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A NOTE ON: MULTI-STEP APPROXIMATION SCHEMES FOR THE FIXED POINTS OF FINITE FAMILY OF ASYMPTOTICALLY PSEUDOCONTRACTIVE MAPPINGS

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Abstract. In this paper, using an analytical technique we obtain a strong convergence for a modified three- step iterative scheme due to Suantai [6] for asymptotically pseudocontractive mappings in real Banach spaces. Our result is an improvement and a correction of Rafiq's [4] results.

1. Introduction

Let E be an arbitrary real Banach Space and let $J : E \to 2^{E^*}$ be the normalized duality mapping defined by

$$J(x) = \{f \in E^* :< x, f >= \|x\|^2 = \|f\|^2\}, \forall x \in E$$

where E^* denotes the dual space of E and $< ., . >$ denotes the generalized duality pairing between E and E^*. The single-valued normalized duality mapping is denoted by j. Let K be a nonempty closed convex subset of E and $T : K \to K$ be a map.

The mapping T is said to be uniformly L- Lipschitzian if there exists a constant $L > 0$ such that

$$\|T^n x - T^n y\| \leq L\|x - y\|$$

for any $x, y \in K$ and $\forall n \geq 1$.

The mapping T is said to be asymptotically pseudocontractive if there exists a sequence $(k_n) \subset [1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ and for any $x, y \in K$ there exists $j(x - y) \in J(x - y)$ such that

$$< T^n x - T^n y, j(x - y) >\leq k_n\|x - y\|^2, \forall n \geq 1.$$

The concept of asymptotically pseudocontractive mappings was introduced by Schu [5].

Ofoedu [3] used the modified Mann iteration process introduced by Schu [5] ,

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n x_n \quad n \geq 0,$$

to obtain a strong convergence theorem for uniformly Lipschitzian asymptotically pseudo-contractive mapping in real Banach space setting. This result itself is a generalization of many of the previous results (see [3] and the references therein).

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Recently, Rafiq [4] employed the iterative scheme introduced by Suantai [6] to establish a strong convergence result for a modified three-step iterative scheme when dealing with asymptotically pseudocontractive mappings. In fact, he proved the following theorem :

Theorem 1.1 ([4]). *Let K be a nonempty closed convex subset of E, $T : K \to K$ be the asymptotically pseudocontractive mapping with $T(K)$ bounded and the sequence $k_n \subset [1, \infty)$, $\lim_{n\to\infty} k_n = 1$ such that $F(T) = \{x \in K : Tx = x\} \neq \phi$. Further let T be uniformly continuous and $\{a_n\}_{n\geq}, \{b_n\}_{n\geq 0}, \{c_n\}_{n\geq 0}, \{a'_n\}_{n\geq 0}$, $\{b'_n\}_{n\geq 0}, \{c'_n\}_{n\geq 0}, \{a''_n\}_{n\geq 0}$ be real sequences in $[0, 1]$; $a_n+b_n+c_n = 1 = a'_n+b'_n+c'_n$ satisfying the following conditions: (i) $\lim_{n\to\infty}(b_n + c'_n) = 0 = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = \lim_{n\to\infty} a''_n$ (ii)$\sum_{n\geq 0}(b_n + a_n) = \infty$.*

For arbitrary $x_0 \in K$, let $\{x_n\}_{n=1}^{\infty}$ be the iterative sequence defined by

$$\begin{aligned} x_{n+1} &= a_n x_n + b_n T^n y_n + c_n T^n z_n \\ y_n &= a'_n x_n + b'_n T^n z_n + c'_n T^n x_n. \\ z_n &= (1 - a''_n) x_n + a'' T^n x_n \end{aligned} \tag{1.1}$$

Suppose for any $\rho \in F(T)$ there exists a strictly increasing function $\psi : [0, \infty) \to [0, \infty)$ with $\psi(0) = 0$ such that

$$< T^n x - \rho, j(x - \rho) > \leq k_n \|x - \rho\|^2 - \psi(\|x - \rho\|)$$

for all $x \in K$. Then $\{x_n\}_{n\geq 0}$ converges strongly to a fixed point of T.

