International Journal of Analysis and Applications
ISSN 2291-8639
Volume 15, Number 1 (2017), 62-74
http://www.etamaths.com

CHARACTERIZATIONS OF ABEL GRASSMANN'S GROUPOIDS BY THE PROPERTIES OF THEIR DOUBLE-FRAMED SOFT IDEALS

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ABSTRACT. In this paper, we introduce the concept of double-framed soft ideals and investigate properties of these ideals in regular, intra-regular, right regular and left regular AG-groupoids. We also characterize intra-regular AG-groupoids in terms of the double-framed soft ideals.

1. Introduction

The uncertainty which appeared in economics, engineering, environmental science, medical science and social science and so many other applied sciences is too complicated to be solved within traditional mathematical framework. Molodtsov [1] introduced the concept of soft set which can be used as a generic mathematical tool for dealing with uncertainties. Molodtsov pointed out several directions for the applications of soft sets. Worldwide, there has been a rapid growth in interest in soft set theory and its applications. Evidence of this can be found in the increasing number of quality articles on soft sets and related topics that have been published in recent years. Maji et al. [2] described the application of soft set to a decision making problem. Maji et al. [3] also studied several operations on soft sets. Jun et al. [4] introduced the notion of soft ordered semigroup. At present, soft set theory is applied to different algebraic structure. We refer the reader to the papers [5–13].

The idea of generalization of a commutative semigroup, (known as left almost semigroup) was introduced by M. A. Kazim and M. Naseeruddin in 1972 (see [15]). Some other names have also been used in literature for left almost semigroups. Cho et al. [16] studied this structure under the name of right modular groupoid. Holgate [17] studied it as left invertive groupoid. Similarly, Stevanovic and Protic [18] called this structure an Abel-Grassmann groupoid (or simply AG-groupoid), which is the primary name under which this structure in known nowadays. There are many important applications of AG-groupoids in the theory of flocks [19].

Recently, Jun et al. extended the notions of union and intersectional soft sets into double-framed soft sets and defined double-framed soft subalgebra of a BCK/BCI-algebra and studied the related properties in [21]. In [14], Jun et al. also defined the concept of a double-framed soft ideal (briefly, DFS ideal) of a BCK/BCI-algebra and gave many valuable results.

In the present paper, we apply the idea given by Jun et al. in [21], to AG-groupoids and introduce the concept of double-framed soft ideals in AG-groupoids and investigate their related properties. The respective examples of these notions are investigated. Intra-regular AG-groupoids are characterized using the DFS ideals of AG-groupoids.

2. Preliminaries

A groupoid $(S, \cdot)$ is called an AG-groupoid if it satisfies the left invertive law:

$$(ab)c = (cb)a \text{ for all } a, b, c \in S.$$

This structure is closely related with a commutative semigroup because if an AG-groupoid contains right identity then it becomes a commutative monoid. An AG-groupoid may or may not contain a left identity. If there exist a left identity in an AG-groupoid then it is unique.

Received 10^{th} May, 2017; accepted 14^{th} July, 2017; published 1^{st} September, 2017.
2010 *Mathematics Subject Classification.* 03C05, 06D72.
Key words and phrases. DFS-set; DFS AG-groupoid; DFS ideal; DFS int-uni product; DFS including set.

Every AG-groupoid S satisfies the medial law: $(ab)(cd) = (ac)(bd)$ for all $a, b, c, d \in S$. Every AG-groupoid S with left identity satisfies the paramedial law: $(ab)(cd) = (db)(ca)$ for all $a, b, c, d \in S$. In an AG-groupoid S with left identity, using the paramedial law, it is easy to prove that $(ab)(cd) = (dc)(ba)$ for all $a, b, c, d \in S$.

In an AG-groupoid S with left identity, we have $a(bc) = b(ac)$ for all $a, b, c \in S$.

An AG-groupoid S is called AG**-groupoid if $x(yz) = y(xz)$ for all $x, y, z \in S$.

Throughout this paper, S will represent an AG-groupoid unless otherwise stated. For nonempty subsets A and B of S we have $AB := \{ab | a \in A \text{ and } b \in B\}$.

A nonempty subset A of an AG-groupoid S is called *sub AG-groupoid* of S if $A^2 \subseteq A$.

A nonempty subset A of an AG-groupoid S is called *left* (resp. *right*) *ideal* of S if $SA \subseteq A$ (resp. $AS \subseteq A$).

If A is both a left and a right ideal of S then it is called a *two-sided ideal* or simply an *ideal* of S.

We denote by $L[a^2], R[a^2]$ and $J[a^2]$, the priciple left ideal, right ideal, two sided ideal of an AG-groupoid S generated by $a^2 \in S$. Note that the principal left ideal, right ideal, two sided ideals of an AG-groupoid S generated by a^2 are equal. That is $L[a^2] = R[a^2] = J[a^2] = Sa^2 = a^2S = Sa^2S = \{sa^2 : s \in S\}$. The reader is invited to read [25, 26]

An AG-groupoid S is called;

i) right regular if for all $a \in S$, there exist $x \in S$ such that $a = (aa)x$.
ii) left regular if for all $a \in S$, there exist $x \in S$ such that $a = x(aa)$.
iii) regular if for all $a \in S$, there exist $x \in S$ such that $a = (ax)a$.
iv) intra-regular if for all $a \in S$, there exist $x, y \in S$ such that $a = (xa^2)y$.

3. Soft set (basic operations)

In [6], Atagun and Sezgin introduced some new operations on soft set theory and defined soft sets in the following way:

Let U be an initial universe, E a set of parameters, $P(U)$ the power set of U and $A \subseteq E$. Then soft set f_A over U is a function defined by: $f_A : E \longrightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here f_A is called an approximate function. A soft set over U can be represented by the set of ordered pairs

$$f_A := \{(x, f_A(x)) : x \in E,\ f_A(x) \in P(U)\}.$$

It is clear that a soft set is a parameterized family of subsets of U. The set of all soft sets over U is denoted by $S(U)$.

Definition 3.1. Let $f_A, f_B \in S(U)$. Then f_A is a soft subset of f_B, denoted by $f_A \widetilde{\subseteq} f_B$ if $f_A(x) \subseteq f_B(x)$ for all $x \in E$. Two soft sets f_A, f_B are said to be equal soft sets if $f_A \widetilde{\subseteq} f_B$ and $f_B \widetilde{\subseteq} f_A$ and is denoted by $f_A \widetilde{=} f_B$.

Definition 3.2. Let $f_A, f_B \in S(U)$. Then the union of f_A and f_B, denoted by $f_A \widetilde{\cup} f_B$, is defined by $f_A \widetilde{\cup} f_B = f_{A \cup B}$, where $f_{A \cup B}(x) = f_A(x) \cup f_B(x)$, for all $x \in E$.

Definition 3.3. Let $f_A, f_B \in S(U)$. Then the intersection of f_A and f_B, denoted by $f_A \widetilde{\cap} f_B$, is defined by $f_A \widetilde{\cap} f_B = f_{A \cap B}$, where $f_{A \cap B}(x) = f_A(x) \cap f_B(x)$, for all $x \in E$.

Definition 3.4. [22] Let $f_A, f_B \in S(U)$. Then the soft product of f_A and f_B, denoted by $f_A \widetilde{\circ} f_B$, is defined by

$$(f_A \widetilde{\circ} f_B)(x) := \begin{cases} \bigcup\limits_{x=yz} \{f_A(y) \cap f_B(z)\} & \text{if } \exists\ y, z \in S \text{ such that } x = yz \\ \emptyset & \text{otherwise.} \end{cases}$$

Throughout this paper, let $E = S$, where S is an AG-groupoid and $A, B, C, \cdots$ are sub AG-groupoids, unless otherwise stated.

Definition 3.5. [21] A *double-framed soft pair* $\langle (\alpha, \beta); A \rangle$ is called a *double-framed soft set* of A over U (briefly, *DFS*-set of A), where α and β are mappings from A to $P(U)$.

The set of all DFS-sets of S over U will be denoted by $DFS(U)$.

For a DFS-set $\langle (\alpha, \beta); A \rangle$ of A and two subsets γ and δ of U, the *γ-inclusive set* and the *δ-exclusive set* of $\langle (\alpha, \beta); A \rangle$, denoted by $i_A(\alpha; \gamma)$ and $e_A(\beta; \delta)$, respectively, are defined as follows:

$$i_A(\alpha; \gamma) := \{x \in A | \alpha(x) \supseteq \gamma\}$$

and

$$e_A(\beta;\delta) := \{x \in A | \beta(x) \subseteq \delta\}$$

respectively. The set

$$DF_A(\alpha,\beta)_{(\gamma,\delta)} := \{x \in A | \alpha(x) \supseteq \gamma, \beta(x) \subseteq \delta\}$$

is called a *double framed soft including set* [21] of $\langle(\alpha,\beta);A\rangle$.
It is clear that $DF_A(\alpha,\beta)_{(\gamma,\delta)} := i_A(\alpha;\gamma) \cap e_A(\beta;\delta)$.

