International Journal of Analysis and Applications
Volume 16, Number 2 (2018), 209-221
URL: https://doi.org/10.28924/2291-8639
DOI: 10.28924/2291-8639-16-2018-209

ON GENERALIZED LOCAL PROPERTY OF $|\mathcal{A};\delta|_k$-SUMMABILITY OF FACTORED FOURIER SERIES

B. B. JENA[1], VANDANA[2,*], S. K. PAIKRAY[1] AND U. K. MISRA[3]

[1] *Department of Mathematics, Veer Surendra Sai University of Technology, Burla 768018, Odisha, India*

[2] *Department of Management Studies, Indian Institute of Technology, Madras, Tamil Nadu-600 036, India*

[3] *Mathematics, National Institute of Science and Technology, Pallur Hills, Golanthara 761008, Odisha, India*

[*] *Corresponding author:* vdrai1988@gmail.com

Abstract. The convergence of Fourier series of a function at a point depends upon the behaviour of the function in the neighborhood of that point and it leads to the local property of Fourier series. In the proposed paper a new result on local property of $|\mathcal{A};\delta|_k$-summability of factored Fourier series has been established that generalizes a theorem of Sarigöl [13] (see [M. A. Sariögol, On local property of $|\mathcal{A}|_k$-summability of factored Fourier series, *J. Math. Anal. Appl.* 188 (1994), 118-127]) on local property of $|\mathcal{A}|_k$-summability of factored Fourier series.

1. Introduction and Motivation

Suppose $\sum a_n$ be a given infinite series with sequence of partial sum (s_n) and let $\mathcal{A} = (a_{nv})$ be a lower triangular matrix with nonzero diagonal entries. Then $\mathcal{A}$ defines the sequence-to-sequence transformation

Received 2017-09-21; accepted 2017-12-07; published 2018-03-07.

2010 *Mathematics Subject Classification.* 40F05; 40D25.

Key words and phrases. Fourier series; lower triangular matrix; $|\mathcal{A};\delta|_k$-summability; local property.

from the sequence $s = (s_n)$ to $\mathcal{A}(s) = (\mathcal{A}_n(s))$, with

$$\mathcal{A}_n(s) = \sum_{v=0}^{n} a_{nv} s_v. \tag{1.1}$$

A series $\sum a_n$ is summable $|\mathcal{A}|_k$ $(k \geq 1)$ if, (see [13])

$$\sum_{n=1}^{\infty} |a_{nn}|^{1-k} |\mathcal{A}_n(s) - \mathcal{A}_{n-1}(s)|^k < \infty, \tag{1.2}$$

and the series $\sum a_n$ is summable $|\mathcal{A}; \delta|_k$ $(k \geq 1)$ if, (see [6])

$$\sum_{n=1}^{\infty} |a_{nn}|^{1-k-\delta k} |\mathcal{A}_n(s) - \mathcal{A}_{n-1}(s)|^k < \infty. \tag{1.3}$$

Let us consider two lower triangular matrices $\bar{\mathcal{A}}$ and $\hat{\mathcal{A}}$ associated with $\mathcal{A}$ as follows:

$$\bar{a}_{nv} = \sum_{r=v}^{n} a_{nr}, \qquad (n, v = 0, 1, 2, ...,)$$

and

$$\hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}. \qquad (n = 1, 2, 3, ...,).$$

In special case, when $\mathcal{A} = (\bar{\mathcal{N}}, p_n)$ then $|\mathcal{A}, \delta|_k$-summability reduces to $|\bar{\mathcal{N}}, p_n; \delta|_k$-summability and for $k = 1$, $(|\bar{\mathcal{N}}, p_n; \delta|)$ is equivalent to $|\mathcal{R}, p_n; \delta|$-summability (see [2]). Also, if we take $A = (C, \alpha)$ with $(\alpha > -1)$, then $|\mathcal{A}, \delta|_k$-summability becomes $|\mathcal{C}, \alpha, (\alpha - 1)(1 - 1/k)\delta|_k$ in Flett's notation. Furthermore, for double absolute factorable summability matrix (see [11]).

