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INT-SOFT INTERIOR HYPERIDEALS OF ORDERED SEMIHYPERGROUPS

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Abstract. The main theme of this paper is to study ordered semihypergroups in the context of int-soft interior hyperideals. In this paper, the notion of int-soft interior hyperideals are studied and their related properties are discussed. We present characterizations of interior hyperideals in terms of int-soft interior hyperideals. The concepts of int-soft hyperideals and int-soft interior hyperideals coincide in a regular as well as in intra-regular ordered semihypergroups. We prove that every int-soft hyperideal is an int-soft interior hyperideal but the converse is not true which is shown with help of an example. Furthermore we characterize simple ordered semihypergroups by means of int-soft hyperideals and int-soft interior hyperideals.

1. Introduction

The real world is inherently uncertain, imprecise, and vague. Various problems in system identification involve characteristics which are essentially nonprobabilistic in nature [19]. In response to this situation, Zadeh [20], introduced fuzzy set theory as an alternative to probability theory. Uncertainty is an attribute of information. In order to suggest a more general framework, the approach to uncertainty is outlined by Zadeh [21]. To solve a complicated problem in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of intuitionistic fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [6]. Maji et al. [22] and Molodtsov [6], suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov [6], introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. Worldwide, there has been a rapid growth in interest in soft set theory and its applications in recent years see [1–5, 9, 10, 16]. The concept of hyperstructure was first introduced by Marty [7], at the 8^{th} Congress of Scandinavian Mathematicians in 1934, when he defined hypergroups and started to analyze its properties. Now, the theory of algebraic hyperstructures has become a well-established branch in algebraic theory and it has extensive applications in many branches of mathematics and applied science. Later on, people have developed the semihypergroups, which are the simplest algebraic hyperstructures having closure and associative properties. A comprehensive review of the theory of hyperstructures can be found in [11–15, 17]. In this paper, we study the concept of int-soft interior hyperideals in ordered semihypergroups and present some related examples of this concept. We show that int-soft hyperideals and int-soft interior hyperideals coincide in regular ordered semihypergroups and intra-regular ordered semihypergroups. We characterize ordered semihypergroups in terms of int-soft hyperideals and int-soft interior hyperideals. Simple ordered

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semihypergroups are characterized by using the notions of int-soft hyperideals and int-soft interior hyperideals.

2. Preliminaries

2.1. **Basic results on ordered semihypergroups.** A hypergroupoid is a nonempty set S equipped with a hyperoperation $\circ$, that is a map $\circ : S \times S \longrightarrow P^*(S)$, where $P^*(S)$ denotes the set of all nonempty subsets of S (see [7]). We shall denote by $x \circ y$, the hyperproduct of elements x, y of S. A hypergroupoid $(S, \circ)$ is called a semihypergroup if $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z \in S$. Let A, B be the nonempty subsets of S. Then the hyperproduct of A and B is defined as $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$. We shall write $A \circ x$ instead of $A \circ \{x\}$ and $x \circ A$ for $\{x\} \circ A$.

Definition 2.1. (see [11]). An algebraic hyperstructure $(S, \circ, \leq)$ is called an ordered semihypergroup (also called po-semihypergroup) if $(S, \circ)$ is a semihypergroup and $(S, \leq)$ is a partially ordered set such that the monotone condition holds as follows:
$a \leq b$ implies that $x \circ a \leq x \circ b$ and $a \circ x \leq b \circ x$ for all $x, a, b \in S$, where, if $A, B \in P^*(S)$ then we say that $A \preceq B$ if for every $a \in A$ there exists $b \in B$ such that $a \leq b$. If $A = \{a\}$ then we write $a \preceq B$ instead of $\{a\} \preceq B$.

Definition 2.2. (see [13]). A nonempty subset A of an ordered semihypergroup $(S, \circ, \leq)$ is called a subsemihypergroup of S if for all $x, y \in A$, implies that $x \circ y \subseteq A$.

Equivalently A nonempty subset A of an ordered semihypergroup $(S, \circ, \leq)$ is called a subsemihypergroup of S if $A \circ A \subseteq A$.

Definition 2.3. (see [11]). Let $(S, \circ, \leq)$ be an ordered semihypergroup and A be a nonempty subset of S. Then A is called a left (resp., right) hyperideal of S if:
(1) $S \circ A \subseteq A$ (resp., $A \circ S \subseteq A$).
(2) If $a \in A$ and $S \ni b \leq a$ then $b \in A$.
If A is both a right hyperideal and a left hyperideal of S, then it is called a hyperideal (or two-sided hyperideal) of S.

Definition 2.4. (see [18]) Let $(S, \circ, \leq)$ be an ordered semihypergroup. A subsemihypergroup A of S is called an interior hyperideal of S if:
(1) $S \circ A \circ S \subseteq A$.
(2) If $a \in A$ and $S \ni b \leq a$ then $b \in A$.
For $A \subseteq S$, we denote $(A] = \{t \in S \mid t \leq h \text{ for some } h \in A\}$.

Lemma 2.1. (see [11]). Let $(S, \circ, \leq)$ be an ordered semihypergroup and A, B are the nonempty subsets of S. Then the following statements hold:
(1) $A \subseteq (A]$.
(2) $A \subseteq B$ implies that $(A] \subseteq (B]$.
(3) $(A] \circ (B] \subseteq (A \circ B]$.
(4) $((A] \circ (B]] = (A \circ B]$.
(5) $((A]] = (A]$.

Definition 2.5. (see [18]). An ordered semihypergroup $(S, \circ, \leq)$ is called *regular* if for each $a \in S$ there exists $x \in S$ such that $a \leq a \circ x \circ a$.

Definition 2.6. (see [18]). An ordered semihypergroup $(S, \circ, \leq)$ is called intra-*regular* if for each $a \in S$ there exist $x, y \in S$ such that $a \leq x \circ a \circ a \circ y$.

