_3
0	0.3	0.4	0.3
g	0.4	0.5	0.5
l	0.7	0.7	0.8
v	0.7	0.7	0.8

It follows that (δ, M) *is an* $(\in, \in \vee q_k)$*-IFSBN of* $\mathcal{R}$.

Definition 3.2. *We declare that for two* $(\in, \in \vee q_k)$*-IFSBNs* (δ, M) *and* (υ, O) *of* $\mathcal{R}$, (δ, M) *is an IFS-sub-BR of* (υ, O). *Additionally,* $(\delta, M) \subseteq (\upsilon, O)$ *is written if*

(i) $M \subseteq O$,

(ii) $\forall r \in \mathcal{R}, a \in M, \delta_a(r) \leq \upsilon_a(r)$ *and* $\delta_a^{'}(r) \geq \upsilon_a^{'}(r)$.

Definition 3.3. *Two* $(\in, \in \vee q_k)$*-IFSBNs* (δ, M) *and* (υ, O) *of* $\mathcal{R}$ *are equal if* $(\delta, M) \subseteq (\upsilon, O)$ *and* $(\upsilon, O) \subseteq (\delta, M)$.

Definition 3.4. *The union of* $(\in, \in \vee q_k)$*-IFSBNs* (δ, M) *and* (υ, O) *of* $\mathcal{R}$ *is* $(\delta, M) \cup (\upsilon, O)$. *An* $(\in, \in \vee q_k)$*-IFSBN* $\gamma : M \cup O \to [0,1]^{\mathcal{R}}$ *is used to describe it, guaranteeing that for each* $a \in M \cup O$,

$$\gamma_a = \begin{cases} \langle r, \delta_a(r), \delta_a'(r)\rangle, & if \quad a \in M - O \\ \langle r, \upsilon_a(r), \upsilon_a'(r)\rangle, & if \quad a \in O - M \\ \langle r, \delta_a(r) \vee \upsilon_a(r), \delta_a'(e) \wedge \upsilon_a'(e)\rangle, & if \quad a \in M \cap O \end{cases}$$

This is demonstrated by $(\gamma, Q) = (\delta, M) \cup (\upsilon, O)$, where $Q = M \cup O$.

Theorem 3.1. *If (δ, M) and (υ, O) are $(\in, \in \vee q_k)$-IFSBNs of $\mathcal{R}$, then $(\delta, M) \cup (\upsilon, O)$ is an $(\in, \in \vee q_k)$-IFSBN of $\mathcal{R}$.*

Proof. For any $a \in M \cup O$ and $r, v \in \mathcal{R}$, we consider the subsequent scenarios.

Case 1. Let $a \in M - O$. Then

$$\begin{aligned} \gamma_a(r+v) &= \delta_a(r+v) \\ &\geq \delta_a(r) \wedge \delta_a(v) \\ &= \gamma_a(r) \wedge \gamma_a(v) \\ \gamma_a(rv) &= \delta_a(rv) \\ &\geq \delta_a(r) \wedge \delta_a(v) \\ &= \gamma_a(r) \wedge \gamma_a(v) \\ \gamma_a'(r+v) &= \delta_a'(r+v) \\ &\leq \delta_a'(r) \vee \delta_a'(v) \\ &= \gamma_a'(r) \vee \gamma_a'(v) \\ \gamma_a'(rv) &= \delta_a'(rv) \\ &\leq \delta_a'(r) \vee \delta_a'(v) \\ &= \gamma_a'(r) \vee \gamma_a'(v). \end{aligned}$$

Case 2. Let $a \in O - M$. Next, by the proof of Case 1, we have

$$\begin{aligned} \gamma_a(r+v) &= \upsilon_a(r+v) \\ &\geq \upsilon_a(r) \wedge \upsilon_a(v) \\ &= \gamma_a(r) \wedge \gamma_a(v) \\ \gamma_a(rv) &= \upsilon_a(rv) \\ &\geq \upsilon_a(r) \wedge \upsilon_a(v) \\ &= \gamma_a(r) \wedge \gamma_a(v) \\ \gamma_a'(r+v) &= \upsilon_a'(r+v) \\ &\leq \upsilon_a'(r) \vee \upsilon_a'(v) \\ &= \gamma_a'(r) \vee \gamma_a'(v) \\ \gamma_a'(rv) &= \upsilon_a'(rv) \end{aligned}$$

$$\begin{aligned} &\leq v_a'(r) \vee v_a'(v) \\ &= \gamma_a'(r) \vee \gamma_a'(v). \end{aligned}$$