We use the notations

$\Delta c_n = c_n - c_{n+1}$ and $\bar{\Delta} c_{n,v} = c_{nv} - c_{n-1,v}$, $c_{-1,0} = 0$, $(n, v = 0, 1, 2, ...,)$.

A sequence (λ_n) is called a convex sequence if,

$\Delta^2(\lambda_n) \geq 0$ for every $n \in Z_+$,

where

$\Delta^2(\lambda_n) = \Delta(\lambda_n) - \Delta(\lambda_{n+1})$ and $\Delta(\lambda_n) = \lambda_n - \lambda_{n+1}$.

Let $f(t) \in L(-\pi, \pi)$ be a 2π periodic function. Without loss of generality let us consider that $a_0 = 0$ in the Fourier series expansion of $f(t)$ that is,

$$\int_{-\pi}^{\pi} f(t)dt = 0. \tag{1.4}$$

Thus the Fourier series expansion of $f(t)$ becomes:

$$f(t) = \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t). \tag{1.5}$$

It is well known that the convergence of the Fourier series at $t = x$ is a local property of f [16] (i.e., it depends only on the behavior of f in an arbitrarily small neighborhood of x) and hence the summability of the Fourier series at $t = x$ by any regular linear summability method is also a local property of f. Moreover, as regards to the approximation of Fourier series of functions see the recent results [9], [10] and [5].

2. Preliminaries

Dealing with Riesz summability and local property of Fourier series, Mohanty [12] has established that $|\mathcal{R}, \log(n), 1|$-summability of a factored Fourier series

$$\sum \frac{\mathcal{A}_n}{\log(n+1)} \tag{2.1}$$

of a function $f(t)$ at any point $t = x$ is a local property of the generating function of $f(t)$ but the summability $|\mathcal{C}, 1|$ of this series is not. Subsequently, replacing the series (2.1) by

$$\sum \frac{\mathcal{A}_n(t)}{(\log\log(n+1))^{\delta}} \ (\delta > 1). \tag{2.2}$$

Matsumoto [7] as obtained a new result on local property of $|\mathcal{R}, p_n, 1|$-summability.

Generalizing the above result Bhatt [1] proved the following theorem:

Theorem 2.1. *Suppose (λ_n) is a convex sequence such that $\sum \frac{\lambda_n}{n}$ is convergent, then the $|\mathcal{R}, \log(n), 1|$-summability of a factored Fourier series $\sum \mathcal{A}_n(t)\lambda_n \log(n)$ at any point $t = x$ is a local property of $f(t)$.*

By replacing the factor $\lambda_n \log(n)$ in a most general form, Mishra [8] has proved the following theorem.

Theorem 2.2. *Suppose (p_n) be a sequence satisfying following conditions:*

$$P_n = O(np_n),$$

$$P_n \Delta p_n = O(p_n p_{n+1}).$$

Then the $|\bar{\mathcal{N}}, p_n|$-summability of a factored Fourier series

$$\sum_{n=1}^{\infty} \mathcal{A}_n(t) \lambda_n P_n (np_n)^{-1} \tag{2.3}$$

at any point $t = x$ is a local property of $f(t)$, where (λ_n) is a convex sequence.

Replacing $|\bar{\mathcal{N}}, p_n|$-summability in Mishra's result, Bor [3] proved a more general form on $|\bar{\mathcal{N}}, p_n|_k$-summability method. Quite recently, Bor [4] introduced the following result on $|\bar{\mathcal{N}}, p_n|_k$-summability of a factored Fourier series at any point $t = x$ as a local property of $f(t)$ under more appropriate conditions then those given in the theorem.

Theorem 2.3. *Let the positive sequence (p_n) and a sequence (λ_n) be such that*

$$\Delta X_n = O(n^{-1});$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \{|\lambda_n|^k + |\lambda_{n+1}|^k\} X_n^{k-1} \leqq \infty;$$

$$\sum_{n=1}^{\infty} (X_n^k + 1) |\Delta \lambda_n| \leqq \infty,$$

where $X_n = (np_n)^{-1} P_n$. Then the $|\mathcal{N}, p_n|_k$-summability of a factored Fourier series $\sum_{n=1}^{\infty} \lambda_n X_n A_n(t)$ at any point $t = x$ is a local property of $f(t)$.

