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TWIN HYPERCUBE FOR INTUITIONISTIC FUZZY SETS AND THEIR APPLICATION IN MEDICINE

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Abstract. In this paper, we present a description of intuitionistic fuzzy sets as vectors in twin hypercube. Finally we verify an application of intuitionistic fuzzy sets in medicine using twin hypercube for intuitionistic fuzzy sets.

1. Introduction

The concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. The intuitionistic fuzzy sets have some applications in different sciences. In the papers [6,9,12,13] has been developed the concept of applications of intuitionistic fuzzy sets in medical diagnosis using from relation between intuitionistic fuzzy sets and symptoms. Also this paper is a generalization of the paper [11] to intuitionistic fuzzy sets. We present some applications of intuitionistic fuzzy sets in diagnosis. Actually by employing twin hypercube for intuitionistic fuzzy sets we determine the kind of patient.

This paper is organized as follows: In first section we recall basic facts about intuitionistic fuzzy sets. In second section we present twin hypercube. In third section we describe distance and entropy of intuitionistic fuzzy sets and fourth section we present intuitionistic fuzzy segments and intuitionistic fuzzy midpoints. In the final section we study an application of intuitionistic fuzzy sets in medicine.

2. Basic facts about intuitionistic fuzzy sets

After the introduction of fuzzy sets by Zadeh [16], several researches consider some generalization of fuzzy sets. As an important generalization of the notion of fuzzy sets on a non-empty set X, Atanassov introduced in [1,3], the concept of intuitionistic fuzzy sets defined on a non-empty set X as an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{2.1}$$

where the functions

$$\mu_A : X \to [0,1], \quad \nu_A : X \to [0,1],$$

denote the degree of membership and the degree of non-membership of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$. If $\nu_A(x) = 1 - \mu_A(x)$, then A is a classical fuzzy set. Such defined objects are studied by many authors and have many interesting applications not only in mathematics (see [5]).

Let A and B be two intuitionistic fuzzy sets in X. Then, the following expressions are defined in [1,3].

(1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$,
(2) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(3) $A^C = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
(4) $A \cap B = \{\langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}\rangle : x \in X\}$,
(5) $A \cup B = \{\langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}\rangle : x \in X\}$,
(6) $\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$,
(7) $\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle : x \in X\}$.

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We recall the following two examples from [7].

Example 2.1. *Consider the universe* $X = \{10, 100, 500, 1000, 1200\}$. *An intutionistic fuzzy set "Large" of* X *denoted by* L *and may be defined by*

$$\begin{aligned} L &= \{\langle 10,\ 0.01,\ 0.9\rangle,\ \langle 100,\ 0.1,\ 0.88\rangle,\ \langle 500,\ 0.4,\ 0.5\rangle, \\ &\quad \langle 1000,\ 0.8,\ 0.1\rangle,\ \langle 1200,\ 1,\ 0\rangle\}. \end{aligned}$$

One may define an intuitionistic fuzzy set "Very Large" (denoted by VL*) as follows:*

$$\mu_{VL}(x) = (\mu_L(x))^2 \quad \text{and} \quad \nu_{VL}(x) = 1 - (1 - \nu_L(x))^2,$$

for all $x \in X$. *Thus,*

$$\begin{aligned} VL &= \{\langle 10,\ 0.0001,\ 0.99\rangle,\ \langle 100,\ 0.01,\ 0.9856\rangle,\ \langle 500,\ 0.16,\ 0.75\rangle, \\ &\quad \langle 1000,\ 0.64,\ 0.19\rangle,\ \langle 1200,\ 1,\ 0\rangle\}. \end{aligned}$$

Example 2.2. *Consider the universe* $\{a_1, a_2, a_3, a_4, a_5, a_6\}$. *Let* A *and* B *be two intuitionistic fuzzy sets of* X *given by*

$$\begin{aligned} A &= \{\langle a_1,\ 0.2,\ 0.6\rangle,\ \langle a_2,\ 0.3,\ 0.7\rangle,\ \langle a_3,\ 1,\ 0\rangle, \\ &\quad \langle a_4,\ 0.8,\ 0.1\rangle,\ \langle a_5,\ 0.5,\ 0.4\rangle\} \end{aligned}$$

and

$$\begin{aligned} B &= \{\langle a_1,\ 0.4,\ 0.4\rangle,\ \langle a_2,\ 0.5,\ 0.2\rangle,\ \langle a_3,\ 0.6,\ 0.2, \\ &\quad \langle a_4,\ 0.1,\ 0.7\rangle,\ \langle a_5,\ 0,\ 1\rangle\}. \end{aligned}$$

Then,

$$\begin{aligned} A^c &= \{\langle a_1,\ 0.6,\ 0.2\rangle,\ \langle a_2,\ 0.7,\ 0.3\rangle,\ \langle a_3,\ 0,\ 1\rangle, \\ &\quad \langle a_4,\ 0.1,\ 0.8\rangle,\ \langle a_5,\ 0.4,\ 0.5\rangle\}, \\ A \cap B &= \{\langle a_1,\ 0.2,\ 0.6\rangle,\ \langle a_2,\ 0.3,\ 0.7\rangle,\ \langle a_3,\ 0.6,\ 0.2\rangle, \\ &\quad \langle a_4,\ 0.1,\ 0.7\rangle,\ \langle a_5,\ 0,\ 1\rangle\}, \\ A \cup B &= \{\langle a_1,\ 0.4,\ 0.4\rangle,\ \langle a_2,\ 0.5,\ 0.2\rangle,\ \langle a_3,\ 1,\ 0\rangle, \\ &\quad \langle a_4,\ 0.8,\ 0.1\rangle,\ \langle a_5,\ 0.5,\ 0.4\rangle\}, \\ \Box A &= \{\langle a_1,\ 0.2,\ 0.8\rangle,\ \langle a_2,\ 0.3,\ 0.7\rangle,\ \langle a_3,\ 1,\ 0\rangle, \\ &\quad \langle a_4,\ 0.8,\ 0.2\rangle,\ \langle a_5,\ 0.5,\ 0.5\rangle\}, \\ \Diamond B &= \{\langle a_1,\ 0.6,\ 0.4\rangle,\ \langle a_2,\ 0.8,\ 0.2\rangle,\ \langle a_3,\ 0.8,\ 0.2\rangle, \\ &\quad \langle a_4,\ 0.3,\ 0.7\rangle,\ \langle a_5,\ 0,\ 1\rangle\}. \end{aligned}$$

For each intuitionistic fuzzy set in X, *we call*

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x),$$

the intuitionistic index of x *in* A. *It is a hesitancy degree of* x *to* A *[2, 4].*

3. Twin hypercube

Similar to Kosko [10], we introduce a geometrical interpretation of intuitionistic fuzzy sets as vectors in twin hypercube. Indeed, for a given set $X = \{x_1, \ldots, x_n\}$, an intuitionistic fuzzy set A is determined just by two mappings $\mu_A : X \longrightarrow I = [0, 1]$ and $\nu_A : X \longrightarrow I = [0, 1]$, and the values $\mu_A(x)$ and $\nu_A(x)$ giving the degree of membership and the degree of non-membership of each element $x \in X$ to the intuitionistic fuzzy set A, respectively. Thus, the set of all intutionistic fuzzy sets in X is determined precisely by two unit hypercubes. One of them is the unit hypercube of membership degree and another one is the unit hypercube of non-membership degree.

