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ON RANDOM COINCIDENCE & FIXED POINTS FOR A PAIR OF MULTI-VALUED & SINGLE-VALUED MAPPINGS

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Abstract. Let (X, d) be a Polish space, $CB(X)$ the family all nonempty closed and bounded subsets of X and (Ω, Σ) be a measurable space. In this paper a pair of hybrid measurable mappings $f : \Omega \times X \to X$ and $T : \Omega \times X \to CB(X)$, satisfying the inequality (2.1) below are introduced and investigated. It is proved that if X is complete, $T(\omega, \cdot)$, $f(\omega, \cdot)$ are continuous for all $\omega \in \Omega$, $T(\cdot, x)$, $f(\cdot, x)$ are measurable for all $x \in X$ and $T(\omega, \xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi : \Omega \to X$ such that $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

1. Introduction

Random nonlinear analysis is an important mathematical discipline which is mainly concerned with the study of random nonlinear operators and their properties and is much needed for the study of various classes of random equations. Of course famously random methods have revolutionized the financial markets. Random fixed point theorems for random contraction mappings on separable complete metric spaces were first proved by Špaček[24] and Hanš[7,8]. The survey article by Bharucha-Reid [1] in 1976 attracted the attention of several mathematicians (see Chang and Huang[2], Hanš[7],[8], Špaček[24], Huang[10], Itoh [11], Liu [14], Papageorgiou [15],[16], Shahzad and Hussain [21],Shahzad and Latif [22], Tan and Yuan [25]) and give wings to this theory. Itoh [11] extended Špaček's and Hanš's theorem to multi-valued contraction mapping . The stochastic version of the well-known Schauder's fixed point theorem was proved by Sehgal and Singh [20].
Let (X, d) be a metric space and $T : X \to X$ a mapping. The class of mappings T satisfying the following contractive conditions:

$$\begin{aligned} d(Tx, Ty) \leq\ & a \max\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\} \\ & + b \max\{d(x, Tx), d(y, Ty)\} + c[d(x, Ty) + d(y, Tx)] \end{aligned} \tag{1.1}$$

for all $x, y \in X$, where a, b, c are non-negative real numbers such that $b > 0\, c > 0$ and $a + b + 2c = 1$, was introduced and investigated by Ćirić [3]. Ćirić proved that in a complete metric space such mappings have a unique fixed point. This class of mappings was further studied by many authors (Ćirić[4],[5], Singh and Mishra[23], and Rhoades et al. [18]).Sehgal and Singh [20] have generalized Ćirić's

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[4] fixed point theorem to a common fixed point theorem of a pair of mappings and presented some application of such theorems to dynamic programming.
In this paper we introduced a new class of nonexpansive type mappings for a pair of multi-valued and single valued mappings which is a stochastic version of Ćirić's [3] fixed point theorem to find the coincidence and fixed points for such class of mappings.

2. Preliminaries

Let (Ω, Σ) be a measurable space with Σ a sigma algebra of subsets of Ω and let (X, d) be a metric space. We denote by 2^X the family of all subsets of X, by $CB(X)$ the family of all nonempty closed and bounded subsets of X and by H the Hausdorff metric on $CB(X)$, induced by the metric d. For any $x \in X$ and $A \subseteq X$, by $d(x, A)$ we denote the distance between $x \text{and} A$, i.e., $d(x, A) = \inf\{d(x, a) : a \in A\}$. A mapping $T : \Omega \to 2^X$ is called $\Sigma-$*measurable* if for any open subset U of X, $T^{-1}(U) = \{\omega : T(\omega) \cap U \neq \phi\} \in \Sigma$.In what follows, when we speak of measurability we will mean $\Sigma-$ measurability. A mapping $f : \Omega \times X \to X$ is called a *random operator* if for any $x \in X, f(\cdot, x)$ is measurable. A mapping $T : \Omega \times X \to CB(X)$ is called a *multi-valued random operator* if for every $x \in X$, $T(\cdot, x)$ is measurable.A mapping $s : \Omega \to X$ is called a *measurable selector* of a measurable multifunction $T : \Omega \to 2^X$ if s is measurable and $s(\omega) \in T(\omega)$ for all $\omega \in \Omega$. A measurable mapping $\xi : \Omega \to X$ is called a *random fixed point* of a random multifunction $T : \Omega \times X \to CB(X)$ if $\xi(\omega) \in T(\omega, \xi(\omega))$ for every $\omega \in \Omega$. A mapping $\xi : \Omega \to X$ is called a *random coincidence* of $T : \Omega \times X \to CB(X)$ and $f : \Omega \times X \to X$ if $f(\omega, \xi(\omega)) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.
The aim of this paper is to prove a stochastic analogue of the Ćirić's [3] fixed point theorem for single valued mappings, extended to a coincidence point theorem for a pair of a random operator $f : \Omega \times X \to X$ and a multi-valued random operator $T : \Omega \times X \to CB(X)$, satisfying the following nonexpansive type condition: for each $\omega \in \Omega$,