We observed some mistakes in the proof of the theorem above. For instance, in equation (10) of Rafiq [4] the author set; $d_n = \|T^n y_n - T^n x_{n+1}\|$, $e_n = \|T^n z_n - T^n x_{n+1}\|$. He further obtained from equations (12) and (14) that $\lim_{n\to\infty} \|y_n - x_{n+1}\| = 0$, $\lim_{n\to\infty} \|z_n - x_{n+1}\| = 0$. And, using the uniform continuity of T, he concluded that $d_n = \lim_{n\to\infty} \|T^n y_n - T^n x_{n+1}\| = 0$, $e_n = \lim_{n\to\infty} \|T^n z_n - T^n x_{n+1}\| = 0$. This conclusion is, however not correct.

For example, let $Tx = 4x \ \ \forall x \in R$ and suppose $x_{n+1} = (1 - \frac{1}{n})$, $y_n = z_n = (1 + \frac{1}{n})$ for all $n \geq 1$, obviously $d_n = \lim_{n\to\infty} \|T^n y_n - T^n x_{n+1}\| \neq 0$, $e_n = \lim_{n\to\infty} \|T^n z_n - T^n x_{n+1}\| \neq 0$. Thus, the result of Rafiq [4] needs to be improve.

In this paper, an improvement and a correction to the main result of Rafiq [4] is presented.

The following lemmas are needed.

Lemma 1.1 [3, 4] . Let E be real Banach Space and $J : E \to 2^{E^*}$ be the normalized duality mapping. Then, for any $x, y \in E$

$$\|x + y\|^2 \leq \|x\|^2 + 2 < y, j(x + y) >, \forall j(x + y) \in J(x + y).$$

Lemma 1.2 [7]. Let $\{\alpha_n\}$ be a non- negative sequence which satisfies the following inequality

$$\alpha_{n+1} \leq (1 - \lambda_n)\alpha_n + \sigma_n,$$

where $\lambda_n \in (0, 1), \forall n \in N$, $\sum_{n=1}^{\infty} \lambda_n = \infty$ and $\sigma_n = o(\lambda_n)$. Then $\lim_{n\to\infty} \alpha_n = 0$.

2. Main results

Theorem 2.1. *Let K be a nonempty closed convex subset of E, $T : K \to K$ be asymptotically pseudocontractive and uniformly Lipschitzian map with Lipschitzian constant $L > 0$ and the sequence $k_n \subset [1, \infty)$, $\lim_{n\to\infty} k_n = 1$ such that $F(T) =$*

$\{x \in K : Tx = x\} \neq \phi$. Let $\{a_n\}_{n\geq 0}, \{b_n\}_{n\geq 0}, \{c_n\}_{n\geq 0}, \{a'_n\}_{n\geq 0}$, $\{b'_n\}_{n\geq 0}, \{c'_n\}_{n\geq 0}, \{a''_n\}_{n\geq 0}$ be real sequences in $[0,1]$ satisfying : (i) $a_n + b_n + c_n = 1 = a'_n + b'_n + c'_n$ (ii) $\lim_{n\to\infty} b_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c_n = 0 = \lim_{n\to\infty} c'_n$ (iii) $\sum_{n\geq 0}(b_n + c_n) = \infty$. For arbitrary $x_0 \in K$, let $\{x_n\}_{n=1}^{\infty}$ be the iterative sequence defined by (1.1). Suppose for any $\rho \in F(T)$ there exists a strictly increasing function $\psi : [0,\infty) \to [0,\infty)$ with $\psi(0) = 0$ such that

$$< T^n x_n - \rho, j(x_n - \rho) > \leq k_n\|x_n - \rho\|^2 - \psi(\|x_n - \rho\|)$$

for all $x \in K$. Then $\{x_n\}_{n\geq 0}$ converges strongly to a fixed point of T.