2.2. **Basic concepts of soft sets.** In what follows, we take $E = S$ as the set of parameters, which is an ordered semihypergroup, unless otherwise specified. From now on, U is an initial universe set, E is a set of parameters, $P(U)$ is the power set of U and $A, B, C... \subseteq E$.

Definition 2.7. (see [6]). A soft set f_A over U is defined as $f_A : E \longrightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Hence f_A is also called an approximation function. A soft set f_A over U can be represented by the set of ordered pairs $f_A = \{(x, f_A(x)) | x \in E,\ f_A(x) \in P(U)\}$. It is clear from Definition 2.7, that a soft set is a parameterized family of subsets of U. Note that the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.8. (see [6])
(i) Let $f_A, f_B \in S(U)$. Then f_A is called a soft subset of f_B, denoted by $f_A \subseteq f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Two soft sets f_A and f_B are said to be equal soft sets if $f_A \subseteq f_B$ and $f_B \subseteq f_A$ and is

denoted by $f_A = f_B$.
(ii) Let $f_A, f_B \in S(U)$. Then the soft union of f_A and f_B, denoted by $f_A \cup f_B = f_{A\cup B}$, is defined by $(f_A \cup f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$.
(iii) Let $f_A, f_B \in S(U)$. Then the soft intersection of f_A and f_B, denoted by $f_A \cap f_B = f_{A\cap B}$, is defined by $(f_A \cap f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$. For $x \in S$, we define

$$A_x = \{(y, z) \in S \times S \mid x \leq y \circ z\}.$$

Definition 2.9. (see [8]). Let f_A and g_B be two soft sets of an ordered semihypergroup S over U. Then, the int-soft product, denoted by $f_A \widetilde{*} g_B$, is defined by

$$f_A \widetilde{*} g_B : S \longrightarrow P(U), x \longmapsto (f_A \widetilde{*} g_B)(x) = \begin{cases} \bigcup\limits_{(y,z)\in A_x} \{f_A(y) \cap g_B(z)\}, & \text{if } A_x \neq \emptyset, \\ \emptyset, & \text{if } A_x = \emptyset, \end{cases}$$

for all $x \in S$.
Definition 2.10. (see [8]). For a nonempty subset A of S the characteristic soft set is defined to be the soft set $\mathcal{S}_\mathcal{A}$ of A over U in which $\mathcal{S}_\mathcal{A}$ is given as follows
$$\mathcal{S}_\mathcal{A} : S \longmapsto P(U). \quad x \longmapsto \begin{cases} U, & \text{if} \quad x \in A \\ \emptyset, & \text{otherwise} \end{cases}$$
For an ordered semihypergroup S, the soft set "$\mathcal{S}_\mathcal{S}$ " of S over U is defined as follows:

$$\mathcal{S}_\mathcal{S} : S \longrightarrow P(U), x \longmapsto \mathcal{S}_\mathcal{S}(x) = U \text{ for all } x \in S.$$

The soft set "$\mathcal{S}_\mathcal{S}$" of an ordered semihypergroup S over U is called the whole soft set of S over U.
Definition 2.11. (see [8]). Let f_A be a soft set of an ordered semihypergroup S over U a subset δ such that $\delta \in P(U)$. The δ-inclusive set of f_A is denoted by $e_A(f_A, \delta)$ and defined to be the set

$$e_A(f_A, \delta) = \{x \in S \mid f_A(x) \supseteq \delta\}.$$

Definition 2.12. (see [8]). A soft set f_A of an ordered semihypergroup S over U is called *an int-soft subsemihypergroup of* S over U if:
$(\forall x, y \in S) \bigcap\limits_{\alpha \in x\circ y} f_A(\alpha) \supseteq f_A(x) \cap f_A(y)$.
Definition 2.13. (see [8]). Let f_A be a soft set of an ordered semihypergroup S over U. Then f_A is called an int-soft left (resp., right) hyperideal of S over U if it satisfies the following conditions:
(1) $(\forall x, y \in S) \bigcap\limits_{\alpha \in x\circ y} f_A(\alpha) \supseteq f_A(y) \left(\text{resp., } \bigcap\limits_{\alpha \in x\circ y} f_A(\alpha) \supseteq f_A(x)\right)$.
(2) $(\forall x, y \in S)\ x \leq y \Longrightarrow f_A(x) \supseteq f_A(y)$.
A soft set f_A over U is called an *int-soft hyperideal* (or int-soft two-sided hyperideal) of S over U if it is both an int-soft left hyperideal and an int-soft right hyperideal of S over U.
Definition 2.14. An int-soft subsemihypergroup f_A of an ordered semihypergroup S over U is called an int-soft interior hyperideal of S over U if it satisfies the following conditions:
(1) $(\forall x, y, a \in S) \bigcap\limits_{\alpha \in x\circ a\circ y} f_A(\alpha) \supseteq f_A(a)$.
(2) $(\forall x, y \in S)\ x \leq y \Longrightarrow f_A(x) \supseteq f_A(y)$.
Example 2.1. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a, b\}$
d	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, b, c\}$

$$\leq := \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, c), (a, d), (b, d), (c, d)\}.$$

Suppose $U = \{p, q, r, s\}$ and $A = \{a, b, c\}$. Let us define $f_A(a) = \{p, q, r, s\}$, $f_A(b) = \{p\}$, $f_A(c) = \{p, q, r\}$ and $f_A(d) = \emptyset$. Then f_A is an int-soft interior hyperideal of S over U.

Example 2.2. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d	e
a	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
b	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
c	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{c\}$	$\{e\}$
d	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{d\}$	$\{e\}$
e	$\{a,b\}$	$\{a,b\}$	$\{c\}$	$\{c\}$	$\{e\}$

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(e,e),(a,c),(a,d),(a,e),(b,c),(b,d),(b,e),(c,d),(c,e)\}.$$

Let $U = \{1,2,3,4\}$ and $A = \{a,b,c,e\}$. Let us define $f_A(a) = \{1,2,3,4\}$, $f_A(b) = \{1,2,3,4\}$, $f_A(c) = \{3,4\}$, $f_A(d) = \emptyset$ and $f_A(e) = \{3,4\}$. Then f_A is an int-soft interior hyperideal of S over U.