Later Sarigöl (see [13]) has proved the following

Theorem 2.4. *Suppose that $\mathcal{A} = (a_{nv})$ is a positive normal matrix satisfying*

$$a_{n-1}, v \geqq a_{nv}, \ (n \leqq v+1)$$

$$\bar{a}_{n,0} = 1 \ (n = 0, 1, 2, ...,)$$

$$\sum_{v=1}^{n-1} a_{vv} \hat{a}_{n,v-1} = O(a_{nn}),$$

$$\Delta x_n = O(n^{-1}),$$

where $X_n = \frac{1}{(na_{nn})}$. If a sequence (λ_n) satisfying following conditions

$$\sum_{n=1}^{\infty} n^{-1}\{|\lambda_n^k| + |\lambda_{n+1}|^k\} X_{n-1}^k \leqq \infty,$$

$$\sum_{n=1}^{\infty} (X_n^k + 1)|\Delta\lambda_n| \leqq \infty.$$

Then the $|\mathcal{A}|_k$-summability of a factored Fourier series $\sum_{n=1}^{\infty} \lambda_n X_n \mathcal{A}_n(t)$ at any point $t = x$ is a local property of $f(t)$.

Again to improve upon and generalize Theorem 2.4, Sulaiman [14] has proved the following theorem for a normal matrix.

Theorem 2.5. *Let $\mathcal{A} = (a_{nv})$ is a normal matrix satisfying*

$$|\hat{a}_{n,v+1}| \leq |a_{nn}|,$$

$$\sum_{n=v+1}^{\infty} |\hat{a}_{n,v+1}| \leq \infty,$$

$$\sum_{v=1}^{n-1} |a_{vv}||\hat{a}_{n,v+1}| = O(|a_{nn}|),$$

$$\Delta X_n = O(\frac{1}{n}),$$

where $X_n = \frac{1}{(na_{nn})}$. *If a sequence* (λ_n) *satisfying the following conditions*

$$\sum_{n=1}^{\infty} n^{-1}\{|\lambda_n^k| + |\lambda_{n+1}|^k\} X_{n-1}^k \leqq \infty,$$

$$\sum_{n=1}^{\infty} X_n |\Delta \lambda_n| \leqq \infty.$$

Then the $|\mathcal{A}|_k$*-summability of a factored Fourier series* $\sum_{n=1}^{\infty} \lambda_n X_n \mathcal{A}_n(t)$ *at any point* $t = x$ *is a local property of* $f(t)$.

3. Main result

In the present paper, we have established a new result on local property of $|\mathcal{A}, \delta|_k$-summability of factored Fourier series $\sum_{n=1}^{\infty} \lambda_n X_n \mathcal{A}_n(t)$ in the form of a theorem as follows.

Theorem 3.1. *Suppose* $\mathcal{A} = (a_{nv})$ *is a positive normal matrix such that*

$$a_{n-1,v} \geq a_{n,v} \ (n \leqq v+1); \tag{3.1}$$

$$\bar{a}_{n,0} = 1 \ (n = 0, 1, ...,); \tag{3.2}$$

$$\sum_{v=1}^{n-1} a_{vv} \hat{a}_{n,v-1} = O(a_{nn}); \tag{3.3}$$

$$\sum_{n=v+1}^{m+1} \hat{a}_{n,v+1} a_{nn}^{-\delta k} = O(v^{\delta k}); \tag{3.4}$$

$$\sum_{n=v+1}^{m+1} a_{nn}^{-\delta k} |\bar{\Delta} a_{nv}| = O(v^{\delta k}),; \tag{3.5}$$

$$\Delta X_n = O(n^{-1}), \tag{3.6}$$

where $X_n = \frac{1}{(na_{nn})}$. *If a sequence* (λ_n) *satisfying the following conditions*

$$\sum_{n=1}^{\infty} n^{-1}\{|\lambda|^k + |\lambda_{n+1}|^k\} X_n^k n^{\delta k} \leqq \infty; \tag{3.7}$$

$$\sum_{n=1}^{\infty} (x_n^k + 1) |\Delta \lambda_n| n^{\delta k} \leqq \infty. \tag{3.8}$$