Proof: From Lemma 1.2, the equation (1.1) and the definition of the asymptotically pseudocontractive and uniformly Lipschitzian map, we have

$$\begin{aligned}
\|x_{n+1} - \rho\|^2 &= \|(1 - b_n - c_n)(x_n - \rho) + b_n(T^n y_n - \rho) + c_n(T^n z_n - \rho)\|^2 \\
&\leq (1 - (b_n + c_n))^2\|x_n - \rho\|^2 \\
&\quad +2\langle b_n(T^n y_n - \rho) + c_n(T^n z_n - \rho), j(x_{n+1} - \rho)\rangle \\
&\leq (1 - (b_n + c_n))^2\|x_n - \rho\|^2 \\
&\quad +2b_n\langle T^n y_n - T^n x_{n+1}, j(x_{n+1} - \rho)\rangle \\
&\quad +2b_n\langle T^n x_{n+1} - \rho, j(x_{n+1} - \rho)\rangle \\
&\quad +2c_n\langle T^n x_{n+1} - \rho, j(x_{n+1} - \rho)\rangle \\
&\quad +2c_n\langle T^n z_n - T^n x_{n+1}, j(x_{n+1} - \rho)\rangle \\
&\leq (1 - (b_n + c_n))^2\|x_n - \rho\|^2
\end{aligned}$$

$$\begin{aligned}
&\quad +2b_n(k_n\|x_{n+1} - \rho\|^2 - \psi(\|x_n - \rho\|)) \\
&\quad +2b_n\|T^n y_n - T^n x_{n+1}\|\|x_{n+1} - \rho\| \\
&\quad +2c_n\|T^n z_n - T^n x_{n+1}\|\|x_{n+1} - \rho\| \\
&\quad +2c_n(k_n\|x_{n+1} - \rho\|^2 - \psi(\|x_n - \rho\|)) \\
&\leq (1 - (b_n + c_n))^2\|x_n - \rho\|^2 + 2k_n(b_n + c_n)\|x_{n+1} - \rho\|^2 \\
&\quad -2(b_n + c_n)\psi(\|x_{n+1} - \rho\|) \\
&\quad +2b_n\|T^n y_n - T^n x_{n+1}\|\|x_{n+1} - \rho\| \\
&\quad +2c_n\|T^n z_n - T^n x_{n+1}\|\|x_{n+1} - \rho\|
\end{aligned}$$

$$\begin{aligned}
&\leq (1 - (b_n + c_n))^2\|x_n - \rho\|^2 + 2k_n(b_n + c_n)\|x_{n+1} - \rho\|^2 \\
&\quad -2(b_n + c_n)\psi(\|x_{n+1} - \rho\|) \\
&\quad +2b_n\|T^n y_n - T^n x_{n+1}\|\|x_{n+1} - \rho\| \\
&\quad +2c_n\|T^n z_n - T^n x_{n+1}\|\|x_{n+1} - \rho\| \\
&\leq (1 - \delta_n)^2\|x_n - \rho\|^2 + 2k_n\delta_n\|x_{n+1} - \rho\|^2 \\
&\quad -2\delta_n\psi(\|x_{n+1} - \rho\|) + 2b_nL\|y_n - x_{n+1}\|\|x_{n+1} - \rho\| \\
&\quad +2c_nL\|z_n - x_{n+1}\|\|x_{n+1} - \rho\|,
\end{aligned} \tag{2.1}$$

