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IDEAL CONVERGENT SEQUENCE SPACES WITH RESPECT TO INVARIANT MEAN AND A MUSIELAK-ORLICZ FUNCTION OVER n-NORMED SPACES

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ABSTRACT. In the present paper we defined $\mathcal{I}$-convergent sequence spaces with respect to invariant mean and a Musielak-Orlicz function $\mathcal{M} = (M_k)$ over n-normed spaces. We also make an effort to study some topological properties and prove some inclusion relation between these spaces.

1. INTRODUCTION AND PRELIMINARIES

Let σ be an injective mapping from the set of the positive integers to itself such that $\sigma^p(n) \neq n$ for all positive integers n and p, where $\sigma^p(n) = \sigma(\sigma^{p-1}(n))$. An invariant mean or a σ-mean is a continuous linear functional defined on the space ℓ_∞ such that for all $x = (x_n) \in \ell_\infty$:

(1) If $x_n \geq 0$ for all n, then $\phi(x) \geq 0$,
(2) $\phi(e) = 1$,
(3) $\phi(Sx) = \phi(x)$, where $Sx = (x_{\sigma(n)})$.

V_σ denotes the set of bounded sequences all of whose invariant means are equal which is also called as the space of σ-convergent sequences. In [26], it is defined by

$$V_\sigma = \Big\{ x \in \ell_\infty : \lim_k t_{kn}(x) = \ell, \quad \text{uniformly in } \ n, \ell = \sigma - \lim x \Big\},$$

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where $t_{kn}(x) = \frac{x_n + x_{\sigma^1(n)} + \cdots + x_{\sigma^k(n)}}{k+1}$.

σ-mean is called a Banach limit if σ is the translation mapping $n \to n+1$. In this case, V_σ becomes the set of almost convergent sequences which is denoted by $\hat{c}$ and defined in [11] as

$$\hat{c} = \Big\{ x \in \ell_\infty : \lim_k d_{kn}(x) \text{ exists uniformly in } n \Big\},$$

where $d_{kn}(x) = \frac{x_n + x_{n+1} + \cdots + x_{n+k}}{k+1}$.

The space of strongly almost converegnt sequences was introduced by Maddox [12] as follow:

$$\hat{c} = \Big\{ x \in \ell_\infty : \lim_k d_{kn}(|x - \ell e|) \text{ exists uniformly in } n \text{ for some } \ell \Big\}.$$

The notion of ideal convergence was first introduced by P. Kostyrko [8] as a generalization of statistical convergence which was further studied in topological spaces by Das, Kostyrko, Wilczynski and Malik see [1]. More applications of ideals can be seen in ([1], [2]). Mursaleen and Sharma [19] continue in this direction and introduced I-convergence of generalized sequences with respect to Musielak-Orlicz function.

A family $\mathcal{I} \subset 2^X$ of subsets of a non empty set X is said to be an ideal in X if

(1) $\phi \in \mathcal{I}$

(2) $A, B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$

(3) $A \in \mathcal{I}$, $B \subset A$ imply $B \in \mathcal{I}$,

while an admissible ideal $\mathcal{I}$ of X further satisfies $\{x\} \in \mathcal{I}$ for each $x \in X$ see [8].

A sequence $(x_n)_{n\in\mathbb{N}}$ in X is said to be $\mathcal{I}$-convergent to $x \in X$, if for each $\epsilon > 0$ the set $A(\epsilon) = \Big\{ n \in \mathbb{N} : ||x_n - x|| \geq \epsilon \Big\}$ belongs to $\mathcal{I}$.

A sequence $(x_n)_{n\in\mathbb{N}}$ in X is said to be $\mathcal{I}$-bounded to $x \in X$ if there exists an $K > 0$ such that $\{n \in \mathbb{N} : |x_n| > K\} \in \mathcal{I}$. For more details about ideal convergence sequence spaces (see [7], [9], [15], [16], [17], [18], [21], [25], [26], [27]) and references therein.