Any intuitionistic fuzzy set A determines a vector

$$P \in I^n \times I^n$$

given by

$$\langle (\mu_a(x_1), \ldots, \mu_A(x_n), (\nu_A(x_1), \ldots, \nu_A(x_n)\rangle.$$

Conversely, any vector $Q = \langle (a_1, \ldots, a_n), (b_1, \ldots, b_n)\rangle$ generates an intuitionistic fuzzy set A defined by

$$\mu_A(x_i) = a_1 \text{ and } \nu_A(x_i) = b_1, \text{ for } i = 1, \ldots, n.$$

Cardinality of intuitionistic fuzzy sets [15]. Let A be an intuitionistic fuzzy set in X. First, we consider the following two cardinalities of an intuitionistic fuzzy set.

(1) The least cardinality of A is equal to the so-called sigma-count (cf. [17, 18]), and is called the $\min \sum Count$ (min-sigma-count):

$$\min \sum Count(A) = \sum_{i=1}^{n} \mu_A(x_i).$$

(2) The biggest cardinality of A, which is possible due to π_A, is called the $\max \sum Count$ (max-sigma-count), and is equal to

$$\max \sum Count(A) = \sum_{i=1}^{n} (\mu_A(x_i) + \pi_A(x_i)).$$

Clearly, for A^C, we have

$$\min \sum Count(A^c) = \sum_{i=1}^{n} \nu_A(x_i),$$
$$\max \sum Count(A^c) = \sum_{i=1}^{n} (\nu_A(x_i) + \pi_A(x_i)).$$

Now, the cardinality of an intuitionistic fuzzy set A is defined as the interval

$$Card(A) = |A| = \Big[\min \sum Count(A), \max \sum Count(A)\Big].$$

Therefore, we have

$$|A| = \Big[\sum_{i=1}^{n} \mu_A(x_i), \sum_{i=1}^{n} (1 - \nu_A(x_i))\Big]. \tag{3.1}$$

Theorem 3.1. *If A and B are two intuitionistic fuzzy sets in X, then*

$$|A \cup B| = |A| + |B| - |A \cap B|. \tag{3.2}$$

Proof. According to Eq. (3.1), we have

$$\begin{aligned}
|A \cup B| &= \Big[\sum_{i=1}^{n} \mu_{A\cup B}(x_i), \sum_{i=1}^{n} (1 - \nu_{A\cup B}(x_i))\Big] \\
&= \Big[\sum_{i=1}^{n} \max\{\mu_A(x_i), \mu_B(x_i)\}, \sum_{i=1}^{n} (1 - \max\{\nu_A(x_i), \nu_B(x_i)\})\Big] \\
&= \Big[\sum_{i=1}^{n} \max\{\mu_A(x_i), \mu_B(x_i)\}, \sum_{i=1}^{n} \min\{1 - \nu_A(x_i), 1 - \nu_B(x_i)\}\Big],
\end{aligned}$$

$$\begin{aligned}
|A \cap B| &= \Big[\sum_{i=1}^{n} \mu_{A\cap B}(x_i), \sum_{i=1}^{n} (1 - \nu_{A\cap B}(x_i))\Big] \\
&= \Big[\sum_{i=1}^{n} \min\{\mu_A(x_i), \mu_B(x_i)\}, \sum_{i=1}^{n} (1 - \min\{\nu_A(x_i), \nu_B(x_i)\})\Big] \\
&= \Big[\sum_{i=1}^{n} \min\{\mu_A(x_i), \mu_B(x_i)\}, \sum_{i=1}^{n} \max\{1 - \nu_A(x_i), 1 - \nu_B(x_i)\}\Big].
\end{aligned}$$

Therefore, we obtain

$$\begin{aligned}
|A \cup B| + |A \cap B| &= \Big[\sum_{i=1}^{n} \Big(\mu_A(x_i) + \mu_B(x_i)\Big), \Big((1 - \nu_A(x_i)) + (1 - \nu_B(x_i))\Big)\Big] \\
&= \Big[\sum_{i=1}^{n} \mu_A(x_i), \sum_{i=1}^{n} (1 - \nu_A(x_i))\Big] + \Big[\sum_{i=1}^{n} \mu_B(x_i), \sum_{i=1}^{n} (1 - \nu_B(x_i))\Big] \\
&= |A| + |B|.
\end{aligned}$$

□

Remark 3.1. *When $i = 1$, for simplicity we use the symbol* $\max Count(A)$ *insteas of* $\max \sum Count(A)$.

4. Distance and entropy of intuitionistic fuzzy sets

The above discussion allows us to consider the following distance between intutionistic fuzzy sets. For two intutionistic fuzzy sets A and B, The Hamming distance [14,15] between A and B are given by

$$d(A,B)=\sum_{i=1}^{n}\Big(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|\Big). \tag{4.1}$$

Each summand in (4) is between 0 and 1. For A,B,C intuitionistic fuzzy sets, we have:

(1) $0\le d(A,B)\le n$,
(2) $d(A,B)=0$ if and only if $A=B$,
(3) $d(A,B)=d(B,A)$,
(4) $d(A,B)\le d(A,C)+d(C,B)$.

A geometric interpretation of intutionistic fuzzy sets is presented in [15]. An intuitionistic fuzzy set is represented by a point in the triangle $A'B'D'$, where $A'(1,0,0)$, $B'(1,0,0)$ and $D'(0,0,1)$. An intutionistic fuzzy set A in X is mapped into triangle $A'B'D'$ in that each element of X corresponds to an element of $A'B'D'$, as an example, a point $x'\in A'B'D'$ corresponding to $x\in X$ is marked the values of $\mu_A(x)$, $\nu_A(x)$ and $\pi_A(x)$.

Let E be a fuzzy set defined on fuzzy sets. Then, E is an entropy measure if it satisfies the four De Luca and Termini axioms [8].

According to a geometric representation of an intuitionistic fuzzy set [15], we have

$$E(A)=\frac{a}{b}, \tag{4.2}$$

where a is the distance (A,A_{near}) from A to the nearest point A_{near} among A' and B', and b is the distance (A,A_{far}) from A to the fareast point A_{far} among A' and B'. The Eq. (4.2) describes the degree of fuzziness for a single point belonging to an intuitionistic fuzzy set. For n points belonging to an intuitionistic fuzzy set we have

$$E(A)=\frac{1}{n}\sum_{i=1}^{n}E(A_i). \tag{4.3}$$

Theorem 4.1. *[15] A generalized entropy measure of an intuitionistic fuzzy set A in X is*

$$\begin{aligned}
E(A) &= \frac{1}{n}\sum_{i=1}^{n}\Big(\frac{\max Count(A_i\cap A_i^c)}{\max Count(A_i\cup A_i^c)}\Big)=\frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}\mu_{A_i\cap A_i^c}(x_j)+\pi_{A_i\cap A_i^c}(x_j)}{\sum_{j=1}^{n}\mu_{A_i\cup A_i^c}(x_j)+\pi_{A_i\cup A_i^c}(x_j)}\\
&= \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}\min\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}+1-\mu_{A_i\cap A_i^c}-\nu_{A_i\cap A_i^c}}{\sum_{j=1}^{n}\max\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}+1-\mu_{A_i\cup A_i^c}-\nu_{A_i\cup A_i^c}}\\
&= \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}\min\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}+1-2\min\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}}{\sum_{j=1}^{n}\max\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}+1-2\max\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}}\\
&= \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{n}-\min\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}+1}{\sum_{j=1}^{n}-\max\{\mu_{A_i}(x_j),\nu_{A_i}(x_j)\}+1}.
\end{aligned}$$

where A_i denotes the single element intutionistic fuzzy set corresponding to the i^{th} element of the universe X. In other words, A_i is the i^{th} component of A.