Then the $|\mathcal{A},\delta|_k$*-summability of a factored Fourier series* $\sum_{n=1}^{\infty}\lambda_n X_n \mathcal{A}_n(t)$ *at any point* $t = x$ *is a local property of* $f(t)$.

Remark 3.1. The element $\hat{a}_{nv} \geqq 0$ for each n, v. In fact, it is easily seen from the positiveness of the matrix, (3.1) and (3.2), that $\hat{a}_{00} = 1$,

$$\hat{a}_{nv} = \bar{a}_{n0} - \bar{a}_{v-1,0} + \sum_{j=0}^{v-1}(a_{n-1,j} - a_{nj})$$

$$= \sum_{j=0}^{v-1}(a_{n-1,j} - a_{nj}) \geqq 0 \ (1 \leqq v \leqq n)$$

and equal to zero otherwise.

In order to prove the above theorem we need the a lemma as follows.

Lemma 3.1. *Suppose that the matrix* $\mathcal{A}$ *and the sequence* (λ_n) *satisfy the conditions of the theorem, and that* (s_n) *is bounded. Then factored Fourier series* $\sum_{n=1}^{\infty}\lambda_n X_n \mathcal{A}_n(t)$ *is summable to* $|\mathcal{A},\delta|_k$ $(k \geqq 1,\ \delta \geqq 0)$.

Proof. Let (T_n) be an $\mathcal{A}-$ transformation of $\sum_{i=1}^{n}\lambda_i X_i A_n(t)$, then

$$T_n = \sum_{i=0}^{n} a_{ni}s_i = \sum_{i=1}^{n} a_{ni}\sum_{v=1}^{i}\lambda_v X_v = \sum_{v=1}^{n}\lambda_v X_v \sum_{i=v}^{n} a_{ni} = \sum_{v=1}^{n}\bar{a}_{nv}\lambda_n X_v$$

$$\bar{\Delta}T_n = T_n - T_{n-1} = \sum_{v=1}^{n}(\bar{a}_{nv} - \bar{a}_{n-1,v})\lambda_v X_v = \sum_{v=1}^{n}\hat{a}_{nv}\lambda_v X_v$$

$$\bar{\Delta}T_n = \sum_{v=1}^{n-1}(\hat{a}_{nv}\lambda_v X_v)s_v + a_{nn}\lambda_n X_n s_n$$

$$\text{but, } \Delta(\hat{a}_{nv}\lambda_v X_v) = \lambda_v X_v \Delta\hat{a}_{nv} + \Delta(\lambda_v X_v)\hat{a}_{n,v+1}$$

$$= \lambda_v X_v \bar{\Delta}a_{nv} + (X_v\Delta\lambda_v + \Delta X_v \lambda_{v+1})\hat{a}_{n,v+1}.$$

$$\bar{\Delta} T_n = \sum_{v=1}^{n-1} \hat{a}_{n,v+1} X_v \Delta\lambda_v s_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \lambda_{v+1} \Delta X_v s_v + \sum_{v=1}^{n-1} \bar{\Delta} a_{nv} \lambda_v X_v s_v + a_{nn} \lambda_n X_n s_n$$

$$= T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4}, \qquad \text{(say)}.$$

To complete the proof, it is sufficient to show that by using Minkowski's inequality

$$\sum_{n=1}^{\infty} a_{nn}^{1-k-\delta k} |T_{n,m}|^k < \infty \ (m = 1, 2, 3, 4).$$