Let $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);B\rangle$ be two double-framed soft sets of A over U. Then the *int-uni soft product* [23] is denoted by $\langle(\alpha,\beta);A\rangle \lozenge \langle(f,g);B\rangle$ and is defined as a double framed soft set $\langle(\alpha\tilde{\circ}f, \beta\tilde{\circ}g);S\rangle$ defined to be a double-framed soft set over U, in which $\alpha\tilde{\circ}f$, and $\beta\tilde{\circ}g$ are soft mappings from S to $P(U)$, given as follows:

$$\alpha\tilde{\circ}f : S \longrightarrow P(U), x \longmapsto \begin{cases} \bigcup_{x=yz} \{\alpha(y) \cap f(z)\} & \text{if } \exists\, y,z \in S \text{ such that } x = yz, \\ \emptyset & \text{otherwise,} \end{cases}$$

$$\beta\tilde{\circ}g : S \longrightarrow P(U), x \longmapsto \begin{cases} \bigcap_{x=yz} \{\beta(y) \cup g(z)\} & \text{if } \exists\, y,z \in S \text{ such that } x = yz, \\ U & \text{otherwise.} \end{cases}$$

One can easily see that the operation "$\lozenge$" is well-defined.

Let $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);B\rangle$ be two double-framed soft sets of A and B respectively over a common universe U. Then $\langle(\alpha,\beta);A\rangle$ is called a *double-framed soft subset* (briefly, DFS subset) of $\langle(f,g);B\rangle$, denoted by $\langle(\alpha,\beta);A\rangle \sqsubseteq \langle(f,g);B\rangle$, if

i) $A \subseteq B$,

ii) $(\forall e \in A) \begin{pmatrix} \alpha \text{ and } f \text{ are identical approximations. i.e. } \alpha(e) \subseteq f(e) \\ \beta^c \text{ and } g^c \text{ are identical approximations. i.e. } \beta(e) \supseteq g(e) \end{pmatrix}$.

For any two DFS sets $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);A\rangle$ of A over U, the DFS *int-uni set* [21] of $\langle(\alpha,\beta);A\rangle$ and $\langle(f,g);A\rangle$, is defined to be a DFS set $\left\langle\left(\alpha\tilde{\cap}f, \beta\tilde{\cup}g\right);A\right\rangle$ where$\alpha\tilde{\cap}f$, and $\beta\tilde{\cup}g$ are mappings given by $\alpha\tilde{\cap}f : A \to P(U), x \to \alpha(x) \cap f(x)$, $\beta\tilde{\cup}g : A \to P(U), x \to \beta(x) \cup g(x)$.
It is denoted by $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);A\rangle = \left\langle\left(\alpha\tilde{\cap}f, \beta\tilde{\cup}g\right);A\right\rangle$.

For a non-empty subset A of S, the DFS set $\mathbf{X}_A$=$(\chi_A, \chi_A^c; A)$ is called the double framed characteristic soft set where

$$\chi_A : S \to P(U), x \to \begin{cases} U \text{ if } x \in A \\ \emptyset \text{ if } x \notin A \end{cases},$$

$$\chi_A^c : S \to P(U), x \to \begin{cases} \emptyset \text{ if } x \in A \\ U \text{ if } x \notin A \end{cases}.$$

We have the following lemmas.

Lemma 3.1. *(cf. [24]) If S is an AG-groupoid then the set $(DFS(U), \lozenge)$ is an AG-groupoid.*

Lemma 3.2. *(cf. [24]) If S is an AG-Groupoid then the medial law holds in $DFS(U)$. That is for $\langle(\alpha,\beta);S\rangle, \langle(f,g);S\rangle, \langle(h,k);S\rangle$ and $\langle(p,q);S\rangle \in DFS(U)$, we have $(\alpha\tilde{\circ}f)\tilde{\circ}(h\tilde{\circ}p) = ((h\tilde{\circ}p)\tilde{\circ}f)\tilde{\circ}\alpha$ and $(\beta\tilde{\circ}g)\tilde{\circ}(k\tilde{\circ}q) = ((k\tilde{\circ}q)\tilde{\circ}g)\tilde{\circ}\beta$.*

Lemma 3.3. *(cf. [24]) If S is an AG-groupoid with left identity then the paramedial law holds in $DFS(U)$. That is for all $\langle(\alpha,\beta);S\rangle, \langle(f,g);S\rangle, \langle(h,k);S\rangle$ and $\langle(p,q);S\rangle \in DFS(U)$, $(\alpha\tilde{\circ}f)\tilde{\circ}(h\tilde{\circ}p) = (p\tilde{\circ}f)\tilde{\circ}(h\tilde{\circ}\alpha)$ and $(\beta\tilde{\circ}g)\tilde{\circ}(k\tilde{\circ}q) = (q\tilde{\circ}g)\tilde{\circ}(k\tilde{\circ}\beta)$*

Lemma 3.4. *Let $\langle(\alpha,\beta);S\rangle, \langle(f,g);S\rangle, \langle(h,k);S\rangle$ and $\langle(p,q);S\rangle \in DFS(U)$ then,*

i) $\langle(\alpha,\beta);S\rangle \diamond (\langle(f,g);S\rangle \sqcap \langle(h,k);S\rangle) = (\langle(\alpha,\beta);S\rangle \diamond \langle(f,g);S\rangle) \sqcap (\langle(\alpha,\beta);S\rangle \diamond \langle(h,k);S\rangle)$.

ii) If $\langle(f,g);S\rangle \sqsubseteq \langle(h,k);S\rangle$ then $\langle(\alpha,\beta);S\rangle \diamond \langle(f,g);S\rangle \sqsubseteq \langle(\alpha,\beta);S\rangle \diamond \langle(h,k);S\rangle$.

iii) If $\langle(\alpha,\beta);S\rangle \sqsubseteq \langle(f,g);S\rangle$ and $\langle(h,k);S\rangle \sqsubseteq \langle(p,q);S\rangle$ then $\langle(\alpha,\beta);S\rangle \diamond \langle(h,k);S\rangle \sqsubseteq \langle(f,g);S\rangle \diamond \langle(p,q);S\rangle$.

Proof. Straightforward. □

Lemma 3.5. *Let A and B be two non empty subsets of an AG-groupoid S then the following properties hold:*
i) If $A \subseteq B$ *then* $\mathbf{X}_A \sqsubseteq \mathbf{X}_B$.
ii) $\mathbf{X}_A \sqcap \mathbf{X}_B = \mathbf{X}_{A\cap B}$.
iii) $\mathbf{X}_A \diamond \mathbf{X}_B = \mathbf{X}_{AB}$.

Proof. Straightforward. □

4. Double-framed soft ideals

In this section, we define *double-framed soft AG-groupoids*, *double-framed soft left* (resp. *right*) *ideal* of S over U and discuss their properties in regular, intra-regular, right regular and left regular AG-groupoids.

Definition 4.1. *[24] Let S be an AG-groupoid and $\langle(\alpha,\beta);A\rangle$ be a DFS-set of A over U. Then $\langle(\alpha,\beta);A\rangle$ is called a double-framed soft AG-groupoid (briefly, DFS AG-groupoid) of A over U if it satisfies $\alpha(xy) \supseteq \alpha(x) \cap \alpha(y)$ and $\beta(xy) \subseteq \beta(x) \cup \beta(y)$ for all $x, y \in A$.*

Example 4.1. *Consider an AG-groupoid $S = \{0,1,2,3,4\}$ with the following multiplication table:*

$\cdot$	0	1	2	3	4
0	4	1	1	2	4
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	2	1
4	4	1	1	1	4

Consider a double-framed soft $\langle(\alpha,\beta);S\rangle$ of S over $U = \mathbb{Z}^-$ defined by:
$\alpha(0) = \{-1\}, \alpha(1) = \{-1,-2,-3,-4,-5,-6\}, \alpha(2) = \{-1,-2,-3\}, \alpha(3) = \{-1,-3\},$
$\alpha(4) = \{-1,-3,-5\}.$
$\beta(0) = \{-1,-2,-3,-4,-5\}, \beta(1) = \{-1,-2\}, \beta(2) = \{-1,-2,-3,-4\},$
$\beta(3) = \{-1,-2,-3,-4,-6\}, \beta(4) = \{-2,-4\}.$
By routine checking it is easy to verify that $\langle(\alpha,\beta);S\rangle$ is a double-framed soft AG-groupoid of S over U.
Again consider $U = \left\{ \begin{bmatrix} x & x \\ 0 & 0 \end{bmatrix} | x \in \mathbb{Z}_3 \right\}$, *the set of all 2×2 matrices with entries from $\mathbb{Z}_3$ be the universal set. Define a double-framed soft set $\langle(f,g);B\rangle$ of S over U as follows:*
$f(0) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \right\} = f(2),\ f(1) = f(3) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\},$
$f(4) = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \right\}.$
$g(0) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \right\} = g(2), g(1) = g(3) \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \right\},$
$g(4) = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} \right\}.$
Since $f(3\cdot 3) = f(2) \not\supseteq f(3) \cap f(3)$ and/or $g(0\cdot 0) = g(4) \not\subseteq g(0) \cup g(0)$. Hence, $\langle(f,g);B\rangle$ is not a DFS AG-groupoid of S over U.