For two intuitionistic fuzzy subsets A and B, it is possible to define a degree of subsethood

$$S(A,B) = \frac{|A \cap B|}{|A|}.$$

For the midpoint M, since $\mu_M(x_j) = \nu_M(x_j) = \frac{1}{2}$, we have

$$E(M) = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j=1}^{n} \frac{1}{2}}{\sum_{j=1}^{n} \frac{1}{2}} = \frac{1}{n}\sum_{i=1}^{n} \frac{\frac{n}{2}}{\frac{n}{2}} = \frac{1}{n}\sum_{i=1}^{n} 1 = \frac{n}{n} = 1.$$

Since for any intutionistic fuzzy subset $A \neq M$, we have $E(A) < 1$, therefore the midpoint M is maximally intutionistic fuzzy set. Now we present the following definition.

The intuitionistic fuzzy subset C is between intuitionistic fuzzy subsets A and B, if $d(A,C) + d(C,B) = d(A,B)$.

The metric segment between A and B is defined as the following:

$$segment(A,B) = \{C : d(A,C) + d(C,B) = d(A,B)\},$$

that is,

$$\sum_{i=1}^{n} |\mu_A(x_i) - \mu_C(x_i)| + |\mu_C(x_i) - \mu_B(x_i)| = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|,$$
$$\sum_{i=1}^{n} |\nu_A(x_i) - \nu_C(x_i)| + |\nu_C(x_i) - \nu_B(x_i)| = \sum_{i=1}^{n} |\nu_A(x_i) - \nu_B(x_i)|.$$

Another concept is the idea of equidistant points:

$$equid(A,B) = \{C : d(A,C) = d(C,B)\},$$

that is,

$$\sum_{i=1}^{n} |\mu_A(x_i) - \mu_C(x_i)| = \sum_{i=1}^{n} |\mu_C(x_i) - \mu_B(x_i)|,$$
$$\sum_{i=1}^{n} |\nu_A(x_i) - \nu_C(x_i)| = \sum_{i=1}^{n} |\nu_C(x_i) - \nu_B(x_i)|.$$

If $C \in segment(A,B)$, and it is equidistant to A and B, then

$$d(A,C) = d(C,B) = \frac{1}{2}d(A,B).$$

Therefore we introduce the set of midpoints between A and B:

$$mid(A,B) = \{C : d(A,C) = d(C,B) = \frac{1}{2}d(A,B)\},$$

that is,

$$\sum_{i=1}^{n} |\mu_A(x_i) - \mu_C(x_i)| = \sum_{i=1}^{n} |\mu_C(x_i) - \mu_B(x_i)| = \frac{1}{2}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|,$$
$$\sum_{i=1}^{n} |\nu_A(x_i) - \nu_C(x_i)| = \sum_{i=1}^{n} |\nu_C(x_i) - \nu_B(x_i)| = \frac{1}{2}\sum_{i=1}^{n} |\nu_A(x_i) - \nu_B(x_i)|.$$

Obviously for any two intuitionistic fuzzy subsets A and B we have, $mid(A,B) \subseteq equid(A,B)$.

The canonical midpoint A and B is denoted by

$$\mu_C(x_i) = \frac{1}{2}(\mu_A(x_i) + \mu_B(x_i)), \quad \nu_C(x_i) = \frac{1}{2}(\nu_A(x_i) + \nu_B(x_i)), \quad i = 1, 2, \dots, n.$$

This canonical midpoint is not the unique midpoint and there are more midpoints. Actually we show C is a midpoint between A and B:

$$\sum_{i=1}^{n} |\mu_A(x_i) - \frac{1}{2}(\mu_A(x_i) + \mu_B(x_i))| = \sum_{i=1}^{n} \frac{1}{2}|\mu_A(x_i) - \mu_B(x_i)|,$$

$$\sum_{i=1}^{n} |\nu_A(x_i) - \frac{1}{2}(\nu_A(x_i) + \nu_B(x_i))| = \sum_{i=1}^{n} \frac{1}{2}|\nu_A(x_i) - \nu_B(x_i)|.$$

Therefore $d(A, C) = \frac{1}{2}d(A, B)$ and similarly $d(C, B) = \frac{1}{2}d(A, B)$, then $d(A, C) = d(C, B) = \frac{1}{2}d(A, B)$.

If A is a crisp subset of X, that is, A is a vertex of the hypercube, then $\mu_A : X \to \{0, 1\}$, $\nu_A : X \to \{0, 1\}$, therefore

$$\begin{aligned}
d(A, M) &= \sum_{i=1}^{n} |\mu_A(x_i) - \frac{1}{2}| + |\nu_A(x_i) - \frac{1}{2}| + |\pi_A(x_i) - (1 - \mu_M(x_i) - \nu_M(x_i))| \\
&= \sum_{i=1}^{n} \frac{1}{2} + \frac{1}{2} = \sum_{i=1}^{n} 1 = n, \ \textit{and} \\
d(A, A^c) &= \sum_{i=1}^{n} |\mu_A(x_i) - \mu_{A^c}(x_i)| + |\nu_A(x_i) - \nu_{A^c}(x_i)| + |\pi_A(x_i) - \pi_{A^c}(x_i)| \\
&= \sum_{i=1}^{n} |\mu_A(x_i) - \nu_A(x_i)| + |\nu_A(x_i) - \mu_A(x_i)| + |\pi_A(x_i) - (1 - \nu_A(x_i) - \mu_A(x_i))| \\
&= \sum_{i=1}^{n} 2|\mu_A(x_i) - \nu_A(x_i)| = \sum_{i=1}^{n} 2 = 2n.
\end{aligned}$$

Similarly $d(A^c, M) = n$, and then

$$d(A, M) = d(A^c, M) = \frac{1}{2}d(A, A^c).$$

Hence

$$\sum_{i=1}^{n} |\mu_A(x_i) - \frac{1}{2}| = \frac{1}{2}\sum_{i=1}^{n} |\mu_A(x_i) - \mu_{A^c}(x_i)| = \frac{n}{2},$$

$$\sum_{i=1}^{n} |\nu_A(x_i) - \frac{1}{2}| = \frac{1}{2}\sum_{i=1}^{n} |\nu_A(x_i) - \nu_{A^c}(x_i)| = \frac{n}{2}.$$

Since $d(A, C) = d(C, B) = \frac{1}{2}d(A, B)$, it follows that $d(A, C) + d(C, B) = \frac{1}{2}d(A, B)$ and so $mid(A, B) \subset segment(A, B)$. Hence, $C \in segment(A, B)$. In the bellow example we describe the definitions

Example 4.1. *Let $X = \{x_1, x_2\}$, $\mu_A = \{0, 0\}, \nu_A = \{1, 1\}, \mu_B = \{1, 1\}, \nu_B = \{0, 0\}$. Then, we have*

$$\frac{\mu_A + \mu_B}{2} = (\frac{1}{2}, \frac{1}{2}), \quad \frac{\nu_A + \nu_B}{2} = (\frac{1}{2}, \frac{1}{2}) \in mid(A, B).$$

If $\mu_C = (c_1, c_2), \nu_C = (c_3, c_4)$ is another midpoint between A and B then:

$d(A, C) = c_1 + c_2 + (1 - c_3) + (1 - c_4)$,

$d(A, B) = \sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| = \sum_{i=1}^{n}(2 + 2) = 4$,

$d(C, B) = 1 - c_1 + 1 - c_2 + c_3 + c_4 \Rightarrow c_1 + c_2 = 1, c_3 + c_4 = 1$.