Using Hölder inequality and (3.1), (3.2), (3.8),

Let

$$I_1 = \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k} |T_{n,1}|^k$$

$$\leqq \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k} \left\{ \sum_{v=1}^{n-1} \hat{a}_{n,v+1} X_v |\Delta\lambda_v||s_v| \right\}^k$$

$$= O(1) \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k} \left\{ \sum_{v=1}^{n-1} \hat{a}_{n,v+1} X_v |\Delta\lambda_v| \right\}^k$$

$$= O(1) \sum_{n=2}^{m+1} a_{nn}^{-\delta k} \sum_{v=1}^{n-1} \hat{a}_{n,v+1} X_v^k |\Delta\lambda_v| \left\{ (a_{nn})^{-1} \sum_{v=1}^{n-1} \hat{a}_{n,v+1} |\Delta\lambda_v| \right\}^{k-1}.$$

Since,

$$\hat{a}_{n,v+1} = \sum_{r=v+1}^{n} (a_{nr} - a_{n-1,r}) = \sum_{r=0}^{n} (a_{n-1,r} - a_{n,r})$$

$$\leqq \sum_{r=0}^{n-1} (a_{n-1,r} - a_{nr}) = \bar{a}_{n-1,0} - \bar{a}_{n0} + a_{nn} = a_{nn}.$$

$$\Rightarrow \sum_{v=1}^{n-1} \hat{a}_{n,v+1} |\Delta\lambda_v| \leqq a_{nn} \sum_{v=1}^{n-1} |\Delta\lambda_v| = O(a_{nn}).$$

$$I_1 = O(1)\sum_{n=2}^{m+1} a_{nn}^{-\delta k}\sum_{v=1}^{n-1}\hat{a}_{n,v+1}X_v^k|\Delta\lambda_v|$$

$$= O(1)\sum_{v=1}^{m} X_v^k|\Delta\lambda_v|\sum_{n=v+1}^{m+1}\hat{a}_{n,v+1}a_{nn}^{-\delta k}$$

$$= O(1)\sum_{v=1}^{m} X_v^k|\Delta\lambda_v|v^{\delta k}$$

$$= O(1).$$

Using Hölder inequality, and (3.3), (3.4), (3.6), (3.7),

$$I_2 = \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k}|T_{n,2}|^k$$

$$\leqq \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k}\left\{\sum_{v=1}^{n-1}\hat{a}_{n,v+1}|\lambda_{v+1}||\Delta x_v||s_v|\right\}^k$$

$$= O(1)\sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k}\left\{\sum_{v=1}^{n-1}\hat{a}_{n,v+1}|\lambda_{v+1}|a_{vv}X_v\right\}^k$$

$$= O(1)\sum_{n=2}^{m+1}(a_{nn})^{-\delta k}\sum_{v=1}^{n-1}\hat{a}_{n,v+1}|\lambda_{v+1}|^k a_{vv}X_v^k\left\{(a_{nn})^{-1}\sum_{v=1}^{n-1}a_{vv}\hat{a}_{n,v+1}\right\}^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}(a_{nn})^{-\delta k}\sum_{v=1}^{n-1}\hat{a}_{n,v+1}|\lambda_{v+1}|^k a_{vv}X_v^k$$

$$= O(1)\sum_{v=1}^{m} a_{vv}X_v^k|\lambda_{v+1}|^k\sum_{n=v+1}^{m+1}a_{nn}^{-\delta k}\hat{a}_{n,v+1}$$

$$= O(1)\sum_{v=1}^{m} a_{vv}X_v^k|\lambda_{v+1}|^k v^{\delta k}$$

$$= O(1)\sum_{v=1}^{m}\frac{1}{v}X_v^k|\lambda_{v+1}|^k v^{\delta k}$$

$$= O(1).$$

Using Hölder inequality, and (3.1), (3.2),

$$I_3 = \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k} |T_{n,3}|^k$$

$$\leqq \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k} \left\{ \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_v| X_v |s_v| \right\}^k$$

$$= O(1) \sum_{n=2}^{m+1} a_{nn}^{1-k-\delta k} \left\{ \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_v| X_v \right\}^k$$

$$= O(1) \sum_{n=2}^{m+1} a_{nn}^{-\delta k} \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_v|^k X_v^k \left\{ (a_{nn})^{-1} \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}| \right\}^{k-1}.$$