$$\begin{aligned} H(T(\omega, x), T(\omega, y)) \leq {} & a(\omega) \max\{d(f(\omega, x), f(\omega, y)), d(f(\omega, y), T(\omega, y))\} \\ & + b(\omega) \max\{d(f(\omega, x), T(\omega, x)), d(f(\omega, y), T(\omega, y)), \\ & \quad d(f(\omega, y), T(\omega, x))\} \\ & + c(\omega)[d(f(\omega, x), T(\omega, y)) + d(f(\omega, y), T(\omega, x))] \end{aligned} \tag{2.1}$$

for every $x, y \in X$, where $a, b, c : \Omega \to [0, 1)$ are measurable mappings such that for all $\omega \in \Omega$

$$b(\omega) > 0 \quad c(\omega) > 0 \tag{2.2}$$

$$a(\omega) + b(\omega) + 2c(\omega) = 1 \tag{2.3}$$

3. Main Results

Now, we give our main results.

Theorem 3.1. *Let (X, d) be a complete metric space, (Ω, Σ) be a measurable space and $T : \Omega \times X \to CB(X)$ & $f : \Omega \times X \to X$ be mappings such that*

(1) *$T(\omega,\cdot)$ and $f(\omega,\cdot)$ are continuous for all $\omega \in \Omega$,*
(2) *$T(\cdot,x)$ and $f(\cdot,x)$ are measurable for all $x \in X$,*
(3) *They satisfy (2.1), where $a(\omega), b(\omega), c(\omega) : \Omega \to X$ satisfy (2.2) and (2.3).*

If $T(\omega,\xi(\omega)) \subseteq f(\omega \times X)$ and $f(\omega \times X) = X$ for each $\omega \in \Omega$, then there is a measurable mapping $\xi : \Omega \to X$ such that $f(\omega,\xi(\omega)) \in T(\omega,\xi(\omega))$ for all $\omega \in \Omega$ (i.e. T and f have a random coincidence point).

Proof. Let $\Psi = \{\xi : \Omega \to X\}$ be a family of measurable mappings. Define a function $g : \Omega \times X \to R^+$ as follows:

$$g(\omega, x) = d(x, T(\omega, x)).$$

Since $x \to T(\omega, x)$ is continuous for all $\omega \in \Omega$, we conclude that $g(\omega,\cdot)$ is continuous for all $\omega \in \Omega$. Also, since $\omega \to T(\omega, x)$ is measurable for all $x \in X$, we conclude that $g(\cdot, x)$ is measurable(see Wagner [26], p 868) for all $\omega \in \Omega$.Thus $g(\omega, x)$ is the Caratheodory function.Therefore, if $\xi : \Omega \to X$ is a measurable mapping, then $\omega \to g(\omega, \xi(\omega))$ is also measurable (see [19]).
Now we shall construct a sequence of measurable mappings $\{\xi_n\}$ in Ψ and a sequence $\{f(\omega, \xi_n(\omega))\}$ in X as follows.Let $\xi_0 \in \Psi$ be arbitrary. Then the multifunction $G : \Omega \to CB(X)$ defined by $G(\omega) = T(\omega, \xi_0(\omega))$ is measurable.
From the Kuratowski-Nardzewski [13] selector theorem there is a measurable selector $\mu_1 : \Omega \to X$ such that $\mu_1(\omega) \in T(\omega, \xi_0(\omega))$ for all $\omega \in \Omega$ Since $\mu_1(\omega) \in T(\omega, \xi_0(\omega)) \subseteq X = f(\omega \times X)$, let $\xi_1 \in \Psi$ be such that $f(\omega, \xi_1(\omega)) = \mu_1$.Thus $f(\omega, \xi_1(\omega)) \in T(\omega, \xi_0(\omega))$ for all $\omega \in \Omega$.
Let $k : \Omega \to (1, \infty)$ defined by