Proposition 2.1. Let $(S, \circ, \leq)$ be an ordered semihypergroup and A be a nonempty subset of S. Then A is an interior hyperideal of S if and only if the characteristic function $\mathcal{S}_{\mathcal{A}}$ is an int-soft interior hyperideal of S over U.

Proof. Suppose that A is an interior hyperideal of S. Let x, y and a be any elements of S. If $a \in A$, then $\mathcal{S}_{\mathcal{A}}(a) = U$. Since A is an interior hyperideal of S, we have $\alpha \in x \circ a \circ y \subseteq S \circ A \circ S \subseteq A$ we have $\mathcal{S}_{\mathcal{A}}(\alpha) = U$. Thus $\bigcap_{\alpha \in x \circ a \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) = U = \mathcal{S}_{\mathcal{A}}(a)$. If $a \notin A$ then $\mathcal{S}_{\mathcal{A}}(a) = \emptyset$. Since $\mathcal{S}_{\mathcal{A}}(\alpha) \supseteq \emptyset = \mathcal{S}_{\mathcal{A}}(a)$. Thus $\bigcap_{\alpha \in x \circ a \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) \supseteq \mathcal{S}_{\mathcal{A}}(a)$. Let $x, y \in S$ with $x \leq y$. If $y \notin A$ then $\mathcal{S}_{\mathcal{A}}(y) = \emptyset$ and so $\mathcal{S}_{\mathcal{A}}(x) \supseteq \emptyset = \mathcal{S}_{\mathcal{A}}(y)$. If $y \in A$ then $\mathcal{S}_{\mathcal{A}}(y) = U$. Since $x \leq y$ and A is an interior hyperideal of S, we have $x \in A$ and thus $\mathcal{S}_{\mathcal{A}}(x) = U = \mathcal{S}_{\mathcal{A}}(y)$. Since A is an interior hyperideal of S. Therefore A is a subsemihypergroup of S. Let $x, y \in S$. Then we have, $\bigcap_{\alpha \in x \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) \supseteq \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(y)$ for every $\alpha \in x \circ y$. Indeed: If $x \circ y \nsubseteq A$, then there exists $\alpha \in x \circ y$ such that $\alpha \notin A$, and we have $\bigcap_{\alpha \in x \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) = \emptyset$. Besides that $x \circ y \nsubseteq A$ implies that $x \notin A$ or $y \notin A$. Then $\mathcal{S}_{\mathcal{A}}(x) = \emptyset$ or $\mathcal{S}_{\mathcal{A}}(y) = \emptyset$ and hence $\bigcap_{\alpha \in x \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) = \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(y)$. Let $x \circ y \subseteq A$. Then $\mathcal{S}_{\mathcal{A}}(\alpha) = U$ for any $\alpha \in x \circ y$. It implies that $\bigcap_{\alpha \in x \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) = U$. Since we have $\mathcal{S}_{\mathcal{A}}(x) \subseteq U$ for any $x \in A$. Thus $\bigcap_{\alpha \in x \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) \supseteq \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(y)$. Therefore $\mathcal{S}_{\mathcal{A}}$ is an int-soft interior hyperideal of S over U.

Conversely, let $\emptyset \neq A \subseteq S$ such that $\mathcal{S}_{\mathcal{A}}$ is an int-soft interior hyperideal of S over U. We claim that $A \circ A \subseteq A$. To prove the claim, let $x, y \in A$. By hypothesis, $\bigcap_{\alpha \in x \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) \supseteq \mathcal{S}_{\mathcal{A}}(x) \cap \mathcal{S}_{\mathcal{A}}(y) = U$ which implies that $\mathcal{S}_{\mathcal{A}}(\alpha) \supseteq U$ for any $\alpha \in x \circ y$. On the other hand $\mathcal{S}_{\mathcal{A}}(x) \subseteq U$ for all $x \in S$. Thus for any $\alpha \in x \circ y$, $\mathcal{S}_{\mathcal{A}}(\alpha) = U$ implies that $\alpha \in A$. It thus follows that $A \circ A \subseteq A$. Let $\alpha \in S \circ A \circ S$, then there exist $x, y \in S$ and $a \in A$ such that $\alpha \in x \circ a \circ y$. Since $\bigcap_{\alpha \in x \circ a \circ y} \mathcal{S}_{\mathcal{A}}(\alpha) \supseteq \mathcal{S}_{\mathcal{A}}(a)$, and $a \in A$ we have $\mathcal{S}_{\mathcal{A}}(a) = U$. Hence for each $\alpha \in S \circ A \circ S$, we have $\mathcal{S}_{\mathcal{A}}(\alpha) = U$, and so $\alpha \in A$. Thus $S \circ A \circ S \subseteq A$. Let $x \in S$ and $y \in A$ be such that $x \leq y$. Then $\mathcal{S}_{\mathcal{A}}(x) \supseteq \mathcal{S}_{\mathcal{A}}(y) = U$, and thus $x \in A$. Therefore A is an interior hyperideal of S.

Proposition 2.2. Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A be an int-soft hyperideal of S over U. Then f_A is an int-soft interior hyperideal of S over U.

Proof. Let $x, a, y \in S$. Since f_A is an int-soft hyperideal of S over U. Then for any $\alpha \in x \circ a \circ y$, we have $\bigcap_{\alpha \in x \circ a \circ y} f_A(\alpha) = \bigcap_{\substack{\alpha \in x \circ \beta \\ \beta \in a \circ y}} f_A(\alpha) \supseteq f_A(\beta) \supseteq \bigcap_{\beta \in a \circ y} f_A(\beta) \supseteq f_A(a)$. Thus $\bigcap_{\alpha \in x \circ a \circ y} f_A(\alpha) \supseteq f_A(a)$. Therefore f_A is an int-soft interior hyperideal of S over U. The converse of Proposition 2.2, is not true in general. We can illustrate it by the following example.