Case 3. Let $a \in M \cup O$. It's an easy proof to follow in this case. Consequently, in any case, as we have

$$\begin{aligned} \gamma_a(r+v) &\geq \gamma_a(r) \wedge \gamma_a(v) \\ \gamma_a(rv) &\geq \gamma_a(r) \wedge \gamma_a(v) \\ \gamma_a'(r+v) &\leq \gamma_a'(r) \vee \gamma_a'(v) \\ \gamma_a'(rv) &\leq \gamma_a'(r) \vee \gamma_a'(v). \end{aligned}$$

In light of this, $(\delta, M) \cup (v, O)$ is an $(\in, \in \vee q_k)$-IFSBN of $\mathcal{R}$. □

Definition 3.5. *The intersection of two $(\in, \in \vee q_k)$-IFSBNs (δ, M) and (v, O) of $\mathcal{R}$ is represented by $(\delta, M) \cap (v, O)$. An $(\in, \in \vee q_k)$-IFSBN $\gamma : M \cup O \to [0,1]^{\mathcal{R}}$ is used to describe it, guaranteeing that for each $a \in M \cup O$,*

$$\gamma_a(r) = \begin{cases} \langle r, \delta_a(r), \delta_a'(r) \rangle, & \text{if } a \in M - O \\ \langle r, v_a(r), v_a'(r) \rangle, & \text{if } a \in O - M \\ \langle r, \delta_a(r) \wedge v_a(r), \delta_a'(r) \vee v_a'(r) \rangle, & \text{if } a \in M \cap O \end{cases}$$

This is demonstrated by $(\gamma, Q) = (\delta, M) \cap (v, O)$, where $Q = M \cup O$.

Theorem 3.2. *If (δ, M) and (v, O) are $(\in, \in \vee q_k)$-IFSBNs of $\mathcal{R}$, then $(\delta, M) \cap (v, O)$ is an $(\in, \in \vee q_k)$-IFSBN of $\mathcal{R}$.*

Proof. The proof is easy to comprehend. □

Definition 3.6. *Let (δ, M) and (v, O) be $(\in, \in \vee q_k)$-IFSBNs of $\mathcal{R}$. Then (δ, M) AND (v, O) is demonstrated by $(\delta, M) \wedge (v, O)$ and it's decided by $(\delta, M) \wedge (v, O) = (\gamma, Q)$, where $Q = M \times O$ and $\gamma : Q \to ([0,1] \times [0,1])^{\mathcal{R}}$ is established as*

$$\gamma(r, v) = \delta(r) \cap v(v), \forall (r, v) \in Q.$$

Theorem 3.3. *If (δ, M) and (v, O) symbolize two $(\in, \in \vee q_k)$-IFSBNs of $\mathcal{R}$, then $(\delta, M) \sqcap (v, O)$ and $(\delta, M) \wedge (v, O)$ are $(\in, \in \vee q_k)$-IFSBNs of $\mathcal{R}$.*

Proof. For all $r, v \in \mathcal{R}$ and $(b, f) \in M \times O$, we have

$$\begin{aligned} \gamma_{(b,f)}(r+v) &= \delta_b(r+v) \cap v_f(r+v) \\ &\geq (\delta_b(r) \wedge \delta_b(v)) \cap (v_f(r) \wedge v_f(v)) \\ &= (\delta_b(r) \cap v_f(r)) \wedge (\delta_b(v) \cap v_f(v)) \\ &= \gamma_{(b,f)}(r) \wedge \gamma_{(b,f)}(v) \\ \gamma_{(b,f)}(rv) &= \delta_b(rv) \cap v_f(rv) \\ &\geq (\delta_b(r) \wedge \delta_b(v)) \cap (v_f(r) \wedge v_f(v)) \end{aligned}$$

$$\begin{aligned} &= (\delta_b(r) \cap \upsilon_f(r)) \wedge (\delta_b(v) \cap \upsilon_f(v)) \\ &= \gamma_{(b,f)}(r) \wedge \gamma_{(b,f)}(v). \end{aligned}$$