Theorem 4.1. *Let $\langle(\alpha,\beta);A\rangle$ be a DFS-set over U. Then $\langle(\alpha,\beta);A\rangle$ is a DFS AG-groupoid over U if and only if*

$$\langle(\alpha,\beta);A\rangle \diamondsuit \langle(\alpha,\beta);A\rangle \sqsubseteq \langle(\alpha,\beta);A\rangle.$$

Proof. Assume that $\langle(\alpha,\beta);A\rangle$ is a DFS AG-groupoid over U. Let $a \in A \subseteq S$. If $(\alpha\widetilde{\circ}\alpha)(a) = \emptyset$ and $(\beta\widetilde{\circ}\beta)(a) = U$, then obviously, $(\alpha\widetilde{\circ}\alpha)(a) \subseteq \alpha(a)$ and $(\beta\widetilde{\circ}\beta)(a) \supseteq \beta(a)$. Suppose that there exist

$x, y \in S$ such that $a = xy$. Then

$$\begin{aligned}(\alpha\tilde{\circ}\alpha)(a) &= \bigcup_{a=xy} \{\alpha(x) \cap \alpha(y)\} \subseteq \bigcup_{a=xy} \alpha(xy) \\ &= \bigcup_{a=xy} \alpha(a) = \alpha(a),\end{aligned}$$

and

$$\begin{aligned}(\beta\tilde{\circ}\beta)(a) &= \bigcap_{a=xy} \{\beta(x) \cup \beta(y)\} \supseteq \bigcap_{a=xy} \beta(xy) \\ &= \bigcap_{a=xy} \beta(a) = \beta(a).\end{aligned}$$

Thus, $(\alpha\tilde{\circ}\alpha)(a) \subseteq \alpha(a)$ and $(\beta\tilde{\circ}\beta)(a) \supseteq \beta(a)$ .Hence $\langle(\alpha, \beta); A\rangle \lozenge \langle(\alpha, \beta); A\rangle \sqsubseteq \langle(\alpha, \beta); A\rangle$.

Conversely, assume that $\langle(\alpha, \beta); A\rangle \lozenge \langle(\alpha, \beta); A\rangle \sqsubseteq \langle(\alpha, \beta); A\rangle$. Hence $\alpha\tilde{\circ}\alpha \subseteq \alpha$ and $\beta\tilde{\circ}\beta \supseteq \beta$. Let $x, y \in A \subseteq S$ and $a = xy$, then we have

$$\begin{aligned}\alpha(xy) = \alpha(a) &\supseteq (\alpha\tilde{\circ}\alpha)(a) \\ &= \bigcup_{a=xy} \{\alpha(x) \cap \alpha(y)\} \supseteq \alpha(x) \cap \alpha(y)\end{aligned}$$

and

$$\begin{aligned}\beta(xy) = \beta(a) &\subseteq (\beta\tilde{\circ}\beta)(a) \\ &= \bigcap_{a=xy} \{\beta(x) \cup \beta(y)\} \subseteq \beta(x) \cup \beta(y).\end{aligned}$$

Hence, $\langle(\alpha, \beta); A\rangle$ is a DFS AG-groupoid over U. □

Theorem 4.2. *For a DFS-set $\langle(\alpha, \beta); A\rangle$ of A, the following are equivalent:*

(1) $\langle(\alpha, \beta); A\rangle$ is a DFS AG-groupoid of A.

(2) The non-empty γ-inclusive set and δ-exclusive set of $\langle(\alpha, \beta); A\rangle$ are sub AG-groupoids of S for any subsets γ and δ of U.

Proof. Suppose that $\langle(\alpha, \beta); A\rangle$ is DFS AG-groupoid of A. Let γ and δ be subsets of U such that $i_A(\alpha;\gamma) \neq \emptyset \neq e_A(\beta;\delta)$. Then there exist $x, a \in A$ such that $\alpha(x) \supseteq \gamma$ and $\beta(a) \subseteq \delta$. Let $p, q \in i_A(\alpha;\gamma)$ then $\alpha(p) \supseteq \gamma$ and $\alpha(q) \supseteq \gamma$. Since $\langle(\alpha, \beta); A\rangle$ is DFS AG-groupoid of A, hence $\alpha(pq) \supseteq \alpha(p) \cap \alpha(q) \supseteq \gamma \cap \gamma = \gamma$. Thus $pq \in i_A(\alpha;\gamma)$ and so $i_A(\alpha;\gamma)$ is sub AG-groupoid of S. Now suppose $v, u \in e_A(\beta;\delta)$ then $\beta(v) \subseteq \delta$ and $\beta(u) \subseteq \delta$. Since $\langle(\alpha, \beta); A\rangle$ is DFS AG-groupoid of A, hence $\beta(vu) \subseteq \beta(v) \cup \beta(u) \subseteq \delta \cup \delta = \delta$. Thus $vu \in e_A(\beta;\delta)$ and so $e_A(\beta;\delta)$ is sub AG-groupoid of S.

Conversely, suppose the non-empty γ-inclusive set and δ-exclusive set of $\langle(\alpha, \beta); A\rangle$ are sub AG-groupoids of S for any subsets γ and δ of U. Let $x, y \in A$ such that $\alpha(x) = \gamma_1$, $\alpha(y) = \gamma_2$, $\beta(x) = \delta_1$, $\beta(y) = \delta_2$. Let us take $\gamma = \gamma_1 \cap \gamma_2$ and $\delta = \delta_1 \cup \delta_2$..Now $\alpha(x) = \gamma_1 \supseteq \gamma_1 \cap \gamma_2 = \gamma$ and so $x \in i_A(\alpha;\gamma)$. Similarly $y \in i_A(\alpha;\gamma)$. By hypothesis, $i_A(\alpha;\gamma)$ is a sub AG-groupoid of S, hence $xy \in i_A(\alpha;\gamma)$ and so $\alpha(xy) \supseteq \gamma = \gamma_1 \cap \gamma_2 = \alpha(x) \cap \alpha(y)$. Also as $\beta(x) = \delta_1 \subseteq \delta_1 \cup \delta_2 = \delta$ then $x \in e_A(\beta;\delta)$. Similarly $y \in e_A(\beta;\delta)$. By hypothesis, $e_A(\beta;\delta)$ is a sub AG-groupoid of S, hence $xy \in e_A(\beta;\delta)$ and so $\beta(xy) \subseteq \delta = \delta_1 \cup \delta_2 = \beta(x) \cup \beta(y)$. Therefore $\langle(\alpha, \beta; A)\rangle$ is a DFS AG-groupoid of A. □

For any DFS-set $\langle(\alpha, \beta); E\rangle$ of E, let $\langle(\alpha^*, \beta^*); E\rangle$ be a DFS-set of E defined by

$$\alpha^* : E \to P(U), \qquad x \to \begin{cases} \alpha(x) \text{ if } x \in i_E(\alpha;\gamma) \\ \eta \text{ otherwise} \end{cases}$$

$$\beta^* : E \to P(U), \qquad x \to \begin{cases} \beta(x) \text{ if } x \in e_E(\beta;\delta) \\ \rho \text{ otherwise} \end{cases}$$

where γ, δ, η and ρ are subsets of U with $\eta \subsetneq \alpha(x)$ and $\rho \supsetneq \beta(x)$.