$$k(\omega) = 1 + \frac{b(\omega)c(\omega)}{2}$$

for all $\omega \in \Omega$.Then $k(\omega)$ is measurable.Since $k(\omega) > 1$ and $f(\omega, \xi_1(\omega))$ is a selector of $T(\omega, \xi_0(\omega))$, from Lemma 2.1 of Papageorgiou [15] there is a measurable selector $\mu_2(\omega) = f(\omega, \xi_2(\omega))$; $\xi_2 \in \Psi$, such that for all $\omega \in \Omega$:

$$f(\omega, \xi_2(\omega)) \in T(\omega, \xi_1(\omega))$$

and

$$d(f(\omega, \xi_1(\omega)), f(\omega, \xi_2(\omega))) \le k(\omega)H(T(\omega, \xi_0(\omega)), T(\omega, \xi_1(\omega)))$$

Similarly, as $f(\omega, \xi_2(\omega))$ is a selector of $T(\omega, \xi_1(\omega))$, there is a measurable selector $\mu_3(\omega) = f(\omega, \xi_3(\omega))$ of $T(\omega, \xi_2(\omega)) \subseteq f(\Omega \times X)$ such that

$$d(f(\omega, \xi_2(\omega)), f(\omega, \xi_3(\omega))) \le k(\omega)H(T(\omega, \xi_1(\omega)), T(\omega, \xi_2(\omega)))$$

Continuing in this way we can construct a sequence of measurable mappings $\mu_n : \Omega \to X$, defined by $\mu_n(\omega) = f(\omega, \xi_n(\omega))$; $\xi_n \in \Psi$, such that for all $\omega \in \Omega$:

$$f(\omega, \xi_{n+1}(\omega)) \in T(\omega, \xi_n(\omega))$$

and

$$d(f(\omega, \xi_n(\omega)), f(\omega, \xi_{n+1}(\omega))) \le k(\omega)H(T(\omega, \xi_{n-1}(\omega)), T(\omega, \xi_n(\omega))) \tag{3.1}$$

Now from (2.1)

(3.2)
$$\begin{aligned}H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))) \leq{}& a(\omega)\max\{d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))),d(f(\omega,\xi_1(\omega)),T(\omega,\xi_1(\omega)))\}\\ &+b(\omega)\max\{d(f(\omega,\xi_0(\omega)),T(\omega,\xi_0(\omega))),d(f(\omega,\xi_1(\omega)),T(\omega,\xi_1(\omega)))\\ &\ ,d(f(\omega,\xi_1(\omega)),T(\omega,\xi_0(\omega)))\}\\ &+c(\omega)[d(f(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))+d(f(\omega,\xi_1(\omega)),T(\omega,\xi_0(\omega)))]\end{aligned}$$

Since $f(\omega,\xi_1(\omega)) \in T(\omega,\xi_0(\omega))$, then

$$\begin{aligned}d(f(\omega,\xi_1(\omega)),T(\omega,\xi_0(\omega)))&=0\\ d(f(\omega,\xi_0(\omega)),T(\omega,\xi_0(\omega)))&\leq d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\\ d(f(\omega,\xi_1(\omega)),T(\omega,\xi_1(\omega)))&\leq H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\end{aligned}$$

Thus from (3.2)

(3.3)
$$\begin{aligned}H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))) \leq{}& a(\omega)\max\{d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))),H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\}\\ &+b(\omega)\max\{d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))),H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\}\\ &+c(\omega)[d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))+H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))]\end{aligned}$$

If we assume that $H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))) > d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))$, then from (3.3) and (2.3), we get

$$\begin{aligned}H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))) <{}& a(\omega)H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\\ &+b(\omega)H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\\ &+2c(\omega)H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\\ ={}&(a(\omega)+b(\omega)+2c(\omega))H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\\ ={}&H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))\end{aligned}$$

a contradiction. Therefore, we have

$$H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega))) \leq d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))$$