We know

$$\sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}| = \sum_{v=1}^{n-1} (a_{n-1,v} - a_{nv})$$

$$= \bar{a}_{n-1,0} - \bar{a}_{n,0} + a_{n0} - a_{n-1,0} + a_{nn}$$

$$= a_{n0} - a_{n-1,0} + a_{nn} \leq a_{nn}.$$

$$I_3 = O(1) \sum_{n=2}^{m+1} a_{nn}^{-\delta k} \sum_{v=1}^{n-1} |\bar{\Delta} a_{nv}||\lambda_v|^k X_v^k$$

$$= O(1) \sum_{v=1}^{m} |\lambda_v|^k X_v^k \sum_{n=v+1}^{m+1} a_{nn}^{-\delta k} |\bar{\Delta} a_{nv}|$$

$$= O(1) \sum_{v=1}^{m} |\lambda_v|^k X_v^k v^{\delta k} a_{vv}$$

$$= O(1).$$

Finally, using (3.7),

$$I_4 = \sum_{n=1}^{\infty} a_{nn}^{1-k-\delta k}|T_{n,4}|$$

$$\leqq \sum_{n=1}^{\infty} a_{nn}^{1-k-\delta k}\left\{a_{nn}|\lambda_n|X_n|s_n|\right\}^k$$

$$= O(1)\sum_{n=1}^{\infty} a_{nn}^{1-k-\delta k}\left\{a_{nn}|\lambda_n|X_n\right\}^k$$

$$= O(1)\sum_{n=1}^{\infty}(a_{nn})^{-\delta k}a_{nn}|\lambda|^k X_n^k$$

$$= O(1)\sum_{n=1}^{\infty}(a_{nn})^{-\delta k}|\lambda|^k X_n^k\frac{1}{n}$$

$$= O(1).$$

Thus the proof of the above Lemma is established.

Proof of the Theorem 3.1. Since the convergence of the Fourier series at a point is a local property of its generating function $f(t)$, the theorem follows by formula from chapter II of the book (see details [17]) and from the above Lemma 3.1.

Applications. Now we apply the theorem to the weighted mean in which $\mathcal{A} = (a_{nv})$ is defined as $a_{nv} = p_v P_n^{-1}$, when $(0 \leqq v \leqq n)$ where $P_n = p_0 + p_1 + ... + p_n$; therefore, it is well known that

$$\bar{a}_{nv} = P_n^{-1}(P_n - P_{v-1})$$

and

$$\hat{a}_{n,v+1} = (P_n P_{n-1})^{-1} p_n P_v.$$

One can now easily verify that taking $\delta = 0$ the conditions of the theorem reduce to those of Theorem 2.3.

We may now ask weather there are some examples (other then weighted mean methods) of matrices $\mathcal{A}$ that satisfy the hypotheses of the theorem. For this, apply the theorem to the Cesàro method of order α with $(0 \leqq \alpha \leqq 1)$ in which $\mathcal{A}$ is given by [15]

$$a_{nv} = \frac{\mathcal{A}_{n-v}^{\alpha-1}}{\mathcal{A}_n^{\alpha}}.$$

It is well known that

$$\bar{a}_{nv} = \frac{\mathcal{A}_{n-v}^{\alpha}}{\mathcal{A}_n^{\alpha}}$$

and

$$\hat{a}_{nv} = \frac{v\mathcal{A}_{n-v}^{\alpha-1}}{n\mathcal{A}_n^{\alpha}}.$$

It is now seen that by taking account of $\mathcal{A}_n^{\alpha} \approx \frac{n^{\alpha}}{\Gamma(\alpha+1)}$ conditions (3.1)-(3.8) are satisfied. Therefore the above theorem is same as the following result.