Therefore

- $d(A, C) = \frac{1}{2}d(A, B) \Rightarrow 1 - c_1 + 1 - c_2 = 1 \Rightarrow c_1 + c_2 = 1$,

- $d(C,B) = \frac{1}{2}d(A,B) \Rightarrow c_3 + c_4 = 1,$

so

$$mid(A,B) = \{C : (c_1, c_2), (c_3, c_4) | c_1 + c_2 = 1, c_3 + c_4 = 1\},$$

and we have an infinite set of midpoints.

If $C \in equid(A,B)$*, then* $c_1 + c_2 = (1 - c_1) + (1 - c_2), c_3 + c_4 = (1 - c_3) + (1 - c_4)$*. Therefore* $c_1 + c_2 = 1, c_3 + c_4 = 1$*. Then* $mid(A,B) = equid(A,B)$*. As we can see these sets coincide. Suppose that* $\mu_C = (\frac{1}{2}, \frac{1}{2})$*,* $\nu_C = (\frac{1}{2}, \frac{1}{2})$*. Then,* $d(A,B) = 2, d(A,C) = 2, d(C,B) = 2$*. Thus,* $C \in equid(A,B)$ *but* $C \notin segment(A,B)$*, since* $d(A,C) + d(C,B) \neq d(A,B)$*, therefore* $equid(A,B) \nsubseteq segment(A,B)$

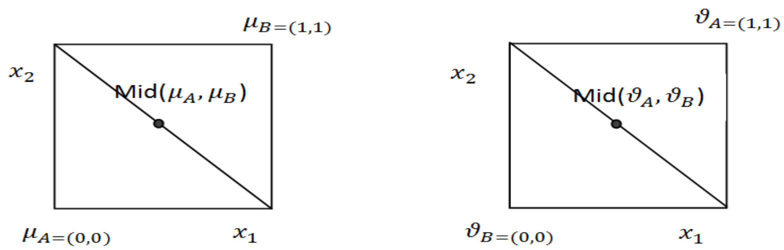

FIGURE 1. Midpoints between $\mu_A = (0,0)$, $\nu_A = (1,1)$ and $\mu_B = (1,1)$, $\nu_B = (0,0)$

Example 4.2. *Now consider* $\mu_A = (0,0)$*,* $\nu_A = (1,1)$*,* $\mu_B = (1,0)$*,* $\nu_B = (0,1)$*, then* $d(A,B) = 2$*,* $C \in mid(A,B)$ *if and only if*

- $c_1 + c_2 = 1 - c_1 + c_2 = 1 \Rightarrow c_1 = \frac{1}{2}, c_2 = \frac{1}{2},$
- $1 - c_3 + 1 - c_4 = c_3 + 1 - c_4 = 1 \Rightarrow c_3 = \frac{1}{2}, c_4 = \frac{1}{2},$

so $mid(A,B) = \Big\{(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})\Big\}$*. Also,* $C \in equid(A,B)$ *if and only if*

- $c_1 + c_2 = 1 - c_1 + c_2 \Rightarrow c_1 = \frac{1}{2},$
- $1 - c_3 + 1 - c_4 = c_3 + 1 - c_4 \Rightarrow c_3 = \frac{1}{2},$

then $equid(A,B) = \Big\{(\frac{1}{2}, c_2), (\frac{1}{2}, c_4)\Big\}$*, hence* $mid(A,B) \subset equid(A,B)$*.*

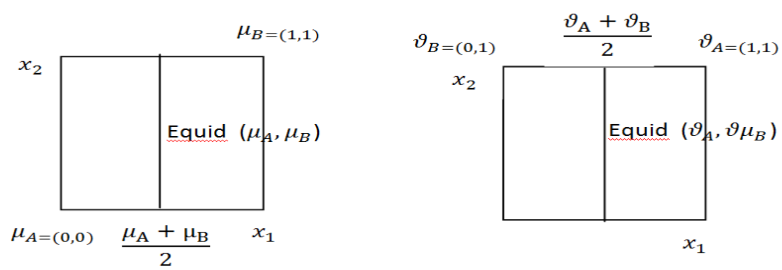

FIGURE 2. Equidistant points between $\mu_A = (0,0)$, $\nu_A = (1,1)$ and $\mu_B = (1,0)$, $\nu_B = (0,1)$

5. Intuitionistic fuzzy segments and intuitionistic fuzzy midpoints

In this section, first we verify the segment between two points in a square and determine the set of midpoints between A and B. Let $\mu_A = (a_1, a_2), \nu_A = (a_3, a_4), \mu_B = (b_1, b_2), \nu_B = (b_3, b_4)$ be intuitionistic fuzzy subsets of the set $X = \{x_1, x_2\}$. If $\mu_C = (c_1, c_2), \nu_C = (c_3, c_4) \in segment(A, B)$, then:

$$|a_1 - c_1| + |a_2 - c_2| + |b_1 - c_1| + |b_2 - c_2| = |a_1 - b_1| + |a_2 - b_2|, \tag{5.1}$$
$$|a_3 - c_3| + |a_4 - c_4| + |b_3 - c_3| + |b_4 - c_4| = |a_3 - b_3| + |a_4 - b_4|. \tag{5.2}$$

So, we obtain $\min\{a_i, b_i\} \leq c_i \leq \max\{a_i, b_i\}, i = 1, 2, 3, 4$.

If $\min\{a_i, b_i\} > c_i$, then

- $|a_1 - c_1| + |b_1 - c_1| > |b_1 - a_1|, \quad |a_2 - c_2| + |b_2 - c_2| > |b_2 - a_2|$, so $|a_1 - c_1| + |a_2 - c_2| + |b_1 - c_1| + |b_2 - c_2| > |b_1 - a_1| + |b_2 - a_2|$ that is contradiction with 5.1, also
- $|a_3 - c_3| + |b_3 - c_3| > |b_3 - a_3|, \quad |a_4 - c_4| + |b_4 - c_4| > |b_4 - a_4|$, therefore $|a_3 - c_3| + |a_4 - c_4| + |b_3 - c_3| + |b_4 - c_4| > |b_3 - a_3| + |b_4 - a_4|$, this relation is contradiction with 5.2.