Since $d(f(\omega,\xi_1(\omega)),T(\omega,\xi_1(\omega))) \leq H(T(\omega,\xi_0(\omega)),T(\omega,\xi_1(\omega)))$, we have

$$d(f(\omega,\xi_1(\omega)),T(\omega,\xi_1(\omega))) \leq d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))$$

Thus by induction we can show that

(3.4) $$H(T(\omega,\xi_n(\omega)),T(\omega,\xi_{n+1}(\omega))) \leq d(f(\omega,\xi_n(\omega)),f(\omega,\xi_{n+1}(\omega)))$$

(3.5) $$d(f(\omega,\xi_n(\omega)),T(\omega,\xi_n(\omega))) \leq d(f(\omega,\xi_{n-1}(\omega)),f(\omega,\xi_n(\omega)))$$

for all $n \geq 1$ and for all $\omega \in \Omega$

From (3.1) and (3.4), we have

(3.6) $$d(f(\omega,\xi_n(\omega)),f(\omega,\xi_{n+1}(\omega))) \leq k(\omega)d(f(\omega,\xi_{n-1}(\omega)),f(\omega,\xi_n(\omega)))$$

From (3.6), we get

(3.7)
$$d(f(\omega,\xi_0(\omega)),f(\omega,\xi_2(\omega))) \le d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) + d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega)))$$
$$\le (1+k(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))$$

From (2.1)

(3.8)
$$\begin{aligned} H(T(\omega,\xi_0(\omega)),T(\omega,\xi_2(\omega))) \le\ & a(\omega)\max\{d(f(\omega,\xi_0(\omega)),f(\omega,\xi_2(\omega))),d(f(\omega,\xi_2(\omega)),T(\omega,\xi_2(\omega)))\} \\ & + b(\omega)\max\{d(f(\omega,\xi_0(\omega)),T(\omega,\xi_0(\omega))),d(f(\omega,\xi_2(\omega)),T(\omega,\xi_2(\omega))) \\ & ,d(f(\omega,\xi_2(\omega)),T(\omega,\xi_0(\omega)))\} \\ & + c(\omega)[d(f(\omega,\xi_0(\omega)),T(\omega,\xi_2(\omega))) + d(f(\omega,\xi_2(\omega)),T(\omega,\xi_0(\omega)))] \end{aligned}$$

Using (3.4), (3.5), (3.6) and (3.7) and by triangle inequality, we get

$$\begin{aligned} d(f(\omega,\xi_2(\omega)),T(\omega,\xi_0(\omega))) &\le H(T(\omega,\xi_1(\omega)),T(\omega,\xi_0(\omega))) \\ &\le d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \\ d(f(\omega,\xi_0(\omega)),T(\omega,\xi_2(\omega))) &\le d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \\ &\quad + d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega))) + d(f(\omega,\xi_2(\omega)),T(\omega,\xi_2(\omega))) \\ &\le (1+k(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) + d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega))) \\ &\le (1+2k(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \end{aligned}$$

Now from (3.8), (3.7),(3.6) and (2.3), we have

$$\begin{aligned} H(T(\omega,\xi_0(\omega)),T(\omega,\xi_2(\omega))) &\le a(\omega)(1+k(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \\ &\quad + b(\omega)k(\omega)d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \\ &\quad + 2c(\omega)(1+k(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \\ &= [1+k(\omega)(a(\omega)+b(\omega)+2c(\omega))-b(\omega)]d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \\ &= [1+k(\omega)-b(\omega)]d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega))) \end{aligned}$$

As $1+k(\omega) < 2k(\omega)$, we have

(3.9) $$H(T(\omega,\xi_0(\omega)),T(\omega,\xi_2(\omega))) \le [2k(\omega)-b(\omega)]d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))$$

From (2.3) and (2.1), we have, as $f(\omega,\xi_2(\omega)) \in T(\omega,\xi_1(\omega))$