where $0 \leq \delta_n = b_n + c_n < 1$.
We note that

$$\begin{aligned}
\|y_n - x_{n+1}\| &= \|x_{n+1} - y_n\| \\
&= \|(1-b_n-c_n)(x_n-y_n) + b_n(T^n y_n - y_n) \\
&\quad + c_n(T^n z_n - y_n)\| \\
&\le (1-b_n-c_n)\|x_n-y_n\| + b_n\|T^n y_n - \rho + \rho - y_n\| \\
&\quad + c_n\|T^n z_n - \rho + \rho - y_n\| \\
&\le (1-b_n-c_n)\|x_n-y_n\| + b_n(1+L)\|y_n-\rho\| \\
&\quad + c_n(L\|z_n-\rho\| + \|y_n-\rho\|) \\
&= (1-b_n-c_n)\|x_n-y_n\| + b_n(1+L)(\|y_n-x_n+x_n-\rho\|) \\
&\quad + c_nL(\|z_n-x_n+x_n-\rho\|) + c_n(\|y_n-x_n+x_n-\rho\|) \\
&\le (1-b_n-c_n)\|x_n-y_n\| \\
&\quad + b_n(1+L)(\|y_n-x_n\| + \|x_n-\rho\|) \\
&\quad + c_nL(\|z_n-x_n\| + \|x_n-\rho\|) \\
&\quad + c_n(\|y_n-x_n\| + \|x_n-\rho\|) \\
&\le (1+b_nL)\|x_n-y_n\| + \delta_n(1+L)\|x_n-\rho\| \\
&\quad + c_nL\|z_n-x_n\|
\end{aligned}$$

$$\begin{aligned}
&= (1+b_nL)\|x_n-y_n\| + \delta_n(1+L)\|x_n-\rho\| \\
&\quad + c_nL(\|z_n-\rho+\rho-x_n\|) \\
&\le (1+b_nL)\|x_n-y_n\| + \delta_n(1+L)\|x_n-\rho\| \\
&\quad + c_nL(\|z_n-\rho\| + \|\rho-x_n\|) \\
&= (1+b_nL)\|x_n-y_n\| + [\delta_n(1+L) + c_nL]\|x_n-\rho\| \\
&\quad + c_nL\|z_n-\rho\| \\
&\le (1+b_nL)\|x_n-y_n\| + [\delta_n(1+L) + c_nL]\|x_n-\rho\| \\
&\quad + c_nL(1+a''_nL)\|x_n-\rho\| \\
&= (1+b_nL)\|x_n-y_n\| + [\delta_n(1+L) \\
&\quad + c_nL(2+a''_nL)]\|x_n-\rho\|.
\end{aligned} \tag{2.2}$$

Observe that

$$\begin{aligned}
\|x_n-y_n\| &= \|y_n-x_n\| \\
&= \|(1-b'_n-c'_n)x_n + b'_nT^nz_n + c'_nT^nx_n - x_n\| \\
&= \|b'_n(T^nz_n-x_n) + c'_n(T^nx_n-x_n)\| \\
&\le b'_nL\|z_n-\rho\| + (b'_n + (1+L)c'_n)\|x_n-\rho\| \\
&\le b'_nL(1+a''_nL)\|x_n-\rho\| + (b'_n + (1+L)c'_n)\|x_n-\rho\| \\
&= [b'_nL(1+a''_nL) + (b'_n + (1+L)c'_n)]\|x_n-\rho\|.
\end{aligned} \tag{2.3}$$

Substituting (2.3) into (2.2) then,

$$\|x_{n+1}-y_n\| \le d_n^1\|x_n-\rho\| \tag{2.4}$$

where $d_n^1 = (1+b_nL)b'_nL(1+a''_nL) + b'_n + (1+L)(c'_n+\delta_n) + c_nL(2+a''_nL)$.
In a similar way