Example 2.3. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a,b\}$
d	$\{a\}$	$\{a\}$	$\{a,b\}$	$\{a\}$

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d),(d,b),(d,c)\}.$$

Suppose $U = \{p,q,r\}$ and $A = \{a,b,d\}$. Let us define $f_A(a) = \{p,q,r\}$, $f_A(b) = \{p\}$, $f_A(c) = \emptyset$ and $f_A(d) = \{p,r\}$. Then f_A is an int-soft interior hyperideal of S over U. This is not an int-soft left hyperideal as $\bigcap\limits_{\alpha\in c\circ d=\{a,b\}} f_A(a)\cap f_A(b) = \{p\} \not\supseteq \{p,r\} = f_A(d)$.

Proposition 2.3. Let $(S,\circ,\leq)$ be a regular ordered semihypergroup and f_A is an int-soft interior hyperideal of S over U. Then f_A is an int-soft hyperideal of S over U.

Proof. Let $x,y\in S$. Since f_A is an int-soft interior hyperideal of S over U. Then $\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha) \supseteq f_A(x)$. Indeed: Since S is regular and $x\in S$, then there exists $z\in S$ such that $x\leq x\circ z\circ x$. Then we have $x\circ y \leq (x\circ z\circ x)\circ y = (x\circ z)\circ(x\circ y)$. So there exist $\alpha\in x\circ y$, $v\in x\circ z$ and $\beta\in v\circ x\circ y$ such that $\alpha\leq\beta$. Since f_A is an int-soft interior hyperideal of S over U, we have $f_A(\alpha)\supseteq f_A(\beta)\supseteq \bigcap\limits_{\beta\in v\circ x\circ y} f_A(\beta)\supseteq f_A(x)$. Thus $\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha)\supseteq f_A(x)$. Therefore f_A is an int-soft right hyperideal of S over U. In a similar way we prove that f_A is an int-soft left hyperideal of S over U.

By Propositions 2.2 and 2.3 we have the following:

Theorem 2.1. In regular ordered semihypergroups the concepts of int-soft hyperideals and int-soft interior hyperideals coincide.

Proposition 2.4. Let $(S,\circ,\leq)$ be an intra-regular ordered semihypergroup and f_A is an int-soft interior hyperideal of S over U. Then f_A is an int-soft hyperideal of S over U.

Proof. Let $a,b\in S$. Then $\bigcap\limits_{u\in a\circ b} f_A(u)\supseteq f_A(a)$. Indeed: Since S is intra-regular and $a\in S$, there exist $x,y\in S$ such that $a\leq x\circ a\circ a\circ y$. Then $a\circ b\leq (x\circ a\circ a\circ y)\circ b = x\circ a\circ(a\circ y\circ b)$. So there exist $u\in a\circ b$, $v\in a\circ y\circ b$ and $\alpha\in x\circ a\circ v$ such that $u\leq\alpha$. Since f_A is an int-soft interior hyperideal of S over U, we have $f_A(u)\supseteq f_A(\alpha)\supseteq \bigcap\limits_{\alpha\in x\circ a\circ v} f_A(\alpha)\supseteq f_A(a)$. Thus $\bigcap\limits_{u\in a\circ b} f_A(u)\supseteq f_A(a)$. Hence f_A is an int-soft right hyperideal of S over U. Similarly we can prove that f_A is an int-soft left hyperideal of S over U. Therefore f_A is an int-soft hyperideal of S over U.

By Propositions 2.2 and 2.4 we have the following:

Theorem 2.2. In intra-regular ordered semihypergroups the concepts of int-soft hyperideals and int-soft interior hyperideals coincide.

Theorem 2.3. Let f_A be a soft set of an ordered semihypergroup S over U and $\delta\in P(U)$. Then f_A is an int-soft interior hyperideal of S over U if and only if each nonempty δ-inclusive set $e_A(f_A,\delta)$ is an interior hyperideal of S.

Proof. Assume that f_A is an int-soft interior hyperideal of S over U. Let $\delta\in P(U)$ such that $e_A(f_A,\delta)\neq\emptyset$. Let $x,y\in e_A(f_A,\delta)$. Then $f_A(x)\supseteq\delta$ and $f_A(y)\supseteq\delta$. By hypothesis, we have $\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha)\supseteq f_A(x)\cap f_A(y)\supseteq\delta\cap\delta=\delta$. Thus for any $\alpha\in x\circ y$, we have $f_A(\alpha)\supseteq\delta$, implies that $\alpha\in e_A(f_A,\delta)$. It follows that $x\circ y\subseteq e_A(f_A,\delta)$. Hence $e_A(f_A,\delta)$ is a subsemihypergroup of S. Let $y\in e_A(f_A,\delta)$ and $x,z\in S$. Then $f_A(y)\supseteq\delta$. Since f_A is an int-soft interior hyperideal of S over U. Thus $\bigcap\limits_{w\in x\circ y\circ z} f_A(w)\supseteq f_A(y)\supseteq\delta$. Hence $f_A(w)\supseteq\delta$ for any $w\in x\circ y\circ z$ implies that $w\in e_A(f_A,\delta)$. Thus $S\circ e_A(f_A,\delta)\circ S\subseteq e_A(f_A,\delta)$. Let $x\in e_A(f_A,\delta)$ and $y\in S$ with $y\leq x$. Then $\delta\subseteq f_A(x)\subseteq f_A(y)$, we get $y\in e_A(f_A,\delta)$. Therefore $e_A(f_A,\delta)$ is an interior hyperideal of S.