(3.10)
$$\begin{aligned} H(T(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega))) \le\ & a(\omega)\max\{d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega))),d(f(\omega,\xi_2(\omega)),T(\omega,\xi_2(\omega)))\} \\ & + b(\omega)\max\{d(f(\omega,\xi_1(\omega)),T(\omega,\xi_1(\omega))),d(f(\omega,\xi_2(\omega)),T(\omega,\xi_2(\omega))) \\ & ,d(f(\omega,\xi_2(\omega)),T(\omega,\xi_1(\omega)))\} \\ & + c(\omega)[d(f(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega))) + d(f(\omega,\xi_2(\omega)),T(\omega,\xi_1(\omega)))] \\ \le\ & [a(\omega)+b(\omega)]\max\{d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega))),d(f(\omega,\xi_2(\omega)),T(\omega,\xi_2(\omega)))\} \\ & + c(\omega)d(f(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega))) \end{aligned}$$

Also by (3.9) since $f(\omega,\xi_1(\omega))\in T(\omega,\xi_0(\omega))$, we have

$$\begin{aligned}d(f(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega)))&\le H(T(\omega,\xi_0(\omega)),T(\omega,\xi_2(\omega)))\\&\le(2k(\omega)-b(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\end{aligned}$$

Thus from (3.10) and (3.6), we have

$$\begin{aligned}H(T(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega)))&\le[a(\omega)+b(\omega)]k(\omega)d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\\&\quad+c(\omega)(2k(\omega)-b(\omega))d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\\&=[k(\omega)(a(\omega)+b(\omega)+2c(\omega))-b(\omega)c(\omega)]d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\end{aligned}$$

implies that

$$H(T(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega)))\le[k(\omega)-b(\omega)c(\omega)]d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\tag{3.11}$$

as $a(\omega)+b(\omega)+2c(\omega)=1$

From (3.1) and (3.11), we have

(3.12)
$$\begin{aligned}d(f(\omega,\xi_2(\omega)),f(\omega,\xi_3(\omega)))&\le k(\omega)H(T(\omega,\xi_1(\omega)),T(\omega,\xi_2(\omega)))\\&\le k(\omega)[k(\omega)-b(\omega)c(\omega)]d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\end{aligned}$$

As $k(\omega)=1+\frac{b(\omega)c(\omega)}{2}$, we have

$$\begin{aligned}k(\omega)[k(\omega)-b(\omega)c(\omega)]&=\Big(1+\frac{b(\omega)c(\omega)}{2}\Big)\Big(1+\frac{b(\omega)c(\omega)}{2}-b(\omega)c(\omega)\Big)\\&=1+\frac{b^2(\omega)c^2(\omega)}{4}\end{aligned}$$

Thus from (3.12)

$$d(f(\omega,\xi_2(\omega)),f(\omega,\xi_3(\omega)))\le\Big(1+\frac{b^2(\omega)c^2(\omega)}{4}\Big)d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))$$

Similarly

$$d(f(\omega,\xi_3(\omega)),f(\omega,\xi_4(\omega)))\le\Big(1+\frac{b^2(\omega)c^2(\omega)}{4}\Big)d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega)))$$

Hence by induction

(3.13)
$$\begin{aligned}d(f(\omega,\xi_n(\omega)),f(\omega,\xi_{n+1}(\omega)))\le\Big(1+\frac{b^2(\omega)c^2(\omega)}{4}\Big)^{[\frac{n}{2}]}\max\{&d(f(\omega,\xi_0(\omega)),f(\omega,\xi_1(\omega)))\\&,d(f(\omega,\xi_1(\omega)),f(\omega,\xi_2(\omega)))\}\end{aligned}$$

where $[\frac{n}{2}]$ stands for the greatest integer not exceeding $\frac{n}{2}$. Also ,since $b(\omega)c(\omega)>0$ for all $\omega\in\Omega$, from (3.13), we have $\{f(\omega,\xi_n(\omega))\}$ is a Cauchy sequence in $f(\omega\times X)$. Since $f(\omega\times X)=X$ is complete, there is a measurable mapping $f(\omega,\xi(\omega))\in f(\omega\times X)$ such that