Theorem 5.1. *The points in the segment between A and B is as follows:*

$$segment(A, B) = \{(c_1, c_2), (c_3, c_4) | \min\{a_i, b_i\} \leq c_i \leq \max\{a_i, b_i\}, i = 1, 2, 3, 4\}.$$

Proof: Suppose that $a_1 \leq b_1$ and $a_3 \geq b_3$ (if $a_1 > b_1$ and $a_3 \leq b_3$ we interchange the roles of μ_A, μ_B and ν_A, ν_B). Now, there are four possibilities, if $\mu_C = (c_1, c_2), \nu_C = (c_3, c_4)$ then:

(1) $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \geq b_3,\ a_4 \leq b_4 \Rightarrow a_1 < c_1 < b_1,\ a_2 < c_2 < b_2,\ b_3 < c_3 < a_3,\ a_4 < c_4 < b_4$,
(2) $a_1 \leq b_1,\ a_2 > b_2,\ a_3 \geq b_3,\ a_4 \leq b_4 \Rightarrow a_1 < c_1 < b_1,\ b_2 < c_2 < a_2,\ b_3 < c_3 < a_3,\ a_4 < c_4 < b_4$,
(3) $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \geq b_3,\ a_4 > b_4 \Rightarrow a_1 < c_1 < b_1,\ a_2 < c_2 < b_2,\ b_3 < c_3 < a_3,\ b_4 < c_4 < a_4$,
(4) $a_1 \leq b_1,\ a_2 > b_2,\ a_3 \geq b_3,\ a_4 > b_4 \Rightarrow a_1 < c_1 < b_1,\ b_2 < c_2 < a_2,\ b_3 < c_3 < a_3,\ b_4 < c_4 < a_4$.

Suppose that $\min\{a_i, b_i\} \leq c_i \leq \max\{a_i, b_i\}$ and we show $C \in segment(A, B)$ that is: $d(A, C) + d(C, B) = d(A, B)$.

Proof: 1. If $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \geq b_3,\ a_4 \leq b_4$, then

$$\begin{aligned} d(A, C) &= c_1 - a_1 + c_2 - a_2 + a_3 - c_3 + c_4 - a_4, \\ d(C, B) &= b_1 - c_1 + b_2 - c_2 + c_3 - b_3 + b_4 - c_4, \\ d(A, C) &= b_1 - a_1 + b_2 - a_2 + a_3 - b_3 + b_4 - a_4. \end{aligned}$$

Therefore $d(A, B) = d(A, C) + d(C, B)$. Similarly in all of cases 2,3,4, $d(A, B) = d(A, C) + d(C, B)$.

Theorem 5.2. *The set of midpoints between A and B has the following possibilities:*

(1) *If either $a_1 = b_1,\ a_3 = b_3$ or $a_2 = b_2,\ a_4 = b_4$, then there is a midpoint given by $mid(A, B) = \{\frac{\mu_A + \mu_B}{2}, \frac{\nu_A + \nu_B}{2}\}$.*
(2) *If $a_1 < b_1,\ a_2 < b_2,\ a_3 > b_3$ and $a_4 < b_4$, then the set of midpoints is as follows. If $\mu_C = (c_1, c_2),\ \nu_C = (c_3, c_4)$, then:*
   - $c_1 + c_2 = \frac{1}{2}(a_1 + a_2 + b_1 + b_2)$,
   - $c_3 + c_4 = \frac{1}{2}(a_3 + a_4 + b_3 + b_4)$,
(3) *If $a_1 < b_1,\ a_2 > b_2,\ a_3 > b_3,\ a_4 > b_4$, then:*
   - $c_1 - c_2 = \frac{1}{2}(a_1 - a_2 + b_1 - b_2)$,
   - $c_3 - c_4 = \frac{1}{2}(a_3 - a_4 + b_3 - b_4)$,
(4) *If $a_1 < b_1,\ a_2 < b_2,\ a_3 > b_3,\ a_4 > b_4$, then:*
   - $c_1 + c_2 = \frac{1}{2}(a_1 + a_2 + b_1 + b_2)$,

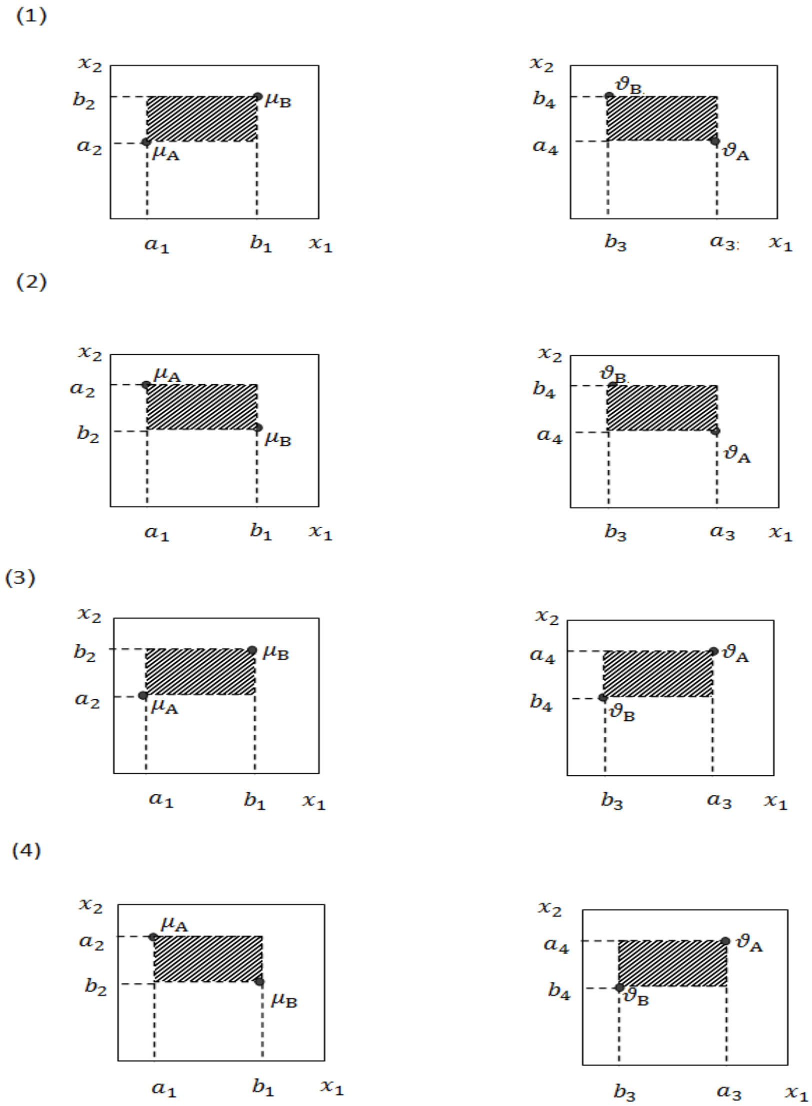

FIGURE 3. Segment between A and B: 1. $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \geq b_3,\ a_4 \leq b_4$, 2. $a_1 \leq b_1,\ a_2 > b_2,\ a_3 \geq b_3,\ a_4 \leq b_4$, 3. $a_1 \leq b_1,\ a_2 \leq b_2,\ a_3 \geq b_3,\ a_4 > b_4$, 4. $a_1 \leq b_1,\ a_2 > b_2,\ a_3 \geq b_3,\ a_4 > b_4$.