Conversely, suppose that $e_A(f_A,\delta)\neq\emptyset$ is an interior hyperideal of S. If $\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha)\subset f_A(x)\cap f_A(y)$ for some $x,y\in S$, then there exists $\delta\in P(U)$ such that $\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha)\subset\delta\subseteq f_A(x)\cap f_A(y)$, which implies that $x,y\in e_A(f_A,\delta)$ and $x\circ y\nsubseteq e_A(f_A,\delta)$. It contradicts the fact that $e_A(f_A,\delta)$ is an interior hyperideal of S. Consequently, $\bigcap\limits_{\alpha\in x\circ y} f_A(\alpha)\supseteq f_A(x)\cap f_A(y)$ for all $x,y\in S$. Next we show that $\bigcap\limits_{\alpha\in x\circ a\circ y} f_A(\alpha)\supseteq f_A(a)$ for all $x,a,y\in S$. Choose $f_A(a)=\delta$, then $a\in e_A(f_A,\delta)$. Since $e_A(f_A,\delta)$ is an interior hyperideal of S, we get $x\circ a\circ y\subseteq e_A(f_A,\delta)$. Then for every $\alpha\in x\circ a\circ y$, we have $f_A(\alpha)\supseteq\delta$ and so $f_A(a)=\delta\subseteq\bigcap\limits_{\alpha\in x\circ a\circ y} f_A(\alpha)$. Let $x,y\in S$ such that $x\leq y$. If $f_A(y)=\delta$ then $y\in e_A(f_A,\delta)$. Since $e_A(f_A,\delta)$ is an interior hyperideal of S, we get $x\in e_A(f_A,\delta)$. So $f_A(x)\supseteq\delta=f_A(y)$. Therefore f_A is an int-soft interior hyperideal of S over U.

Example 2.4. Let $(S, \circ, \leq)$ be an ordered semihypergroup where the hyperoperation and the order relation are defined by:

$\circ$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a,d\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(a,d)\}.$$

Then the interior hyperideals of S are $\{a\}$, $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{a,b,d\}$, $\{a,c,d\}$ and S. Suppose $U = \{e_1,e_2,e_3,e_4\}$ and $A = \{a,b,d\}$. Let us define $f_A(a) = \{e_1,e_2,e_3,e_4\}$, $f_A(b) = \{e_1,e_3\}$, $f_A(c) = \emptyset$ and $f_A(d) = \{e_1,e_4\}$. Then

$$e_A(f_A,\delta) = \begin{cases} \{a,b,d\} & \text{if } \delta = \{e_1\} \\ \{a\} & \text{if } \delta = \{e_2\} \\ \{a,b\} & \text{if } \delta = \{e_3\} \\ \{a,d\} & \text{if } \delta = \{e_4\} \\ \{a\} & \text{if } \delta = \{e_1,e_2\} \\ \{a,b\} & \text{if } \delta = \{e_1,e_3\} \\ \{a,d\} & \text{if } \delta = \{e_1,e_4\} \\ \{a\} & \text{if } \delta = \{e_2,e_3\} \\ \{a\} & \text{if } \delta = \{e_2,e_4\} \\ \{a\} & \text{if } \delta = \{e_3,e_4\} \\ \{a\} & \text{if } \delta = \{e_1,e_2,e_3\} \\ \{a\} & \text{if } \delta = \{e_1,e_2,e_4\} \\ \{a\} & \text{if } \delta = \{e_1,e_3,e_4\} \\ \{a\} & \text{if } \delta = \{e_2,e_3,e_4\} \\ \{a\} & \text{if } \delta = U \end{cases}$$

So by Theorem 2.3, f_A is an int-soft interior hyperideal of S over U.

Theorem 2.4. Let $\{f_{A_i} \mid i \in I\}$ be a family of int-soft interior hyperideals of an ordered semihypergroup S over U. Then $f_A = \bigcap_{i\in I} f_{A_i}$ is an int-soft interior hyperideal of S over U where $\left(\bigcap_{i\in I} f_{A_i}\right)(x) = \bigcap_{i\in I}(f_{A_i}(x))$.

Proof. Let $x, y \in S$. Then, since each f_{A_i} $(i \in I)$ is an int-soft interior hyperideals of S over U, so $\bigcap_{\alpha\in x\circ y} f_{A_i}(\alpha) \supseteq f_{A_i}(x) \cap f_{A_i}(y)$. Thus for any $\alpha \in x \circ y$, $f_{A_i}(\alpha) \supseteq f_{A_i}(x) \cap f_{A_i}(y)$, and we have $f_A(\alpha) = \left(\bigcap_{i\in I} f_{A_i}\right)(\alpha) = \bigcap_{i\in I}(f_{A_i}(\alpha)) \supseteq \bigcap_{i\in I}(f_{A_i}(x) \cap f_{A_i}(y)) = \left(\bigcap_{i\in I}(f_{A_i}(x))\right) \cap \left(\bigcap_{i\in I}(f_{A_i}(y))\right) = \left(\bigcap_{i\in I} f_{A_i}\right)(x) \cap \left(\bigcap_{i\in I} f_{A_i}\right)(y) = f_A(x) \cap f_A(y)$, which implies that $\bigcap_{\alpha\in x\circ y} f_A(\alpha) \supseteq f_A(x) \cap f_A(y)$. Let $a, x, y \in S$ and $\bigcap_{\beta\in x\circ a\circ y} f_{A_i}(\beta) \supseteq f_{A_i}(a)$. Thus for any $\beta \in x \circ a \circ y$, $f_{A_i}(\beta) \supseteq f_{A_i}(a)$. Then $f_A(\beta) = \left(\bigcap_{i\in I} f_{A_i}\right)(\beta) = \bigcap_{i\in I}(f_{A_i}(\beta)) \supseteq \bigcap_{i\in I}(f_{A_i}(a)) = \left(\bigcap_{i\in I} f_{A_i}\right)(a) = f_A(a)$. Thus $\bigcap_{\beta\in x\circ a\circ y} f_A(\beta) \supseteq f_A(a)$. Furthermore, if $x \leq y$, then $f_A(x) \supseteq f_A(y)$. Indeed: Since every f_{A_i} $(i \in I)$ is an int-soft interior hyperideal of S over U, it can be obtained that $f_{A_i}(x) \supseteq f_{A_i}(y)$ for all $i \in I$. Thus $f_A(x) = \left(\bigcap_{i\in I} f_{A_i}\right)(x) = \bigcap_{i\in I}(f_{A_i}(x)) \supseteq \bigcap_{i\in I}(f_{A_i}(y)) = \left(\bigcap_{i\in I} f_{A_i}\right)(y) = f_A(y)$. Thus f_A is an int-soft interior hyperideals of S over U.