In addition to this, we have

$$\begin{aligned} \gamma'_{(b,f)}(r+v) &\leq \gamma'_{(b,f)}(r) \vee \gamma'_{(b,f)}(v) \\ \gamma'_{(b,f)}(rv) &\leq \gamma'_{(b,f)}(r) \vee \gamma'_{(b,f)}(v). \end{aligned}$$

The proofs for $(\delta, M) \sqcap (\upsilon, O)$ are comparable. □

4. $(\in, \in \vee q_k)$-Intuitionistic Fuzzy Soft Ideals

Definition 4.1. *Let (δ, M) represents an IFSS of $\mathcal{R}$. Then (δ, M) is an $(\in, \in \vee q_k)$-IFSI of $\mathcal{R}$ if $\forall a \in M, r, v, h \in \mathcal{R}$,*

(i) $\delta_b(r+v) \geq U\{\delta_b(r), \delta_b(v), (1-k)/2\}$ and $\delta'_b(r+v) \leq V\{\delta'_b(r), \delta'_b(v), (1-k)/2\}$,
(ii) $\delta_b(v+r-v) \geq U\{\delta_b(r), (1-k)/2\}$ and $\delta'_b(v+r-v) \leq V\{\delta'_b(r), (1-k)/2\}$,
(iii) $\delta_b(rv) \geq V\{\delta_b(r), (1-k)/2\}$ and $\delta'_b(rv) \leq U\{\delta'_b(r), (1-k)/2\}$,
(iv) $\delta_b(r(v+h)-rv) \geq U\{\delta_b(h), (1-k)/2)\}$ and $\delta'_b(r(v+h)-rv) \leq V\{\delta'_b(h), (1-k)/2\}$.

Theorem 4.1. *If (δ, M) and (υ, O) are $(\in, \in \vee q_k)$-IFSIs of $\mathcal{R}$, then $(\delta, M) \wedge (\upsilon, O)$ and $(\delta, M) \sqcap (\upsilon, O)$ are $(\in, \in \vee q_k)$-IFSIs of $\mathcal{R}$.*

Proof. Let us take $(\delta, M) \wedge (\upsilon, O) = (\gamma, Q)$ respectively, where $Q = M \times O$ and $I(r, v) = E(r) \cap G(v), \forall (r, v) \in Q$ from the definition. Since (δ, M) and (υ, O) are two $(\in, \in \vee q_k)$-IFSIs of $\mathcal{R}$, we have $\forall b, f \in \mathcal{R}$,

$$\begin{aligned} \gamma_{(b,f)}(r+v) &= U\{\delta_a(r+v), \upsilon_d(r+v)\} \\ &\geq U\{U\{\delta_a(r), \delta_a(v)\}, U\{\upsilon_d(r), \upsilon_d(v)\}\} \\ &= U\{\gamma_{(b,f)}(r), \gamma_{(b,f)}(v)\} \\ \gamma'_{(b,f)}(r+v) &= V\{\delta'_b(r+v), \upsilon'_f(r+v)\} \\ &\leq V\{V\{\delta'_b(r), \delta'_b(v)\}, V\{\upsilon_f(r), \upsilon_f(v)\}\} \\ &= V\{\gamma_{(b,f)}(r), \gamma_{(b,f)}(v)\} \\ \gamma_{(b,f)}(rv) &= U\{\delta_b(rv), \upsilon_f(rv)\} \\ &\geq U\{V\{\delta_b(r), \delta_b(v)\}, M\{\upsilon_f(r), \upsilon_f(v)\}\} \\ &= V\{U\{\delta_b(r), \upsilon_f(r)\}, U\{\delta_b(v), \upsilon_f(v)\}\} \\ &= V\{\gamma_{(b,f)}(r), \gamma_{(b,f)}(v)\} \\ \gamma'_{(b,f)}(rv) &= V\{\delta'_b(rv), \upsilon'_f(rv)\} \\ &\leq V\{U\{\delta'_b(r), \delta'_f(v)\}, U\{\upsilon'_f(r), \upsilon'_f(v)\}\} \\ &= U\{V\{\delta'_b(r), \upsilon'_f(r)\}, V\{\delta'_b(v), \upsilon'_f(v)\}\} \end{aligned}$$

$$= \quad U\{\gamma'_{(b,f)}(r), \gamma'_{(b,f)}(v)\}.$$

As such, $(\delta, M) \wedge (v, O)$ is an $(\in, \in \vee q_k)$-IFSI of $\mathcal{R}$. Similarly, $(\delta, M) \sqcap (v, O)$ is proved. □