Theorem 4.3. *If $\langle(\alpha, \beta); A\rangle$ is a DFS AG-groupoid of A over U then so is $\langle(\alpha^*, \beta^*); A\rangle$.*

Proof. Suppose that $\langle(\alpha,\beta)\,;A\rangle$ is a DFS AG-groupoid of A over U then non-empty γ-inclusive set and δ-exclusive set of $\langle(\alpha,\beta)\,;A\rangle$ are sub AG-groupoids of S for any subsets γ and δ of U. Let $x,y\in A$. If $x,y\in i_A(\alpha;\gamma)$ then $xy\in i_A(\alpha;\gamma)$ and hence $\alpha^*(xy)=\alpha(xy)\supseteq\alpha(x)\cap\alpha(y)=\alpha^*(x)\cap\alpha^*(y)$. If $x\notin i_A(\alpha;\gamma)$ or $y\notin i_A(\alpha;\gamma)$ then $\alpha^*(x)=\eta$ or $\alpha^*(y)=\eta$. Hence $\alpha^*(xy)\supseteq\eta=\alpha^*(x)\cap\alpha^*(y)$.

Now if $x,y\in e_A(\beta;\delta)$ then $xy\in e_A(\beta;\delta)$ and hence $\beta^*(xy)=\beta(xy)\subseteq\beta(x)\cup\beta(y)=\beta^*(x)\cup\beta^*(y)$. If $x\notin e_A(\beta;\delta)$ or $y\notin e_A(\beta;\delta)$ then $\beta^*(x)=\rho$ or $\beta^*(y)=\rho$. Hence $\beta^*(xy)\subseteq\rho=\beta^*(x)\cup\beta^*(y)$. Therefore $\langle(\alpha^*,\beta^*);A\rangle$ is a DFS AG-groupoid of A. □

The converse of this theorem is not true in general.

Example 4.2. *Suppose there are ten patients in the initial universe U given by:* $U=\{p_1,p_2,p_3,p_4,p_5,p_6,p_7,p_8,p_9,p_{10}\}$.

Let $E=\{e_1,e_2,e_3,e_4\}$ be set of parameters showing status of patients in which
e_1 stands for the parameter "chest pain"
e_2 stands for the parameter "head ache"
e_3 stands for the parameter "tooth ache"
e_4 stands for the parameter "back pain"
with the following multiplication table

$\cdot$	e_1	e_2	e_3	e_4
e_1	e_3	e_3	e_3	e_4
e_2	e_4	e_4	e_3	e_3
e_3	e_4	e_4	e_4	e_4
e_4	e_4	e_4	e_4	e_4

Define a DFS set $\langle(\alpha,\beta);E\rangle$ by

$$\alpha:E\longrightarrow P(U),\qquad x\longrightarrow\begin{cases}\{p_1,p_2,p_3,p_5\}\text{ if }x=e_1\\\{p_1,p_2,p_3,p_4,p_5\}\text{ if }x=e_2\\\{p_1,p_3,p_5,p_7\}\text{ if }x=e_3\\\{p_1,p_3,p_5,p_7,p_9\}\text{ if }x=e_4\end{cases}$$

$$\beta:E\longrightarrow P(U),\qquad x\longrightarrow\begin{cases}\{p_1,p_3\}\text{ if }x=e_1\\\{p_1,p_3,p_5\}\text{ if }x=e_2\\\{p_1,p_2\}\text{ if }x=e_3\\\{p_1,p_2\}\text{ if }x=e_4\end{cases}$$

then $i_E(\alpha;\gamma)=\{e_3,e_4\}$ for $\gamma=\{p_1,p_3,p_5,p_7\}$ and $e_E(\beta;\delta)=\{e_3,e_4\}$ for $\delta=\{p_1,p_2\}$. According to the definition, we have $\langle(\alpha^*,\beta^*);E\rangle$ is defined as

$$\alpha^*:E\longrightarrow P(U),\qquad x\longrightarrow\begin{cases}\{p_1,p_3\}\text{ if }x=e_1\\\{p_1,p_3\}\text{ if }x=e_2\\\{p_1,p_3,p_5,p_7\}\text{ if }x=e_3\\\{p_1,p_3,p_5,p_7,p_9\}\text{ if }x=e_4\end{cases}$$

$$\beta^*:E\longrightarrow P(U),\qquad x\longrightarrow\begin{cases}\{p_1,p_2,p_3,p_4,p_5\}\text{ if }x=e_1\\\{p_1,p_2,p_3,p_4,p_5\}\text{ if }x=e_2\\\{p_1,p_2\}\text{ if }x=e_3\\\{p_1,p_2\}\text{ if }x=e_4\end{cases}$$

By routine checking, we have $\langle(\alpha^*,\beta^*);E\rangle$ is DFS AG-groupoid. But $\langle(\alpha,\beta);E\rangle$ is not DFS AG-groupoid because $\alpha(e_3)=\alpha(e_1e_1)\nsupseteq\alpha(e_1)\cap\alpha(e_2)$ or $\beta(e_3)=\beta(e_1e_1)\nsubseteq\beta(e_1)\cup\beta(e_1)$.

Theorem 4.4. *Let A be a nonempty subset of an AG-groupoid S. Then A is a sub AG-groupoid of S if and only if the DFS-set $\mathbf{X}_A=\langle(\chi_A,\chi_A^c)\,;A\rangle$ is a DFS AG-groupoid of S over U.*

Proof. Straightforward. □

Let $\langle(\alpha,\beta)\,;A\rangle$ and $\langle(\alpha,\beta)\,;B\rangle$ be two DFS-sets over U then $(\alpha_\wedge,\beta_\vee)$-product of $\langle(\alpha,\beta)\,;A\rangle$ and $\langle(\alpha,\beta)\,;B\rangle$ is defined [21] to be a DFS-set $\langle(\alpha_{A\wedge B},\beta_{A\vee B});A\times B\rangle$ over U in which

$$\alpha_{A\wedge B}: A\times B \to P(U), \qquad (x,y)\to \alpha(x)\cap\alpha(y)$$

$$\beta_{A\vee B}: A\times B \to P(U), \qquad (x,y)\to \beta(x)\cup\beta(y)$$

Theorem 4.5. *For any AG-groupoids E and F as set of parameters, let $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ be DFS AG-groupoids of E and F respectively. Then $(\alpha_\wedge,\beta_\vee)$-product of $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ is a DFS AG-groupoid of $E\times F$.*

Proof. We note that $E\times F$ is also an AG-groupoid with the operation $(a,b)*(c,d)=(ac,bd)$ for all $(a,b),(c,d)\in E\times F$.
Let $(u,v),(s,t)\in E\times F$, we have $\alpha_{E\wedge F}((u,v)*(s,t))=\alpha_{E\wedge F}(us,vt)=\alpha(us)\cap\alpha(vt)$
$\supseteq \alpha(u)\cap\alpha(s)\cap\alpha(v)\cap\alpha(t)=\alpha(u)\cap\alpha(v)\cap\alpha(s)\cap\alpha(t)=\alpha_{E\wedge F}(u,v)\cap\alpha_{E\wedge F}(s,t)$,
and
$\beta_{E\vee F}((u,v)*(s,t))=\beta_{E\vee F}(us,vt)=\beta(us)\cup\beta(vt)\subseteq\beta(u)\cup\beta(s)\cup\beta(v)\cup\beta(t)$
$=\beta(u)\cup\beta(v)\cup\beta(s)\cup\beta(t)=\beta_{E\vee F}(u,v)\cup\beta_{E\vee F}(s,t)$.
Hence $(\alpha_\wedge,\beta_\vee)$-product of $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ is a DFS AG-groupoid of $E\times F$. □

Definition 4.2. *A DFS-set $\langle(\alpha,\beta);A\rangle$ of A over U is called a double-framed soft left (resp. right) ideal (briefly, DFS left (right) ideal) of A over U if it satisfies:*

$$\alpha(ab)\supseteq\alpha(b)\ (\text{ resp. } \alpha(ab)\supseteq\alpha(a))$$
$$\text{and } \beta(ab)\subseteq\beta(b)\ (\text{resp. } \beta(ab)\subseteq\beta(a)) \text{ for all } a,b\in A.$$

A DFS-set $\langle(\alpha,\beta);A\rangle$ of A over U is called a double-framed soft two-sided ideal (briefly, DFS two-sided ideal) of A over U if it is both a DFS left and a DFS right ideal of A over U.