Let $A = A_{ij}$ be an infinite matrix of complex numbers a_{ij}, where $i, j, \in \mathbb{N}$. We write $Ax = (A_i(x))$ if $A_i(x) = \sum_{j=1}^{\infty} a_{ij} x_j$ converges for each $i \in \mathbb{N}$. Throughout the paper, by $t_{kn}(Ax)$, we mean

$$t_{kn}(Ax) = \frac{A_n(x) + A_{\sigma^1(n)}(x) + \cdots , A_{\sigma^k(n)}(x)}{k+1}, \text{ for all } k, n \in \mathbb{N}.$$

A sequence space X is called as solid (or normal) if $(\alpha_k x_k) \in X$ whenever $(x_k) \in X$ and (α_k) is a sequence of scalars such that $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$.

Let X be a sequence space and $K = \{k_1 < k_2 < \cdots\} \subseteq \mathbb{N}$. The sequence space $Z_K^X = \{(x_{kn}) \in w : (x_n) \in X\}$ is called K-step space of X.

A canonical preimage of a sequence $(x_{kn}) \in Z_K^X$ is a sequence $(y_n) \in w$ defined by

$$y_n = \begin{cases} x_n, & \text{if } n \in \mathbb{N}; \\ 0, & \text{otherwise.} \end{cases}$$

A sequence space X is monotone if it contains the canonical preimages of all its step spaces.

An Orlicz function M is a function, which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \longrightarrow \infty$ as $x \longrightarrow \infty$.

Lindenstrauss and Tzafriri [10] used the idea of Orlicz function to define the following sequence space. Let w be the space of all real or complex sequences $x = (x_k)$, then

$$\ell_M = \Big\{x \in w : \sum_{k=1}^{\infty} M\Big(\frac{|x_k|}{\rho}\Big) < \infty\Big\}$$

which is called as an Orlicz sequence space. The space ℓ_M is a Banach space with the norm

$$||x|| = \inf\Big\{\rho > 0 : \sum_{k=1}^{\infty} M\Big(\frac{|x_k|}{\rho}\Big) \leq 1\Big\}.$$

It is shown in [10] that every Orlicz sequence space ℓ_M contains a subspace isomorphic to $\ell_p (p \geq 1)$. The Δ_2-condition is equivalent to $M(Lx) \leq kLM(x)$ for all values of $x \geq 0$, and for $L > 1$.

A sequence $\mathcal{M} = (M_k)$ of Orlicz function is called a Musielak-Orlicz function see ([13],[20]). A sequence $\mathcal{N} = (N_k)$ defined by

$$N_k(v) = \sup\{|v|u - (M_k) : u \geq 0\},\ k = 1, 2, \cdots$$

is called the complementary function of a Musielak-Orlicz function $\mathcal{M}$. For a given Musielak-Orlicz function $\mathcal{M}$, the Musielak-Orlicz sequence space $t_{\mathcal{M}}$ and its subspace $h_{\mathcal{M}}$ are defined as follows

$$t_{\mathcal{M}} = \Big\{x \in w : I_{\mathcal{M}}(cx) < \infty \text{ for some } c > 0\Big\},$$

$$h_{\mathcal{M}} = \Big\{x \in w : I_{\mathcal{M}}(cx) < \infty \text{ for all } c > 0\Big\},$$

where $I_{\mathcal{M}}$ is a convex modular defined by

$$I_{\mathcal{M}}(x) = \sum_{k=1}^{\infty} M_k(x_k), x = (x_k) \in t_{\mathcal{M}}.$$

We consider $t_{\mathcal{M}}$ equipped with the Luxemburg norm

$$||x|| = \inf\Big\{k > 0 : I_{\mathcal{M}}\Big(\frac{x}{k}\Big) \leq 1\Big\}$$

or equipped with the Orlicz norm

$$||x||^0 = \inf\Big\{\frac{1}{k}\Big(1 + I_{\mathcal{M}}(kx)\Big) : k > 0\Big\}.$$

For more details about sequence spaces defined by Orlicz function see ([22], [23], [24]) and reference therein.