Theorem 4.2. *If (δ, M) and (v, O) are $(\in, \in \vee q_k)$-IFSIs of $\mathcal{R}$, then $(\delta, M) \cap (v, O)$ is an $(\in, \in \vee q_k)$-IFSI of $\mathcal{R}$.*

Proof. For any $(r, v) \in \mathcal{R}$ and $b \in Q$, consider the following situations:

Case 1. Let $a \in M - O$. Then

$$\begin{aligned}
\gamma_b(r+v) &= \delta_b(r+v) \\
&\geq U\{\delta_b(r), \delta_b(v)\} \\
&= U\{\gamma_b(r), \gamma_b(v)\} \\
\gamma'_b(r+v) &= \delta'_b(r+v) \\
&\leq V\{\delta'_b(r), \delta'_b(v)\} \\
&= V\{\gamma'_b(r), \gamma'_b(v)\} \\
\gamma_b(rv) &= \delta_b(rv) \\
&\geq V\{\delta_b(r), \delta_b(v)\} \\
&= V\{\gamma_b(r), \gamma_b(v)\} \\
\gamma'_b(rv) &= \delta'_b(rv) \\
&\leq U\{\delta'_b(r), \delta'_b(v)\} \\
&= U\{\gamma'_b(r), \gamma'_b(v)\}.
\end{aligned}$$

Case 2. Let $a \in O - M$. Then

$$\begin{aligned}
\gamma_b(r+v) &= v_b(r+v) \\
&\geq U\{v_b(r), v_b(v)\} \\
&= U\{\gamma_b(r), \gamma_b(v)\} \\
\gamma'_b(r+v) &= v'_b(r+v) \\
&\leq V\{v'_b(r), v'_b(v)\} \\
&= V\{\gamma'_b(r), \gamma'_b(v)\} \\
\gamma_b(rv) &= v_b(rv) \\
&\geq V\{v_b(r), v_b(v)\} \\
&= V\{\gamma_b(r), \gamma_b(v)\} \\
\gamma'_b(rv) &= v'_b(rv) \\
&\leq U\{v'_b(r), v'_b(v)\} \\
&= U\{\gamma'_b(r), \gamma'_b(v)\}.
\end{aligned}$$

Case 3. Let $b \in M \cap O$. Then

$$\begin{aligned}\gamma_b(r+v) &= U\{\delta_b(r+v), v_b(r+v)\}\\ &\geq U\{U\{\delta_b(r), \delta_b(v)\}, U\{v_b(r), v_b(v)\}\}\\ &\geq U\{U\{\delta_b(r), v_b(r)\}, U\{\delta_b(v), v_b(v)\}\}\\ &= U\{\gamma_b(r), \gamma_b(v)\}\\ \gamma_b^{'}(r+v) &= V\{\delta_b^{'}(r+v), v_b^{'}(r+v)\}\\ &\leq V\{V\{\delta_b^{'}(r), \delta_b^{'}(v)\}, V\{v_b^{'}(r), v_b^{'}(v)\}\}\\ &\leq V\{V\{\delta_b^{'}(r), v_b^{'}(r)\}, V\{\delta_b^{'}(v), v_b^{'}(v)\}\}\\ &= V\{\gamma_b^{'}(r), \gamma_b^{'}(v)\}.\end{aligned}$$

Similarly, $\gamma_b(rv) \geq V\{\gamma_b(r), \gamma_b(v)\}$ and $\gamma_b^{'}(rv) \leq U\{\gamma_b^{'}(r), \gamma_b^{'}(v)\}$. Thus, $(\delta, M) \cap (v, O)$ is an $(\in, \in \vee q_k)$-IFSI of $\mathcal{R}$. □