Corollary 3.1. *Let $k \geq 1$ and $0 \leq \alpha \leq 1$. If (λ_n) a convex sequence satisfying following conditions:*

$$\sum_{n=1}^{\infty} n^{\alpha k-\alpha-k}\{|\lambda|^k + |\lambda_{n+1}|^k\}n^{\delta k} \leqq \infty,$$

$$\sum_{n=1}^{\infty} |\Delta\lambda_n| n^{\delta k} \leqq \infty.$$

Then the $|C, \alpha, (\alpha-1)(1-\frac{1}{k})\delta|_k$ summability of a factored Fourier series $\sum_{n=1}^{\infty} \lambda_n X_n \mathcal{A}_n(t)$ with $X_n = \frac{\mathcal{A}^{\alpha}}{n}$ at any point $t = x$ is a local property of the generating function $f(t)$.

4. Conclusion

The result obtained here is more general in the sense that, by substituting $\delta = 0$, the $|\mathcal{A}; \delta|_k$-summability reduces to $|\mathcal{A}|_k$-summability.

Acknowledgment

The authors would like to express their heartfelt thanks to the editors and anonymous referees for their most valuable comments and constructive suggestions which leads to the significant improvement of the earlier version of the manuscript.

References

[1] S. N. Bhatt, An aspect of local property of $|R, log, 1|$ summability of the factored Fourier series, Proc. Natl. Inst. India 26 (1960), 69-73.

[2] H. Bor, A note on local property of factored Fourier series, J. Non. Anal. 64 (2006), 513-517.

[3] H. Bor, Local property of $|\bar{N}, p_n|_k$-summability of factored Fourier series, Bull. Inst. Math. Acad. Sinica 17 (1989), 165-170.

[4] H. Bor, On the local property of $|\bar{N}, p_n|_k$-summability of factored Fourier series, J. Math. Anal. 163 (1992), 220-226.

[5] Deepmala, Piscoran Laurian-Ioan, Approximation of signals (functions) belonging to certain Lipschitz classes by almost Riesz means of its Fourier series, J. Inequal. Appl. 2016 (2016), Art. ID 163.

[6] T. M. Fleet, On an extension of absolute summability and some theorems of Littlewood and paley, Proc. London Math. Soc. 37 (1957), 113-141.

[7] K. Matsumoto, Local property of the summability $|\mathcal{R}, p_n, 1|$, Tohoku Math. J 8 (1956), 114-124.

[8] K. N. Mishra, Multipliers for $|\bar{\mathcal{N}}, p_n|$-summability of Fourier series, Bull.Inst. Math. Acad. Sinica 14 (1984), 431-438.

[9] V. N. Mishra, Some Problems on Approximations of Functions in Banach Spaces, Ph.D. Thesis (2007), Indian Institute of Technology, Roorkee 247 667, Uttarakhand, India.

[10] V. N. Mishra, L. N. Mishra, Trigonometric Approximation of Signals (Functions) in $L_p(p \geq 1)$- norm, Int. J. Contem. Math. Sci. 7 (2012), 909-918.

[11] V. N. Mishra, S. K. Paikray, P. Palo, P. N. Samanta, M. Misra, U. K. Misra, On double absolute factorable matrix summability, Tbilisi Math. J. 10 (2017), 29-44.

[12] R. Mohanty, On the summability $|\mathcal{R}, logw, 1|$ of Fourier series, J. London Math. soc. 25 (1950), 67-72.

[13] M. A. Sariögol, On local property of $|\mathcal{A}|_k$-summability of factored Fourier series, J. Math. Anal. Appl. 188 (1994), 118-127.

[14] W. T. Sulaiman, On local property of absolute weighted mean summability of Fourier series, Bull. Math. Anal. Appl. 4 (2011), 163-168.

[15] D. S Yu and P. Zhou, A new class of matrices and its applications to absolute summability factor theorems, Math. Comput. Model. 57 (2013), 401-412.

[16] E. C. Tichmarsh, The Theory of Functions, Oxford University Press, London, (1961).

[17] A. Zygmund, Trigonometric series, vol. I, Cambridge Univ. Press, Cambridge, (1959).