$$\lim_{n\to\infty}f(\omega,\xi_n(\omega))=f(\omega,\xi(\omega))\tag{3.14}$$

Again by triangle inequality and (2.1), we get

$$\begin{aligned} d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega))) &\leq d(f(\omega,\xi(\omega)),f(\omega,\xi_{n+1}(\omega)))+d(f(\omega,\xi_{n+1}(\omega)),T(\omega,\xi(\omega)))\\ &\leq d(f(\omega,\xi(\omega)),f(\omega,\xi_{n+1}(\omega)))+H(T(\omega,\xi_n(\omega)),T(\omega,\xi(\omega)))\\ &\leq d(f(\omega,\xi(\omega)),f(\omega,\xi_{n+1}(\omega)))\\ &+a(\omega)\max\{d(f(\omega,\xi_n(\omega)),f(\omega,\xi(\omega))),d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega)))\}\\ &+b(\omega)\max\{d(f(\omega,\xi_n(\omega)),T(\omega,\xi_n(\omega))),d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega)))\\ &\quad ,d(f(\omega,\xi(\omega)),T(\omega,\xi_n(\omega)))\}\\ &+c(\omega)[d(f(\omega,\xi_n(\omega)),T(\omega,\xi(\omega)))+d(f(\omega,\xi(\omega)),T(\omega,\xi_n(\omega)))] \end{aligned}$$

Thus

(3.15)

$$\begin{aligned} d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega))) &\leq d(f(\omega,\xi(\omega)),f(\omega,\xi_{n+1}(\omega)))\\ &+a(\omega)\max\{d(f(\omega,\xi_n(\omega)),f(\omega,\xi(\omega))),d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega)))\}\\ &+b(\omega)\max\{d(f(\omega,\xi_n(\omega)),f(\omega,\xi_{n+1}(\omega))),d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega)))\\ &\quad ,d(f(\omega,\xi(\omega)),f(\omega,\xi_{n+1}(\omega)))\}\\ &+c(\omega)[d(f(\omega,\xi_n(\omega)),T(\omega,\xi(\omega)))+d(f(\omega,\xi(\omega)),f(\omega,\xi_{n+1}(\omega)))] \end{aligned}$$

Taking limit as $n\to\infty$, we have

$$\begin{aligned} d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega))) &\leq [a(\omega)+b(\omega)+c(\omega)]d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega)))\\ &=[1-c(\omega)]d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega))) \end{aligned}$$

implies that $d(f(\omega,\xi(\omega)),T(\omega,\xi(\omega)))=0$, as $1-c(\omega)<1$ and for $\omega\in\Omega$. Hence as $T(\omega,\xi(\omega))$ is closed $f(\omega,\xi(\omega))\in T(\omega,\xi(\omega))$, for all $\omega\in\Omega$. □

Remark 3.2. *If in Theorem 3.1., $f(\omega,x)=x$ for all $(\omega,x)\in\Omega\times X$, then we get the following random fixed point theorem.*

Corollary 3.3. *Let (X,d) be a separable complete metric space. (Ω,Σ) be a measurable space and let a mapping $T:\Omega\times X\to CB(X)$ be such that $T(\omega,\cdot)$ is continuous for all $\omega\in\Omega$, $T(\cdot,x)$ is measurable for all $x\in X$ and*

$$\begin{aligned} H(T(\omega,x),T(\omega,y)) &\leq a(\omega)\max\{d(x,y),d(x,T(\omega,y))\}\\ &+b(\omega)\max\{d(x,T(\omega,x)),d(y,T(\omega,y)),d(y,T(\omega,x))\}\\ &+c(\omega)[d(x,T(\omega,y))+d(y,T(\omega,x))] \end{aligned} \tag{3.16}$$

for every $x,y\in X$, where $a,b,c:\Omega\to(0,1)$ are measurable mappings satisfying (2.2) and (2.3). Then there is a measurable mapping $\xi:\Omega\to X$ such that $\xi(\omega)\in T(\omega,\xi(\omega))$ for all $\omega\in\Omega$.

Corollary 3.4. *([6], Corollary 1) Let (X,d) be a separable complete metric space. (Ω,Σ) be a measurable space and let a mapping $T:\Omega\times X\to CB(X)$ be such that $T(\omega,\cdot)$ is continuous for all $\omega\in\Omega$, $T(\cdot,x)$ is measurable for all $x\in X$ and*

$$\begin{aligned} H(T(\omega,x),T(\omega,y)) &\leq a(\omega)\max\{d(x,y),d(x,T(\omega,x)),d(y,T(\omega,y))\\ &\quad ,\frac{1}{2}[d(x,T(\omega,y))+d(y,T(\omega,x))]\}\\ &+b(\omega)\max\{d(x,T(\omega,x)),d(y,T(\omega,y))\}\\ &+c(\omega)[d(x,T(\omega,y))+d(y,T(\omega,x))] \end{aligned} \tag{3.17}$$

for every $x, y \in X$, where $a, b, c : \Omega \to (0,1)$ are measurable mappings satisfying (2.2) and (2.3). Then there is a measurable mapping $\xi : \Omega \to X$ such that $\xi(\omega) \in T(\omega, \xi(\omega))$ for all $\omega \in \Omega$.