Theorem 4.3. *If (δ, M) and (v, O) are $(\in, \in \vee q_k)$-IFSIs of $\mathcal{R}$, then $(\delta, M) \circ (v, O)$ is an $(\in, \in \vee q_k)$-IFSI of $\mathcal{R}$.*

Proof. For any $r, v \in \mathcal{R}$ and $b \in M \cup O$, analyze the subsequent situations:

Case 1. Let $a \in M - O$. Then

$$\begin{aligned}(\delta \circ v)_b(r+v) &= \delta_b(r+v)\\ &\geq U\{\delta_b(r), \delta_b(v)\}\\ &= U\{(\delta \circ v)_b(r), (\delta \circ v)_b(v)\}\\ (\delta \circ v)_b^{'}(r+v) &= \delta_b^{'}(r+v)\\ &\leq V\{\delta_b^{'}(r), \delta_b^{'}(v)\}\\ &= V\{(\delta \circ v)_b^{'}(r), (\delta \circ v)_b^{'}(v)\}\\ (\delta \circ v)_b(rv) &= \delta_b(rv)\\ &\geq V\{\delta_b(r), \delta_b(v)\}\\ &= V\{(\delta \circ v)_b(r), (\delta \circ v)_b(v)\}\\ (\delta \circ v)_b^{'}(rv) &= \delta_b^{'}(rv)\\ &\geq U\{\delta_b^{'}(r), \delta_b^{'}(v)\}\\ &= U\{(\delta \circ v)_b^{'}(r), (\delta \circ v)_b^{'}(v)\}.\end{aligned}$$

Case 2. Let $a \in O - M$. This incident is similar to Case 1.

Case 3. Let $a \in M \cap O$. Then

$$(\delta \circ v)_b(r) = \sup_{c=c_1c_2} U\{\delta_b(c_1), v_b(c_2)\}$$

$$
\begin{aligned}
&\leq \sup_{cp=c_1c_2p} U\{\delta_b(c_1p), v_b(c_2p)\}\\
&\leq \sup_{cp=ut} U\{\delta_b(u), v_b(t)\}\\
&= (\delta \circ v)_b(rv).
\end{aligned}
$$

In the same way, we can write $(\delta \circ v)_b(v) \leq (\delta \circ v)_b(rv)$. Consequently $(\delta \circ v)_b(rv) \geq V\{\delta \circ v)_b(r), (\delta \circ v)_b(v)\}$. Also,

$$
\begin{aligned}
(\delta \circ v)'_b(r) &= \inf_{c=c_1c_2} V\{\delta'_b(c_1), v'_b(c_2\}\\
&\geq \inf_{cp=c_1c_2p} V\{\delta'_b(c_1p), \delta'_b(c_2p)\}\\
&\geq \inf_{cp=ut} V\{\delta'_b(u), v'_b(t)\}\\
&= (\delta \circ v)'_b(rv).
\end{aligned}
$$

In the same way, we can write $(\delta \circ v)'_b(v) \geq (\delta \circ v)'_b(rv)$. Consequently $(\delta \circ v)'_b(rv) \leq U\{(\delta \circ v)'_b(r), (\delta \circ v)'_b(v)\}$. The proof is then finished. □

5. $(\in, \in \vee q_k)$-Idealistic Intuitionistic Fuzzy Soft Boolean Near-Rings

Definition 5.1. *Let (δ, M) be an $(\in, \in \vee q_k)$-IFSBN of $\mathcal{R}$. (δ, M) after that referred to as an $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$ if $\delta(b)$ is an $(\in, \in \vee q_k)$-IIFSI of $\mathcal{R}$, $\forall\, b \in Supp(\delta, M)$, i.e., $\forall r, v, h \in \mathcal{R}$,*

(i) $\delta_b(r+v) \geq U\{\delta_b(r), \delta_b(v), (1-k)/2\}$ and $\delta'_b(r+v) \leq V\{\delta'_b(r), \delta'_b(v), (1-k)/2\}$,
(ii) $\delta_b(-r) \geq U\{\delta_b(r), (1-k)/2\}$ and $\delta'_b(-r) \leq V\{\delta'_b(r), (1-k)/2\}$,
(iii) $\delta_a(r) \geq U\{\delta_b(v+r-v), (1-k)/2\}$ and $\delta'_b(r) \leq V\{\delta'_b(v+r-v), (1-k)/2\}$,
(iv) $\delta_b(rv) \geq U\{\delta_b(v), (1-k)/2\}$ and $\delta'_b(rv) \leq V\{\delta'_b(v), (1-k)/2\}$,
(v) $\delta_b((r+h)v - rv) \geq U\{\delta_b(h), (1-k)/2)\}$ and $\delta'_b((r+h)v - rv) \leq V\{\delta'_b(h), (1-k)/2\}$.