$$(2.5)\quad \begin{aligned} \|z_n - x_{n+1}\| &= \|x_{n+1} - z_n\| \\ &= \|(1-b_n-c_n)(x_n - z_n) + b_n(T^ny_n - z_n) \\ &\quad + c_n(T^nz_n - z_n)\| \\ &\leq (1-b_n-c_n)\|x_n - z_n\| + b_nL\|y_n - \rho\| + b_n\|z_n - \rho\| \\ &\quad + c_n(1+L)\|z_n - \rho\| \\ &= (1-\delta_n)\|x_n - z_n\| + b_nL\|y_n - x_n + x_n - \rho\| \\ &\quad + b_n\|z_n - \rho\| + c_n(1+L)\|z_n - \rho\| \\ &\leq (1-\delta_n)(\|x_n - \rho\| + \|z_n - \rho\|) + b_nL\|y_n - x_n\| \\ &\quad + b_nL\|x_n - \rho\| + b_n\|z_n - \rho\| + c_n(1+L)\|z_n - \rho\| \\ &\leq (1-\delta_n)(\|x_n - \rho\| + (1+a''_nL)\|x_n - \rho\|) \\ &\quad + b_nL[b'_nL(1+a''_nL) + (b'_n + (1+L)c'_n)]\|x_n - \rho\| \\ &\quad + b_nL\|x_n - \rho\| + b_n(1+a''_nL)\|x_n - \rho\| \\ &\quad + c_n(1+L)(1+a''_nL)\|x_n - \rho\| \\ &\leq c_n(\|x_n - \rho\| + (1+a''_nL)\|x_n - \rho\|) \\ &\quad + b_nL[b'_nL(1+a''_nL) + (b'_n + (1+L)c'_n)]\|x_n - \rho\| \\ &\quad + b_nL\|x_n - \rho\| + b_n(1+a''_nL)\|x_n - \rho\| \\ &\quad + c_n(1+L)(1+a''_nL)\|x_n - \rho\| \\ &= d_n^2\|x_n - \rho\|, \end{aligned}$$

where

$$(2.6)\quad \begin{aligned} d_n^2 &= c_n(2+a''_nL) + b_n(1 + L[b'_nL(1+a''_nL) + (b'_n + (1+L)c'_n)]) \\ &\quad + (1+a''_nL)(b_n + c_nL(1+L)). \end{aligned}$$

Substituting (2.3) and (2.5) into (2.1) we have the equation that follows
(2.7)

$$\begin{aligned} \|x_{n+1} - \rho\|^2 &= (1-\delta_n)^2\|x_n - \rho\|^2 + 2k_n\delta_n\|x_{n+1} - \rho\|^2 \\ &\quad - 2\delta_n\psi(\|x_{n+1} - \rho\|) + 2b_nLd_n^1\|x_n - \rho\|\|x_{n+1} - \rho\| \\ &\quad + 2c_nLd_n^2\|x_n - \rho\|\|x_{n+1} - \rho\| \\ &\leq (1-\delta_n)^2\|x_n - \rho\|^2 + 2k_n\delta_n\|x_{n+1} - \rho\|^2 \\ &\quad - 2\delta_n\psi(\|x_{n+1} - \rho\|) + b_nLd_n^1(\|x_n - \rho\|^2 + \|x_{n+1} - \rho\|^2) \\ &\quad + c_nLd_n^2(\|x_n - \rho\|^2 + \|x_{n+1} - \rho\|^2) \\ &\leq (1-\delta_n)^2\|x_n - \rho\|^2 + 2k_n\delta_n\|x_{n+1} - \rho\|^2 \\ &\quad - 2\delta_n\psi(\|x_{n+1} - \rho\|) + \delta_nLd_n^1(\|x_n - \rho\|^2 + \|x_{n+1} - \rho\|^2) \\ &\quad + \delta_nLd_n^2(\|x_n - \rho\|^2 + \|x_{n+1} - \rho\|^2) + 2\delta_nLd_n^2. \end{aligned}$$

Setting,

$$(2.8)\quad \begin{aligned} A_n &= (1 - (2k_n\delta_n + \delta_nL(d_n^1 + d_n^2))) \\ B_n &= ((1-\delta_n)^2 + \delta_nL(d_n^1 + d_n^2)) \end{aligned}$$

$$(2.9)\quad \begin{aligned} C_n &= (2(1-k_n) - \delta_n - 2L(d_n^1 + d_n^2)) \\ D_n &= 2Ld_n^2. \end{aligned}$$