- $c_3 - c_4 = \frac{1}{2}(a_3 - a_4 + b_3 - b_4)$,

(5) *If* $a_1 < b_1,\ a_2 > b_2,\ a_3 > b_3,\ a_4 < b_4$, *then:*
- $c_1 - c_2 = \frac{1}{2}(a_1 - a_2 + b_1 - b_2)$,
- $c_3 + c_4 = \frac{1}{2}(a_3 + a_4 + b_3 + b_4)$,

Proof: If $C \in mid(A, B)$, then $d(A, C) = d(C, B) = \frac{1}{2}d(A, B)$. So, we have:

- $d(A, C) = d(C, B)$, therefore:
  $|a_1 - c_1| + |a_2 - c_2| + |a_3 - c_3| + |a_4 - c_4| = |b_1 - c_1| + |b_2 - c_2| + |b_3 - c_3| + |b_4 - c_4|$,

- $d(A,C) = \frac{1}{2}d(A,B)$, then we have:
  $|a_1 - c_1| + |a_2 - c_2| + |a_3 - c_3| + |a_4 - c_4| = \frac{1}{2}(|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3| + |a_4 - b_4|)$,
- $d(C,B) = \frac{1}{2}d(A,B)$, so we conclude:
  $|b_1 - c_1| + |b_2 - c_2| + |b_3 - c_3| + |b_4 - c_4| = \frac{1}{2}(|a_1 - b_1| + |a_2 - b_2| + |a_3 - b_3| + |a_4 - b_4|)$,

such that

$$|a_1 - c_1| + |a_2 - c_2| = |b_1 - c_1| + |b_2 - c_2| = \frac{1}{2}(|a_1 - b_1| + |a_2 - b_2|), \quad (5.3)$$

$$|a_3 - c_3| + |a_4 - c_4| = |b_3 - c_3| + |b_4 - c_4| = \frac{1}{2}(|a_3 - b_3| + |a_4 - b_4|). \quad (5.4)$$

(1) Suppose that $a_1 = b_1$ and $a_3 = b_3$. Then we have $c_1 = a_1 = b_1$ and $c_3 = a_3 = b_3$. By using the above equalities we obtain
  - $|a_2 - c_2| = |b_2 - c_2|$,
  - $|a_4 - c_4| = |b_4 - c_4| = \frac{1}{2}|a_4 - b_4|$.

  Then from the above equalities we conclude $c_2 = \frac{1}{2}(a_2 + b_2)$ and $c_4 = \frac{1}{2}(a_4 + b_4)$.
  Similarly if $a_2 = b_2$ and $a_4 = b_4$ then we have $c_2 = a_2 = b_2$ and $c_4 = a_4 = b_4$. Therefore, $c_1 = \frac{1}{2}(a_1 + b_1),\ c_3 = \frac{1}{2}(a_3 + b_3)$.

FIGURE 4. Unique midpoint between A and B.

(2) Now, suppose that $a_1 < b_1,\ a_2 < b_2,\ a_3 > b_3,\ a_4 < b_4$. Then, by using 5.3 we find:
  - $c_1 - a_1 + c_2 - a_2 = b_1 - c_1 + b_2 - c_2$,
  - $c_1 - a_1 + c_2 - a_2 = \frac{1}{2}(b_1 - a_1 + b_2 - a_2)$,
  - $b_1 - c_1 + b_2 - c_2 = \frac{1}{2}(b_1 - a_1 + b_2 - a_2)$.

  So, $c_1 + c_2 = \frac{1}{2}(a_1 + a_2 + b_1 + b_2)$, also by 5.3 we have:
  - $c_3 - a_3 + c_4 - a_4 = b_3 - c_3 + b_4 - c_4$,
  - $c_3 - a_3 + c_4 - a_4 = \frac{1}{2}(a_3 - b_3 + b_4 - a_4)$,
  - $b_3 - c_3 + b_4 - c_4 = \frac{1}{2}(a_3 - b_3 + b_4 - a_4)$.

  Therefore, we obtain $c_3 + c_4 = \frac{1}{2}(a_3 + a_4 + b_3 + b_4)$.

(3) If $a_1 < b_1,\ a_2 > b_2,\ a_3 > b_3,\ a_4 > b_4$, then by using 5.3 we obtain:
  - $c_1 - a_1 + a_2 - c_2 = b_1 - c_1 + c_2 - b_2$,
  - $c_1 - a_1 + a_2 - c_2 = \frac{1}{2}(b_1 - a_1 + a_2 - b_2)$,
  - $b_1 - c_1 + c_2 - b_2 = \frac{1}{2}(b_1 - a_1 + a_2 - b_2)$.

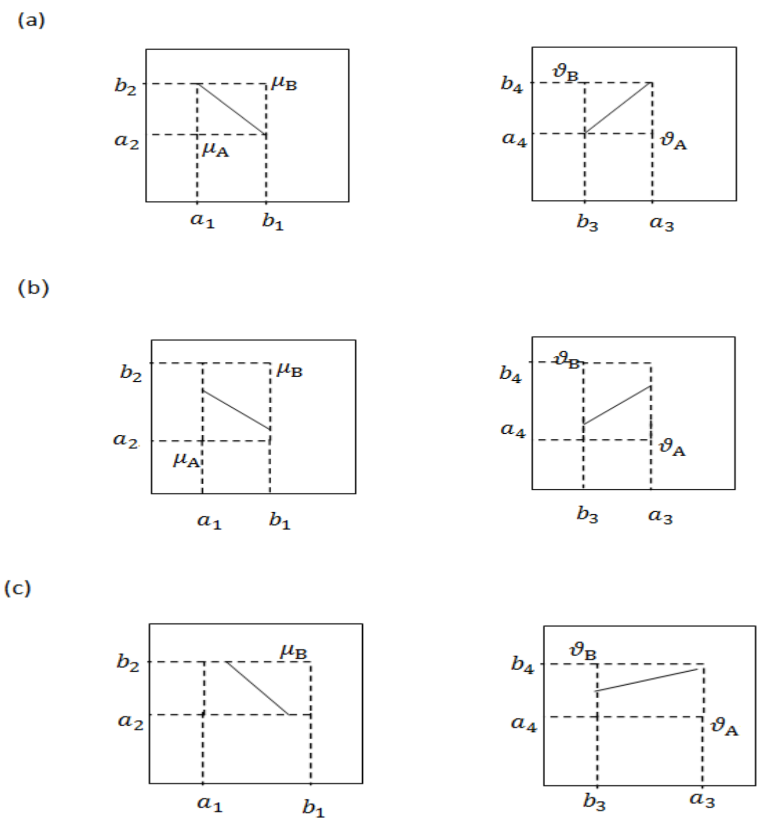

FIGURE 5. Midpoints between A and B: (a) $b_1 - a_1 = b_2 - a_2,\ b_3 - a_3 = b_4 - a_4,$ (b) $b_1 - a_1 < b_2 - a_2,\ a_3 - b_3 < b_4 - a_4,$ (c) $b_1 - a_1 > b_2 - a_2,\ a_3 - b_3 > b_4 - a_4$.