Example 5.1. *The nonempty set $\mathcal{R} = \{0, g, l, v\}$ can be subjected to the binary operations $+$ and $\cdot$ in the following terms:*

$+$	0	g	l	v
0	0	g	l	v
g	g	0	v	l
l	l	v	0	g
v	v	l	g	0

$\cdot$	0	g	l	v
0	0	0	0	0
g	0	g	0	g
l	0	0	l	l
v	0	g	l	v

Then $(\mathcal{R}, +, .)$ is a BN. Define an IFSS (δ, M) over $\mathcal{R}$ by letting $M = \{e_1, e_2, e_3\}$ be the parameters.

$+$	e_1	e_2	e_3
0	0.2	0.4	0.3
g	0.2	0.4	0.3
l	0.1	0.3	0.2
v	0.1	0.3	0.2

$\cdot$	e_1	e_2	e_3
0	0.3	0.4	0.3
g	0.4	0.5	0.3
l	0.7	0.7	0.2
v	0.7	0.7	0.2

It follows that (δ, M) is an $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$.

Theorem 5.1. *If (δ, M) and (υ, O) are two $(\in, \in \vee q_k)$-IIFSBNs of $\mathcal{R}$, then $(\delta, M) \sqcap (\upsilon, O)$ is a part of the $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$, in the event that it isn't null.*

Proof. Let $(\gamma, Q) = (\delta, M) \cap (\upsilon, O), \forall b \in Q, I(b) = E(b) \cap G(b)$. Suppose (γ, Q) isn't null, so there exists $b \in Supp(\gamma, Q)$ such that $\gamma_b = \delta_b \cap \upsilon_b \neq \emptyset$. That is, $\gamma_b(r) = \delta_b(r) \wedge \upsilon_b(r)$ and $\gamma'_b(r) = \delta'_b(r) \vee \upsilon'_b(r), \forall r \in \mathcal{R}$. Since (γ, Q) is an $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$, we have $\forall r, v, h \in \mathcal{R}$,

(i) $\delta_b(r+v) \geq U\{\delta_b(r), \delta_b(v), (1-k)/2\}$ and $\delta'_b(r+v) \leq V\{\delta'_b(r), \delta'_b(v), (1-k)/2\}$,
(ii) $\delta_b(-r) \geq U\{\delta_b(r), (1-k)/2\}$ and $\delta'_b(-r) \leq V\{\delta'_b(r), (1-k)/2\}$,
(iii) $\delta_b(r) \geq U\{\delta_b(v+r-v), (1-k)/2\}$ and $\delta'_b(r) \leq V\{\delta'_b((v+r-v)), (1-k)/2\}$,
(iv) $\delta_b(rv) \geq U\{\delta_b(v), (1-k)/2\}$ and $\delta'_b(rv) \leq V\{\delta'_b(v), (1-k)/2\}$,
(v) $\delta_b((r+h)v - rv) \geq U\{\delta_b(h), (1-k)/2)\}$ and $\delta'_b((r+h)v - rv) \leq V\{\delta'_r(h), (1-k)/2\}$.