Example 4.3. *There are six women patients in the initial universe set U given by*

$$U:=\{p_1,p_2,p_3,p_4,p_5,p_6\}.$$

Let $S:=\{e_0,e_1,e_2\}$ be the set of status of each patient in U with the following type of disease
e_0 stands for the parameter "headache",
e_1 stands for the parameter "chest pain",
e_2 stands for the parameter "mental depression",
with the following binary operation $*$ given in the Cayley table:

$*$	e_0	e_1	e_2
e_0	e_0	e_0	e_0
e_1	e_2	e_2	e_2
e_2	e_0	e_0	e_0

Then $(S,*)$ is an AG-groupoid. Consider a DFS-set $\langle(\alpha,\beta);S\rangle$ over U as follows:

$$\alpha: S\longrightarrow P(U), x\longmapsto\begin{cases}\{p_1,p_2,p_3\} & \text{if } x=e_0,\\ \{p_2,p_3\} & \text{if } x=e_1,\\ \{p_1,p_2,p_3\} & \text{if } x=e_2,\end{cases}$$

and

$$\beta: S\longrightarrow P(U), x\longmapsto\begin{cases}\{p_2,p_4\} & \text{if } x=e_0,\\ \{p_1,p_2,p_3,p_4\} & \text{if } x=e_1,\\ \{p_1,p_2,p_4\} & \text{if } x=e_2.\end{cases}$$

Then one can easily show that $\langle(\alpha,\beta);S\rangle$ is a DFS ideal over U. However, if we define another double-framed soft set $\langle(f,g);S\rangle$ such that

$$f: S\longrightarrow P(U), x\longmapsto\begin{cases}\{p_1,p_2,p_6\} & \text{if } x=e_0\\ \{p_1\} & \text{if } x=e_1\\ \{p_2,p_4,p_6\} & \text{if } x=e_2\end{cases}$$

and

$$g : S \longrightarrow P(U), x \longmapsto \begin{cases} \{p_2, p_4, p_6\} & \text{if } x = e_0 \\ \{p_1, p_6\} & \text{if } x = e_1 \\ \{p_1, p_2, p_3\} & \text{if } x = e_2 \end{cases}$$

.

Then $\langle (f, g) ; S \rangle$ *is not DFS ideal of* S *over* U*, because*

$$f(e_2 * e_0) = f(e_0) = \{p_1, p_2, p_6\} \not\supseteq \{p_2, p_4, p_6\} = f(e_2)$$

and/or

$$g(e_2 * e_0) = f(e_0) = \{p_2, p_4, p_6\} \not\subseteq \{p_1, p_2, p_3, p_4\} = g(e_2).$$

Proposition 4.1. *Let* $\langle (\alpha, \beta) ; A \rangle$ *be a DFS-set over* U*. Then* $\langle (\alpha, \beta) ; A \rangle$ *is a DFS ideal of* S *over* U *if and only if* $\alpha(xy) \supseteq \alpha(x) \cup \alpha(y)$ *and* $\beta(xy) \subseteq \beta(x) \cap \beta(y)$ *for all* $x, y \in S$*.*

Proof. Let $\langle (\alpha, \beta) ; A \rangle$ be a DFS ideal of S over U. Then $\alpha(xy) \supseteq \alpha(y), \alpha(xy) \supseteq \alpha(x)$ and $\beta(xy) \subseteq \beta(y), \beta(xy) \subseteq \beta(x)$ for all $x, y \in S$. Thus, $\alpha(xy) \supseteq \alpha(x) \cup \alpha(y)$ and $\beta(xy) \subseteq \beta(x) \cap \beta(y)$ for all $x, y \in S$.

Conversely, suppose that $\alpha(xy) \supseteq \alpha(x) \cup \alpha(y)$ and $\beta(xy) \subseteq \beta(x) \cap \beta(y)$ for all $x, y \in S$. Then $\alpha(xy) \supseteq \alpha(x) \cup \alpha(y) \supseteq \alpha(x), \alpha(y)$ and $\beta(xy) \subseteq \beta(x) \cap \beta(y) \subseteq \beta(x), \beta(y)$. Hence $\langle (\alpha, \beta) ; A \rangle$ is a DFS ideal of S over U. □

Proposition 4.2. *Let* $\langle (\alpha, \beta) ; A \rangle$ *be a DFS-set over* U*. If* $\langle (\alpha, \beta) ; A \rangle$ *is a DFS left (resp., right or two-sided) ideal over* U*. Then* $\langle (\alpha, \beta) ; A \rangle$ *is a DFS AG-groupoid over* U*.*

Proof. Straightforward. □

Proposition 4.3. *If* S *is an AG-groupoid with left identity* e *then every DFS right ideal is DFS ideal.*

Proof. Let $\langle (\alpha, \beta) ; A \rangle$ be a DFS right ideal of A over U. Now let $x, y \in A$, then $\alpha(xy) = \alpha((ex)y) = \alpha((yx)e) \supseteq \alpha(yx) \supseteq \alpha(y)$ and $\beta(xy) = \beta((ex)y) = \beta((yx)e) \subseteq \beta(yx) \subseteq \beta(y)$. Hence $\alpha(xy) \supseteq \alpha(y)$ and $\beta(xy) \subseteq \beta(y)$ for all $x, y \in A$. Thus $\langle (\alpha, \beta) ; A \rangle$ is DFS left ideal and hence $\langle (\alpha, \beta) ; A \rangle$ is DFS ideal of A over U. □

The converse of the above theorem is not true in general.

Example 4.4. *Let* $S = \{1, 2, 3, 4\}$ *with the following multiplication table:*

$\cdot$	1	2	3	4
1	2	2	4	4
2	2	2	2	2
3	1	2	3	4
4	1	2	1	2

It is easy to see that 3 *is left identity in* S*.*
Consider a DFS-set $\langle (\alpha, \beta) ; S \rangle$ *over* $U = \mathbb{Z}$ *as follows:*

$$\alpha : S \longrightarrow P(U), x \longmapsto \begin{cases} 4\mathbb{Z} \text{ if } x = 1, \\ \mathbb{Z} \text{ if } x = 2, \\ 8\mathbb{Z} \text{ if } x = 3, \\ 2\mathbb{Z} \text{ if } x = 4 \end{cases}$$

and

$$\beta : S \longrightarrow P(U), x \longmapsto \begin{cases} 8\mathbb{Z} \text{ if } x = 1, \\ 16\mathbb{Z} \text{ if } x = 2, \\ \mathbb{Z} \text{ if } x = 3, \\ 4\mathbb{Z} \text{ if } x = 4 \end{cases}$$

Then one can easily show that $\langle (\alpha, \beta) ; S \rangle$ *is a DFS left ideal over* U*. However,* $\langle (\alpha, \beta); S \rangle$ *is not DFS right ideal over* U *because* $\alpha(1) = \alpha(41) \not\supseteq \alpha(4)$ *and/or* $\beta(4) = \beta(14) \not\subseteq \beta(1)$*.*

Proposition 4.4. *(cf. [24]) Let A be a nonempty subset of an AG-groupoid S. Then A is an ideal of S if and only if the DFS-set $\mathbf{X}_A = \langle(\chi_A, \chi_A^c)\,; A\rangle$ is a DFS ideal of S over U.*

Proof. Straightforward □

Theorem 4.6. *(cf. [24]) A DFS set $\langle(\alpha,\beta);A\rangle$ is DFS left (resp. right) ideal of A over U if and only if $\mathbf{X}_A \diamond \langle(\alpha,\beta);A\rangle \sqsubseteq \langle(\alpha,\beta);A\rangle$ (resp. $\langle(\alpha,\beta);A\rangle \diamond \mathbf{X}_A \sqsubseteq \langle(\alpha,\beta);A\rangle$).*

Proof. Straightforward □

Theorem 4.7. *If $\langle(\alpha,\beta)\,;S\rangle$ is a DFS left (resp. right) ideal of S over U then so is $\langle(\alpha^*,\beta^*);S\rangle$.*

Proof. Suppose that $\langle(\alpha,\beta)\,;S\rangle$ is a DFS left ideal of S over U then non-empty γ-inclusive set and δ-exclusive set of $\langle(\alpha,\beta)\,;S\rangle$ are left ideals of S for any subsets γ and δ of U. Let $a,b\in S$. If $b\in i_S(\alpha;\gamma)$ then $ab\in i_S(\alpha;\gamma)$. Thus $\alpha^*(ab)=\alpha(ab)\supseteq\alpha(b)=\alpha^*(b)$. If $b\notin i_S(\alpha;\gamma)$ then $ab\in i_S(\alpha;\gamma)$ or $ab\notin i_S(\alpha;\gamma)$. If $ab\in i_S(\alpha;\gamma)$ then $\alpha^*(ab)=\alpha(ab)\supset\eta=\alpha^*(b)$. If $ab\notin i_S(\alpha;\gamma)$ then $\alpha^*(ab)=\eta=\alpha^*(b)$. In either case $\alpha^*(ab)\supseteq\alpha^*(b)$ for all $a,b\in S$.