Lemma 2.2. Let S be an ordered semihypergroup and f_A is a soft set of S over U. If f_A is an int-soft subsemihypergroup of S over U such that

$$x \leq y \Longrightarrow f_A(x) \supseteq f_A(y), \ \forall x, y \in S,$$

then $f_A \widetilde{*} f_A \widetilde{\subseteq} f_A$. Conversely if $f_A \widetilde{*} f_A \widetilde{\subseteq} f_A$, then f_A is an int-soft subsemihypergroup of S over U.
Proof. Let $x \in S$. If $A_x = \emptyset$, then $(f_A \widetilde{*} f_A)(x) = \emptyset \subseteq f_A(x)$. If $A_x \neq \emptyset$, then $(b,c) \in A_x$ such that $x \leq b \circ c$. This means that there exists $\alpha \in b \circ c$ such that $x \leq \alpha$.

$$\begin{aligned}
(f_A \widetilde{*} f_A)(x) &= \bigcup_{(b,c)\in A_x} \{f_A(b) \cap f_A(c)\} \\
&\subseteq \bigcup_{(b,c)\in A_x} f_A(\alpha) \\
&\subseteq \bigcup_{(b,c)\in A_x} f_A(x) \\
&= f_A(x).
\end{aligned}$$

Thus $f_A \widetilde{*} f_A \subseteq f_A$.
Conversely, if $f_A \widetilde{*} f_A \subseteq f_A$, then for all $x, y \in S$ and $\alpha \in x \circ y$. We have

$$\begin{aligned}
f_A(\alpha) &\supseteq (f_A \widetilde{*} f_A)(\alpha) \\
&= \bigcup_{(x,y)\in A_\alpha} \{f_A(x) \cap f_A(y)\} \\
&\supseteq \{f_A(x) \cap f_A(y)\} \\
f_A(\alpha) &\supseteq \{f_A(x) \cap f_A(y)\}.
\end{aligned}$$

Hence $\bigcap_{\alpha\in x\circ y} f_A(\alpha) \supseteq \{f_A(x) \cap f_A(y)\}$. Thus f_A is an int-soft subsemihypergroup of S over U.
Theorem 2.5. Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A be a soft set of S over U. Then f_A is an int-soft interior hyperideal of S over U if and only if $f_A \widetilde{*} f_A \subseteq f_A$ and $\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S} \subseteq f_A$.
Proof. Let f_A be an int-soft interior hyperideal of S over U. Then $(\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(a) \subseteq f_A(a)$ for all $a \in S$. Indeed: If $(\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(a) = \emptyset$, clearly, $(\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(a) \subseteq f_A(a)$. Let $(\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(a) \neq \emptyset$. Then we can prove that $(\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(a) \subseteq f_A(a)$. In fact, let $(x, y) \in A_a$ and $(p, q) \in A_x$, i.e., $a \leq x \circ y$ and $x \leq p \circ q$. Then $a \leq p \circ q \circ y$, and there exists $u \in p \circ q \circ y$ such that $a \leq u$. Since f_A is an int-soft interior hyperideal of S over U. Then $f_A(a) \supseteq f_A(u) \supseteq \bigcap_{u\in p\circ q\circ y} f_A(u) \supseteq f_A(q)$. Thus

$$\begin{aligned}
(\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(a) &= \bigcup_{(x,y)\in A_a} \{(\mathcal{S}_\mathcal{S} \widetilde{*} f_A)(x) \cap \mathcal{S}_\mathcal{S}(y)\} \\
&= \bigcup_{(x,y)\in A_a} \{(\mathcal{S}_\mathcal{S} \widetilde{*} f_A)(x) \cap U\} \\
&= \bigcup_{(x,y)\in A_a} (\mathcal{S}_\mathcal{S} \widetilde{*} f_A)(x) \\
&= \bigcup_{(x,y)\in A_a} \left\{ \bigcup_{(p,q)\in A_x} (\mathcal{S}_\mathcal{S}(p) \cap f_A(q)) \right\} \\
&= \bigcup_{(x,y)\in A_a} \left\{ \bigcup_{(p,q)\in A_x} (U \cap f_A(q)) \right\} \\
&= \bigcup_{(x,y)\in A_a} \bigcup_{(p,q)\in A_x} (f_A(q)) \\
&\subseteq f_A(a).
\end{aligned}$$

Thus $\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S} \subseteq f_A$.
Conversely, for any $x, y, z \in S$, let $\alpha \in x \circ y \circ z$. Then, there exists $u \in x \circ y \subseteq (x \circ y]$ such that

$\alpha \in u \circ z \subseteq (u \circ z]$, and we have $(x, y) \in A_u$, $(u, z) \in A_\alpha$. Since $\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S} \subseteq f_A$, we have

$$\begin{aligned}
f_A(\alpha) &\supseteq (\mathcal{S}_\mathcal{S} \widetilde{*} f_A \widetilde{*} \mathcal{S}_\mathcal{S})(\alpha) \\
&= \bigcup_{(p,q) \in A_\alpha} [\{\mathcal{S}_\mathcal{S} \widetilde{*} f_A\}(p) \cap \mathcal{S}_\mathcal{S}(q)] \\
&\supseteq \{(\mathcal{S}_\mathcal{S} \widetilde{*} f_A)(u) \cap \mathcal{S}_\mathcal{S}(z)\} \\
&= \{(\mathcal{S}_\mathcal{S} \widetilde{*} f_A)(u) \cap U\} \\
&= (\mathcal{S}_\mathcal{S} \widetilde{*} f_A)(u) \\
&= \bigcup_{(s,t) \in A_u} [\mathcal{S}_\mathcal{S}(s) \cap f_A(t)] \\
&\supseteq \{\mathcal{S}_\mathcal{S}(x) \cap f_A(y)\} \\
&= \{U \cap f_A(y)\} \\
&= f_A(y).
\end{aligned}$$

It thus follows that $\bigcap_{\alpha \in x \circ y \circ z} f_A(\alpha) \supseteq f_A(y)$. The rest of the proof is a consequence of the Lemma 2.2.