Suppose we set $inf_{n\geq N}\frac{\psi(\|x_{n+1}-\rho\|)}{1+\|x_{n+1}-\rho\|^2} = r$. Then $r = 0$. If it is not the case, we assume that $r > 0$. Let $0 < r < min\{1, r\}$, then $\frac{\psi(\|x_{n+1}-\rho\|)}{1+\|x_{n+1}-\rho\|^2} \geq r$, i.e.,

$$(2.10)\quad \psi(\|x_{n+1} - \rho\|) \geq r + r\|x_{n+1} - \rho\|^2 \geq r\|x_{n+1} - \rho\|^2.$$

Since $\lim_{n\to\infty} k_n\delta_n = 0$, there exists a natural number N_0 such that

$$\frac{1}{2} < A_n < 1,$$

for all $n > N_0$.
Thus equation (2.7) becomes,

$$\begin{aligned} \|x_{n+1} - p\|^2 &\leq \tfrac{B_n}{A_n}\|x_n - p\|^2 - 2\delta_n \tfrac{\psi(\|x_{n+1}-\rho\|)}{A_n} + \tfrac{\delta_n D_n}{A_n} \\ &\leq (1 - \delta_n C_n)\|x_n - p\|^2 - 2\delta_n\psi(\|x_{n+1} - \rho\|) + 2\delta_n D_n. \end{aligned} \tag{2.11}$$

Substituting (2.10) into (2.11), we have

$$\begin{aligned} \|x_{n+1} - p\|^2 &\leq \tfrac{1-\delta_n C_n}{1+2\delta_n r}\|x_n - p\|^2 + \tfrac{2\delta_n D_n}{1+2\delta_n r} \\ &\leq (1 - \delta_n \tfrac{(C_n+2r)}{1+2\delta_n r})\|x_n - p\|^2 + \tfrac{2\delta_n D_n}{1+2\delta_n r} \end{aligned} \tag{2.12}$$

Since $\lim_{n\to\infty} \delta_n = \lim_{n\to\infty} C_n = 0$, we choose $N_1 > N_0$ such that $\frac{(C_n+2r)}{1+2\delta_n r} > r$, for all $n > N_1$. It follows from (2.12) that

$$\|x_{n+1} - p\|^2 \leq (1 - \delta_n r)\|x_n - p\|^2 + \tfrac{2\delta_n D_n}{1+2\delta_n r} \tag{2.13}$$

for all $n > N_1$. If we set $b_n = \|x_n - \rho\|$, It follows from Lemma 1.2 that, $\lim_{n\to\infty} b_n = 0$, which is a contradiction. Thus, there exists an infinite subsequence such that $\lim_{n\to\infty} b_{nj_0+1} = 0$. Next, we prove that $\lim_{n\to\infty} b_{nj_0+m} = 0$ by induction. Let $\forall\ \epsilon \in (0,1)$, choose $n_{j_0} > N$ such that $b_{nj_0+1} < \epsilon, C_{nj_0+1} > \frac{\psi(\epsilon)}{4}, D_{nj_0+1} < \frac{\psi(\epsilon)}{2}$. First, we want to prove $b_{nj_0+2} < \epsilon$. Suppose it is not the case. Then $b_{nj_0+2} \geq \epsilon$, this implies $\psi(b_{nj_0+2}) \geq \psi(\epsilon)$. Using (2.11) we now obtain the following

$$\begin{aligned} b^2_{nj_0+2} &\leq b^2_{nj_0+1} - \delta_n \tfrac{\psi(\epsilon)}{4}\epsilon^2 - 2\delta_n\psi(\epsilon) + 2\delta_n \tfrac{\psi(\epsilon)}{2} \\ &\leq b^2_{nj_0+1} - \delta_n\psi(\epsilon) \\ &< \epsilon^2, \end{aligned} \tag{2.14}$$

which is a contracdiction. Hence $b_{nj_0+2} < \epsilon$ holds and inductively we can show that $b_{nj_0+i} < \epsilon, \quad \forall i \geq 1$ holds. This implies that $\lim_{n\to\infty} b_n = 0$, i.e., $\lim_{n\to\infty} \|x_n - \rho\| = 0$.

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