Now if $b\in e_S(\beta;\delta)$ then $ab\in e_S(\beta;\delta)$ and hence $\beta^*(ab)=\beta(ab)\subseteq\beta(b)=\beta^*(b)$. If $b\notin e_S(\beta;\delta)$ then $ab\in e_S(\beta;\delta)$ or $ab\notin e_S(\beta;\delta)$. If $ab\in e_S(\beta;\delta)$ then $\beta^*(ab)=\beta(ab)\subset\rho=\beta^*(b)$. If $ab\notin e_S(\beta;\delta)$ then $\beta^*(ab)=\rho=\beta^*(b)$. In either case $\beta^*(ab)\subseteq\beta^*(b)$. Therefore $\langle(\alpha^*,\beta^*);S\rangle$ is a DFS left ideal of S. over U.
In a similar fashion, we can prove the result for DFS right ideal. □

The converse of the above theorem is not true in general.

Example 4.5. *Suppose $U=\mathbb{Z}$ and $S=\{0,1,2\}$ with the following multiplication table*

$\cdot$	0	1	2
0	0	0	0
1	2	2	2
2	0	0	0

Then $(S,\cdot)$ an AG-groupoid. Consider a DFS $\langle(\alpha,\beta);S\rangle$ over U as follows:

$$\alpha : S\to P(U),\qquad x\longmapsto\begin{cases}4\mathbb{Z} \text{ if } x=0\\ 6\mathbb{Z} \text{ if } x=1\\ 4\mathbb{Z} \text{ if } x=2\end{cases}$$

and

$$\beta : S\to P(U),\qquad x\longmapsto\begin{cases}16\mathbb{Z} \text{ if } x=0\\ 6\mathbb{Z}\text{if } x=1\\ 16\mathbb{Z} \text{ if } x=2\end{cases}$$

Then for $\gamma=\delta=4\mathbb{Z}$ we have $i_S(\alpha;\gamma)=e_S(\beta;\delta)=\{0,2\}$.
Now define $\langle(\alpha^*,\beta^*);S\rangle$ as follows:

$$\alpha^* : S\to P(U),\qquad x\longmapsto\begin{cases}4\mathbb{Z} \text{ if } x=0\\ 12\mathbb{Z} \text{ if } x=1\\ 4\mathbb{Z} \text{ if } x=2\end{cases}$$

and

$$\beta^* : S\to P(U),\qquad x\longmapsto\begin{cases}16\mathbb{Z} \text{ if } x=0\\ \mathbb{Z} \text{ if } x=1\\ 16\mathbb{Z} \text{ if } x=2\end{cases}$$

Routine calculations shows that $\langle(\alpha^*,\beta^*);S\rangle$ is a DFS left ideal over U. But $\langle(\alpha,\beta);S\rangle$ is not DFS left ideal over U since $\alpha(0)=\alpha(01)\not\supseteq\alpha(1)$ and/or $\beta(0)=\beta(01)\not\subseteq\beta(1)$.

Theorem 4.8. *For any AG-groupoids E and F as set of parameters, let $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ be DFS left (resp. right) ideals of E and F respectively. Then $(\alpha_\wedge,\beta_\vee)$-product of $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ is a DFS left (resp. right) ideal of $E\times F$.*

Proof. By definition, the $(\alpha_\wedge,\beta_\vee)$-product of $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ is a DFS $\langle(\alpha_{E\wedge F},\beta_{E\vee F});E\times F\rangle$ in which
$\alpha_{E\wedge F}: E\times F\to P(U),\ (x,y)\to\alpha(x)\cap\alpha(y)$ and $\beta_{E\vee F}: E\times F\to P(U),\ (x,y)\to\beta(x)\cup\beta(y)$.
We note that $E\times F$ is also an AG-groupoid with the operation $(a,b)*(c,d)=(ac,bd)$ for all $(a,b),(c,d)\in E\times F$.
Let $(u,v),(s,t)\in E\times F$, we have $\alpha_{E\wedge F}((u,v)*(s,t))=\alpha_{E\wedge F}(us,vt)=\alpha(us)\cap\alpha(vt)\supseteq\alpha(s)\cap\alpha(t)=\alpha_{E\wedge F}(s,t)$,
and $\beta_{E\vee F}((u,v)*(s,t))=\beta_{E\vee F}(us,vt)=\beta(us)\cup\beta(vt)\subseteq\beta(s)\cup\beta(t)=\beta_{E\vee F}(s,t)$. Hence $(\alpha_\wedge,\beta_\vee)$-product of $\langle(\alpha,\beta);E\rangle$ and $\langle(\alpha,\beta);F\rangle$ is a DFS left ideal of $E\times F$. □

Theorem 4.9. *A DFS-set of a right regular AG-groupoid S is DFS left ideal iff it is a DFS right ideal of S over U.*

Proof. Let S be a right regular AG-groupoid and let $\langle(\alpha,\beta);A\rangle$ be a DFS-left ideal of A over U. Now let $a,b\in A\subseteq S$, so $a\in S$. But since S is right regular so there exist an element x such that $a=(aa)x$. Now $\alpha(ab)=\alpha(((aa)x)b)=\alpha((bx)(aa))\supseteq\alpha(aa)\supseteq\alpha(a)$ and $\beta(ab)=\beta(((aa)x)b)=\beta((bx)(aa))\subseteq\beta(aa)\subseteq\beta(a)$. Hence $\langle(\alpha,\beta);A\rangle$ is DFS-right ideal of A over U.

Conversely, let $\langle(\alpha,\beta);A\rangle$ be a DFS-right ideal of A over U. Take $a,b\in A\subseteq S$, so $a\in S$. But since S is right regular so there exist an element x such that $a=(aa)x$. Now $\alpha(ab)=\alpha(((aa)x)b)=\alpha((bx)(aa))=\alpha((ba)(xa))\supseteq\alpha(ba)\supseteq\alpha(b)$ and $\beta(ab)=\beta(((aa)x)b)=\beta((bx)(aa))=\beta((ba)(xa))\subseteq\beta(ba)\subseteq\beta(b)$. Hence $\langle(\alpha,\beta);A\rangle$ is DFS-left ideal of A over U. □

Proposition 4.5. *A DFS-set of an intra-regular AG-groupoid S is a DFS right ideal if and only if it is a DFS left ideal.*

Proof. Let $\langle(\alpha,\beta);A\rangle$ be a DFS right ideal of A over U. Let $a,b\in A$. Since $a\in S$ and S is intra-regular AG-groupoid so there exists $x,y\in S$ such that $a=(xa^2)y$. Then we have
$\alpha(ab)=\alpha(((xa^2)y)b)=\alpha((by)(xa^2))\supseteq\alpha(by)\supseteq\alpha(b)$. Also $\beta(ab)=\beta(((xa^2)y)b)=\beta((by)(xa^2))\subseteq\beta(by)\subseteq\beta(b)$. Hence $\langle(\alpha,\beta);A\rangle$ is DFS left ideal of A over U.

Conversely, assume that $\langle(\alpha,\beta);A\rangle$ is a DFS left ideal of A over U. Now $\alpha(ab)=\alpha(((xa^2)y)b)=\alpha((by)(xa^2))\supseteq\alpha(xa^2)\supseteq\alpha(a^2)\supseteq\alpha(a)$. Also $\beta(ab)=\beta(((xa^2)y)b)=\beta((by)(xa^2))\subseteq\beta(xa^2)\subseteq\beta(a^2)\subseteq\beta(a)$. Hence $\langle(\alpha,\beta);A\rangle$ is DFS right ideal of A over U. □

Proposition 4.6. *A DFS right ideal of a regular AG-groupoid S is a DFS left ideal of S.*

Proof. Let $\langle(\alpha,\beta);S\rangle$ be a DFS right ideal of a regular AG-groupoid S. Let $x,y\in S$. Since $x\in S$ and S is regular so there exist $a\in S$ such that $x=(xa)x$. Thus,

$\alpha(xy)=\alpha(((xa)x)y)=\alpha((yx)(xa))\supseteq\alpha(yx)\supseteq\alpha(y)$ and $\beta(xy)=\beta(((xa)x)y)=\beta((yx)(xa))\subseteq\beta(yx)\subseteq\beta(y)$. Hence $\langle(\alpha,\beta);S\rangle$ is DFS left ideal. □

Proposition 4.7. *Every DFS right ideal of a regular AG-groupoid S is idempotent.*

Proof. Let $\langle(\alpha,\beta);S\rangle$ be a DFS right ideal of a regular AG-groupoid S. Then $\langle(\alpha,\beta);S\rangle\diamond\langle(\alpha,\beta);S\rangle\sqsubseteq\langle(\alpha,\beta);S\rangle\diamond\mathbf{X}_S\sqsubseteq\langle(\alpha,\beta);S\rangle$.
Now we show that $\langle(\alpha,\beta);S\rangle\sqsubseteq\langle(\alpha,\beta);S\rangle\diamond\langle(\alpha,\beta);S\rangle$. Since S is regular, so for any $x\in S$, there exist an element $y\in S$ such that $x=(xy)x$.
We have $(\alpha\tilde{\circ}\alpha)(x)=\bigcup\limits_{x=ab}\{\alpha(a)\cap\alpha(b)\}\supseteq\alpha(xy)\cap\alpha(x)\supseteq\alpha(x)\cap\alpha(x)=\alpha(x)$
and $(\beta\tilde{\circ}\beta)(x)=\bigcap\limits_{x=ab}\{\beta(a)\cup\beta(b)\}\subseteq\beta(xy)\cup\beta(x)\subseteq\beta(x)\cup\beta(x)=\beta(x)$.
Hence $\langle(\alpha,\beta);S\rangle\sqsubseteq\langle(\alpha,\beta);S\rangle\diamond\langle(\alpha,\beta);S\rangle$ and so $\langle(\alpha,\beta);S\rangle=\langle(\alpha,\beta);S\rangle\diamond\langle(\alpha,\beta);S\rangle$, which is the desired result.