3. Characterizations of simple ordered semihypergroups in terms of int-soft hyperideals and int-soft interior hyperideals

Definition 3.1. (see [18]). An ordered semihypergroup $(S, \circ, \leq)$ is called simple if it has no a proper hyperideal.

Lemma 3.1. (see [18]). An ordered semihypergroup $(S, \circ, \leq)$ is a simple ordered semihypergroup if and only if for every $a \in S$, $(S \circ a \circ S] = S$.

Let $(S, \circ, \leq)$ is an ordered semihypergroup and $a \in S$, and f_A be a soft set of S over U we denote by I_a the subset of S defines as follows:

$$I_a = \{b \in S \mid f_A(b) \supseteq f_A(a)\}.$$

Proposition 3.1. Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A is an int-soft right hyperideals of S over U. Then the set I_a is a right hyperideal of S for every $a \in S$.

Proof. Let $a \in S$. First of all $\emptyset \neq I_a \subseteq S$. Since $a \in I_a$. Let $b \in I_a$ and $s \in S$. Then $b \circ s \subseteq I_a$. Indeed: Since f_A is an int-soft right hyperideals of S over U and $b, s \in S$, we have $\bigcap_{\alpha \in b \circ s} f_A(\alpha) \supseteq f_A(b)$. Since $b \in I_a$, we have $f_A(b) \supseteq f_A(a)$. Thus $\bigcap_{\alpha \in b \circ s} f_A(\alpha) \supseteq f_A(a)$, implies that $f_A(\alpha) \supseteq f_A(a)$, so $\alpha \in I_a$ and hence $b \circ s \subseteq I_a$. Let $b \in I_a$ and $S \ni s \leq b$. Then $s \in I_a$. Indeed: Since f_A is an int-soft right hyperideals of S over U, $b, s \in S$ and $s \leq b$, we have $f_A(s) \supseteq f_A(b)$. Since $b \in I_a$, we have $f_A(b) \supseteq f_A(a)$. Then $f_A(s) \supseteq f_A(a)$, so $s \in I_a$.

In a similar way we prove the following:

Proposition 3.2. Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A is an int-soft left hyperideals of S over U. Then the set I_a is a left hyperideal of S for every $a \in S$.

By Propositions 3.1 and 3.2 we have the following:

Proposition 3.3. Let $(S, \circ, \leq)$ be an ordered semihypergroup and f_A is an int-soft hyperideals of S over U. Then the set I_a is a hyperideal of S for every $a \in S$.

Theorem 3.1. (see [8]). Let $(S, \circ, \leq)$ be an ordered semihypergroup and $\emptyset \neq I \subseteq S$. Then I is a hyperideal of S if and only if the characteristic function $\mathcal{S}_\mathcal{I}$ is an int-soft hyperideals of S over U.

Theorem 3.2. An ordered semihypergroup $(S, \circ, \leq)$ is a simple ordered semihypergroup if and only if every int-soft hyperideal of S over U is a constant function.

Proof. Assume that S is a simple ordered semihypergroup. Let f_A is an int-soft hyperideal of S over U and $a, b \in S$. By Proposition 3.3, we obtain I_a is a hyperideal of S. By assumption, this implies that $I_a = S$. Then $b \in I_a$, that is $f_A(b) \supseteq f_A(a)$. By symmetry we get $f_A(a) \supseteq f_A(b)$. Therefore $f_A(a) = f_A(b)$.

Conversely, we assume that for every int-soft hyperideal of S over U is a constant function. Let I be a hyperideal of S and $x \in S$. By Theorem 3.1, we obtain the characteristic function $\mathcal{S}_\mathcal{I}$ is an int-soft hyperideal of S over U. By assumption, $\mathcal{S}_\mathcal{I}$ is a constant function, that is $\mathcal{S}_\mathcal{I}(x) = \mathcal{S}_\mathcal{I}(b)$ for every

$b \in S$. Let $a \in I$. Then $\mathcal{S}_{\mathcal{I}}(x) = \mathcal{S}_{\mathcal{I}}(a) = U$, and so $x \in I$. Therefore $S \subseteq I$.

Theorem 3.3. Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then S is a simple ordered semihypergroup if and only if every int-soft interior hyperideal of S over U is a constant function.

Proof. Assume that S is a simple ordered semihypergroup. Let f_A be an int-soft interior hyperideal of S over U and $a, b \in S$. By Lemma 3.1, we have $S = (S \circ b \circ S]$. Since $a \in S$, we have $a \in (S \circ b \circ S]$. Then there exist $x, y \in S$ such that $a \leq x \circ b \circ y$, i.e., there exists $\alpha \in x \circ b \circ y$ such that $a \leq \alpha$. Since f_A is an int-soft interior hyperideal of S over U, we have $f_A(a) \supseteq f_A(\alpha) \supseteq \bigcap_{\alpha \in x \circ b \circ y} f_A(\alpha) \supseteq f_A(b)$. Hence $f_A(a) \supseteq f_A(b)$. By symmetry we can prove that $f_A(b) \supseteq f_A(a)$. Therefore $f_A(a) = f_A(b)$.

Conversely, assume that every int-soft interior hyperideal of S over U is a constant function. Let f_A is an int-soft hyperideal of S over U. Then f_A is an int-soft interior hyperideal of S over U. By assumption f_A is a constant function. By Theorem 3.2, S is a simple ordered semihypergroup.