Additionally, the fuzzy sets $\upsilon_b(r)$ and $\upsilon'_b(r)$ share the same properties. Next, we have

$$\begin{aligned} \gamma_b(r+v) &= (\delta_b \wedge \upsilon_b)(r+v) \\ &= \delta_b(r+v) \wedge \upsilon_b(r+v) \\ &\geq (\delta_b(r) \wedge \delta_b(v)) \wedge (\upsilon_b(r) \wedge \upsilon_b(v)) \\ &= (\delta_b(r) \wedge \upsilon_b(r) \wedge (\delta_b(v) \wedge \upsilon_b(v)) \\ &= (\delta_b \wedge \upsilon_b)(r) \wedge (\delta_b \wedge \upsilon_b)(v) \\ &= \gamma_b(r) \wedge \gamma_b(v). \end{aligned}$$

Likewise, we obtain

$$\gamma'_b(r+v) \leq \gamma'_b(r) \vee \gamma'_b(v).$$

Let's now demonstrate that

$$\begin{aligned} \gamma_b[(r+h)v - rv] &= \delta_b[(r+h)v - rv] \wedge \upsilon_b[(r+h)v - rv] \\ &\geq \delta_b(h) \wedge \upsilon_b(h) \\ &= \gamma_b(h) \\ \gamma'_b[(r+h)v - rv] &= \delta'_b[(r+h)v - rv] \vee \upsilon'_b[(r+h)v - rv] \\ &\leq \delta'_b(h) \vee \upsilon'_b(h) \\ &= \gamma'_b(h). \end{aligned}$$

For every $r, h \in \mathcal{R}$, the other equalities are demonstrated in a similar manner. As a result, $(\delta, M) \cap (\upsilon, O)$ is an $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$, as desired. □

Theorem 5.2. *If (δ, M) and (υ, O) are $(\in, \in \vee q_k)$-IIFSBNs of $\mathcal{R}$, then $(\delta, M) \wedge (\upsilon, O)$ is an $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$.*

Proof. Let $(\delta, M) \wedge (\upsilon, O) = (\gamma, Q)$, where $\gamma_{(b,b')} = \delta_b \cap \upsilon_{b'}, \forall (b, b') \in Q \times Q$. Let $(b, b') \in Supp(\gamma, Q)$. Then $I(b, b') = \delta(b) \cap \upsilon(b') \neq \emptyset$. For simplicity, we only show that $\gamma_{(b,b')}(rv) \geq \gamma_{(b,b')}(v)$ and

$\gamma'_{(b,b')}(rv) \leq \gamma'_{(b,b')}(v), \forall r, v \in \mathcal{R}$. Let $r, v \in \mathcal{R}$. Then

$$\begin{aligned} \gamma_{(b,b')}(rv) &= \delta_b(rv) \wedge v_{b'}(rv) \\ &\geq \delta_b(v) \wedge v_{b'}(v) \\ &= \gamma_{b,b'}(v) \\ \gamma'_{(b,b')}(rv) &= \delta'_b(rv) \vee v'_{b'}(rv) \\ &\leq \delta'_b(v) \vee v'_{b'}(v) \\ &= \gamma'_{(b,b')}(v). \end{aligned}$$

It is simple to satisfy the remaining equalities. Here, it is shown that $(\delta, M) \wedge (v, O)$ is an $(\in, \in \vee q_k)$-IIFSBN of $\mathcal{R}$. □

6. Conclusion

This study presents a comprehensive framework for $(\in, \in \vee q_k)$-intuitionistic fuzzy soft Boolean near-rings ($(\in, \in \vee q_k)$-IFSBNs), $(\in, \in \vee q_k)$-intuitionistic fuzzy soft ideals ($(\in, \in \vee q_k)$-IFSIs), and $(\in, \in \vee q_k)$-idealistic intuitionistic fuzzy soft Boolean near-rings ($(\in, \in \vee q_k)$-IIFSBNs), extending the algebraic structures of fuzzy and soft set theories to Boolean near-rings. We establish key properties, operations, and theorems that define and validate these structures, demonstrating their mathematical consistency and applicability. The introduction of $(\in, \in \vee q_k)$-IFSIs further enhances the algebraic framework by incorporating idealistic properties, leading to a refined approach for handling algebraic uncertainty. Moreover, the development of $(\in, \in \vee q_k)$-IIFSBNs ensures a more structured and comprehensive representation of idealistic intuitionistic fuzzy soft elements in Boolean near-rings. These contributions lay the groundwork for further exploration in uncertainty modeling, computational intelligence, and algebraic system generalizations. Future research may focus on the integration of these structures with lattice theory, category theory, and real-world decision-making models, reinforcing the broader impact of intuitionistic fuzzy soft algebra in both theoretical and applied contexts.

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