□

Proposition 4.8. *Let $\langle(\alpha,\beta);S\rangle$ be a DFS right ideal and $\langle(f,g);S\rangle$ a DFS left ideal of S over U, respectively. Then $\langle(\alpha,\beta);S\rangle\Diamond\langle(f,g);S\rangle\sqsubseteq\langle(\alpha,\beta);S\rangle\sqcap\langle(f,g);S\rangle$.*

Proof. Let $\langle(\alpha,\beta);S\rangle$ be a DFS right ideal and $\langle(f,g);S\rangle$ be DFS left ideal of S over U. Then $\langle(\alpha,\beta);S\rangle \sqsubseteq \mathbf{X}_S$ and $\langle(f,g);S\rangle \sqsubseteq \mathbf{X}_S$ always true.
We have $\langle(\alpha,\beta);S\rangle \diamond \langle(f,g);S\rangle \sqsubseteq \langle(\alpha,\beta);S\rangle \diamond \mathbf{X}_S \sqsubseteq \langle(\alpha,\beta);S\rangle$
and $\langle(\alpha,\beta);S\rangle \diamond \langle(f,g);S\rangle \sqsubseteq \mathbf{X}_S \Diamond \langle(f,g);S\rangle \sqsubseteq \langle(f,g);S\rangle$. It follows that $\langle(\alpha,\beta);S\rangle \diamond \langle(f,g);S\rangle \sqsubseteq \langle(\alpha,\beta);S\rangle \sqcap \langle(f,g);S\rangle$. □

Definition 4.3. *A DFS $\langle(\alpha,\beta);A\rangle$ of A over U is called DFS semiprime if $\alpha(a) \supseteq \alpha(a^2)$ and $\beta(a) \subseteq \beta(a^2)$ for all $a \in A$.*

Theorem 4.10. *For a non empty subset A of an AG-groupoid S, the following conditions are equivalent:*
i) A is semiprime.
ii) The DFS characteristics function $\mathbf{X}_A$ is DFS semiprime.

Proof. (i)⇒(ii). Assume that A is semiprime. Let $a \in A$. If $a^2 \in A$ then $a \in A$ since A is semiprime. Thus $\chi_A(a) = U = \chi_A(a^2)$ and $\chi_A^c(a) = \emptyset = \chi_A^c(a^2)$.
If $a^2 \notin A$ then $\chi_A(a^2) = \emptyset \subseteq \chi_A(a)$ and $\chi_A^c(a^2) = U \supseteq \chi_A^c(a)$. Hence $\mathbf{X}_A$ is DFS semiprime.
(ii)⇒(i). Assume $\mathbf{X}_A$ is DFS semiprime. Let $a^2 \in A$. Then $U = \chi_A(a^2)$. But $\chi_A(a) \supseteq \chi_A(a^2) = U$. Hence $\chi_A(a) = U$ and so $a \in A$.
Also $\chi_A^c(a) \subseteq \chi_A^c(a^2) = \emptyset$, so $\chi_A^c(a) = \emptyset$. Thus $a \in A$. Hence A is semiprime. □

Proposition 4.9. *For any DFS AG-groupoid $\langle(\alpha,\beta);A\rangle$ of A over U, the following conditions are equivalent:*
i) $\langle(\alpha,\beta);A\rangle$ is DFS semiprime.
ii) $\alpha(a) = \alpha(a^2)$ and $\beta(a) = \beta(a^2)$ for all $a \in A$.

Proof. (i)⇒(ii). Assume $\langle(\alpha,\beta);A\rangle$ is DFS semiprime and let $a \in A$. Now $\alpha(a) \supseteq \alpha(a^2) = \alpha(aa) \supseteq \alpha(a)\cap\alpha(a) \supseteq \alpha(a)$, so $\alpha(a) = \alpha(a^2)$. Also $\beta(a) \subseteq \beta(a^2) = \beta(aa) \subseteq \beta(a)\cup\beta(a) = \beta(a)$, so $\beta(a) = \beta(a^2)$.
(ii)⇒(i). It is obvious. □

Proposition 4.10. *For an AG-groupoid S with left identity e, the following conditions are equivalent:*
i) S is left regular.
ii) Every DFS left ideal of S is DFS semiprime.

Proof. (i)⇒(ii). Assume that S is left regular. Let $\langle(\alpha,\beta);S\rangle$ is DFS left ideal of S. Let $a \in S$. Since S is left regular so there exist $x \in S$ such that $a = x(aa)$.
Now $\alpha(a) = \alpha(x(aa)) \supseteq \alpha(aa) = \alpha(a^2)$ and $\beta(a) = \beta(x(aa)) \subseteq \beta(aa) = \beta(a^2)$. Thus $\langle(\alpha,\beta);S\rangle$ is DFS semiprime.
(ii)⇒(i). Assume (ii) holds. Since Sa^2 is left ideal so $\mathbf{X}_{Sa^2} = \left\langle(\chi_{Sa^2},\chi_{Sa^2}^c);Sa^2\right\rangle$ is DFS left ideal and so by hypothesis $\mathbf{X}_{Sa^2} = \left\langle(\chi_{Sa^2},\chi_{Sa^2}^c);Sa^2\right\rangle$ is DFS semiprime.
Since S is AG-groupoid with left identity so $a^2 \in Sa^2$ and hence $U = \chi_{Sa^2}(a^2) \supseteq \chi_{Sa^2}(a) \supseteq \chi_{Sa^2}(a^2)$. Thus $\chi_{Sa^2}(a) = U$ Hence $a \in Sa^2$.
In the other case $\emptyset = \chi_{Sa^2}^c(a^2) \subseteq \chi_{Sa^2}^c(a) \subseteq \chi_{Sa^2}^c(a^2)$. So $\chi_{Sa^2}^c(a) = \emptyset$ which imply $a \in Sa^2$. Hence in any case $a \in Sa^2$ and so S is left regular. □

5. Characterizations of intra-regular AG-groupoids in terms of DFS ideals

In this section, we give some characterizations of intra-regular AG-groupoids using their DFS ideals.

Proposition 5.1. *For an AG groupoid S, the following conditions are equivalent:*
i) S is intra-regular.
ii) Every DFS ideal $\langle(\alpha,\beta);A\rangle$ is DFS soft semiprime.
iii) $\alpha(a) = \alpha(a^2)$ and $\beta(a) = \beta(a^2)$ for every DFS ideal $\langle(\alpha,\beta);A\rangle$ for all $a \in A$.

Proof. (i)⇒(iii). Suppose that S is intra-regular. Let $\langle(\alpha,\beta);A\rangle$ is a DFS ideal which is semiprime. Take $a \in A \subseteq S$, so there exist $x, y \in S$ such that $a = (xa^2)y$. Thus,
$\alpha(a) = \alpha((xa^2)y) \supseteq \alpha(xa^2) = \alpha(x(aa)) \supseteq \alpha(aa) \supseteq \alpha(a)$ and so $\alpha(a) = \alpha(a^2)$.
Now $\beta(a) = \beta((xa^2)y) \subseteq \beta(xa^2) = \beta(x(aa)) \subseteq \beta(aa) \subseteq \beta(a)$ and so $\beta(a) = \beta(a^2)$.
(iii)⇒(i). Assume that for every DFS ideal $\langle(\alpha,\beta);A\rangle$ of A over U, we have $\alpha(a) = \alpha(a^2)$ and

$\beta(a) = \beta(a^2)$ for all $a \in A$.
Since $J[a^2]$ is an ideal of S, so $\mathbf{X}_{J[a^2]}$ is DFS ideal of S. Since $a^2 \in J[a^2]$, we have
$\chi_{J[a]}(a) = \chi_{J[a^2]}(a^2) = U$. Thus $a \in J[a^2] = (Sa^2)S$. Also $\chi^c_{J[a^2]}(a) = \chi^c_{J[a^2]}(a^2) = \emptyset$. In this case, too, $a \in J[a^2] = (Sa^2)S$. Hence S is intra-regular.
(iii)⇒(ii). Obvious.
(ii)⇒(iii). Let $\langle(\alpha,\beta);A\rangle$ is a DFS ideal which is semiprime. Now $\alpha(a) \supseteq \alpha(a^2) = \alpha(aa) \supseteq \alpha(a)$. Thus $\alpha(a) = \alpha(a^2)$.
Also $\beta(a) \subseteq \beta(a^2) = \beta(aa) \subseteq \beta(a)$. Thus $\beta(a) = \beta(a^2)$. This completes the proof. □

Theorem 5.1. *For an AG-groupoid S with left identity e, the following conditions are equivalent.*
i) S is intra-regular.
ii) $L \cap R \subseteq LR$ for every left ideal L and every right ideal R of S and R is semiprime.
iii) $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);B\rangle \sqsubseteq \langle(\alpha,\beta);A\rangle \diamond \langle(f,g);B\rangle$ .for every DFS left ideal $\langle(\alpha,\beta);A\rangle$ and every DFS right ideal $\langle(f,g);B\rangle$ and $\langle(f,g);B\rangle$ is DFS semiprime.