Corollary 3.1. Let $(S, \circ, \leq)$ be an intra-regular ordered semihypergroup. Then every int-soft interior hyperideal of S over U is a constant function.

As a consequence of Lemma 3.1, Theorem 3.2, and Theorem 3.3, we present characterizations of a simple ordered semihypergroup as the following theorem.

Theorem 3.4. Let $(S, \circ, \leq)$ be an ordered semihypergroup. Then the following statements are equivalent:

(1) S is a simple ordered semihypergroup.
(2) $S = (S \circ a \circ S]$ for every $a \in S$.
(3) Every int-soft hyperideal of S over U is a constant function.
(4) Every int-soft interior hyperideal of S over U is a constant function.

Proposition 3.4. Let $(S, \circ, \leq)$ be an intra-regular ordered semihypergroup. Then for every interior hyperideals A and B of S we have

(1) $(A \circ A] = A$.
(2) $(A \circ B] = (B \circ A]$.

Proof. (1). Let S be an intra-regular ordered semihypergroup and A, B are the interior hyperideals of S. Let $a \in A$. Since S is intra-regular, there exist $x, y \in S$ such that $a \leq x \circ a \circ a \circ y = (x \circ a) \circ (a \circ y) \leq x \circ (x \circ a \circ a \circ y) \circ (x \circ a \circ a \circ y) \circ y = ((x \circ x \circ a) \circ a \circ (y)) \circ ((x \circ a) \circ a \circ (a \circ y \circ y)) \subseteq (S \circ A \circ S) \circ (S \circ A \circ S) \subseteq A \circ A \Longrightarrow a \in (A \circ A] \Longrightarrow A \subseteq (A \circ A]$.

For the reverse inclusion, let $a \in (A \circ A]$, then $a \leq a_1 \circ a_2$ for some $a_1, a_2 \in A$. Then $a \leq x \circ a \circ a \circ y = (x \circ a) \circ (a \circ y) \leq x \circ (a_1 \circ a_2) \circ (a_1 \circ a_2) \circ y = (x \circ a_1 \circ a_2) \circ a_1 \circ (a_2 \circ y) \subseteq S \circ A \circ S \subseteq A \Longrightarrow a \in (A] = A \Longrightarrow (A \circ A] \subseteq A$. Thus $(A \circ A] = A$.

(2). Let A and B be interior hyperideals of S. Then $(A \circ B] = (B \circ A]$. Indeed: By (1) we have $(A \circ B] = ((A \circ B] \circ (A \circ B]] = (((A \circ B] \circ (A \circ B]) \circ ((A \circ B] \circ (A \circ B])]$
$\subseteq (((A \circ B) \circ (A \circ B)] \circ ((A \circ B) \circ (A \circ B)]] = (((A) \circ B \circ (A \circ B)] \circ ((A \circ B) \circ A \circ (B)]]$
$\subseteq ((S \circ B \circ S] \circ (S \circ A \circ S]] \subseteq ((B] \circ (A]] = (B \circ A] \Longrightarrow (A \circ B] \subseteq (B \circ A]$. By symmetry we have $(B \circ A] \subseteq (A \circ B]$. Thus $(A \circ B] = (B \circ A]$.

Proposition 3.5. Let $(S, \circ, \leq)$ be an intra-regular ordered semihypergroup and f_A is an int-soft interior hyperideal of S over U. Then for every $a \in S$ such that $a \circ a \leq a$, we have the following

(1) $\bigcap_{v \in a \circ a} f_A(v) = f_A(a)$.
(2) $\bigcap_{\alpha \in a \circ b} f_A(\alpha) = \bigcap_{\beta \in b \circ a} f_A(\beta)$.

Proof. (1). Let S be an intra-regular ordered semihypergroup and f_A is an int-soft interior hyperideal of S over U and $a \in S$. Then $\bigcap_{v \in a \circ a} f_A(v) = f_A(a)$. Indeed: Since S is intra-regular and $a \in S$, there exist $x, y \in S$ such that $a \leq x \circ a \circ a \circ y$ for some $x, y \in S$. So there exist $v \in a \circ a$ and $z \in x \circ v \circ y$ such that $a \leq z$. Then $f_A(a) \supseteq f_A(z) \supseteq \bigcap_{z \in x \circ v \circ y} f_A(z) \supseteq f_A(v)$. Hence $f_A(a) \supseteq \bigcap_{v \in a \circ a} f_A(v)$. Since $a \circ a \leq a$ so there is $v \in a \circ a$ such that $v \leq a$. Then we have $f_A(v) \supseteq f_A(a)$. Thus $\bigcap_{v \in a \circ a} f_A(v) \supseteq f_A(a)$. Therefore $\bigcap_{v \in a \circ a} f_A(v) = f_A(a)$.

(2). Suppose $a, b \in S$. Let $\alpha \in a \circ b$ and $\beta \in b \circ a$. Then we have $\bigcap_{\alpha \in a \circ b} f_A(\alpha) = \bigcap_{\beta \in b \circ a} f_A(\beta)$. Indeed: By (1) we have $f_A(\alpha) = \bigcap_{u \in \alpha \circ \alpha} f_A(u) \supseteq \bigcap_{u \in a \circ b \circ a \circ b} f_A(u) = \bigcap_{u \in a \circ (b \circ a) \circ b} f_A(u) = \bigcap_{\substack{u \in a \circ \beta \circ b \\ \beta \in b \circ a}} f_A(u) \supseteq f_A(\beta) \supseteq \bigcap_{\beta \in b \circ a} f_A(\beta)$. It follows that $\bigcap_{\alpha \in a \circ b} f_A(\alpha) \supseteq \bigcap_{\beta \in b \circ a} f_A(\beta)$. By symmetry it can be shown that $\bigcap_{\beta \in b \circ a} f_A(\beta) \supseteq \bigcap_{\alpha \in a \circ b} f_A(\alpha)$. Hence $\bigcap_{\alpha \in a \circ b} f_A(\alpha) = \bigcap_{\beta \in b \circ a} f_A(\beta)$.

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