Thus, $c_1 - c_2 = \dfrac{1}{2}(a_1 - a_2 + b_1 - b_2)$, also by using the relation 5.4 we have:

- $c_3 - a_3 + a_4 - c_4 = b_3 - c_3 + c_4 - b_4$,
- $c_3 - a_3 + a_4 - c_4 = \dfrac{1}{2}(a_3 - b_3 + a_4 - b_4)$,
- $b_3 - c_3 + c_4 - b_4 = \dfrac{1}{2}(a_3 - b_3 + a_4 - b_4)$.

So, we conclude $c_3 - c_4 = \dfrac{1}{2}(a_3 - a_4 + b_3 - b_4)$.

(4) Let $a_1 < b_1,\ a_2 < b_2,\ a_3 \geq b_3,\ a_4 > b_4$. According to the case 2, if $a_1 < b_1,\ a_2 < b_2$, we have $c_1 + c_2 = \dfrac{1}{2}(a_1 + a_2 + b_1 + b_2)$.

Now, if $a_3 > b_3,\ a_4 > b_4$, we find $c_3 - c_4 = \dfrac{1}{2}(a_3 - a_4 + b_3 - b_4)$.

(5) Let $a_1 < b_1,\ a_2 > b_2,\ a_3 > b_3,\ a_4 < b_4$. According to the case 3, if $a_1 < b_1,\ a_2 > b_2$, we obtain $c_1 - c_2 = \dfrac{1}{2}(a_1 - a_2 + b_1 - b_2)$.

Also, if $a_3 > b_3,\ a_4 < b_4$, we have $c_3 + c_4 = \dfrac{1}{2}(a_3 + a_4 + b_3 + b_4)$.

Now in this section we determine the segment between two points μ_A, ν_A and μ_B, ν_B and the set of midpoints between μ_A, ν_A and μ_B, ν_B in a hypercube I^3.

As in the two-dimensional hypercube I^2, we find the following result.

Theorem 5.3. *The points in the segment between* $\mu_A = (a_1, a_2, a_3), \nu_A = (a_4, a_5, a_6)$ *and* $\mu_B = (b_1, b_2, b_3), \nu_B = (b_4, b_5, b_6)$ *are given by*

$$segment(A, B) = \{C = (c_1, c_2, c_3),\ (c_4, c_5, c_6) : \min\{a_i, b_i\} \leq c_i \leq \max\{a_i, b_i\},\ i = 1, 2, 3, 4, 5, 6\}.$$

Example 5.1. *Suppose the first variable* x_1 *depends on several blood tests, the second one* x_2 *on cardiac tests, and the third variable* x_3 *on some non-invasive vascular tests. For example, patient 1 is the point* $P_1 = (0.9, 0.2, 0.9),\ (0.1, 0.8, 0.1)$ *in twin three dimensional hypercube and patient 2 is* $P_2 = (0.7, 0.8, 0.5), (0.3, 0.2, 0.5)$*, patient 3 is* $P_3 = (0.8, 0.2, 0.9), (0.2, 0.8, 0.1)$*, then*

$$segment(P_1, P_2) = \{(c_1, c_2, c_3), (c_4, c_5, c_6) : 0.7 \leq c_1 \leq 0.9,\ 0.2 \leq c_2 \leq 0.8,$$
$$0.5 \leq c_3 \leq 0.9,\ 0.1 \leq c_4 \leq 0.3,\ 0.2 \leq c_5 \leq 0.8,\ 0.1 \leq c_6 \leq 0.5\}.$$

This segment is represented in grey in Figure 6. The point $P_3 = (0.8, 0.2, 0.9),\ (0.2, 0.8, 0.1)$ *belongs to the segment between* $P_1,\ P_2$*. Indeed* $d(P_1, P_2) = 2.4,\ d(P_1, P_2) = 0.2,\ d(P_3, P_2) = 2.2$ *and then* $d(P_1, P_2) = d(P_1, P_3) + d(P_3, P_2)$*.*

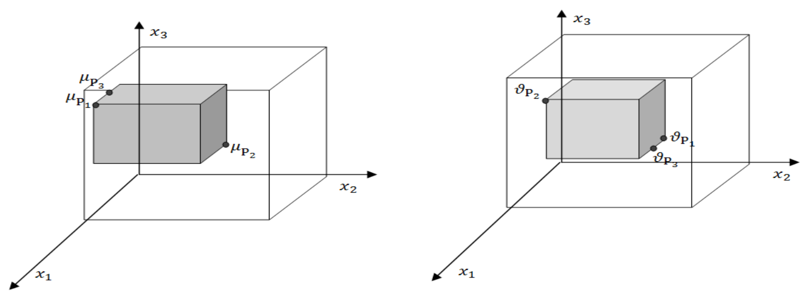

FIGURE 6. Segment between P_1 and P_2

Now, consider the intuitionistic fuzzy set $\mu_{P_1} = (0.7, 0.8, 0.5), \nu_{P_1} = (0.3, 0.2, 0.5)$ and $\mu_{P_2} = (0.9, 0.2, 0.9),\ \nu_{P_2} = (0.1, 0.8, 0.1)$. Let $P_3 = (c_1, c_2, c_3),\ (c_4, c_5, c_6)$ be midpoint between μ_{P_1}, ν_{P_1} and μ_{P_2}, ν_{P_2}. Then, $d(P_2, P_3) = \frac{1}{2} d(P_1, P_2)$. Therefore,

- $c_1 - 0.7 + 0.8 - c_2 + c_3 - 0.5 = 0.6,$
- $0.3 - c_4 + c_5 - 0.2 + 0.5 - c_6 = 0.6,$

also $d(P_1, P_3) = \frac{1}{2} d(P_1, P_2)$, so

- $0.9 - c_1 + c_2 - 0.2 + 0.9 - c_3 = 0.6,$
- $c_4 - 0.1 + 0.8 - c_5 + c_6 - 0.1 = 0.6,$

Therefore we obtain: $c_1 - c_2 + c_3 = 1, c_4 - c_5 + c_6 = 0$ such that $c_1, c_2, c_3, c_4, c_5, c_6$ have to satisfy $0.7 \leq c_1 \leq 0.9,\ 0.2 \leq c_2 \leq 0.8,\ 0.5 \leq c_3 \leq 0.9,\ 0.1 \leq c_4 \leq 0.3,\ 0.2 \leq c_5 \leq 0.8,\ 0.1 \leq c_6 \leq 0.5.$

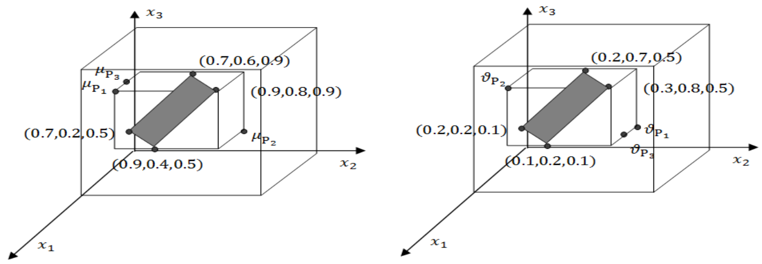

FIGURE 7. Shows the set of midpoints between μ_A, ν_A and μ_B, ν_B. Note that $P_3 = (0.8, 0.2, 0.9), (0.2, 0.8, 0.1) \in segment(A, B)$ but $P_3 \notin mid(A, B)$