Proof. (i)⇒(iii). Assume that S is intra-regular. Let $\langle(\alpha,\beta);A\rangle$ is DFS left ideal and $\langle(f,g);B\rangle$ is DFS right ideal over U. Since S is intra-regular, so for $a \in S$, there exist elements x, y in S such that $a = (xa^2)y = ((x(aa))y) = ((a(xa))y) = (y(xa))a = (y(xa))(ea) = (ye)((xa)a) = (xa)((ye)a) = (xa)((ae)y)$.
Now $(\alpha\tilde{\circ}f)(a) = \bigcup_{a=pq}\{\alpha(p) \cap f(q)\} \supseteq \alpha(xa) \cap f((ae)y) \supseteq \alpha(a) \cap f(ae) \supseteq \alpha(a) \cap f(a)$
and $(\beta\tilde{\circ}g)(a) = \bigcap_{a=pq}\{\beta(p) \cup g(q)\} \subseteq \beta(xa) \cup g((ae)y) \subseteq \beta(a) \cup g(ae) \subseteq \beta(a) \cup g(a)$.
Hence $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);B\rangle \sqsubseteq \langle(\alpha,\beta);A\rangle \diamond \langle(f,g);B\rangle$.
Now $f(a) = f((xa^2)y) = f((xa^2)(ey)) = f((ye)(a^2x)) = f(a^2((ye)x)) \supseteq f(a^2)$
and $g(a) = g((xa^2)y) = g((xa^2)(ey)) = g((ye)(a^2x)) = g(a^2((ye)x)) \subseteq g(a^2)$.
Thus $\langle(f,g);B\rangle$ is DFS semiprime.
(iii)⇒(ii). Assume that (iii) holds. Let L and R be left ideal and right ideal of S respectively Then $\mathbf{X}_L$ is DFS left ideal and $\mathbf{X}_R$ is DFS right ideal. Thus by hypothesis $\mathbf{X}_L \sqcap \mathbf{X}_R \sqsubseteq \mathbf{X}_{LR}$ and $\mathbf{X}_R$ is DFS semiprime. Let $a \in L \cap R$ then $a \in L$ and $a \in R$. Hence $U = \chi_{L\cap R}(a) = (\chi_L \cap \chi_R)(a) \subseteq (\chi_L \circ \chi_R)(a) = \chi_{LR}(a)$. That is $\chi_{LR}(a) = U$ and so $a \in LR$.
In the other case $\emptyset = \chi^c_{L\cap R}(a) = (\chi^c_L \cup \chi^c_R)(a) \supseteq (\chi^c_L \circ \chi^c_R)(a) = \chi^c_{LR}(a)$. That is $\chi^c_{LR}(a) = \emptyset$ and so $a \in LR$. Hence in any case $L \cap R \subseteq LR$. By Theorem 4.10, since $\mathbf{X}_R$ is DFS semiprime, so R is semiprime.
(ii)⇒(i). Assume that (ii) holds. We prove S is intra-regular. Let $a \in S$. Then $a = ea \in Sa$, where Sa is left ideal of S and $a^2 \in a^2S$ and a^2S is right ideal of S.
By hypothesis a^2S is semiprime and so $a \in a^2S$. Thus $a \in Sa \cap a^2S \subseteq (Sa)(a^2S) = (Sa^2)(aS) \subseteq (Sa^2)(SS) \subseteq (Sa^2)S$. Hence S is intra-regular. □

Lemma 5.1. *For an AG groupoid S with left identity, the following conditions are equivalent:*
i) S is intra-regular.
ii) $R \cap L = RL$ for every left ideal L and right ideal R of S and R is semiprime.

Proof. Proof is available in [25]. □

Theorem 5.2. *For an AG-groupoid S with left identity e, the following conditions are equivalent:*
i) S is intra-regular.
ii) $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);B\rangle = \langle(\alpha,\beta);A\rangle \diamond \langle(f,g);B\rangle$ .for every DFS right ideal $\langle(\alpha,\beta);A\rangle$ and every DFS left ideal $\langle(f,g);B\rangle$ and $\langle(\alpha,\beta);A\rangle$ is DFS semiprime.

Proof. (i)⇒(ii). Let $\langle(\alpha,\beta);A\rangle$ is DFS right ideal and $\langle(f,g);B\rangle$ DFS left ideal of S over U.
By Proposition 4.8, $\langle(\alpha,\beta);A\rangle \diamond \langle(f,g);B\rangle \sqsubseteq \langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);B\rangle$.
Next we have since S is intra-regular, so for each $a \in S$, there exist $x, y \in S$ such that $a = (xa^2)y$.
Thus $a = (xa^2)y = (x(aa))y = (a(xa))y = (y(xa))a = ((ey)(xa))a = ((ax)(ye))a$.
Hence $(\alpha\tilde{\circ}f)(a) = \bigcup_{a=pq}\{\alpha(p) \cap \beta(q)\} \supseteq \alpha((ax)(ye)) \cap \beta(a) \supseteq \alpha(ax) \cap \beta(a) \supseteq \alpha(a) \cap \beta(a)$

and $(\beta\widetilde{\circ}g)(a) = \bigcap_{a=pq}\{\beta(p)\cup g(q)\} \subseteq \beta((ax)(ye)) \cup g(a) \subseteq \beta(ax)\cup g(a) \subseteq \beta(a)\cup g(a)$,

and so $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);B\rangle \sqsubseteq \langle(\alpha,\beta);A\rangle \diamond \langle(f,g);B\rangle$. Thus $\langle(\alpha,\beta);A\rangle \sqcap \langle(f,g);B\rangle = \langle(\alpha,\beta);A\rangle \diamond \langle(f,g);B\rangle$.

Also $\alpha(a) = \alpha((xa^2)y) = \alpha((xa^2)(ey)) = \alpha((ye)(a^2x)) = \alpha(a^2((ye)x)) \supseteq \alpha(a^2)$,

and $\beta(a) = \beta((xa^2)y) = \beta((xa^2)(ey)) = \beta((ye)(a^2x)) = \beta(a^2((ye)x)) \subseteq \beta(a^2)$. Thus $\langle(\alpha,\beta);A\rangle$ is DFS semiprime.

(ii)⇒(i). Assume (ii) holds. Let L be a left ideal and R be a right ideal of S. Then $\mathbf{X}_L$ is DFS left ideal and $\mathbf{X}_R$ is DFS right ideal. By hypothesis, $\mathbf{X}_R \sqcap \mathbf{X}_L = \mathbf{X}_R \diamond \mathbf{X}_L$ and $\mathbf{X}_R$ is DFS semiprime. Since $\mathbf{X}_R$ is DFS semiprime, so by Theorem 4.10, R is semiprime.

Let $a \in R\cap L$, then $a\in R$ and $a \in L$. Hence $U = \chi_{R\cap L}(a) = \chi_R(a)\cap\chi_L(a) = (\chi_R\widetilde{\circ}\chi_L)(a) = \chi_{RL}(a)$, so $a\in RL$. Also $\emptyset = \chi^c_{R\cap L}(a) = \chi^c_R(a)\cap\chi^c_L(a) = (\chi^c_R\widetilde{\circ}\chi^c_L)(a) = \chi^c_{RL}(a)$, so $a \in RL$. In any case $R\cap L\subseteq RL$. The other inclusion $RL \subseteq R\cap L$ is obvious, since S is intra-regular.

Thus $R\cap L = RL$. This along with R is semiprime implies that S is intra-regular. □

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