Remark 3.5. *The nonexpansive type condition (3.16) includes (3.17) if we set*

$$m(x,y) = \max\{d(x,y), d(x,T(\omega,x)), d(y,T(\omega,y)), \frac{1}{2}[d(x,T(\omega,y)) + d(y,T(\omega,x))]\}$$

For each x, y such that $m(x,y) = d(x,y)$ and $a(\omega), b(\omega), c(\omega) : \Omega \to (0,1)$
For each x, y such that $m(x,y) = \max\{d(x,T(\omega,x)), d(y,T(\omega,y))\}$,define $a(\omega) = 0, b(\omega) = a(\omega) + b(\omega), c(\omega) = c(\omega)$.
For each x, y such that $m(x,y) = \frac{1}{2}[d(x,T(\omega,y)) + d(y,T(\omega,x))]$, define $a(\omega) = 0, b(\omega) = b(\omega), c(\omega) = a(\omega) + 2c(\omega)$.
Thus Corollary (3.3) is an extension of Corollary (3.4).

Finally, we give a simple example in support of Theorem 3.1. and Corollary 3.3 which shows that these results are actually an improvement of the result of Itoh[11].

Example 3.6. *Let (X,d) be any measurable space and $K = \{0,1,2,4,6\}$ be the subset of the real line. Let the mappings $f : \Omega \times K \to K$ and $T : \Omega \times K \to K$ be defined such that for each $\omega \in \Omega$:*

$f(\omega,0) = 2$	$f(\omega,1) = 4$	$f(\omega,2) = 6$	$f(\omega,4) = 0$	$f(\omega,6) = 1$
$T(\omega,0) = 1$	$T(\omega,1) = 2$	$T(\omega,2) = 4$	$T(\omega,4) = 0$	$T(\omega,6) = 0$

Then for $x = 1$ and $y = 2$, we have

$$\begin{aligned} d(T(\omega,1), T(\omega,2)) &= \frac{4}{5}\max\{\|4-6\|, \|6-4\|\} \\ &+ \frac{1}{20}\max\{\|4-6\|, \|6-4\|, \|6-2\|\} \\ &+ \frac{1}{20}[\|4-4\| + \|6-2\|] \\ &= \frac{4}{5}.2 + \frac{1}{20}.4 + \frac{1}{20}.4 \\ &= 2 \end{aligned}$$

Thus, for $x = 1$ and $y = 2$, f and T satisfy (2.1) with $a(\omega) = \frac{4}{5}, b(\omega) = \frac{1}{20}$ and $c(\omega) = \frac{1}{20}$. It is easy to show that f and T satisfy (2.1) for all $x, y \in K$ with the same $a(\omega), b(\omega)$ and $c(\omega)$. Also, the rest of the assumptions of Theorem 3.1 is satisfied and for $\xi(\omega) = 4$, we have

$$f(\omega, \xi(\omega)) = 0 = T(\omega, \xi(\omega))$$

Note that T does not satisfy (3.16) either, as for instance, for $x = 2$ and $y = 4$, we have

$$\begin{aligned} a(\omega)\max\{\|2-4\|, \|2-0\|\} + b(\omega)\max\{\|2-4\|, \|4-0\|, \|4-4\|\} \\ +c(\omega)[\|2-0\| + \|4-4\|] = 2a(\omega) + 4b(\omega) + 2c(\omega) \\ < 4[a(\omega) + b(\omega) + 2c(\omega)] = 4 = d(T(\omega,2), T(\omega,4)) \end{aligned}$$

Remark 3.7. *Our Theorem 3.1 generalizes and extends the corresponding fixed point theorems for nonexpansive type single valued mapping of Ćirić [3] and Rhoades[17].*

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