6. An application of intuitionistic fuzzy sets in medicine

We now state a complete description of the medical problem. As in the paper [11], we consider the following intuitionistic fuzzy variables: smoking and alcohol drinking. Let $X = \{x_1, x_2\}$, for a nonsmoker we consider the degree of membership is 0 and the degree of nonmembership is 1, this point is the ideal situation for your health. Also for example if you smoke six cigarettes per day we say that your degree of a smoker is 0.8, and we suppose the degree of nonmembership is 0.2. If the consumption of cigarettes is 10 or more, the degree of membership is 1 and the degree of nonmembership is 0. Now for other intuitionistic fuzzy variables, if you drink no alcohol, the degree of membership is 0, and the the degree of nonmembership is 1. If you drink more than 75 cc of alcohol per day, the degree of alcoholism is 1 and the degree of non alcoholism is 0, for 25 cc per day, the degree of membership could be 0.4 and the degree of nonmembership is 0.6 and for 50 cc per day the degree of membership is 0.8 and the degree of nonmembership is 0.2.

Therefore the intuitionistic fuzzy set $\mu_A = (0, 0), \nu_A = (1, 1)$ corresponds to a non smoker and teetotaler, also the intuitionistic fuzzy set $\mu_B = (1, 0), \nu_B = (0, 1)$ showes a heavy smoker but a non teetotaler and the intuitionistic fuzzy set $\mu_C = (0.8, 1), \nu_C = (0.2, 0)$ represents that person smokes about six cigarettes per day and 75 cc of alcohol per day. According to the above text, $\mu_A = (0, 0), \nu_A = (1, 1)$ is the ideal situation for your health that is difficult to achieve. Also for $\mu_B = (1, 1), \nu_B = (0, 0)$ your physician has suggested you to reduce your consumption of cigarettes and alcohol by half, therefore you may achieve a midpoint between intuitionistic fuzzy set μ_A, ν_A and μ_B, ν_B. The intuitionistic fuzzy set $\mu_M = \dfrac{\mu_A + \mu_B}{2} = (0.5, 0.5), \nu_M = \dfrac{\nu_A + \nu_B}{2} = (0.5, 0.5)$ is a moderate number that you consume four cigarettes per day and 30 cc alcohol per day.

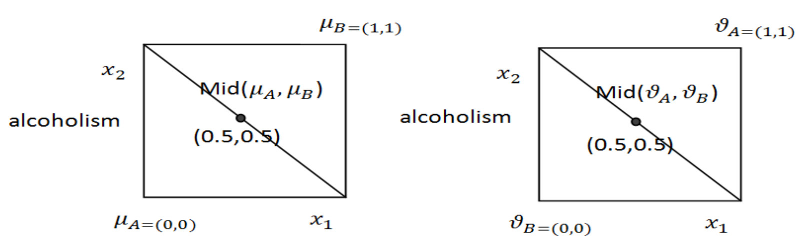

FIGURE 8. midpoint between a non smoker and teetotaler $\mu_A = (0, 0), \nu_A = (1, 1)$ and a heavy smoker and heavy drinker $\mu_B = (1, 1), \nu_B = (0, 0)$.

According to the previous sections, a midpoint between μ_A, ν_A and μ_B, ν_B is any intuitionistic fuzzy set $(a_1, a_2),\ (a_3, a_4)$ with $a_1 + a_2 = 1,\ a_3 + a_4 = 1$, therefore the points $\mu_A = (0.2, 0.8), \nu_A = (0.8, 0.2)$

and $\mu_B = (0.7, 0.3), \nu_B = (0.3, 0.7)$ are also valid midpoints. Any intuitionistic fuzzy set with $\nu = 1-\mu$ is on the line $a_1 + a_2 = 1,\ a_3 + a_4 = 1$. If we calculate the distance of any intuitionistic fuzzy points that is on the line $a_1 + a_2 = 1,\ a_3 + a_4 = 1$ to point $M = (0.5, 0.5),\ (0.5, 0.5)$, the point that has the least distance, has better condition.

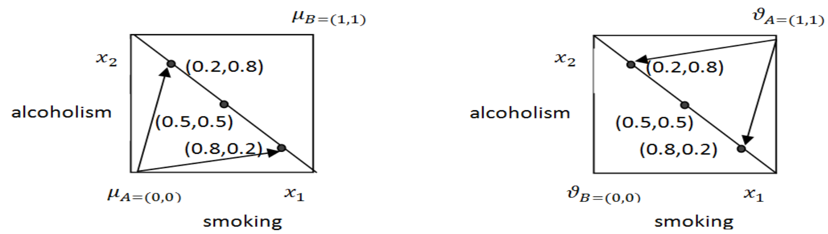

Figure 9

Example 6.1. *Consider the intuitionistic fuzzy sets* $A = (0.3, 0.7), (0.7, 0.3), B = (0.4, 0.6), (0.6, 0.4), C = (0.1, 0.9), (0.9, 0.1)$, *by above description we find the best point:*

The midpoint is $M = (0.5, 0.5), (0.5, 0.5)$, *therefore* $d(A, M) = 0.2+0.2+0.2+0.2 = 0.8,\ d(B, M) = 0.4,\ d(C, M) = 1.2$, *that* C *has the most distance and* B *has the lest distance. Therefore the best point is* B *and the worst point is* C.

Any intuitionistic fuzzy set with $\nu \neq 1 - \mu$ is not on the line $a_1 + a_2 = 1,\ a_3 + a_4 = 1$. For foundation the best point, we calculate distance of points to ideal point, that is $A = (0, 0), (1, 1)$. Any point that it's distance is the least from ideal point, this point is the best point.

Example 6.2. *Consider the intuitionistic fuzzy sets* $A = (0.3, 0.8), (0.5, 0.1),\ B = (0.3, 0.8), (0.7, 0.2)$ *and* $C = (0.3, 0.8), (0.6, 0.1)$. *For point* A *the degree of membership of smoking is 0.3 and the degree of nonmembership is 0.5 and the degree of membership of alcoholism is 0.8 and the degree of non-membership is 0.1. We calculate distances* A, B, C *to ideal point, that is* $I = (0, 0), (1, 1)$, *therefore* $d(A, I) = 2.5,\ d(B, I) = 2.2,\ d(C, I) = 2.4$. *As we can see the distance of* B *to ideal point is minimum and it is the best point, also the distance of* A *to point* I *is maximum, and it is the worst point.*

Example 6.3. *Suppose that* $\mu_{P_1} = (0.1, 0.6, 0.9),\ \nu_{P_1} = (0.9, 0.4, 0.1),\ \mu_{P_2} = (0.4, 0.4, 0.9),\ \nu_{P_2} = (0.6, 0.6, 0.1),\ \mu_{P_3} = (1, 0.4, 0.5),\ \nu_{P_3} = (0, 0.6, 0.5)$. *Now, we find distances* P_1, P_2, P_3 *to ideal point. By consideration* $\mu_I = (0, 0, 0),\ \nu_I = (1, 1, 1)$, *then* $d(P_1, I) = 2.8,\ d(P_2, I) = 2,\ d(P_3, I) = 2.4$. *Therefore the best point is* P_2 *and the worst point is* P_1.

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