The concept of 2-normed spaces was initially developed by Gähler[3] in the mid of 1960's, while that of

n-normed spaces one can see in Misiak [14]. Since then, many others have studied this concept and obtained various results, see Gunawan ([4],[5]) and Gunawan and Mashadi [6]. Let $n \in \mathbb{N}$ and X be a linear space over the field $\mathbb{K}$, where $\mathbb{K}$ is field of real or complex numbers of dimension d, where $d \geq n \geq 2$. A real valued function $||\cdot, \cdots, \cdot||$ on X^n satisfying the following four conditions:

(1) $||x_1, x_2, \cdots, x_n|| = 0$ if and only if $x_1, x_2, \cdots, x_n$ are linearly dependent in X;

(2) $||x_1, x_2, \cdots, x_n||$ is invariant under permutation;

(3) $||\alpha x_1, x_2, \cdots, x_n|| = |\alpha| \ \ ||x_1, x_2, \cdots, x_n||$ for any $\alpha \in \mathbb{K}$, and

(4) $||x + x', x_2, \cdots, x_n|| \leq ||x, x_2, \cdots, x_n|| + ||x', x_2, \cdots, x_n||$

is called a n-norm on X, and the pair $(X, ||\cdot, \cdots, \cdot||)$ is called a n-normed space over the field $\mathbb{K}$.
For example, we may take $X = \mathbb{R}^n$ being equipped with the Euclidean n-norm $||x_1, x_2, \cdots, x_n||_E =$ the volume of the n-dimensional parallelopiped spanned by the vectors $x_1, x_2, \cdots, x_n$ which may be given explicitly by the formula

$$||x_1, x_2, \cdots, x_n||_E = |\det(x_{ij})|,$$

where $x_i = (x_{i1}, x_{i2}, \cdots, x_{in}) \in \mathbb{R}^n$ for each $i = 1, 2, \cdots, n$. Let $(X, ||\cdot, \cdots, \cdot||)$ be a n-normed space of dimension $d \geq n \geq 2$ and $\{a_1, a_2, \cdots, a_n\}$ be linearly independent set in X. Then the following function $||\cdot, \cdots, \cdot||_\infty$ on X^{n-1} defined by

$$||x_1, x_2, \cdots, x_{n-1}||_\infty = \max\{||x_1, x_2, \cdots, x_{n-1}, a_i|| : i = 1, 2, \cdots, n\}$$

defines an $(n-1)$-norm on X with respect to $\{a_1, a_2, \cdots, a_n\}$.
A sequence (x_k) in a n-normed space $(X, ||\cdot, \cdots, \cdot||)$ is said to converge to some $L \in X$ if

$$\lim_{k \to \infty} ||x_k - L, z_1, \cdots, z_{n-1}|| = 0 \ \text{ for every } \ z_1, \cdots, z_{n-1} \in X.$$

A sequence (x_k) in a n-normed space $(X, ||\cdot, \cdots, \cdot||)$ is said to be Cauchy if

$$\lim_{k,p \to \infty} ||x_k - x_p, z_1, \cdots, z_{n-1}|| = 0 \ \text{ for every } \ z_1, \cdots, z_{n-1} \in X.$$

If every cauchy sequence in X converges to some $L \in X$, then X is said to be complete with respect to the n-norm. Any complete n-normed space is said to be n-Banach space.

In the present paper, we define some new sequence spaces by using the concept of ideal convergence, invariant mean, Musielak-Orlicz function, n-normed and A transform as follows:
$\mathcal{I} - c_0^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) =$

$$\Big\{x \in w : \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq \epsilon\Big\} \in \mathcal{I}, \ \text{ for all } \ n \in \mathbb{N}\Big\},$$

$\mathcal{I}-c^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)=$

$$\Big\{x\in w:\Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}(A(x)-L)}{\rho},z_1,\cdots,z_{n-1}||\Big)\Big]^{p_k}\geq\epsilon\Big\}\in\mathcal{I},\ \text{for all}\ n\in\mathbb{N}\ \&\ \text{for some}\ L\in\mathbb{C}\Big\},$$

$\mathcal{I}-\ell_{\infty}^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)=$

$$\Big\{x\in w:\exists\ K>0\ \text{such that}\ \Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho},z_1,\cdots,z_{n-1}||\Big)\Big]^{p_k}\geq K\Big\}\in\mathcal{I},\ \text{for all}\ n\in\mathbb{N}\Big\}.$$

If we take $p=(p_k)=1$, we get the spaces

$\mathcal{I}-c_0^{\sigma}(A,\mathcal{M},||\cdot,\cdots,\cdot||)=$

$$\Big\{x\in w:\Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho},z_1,\cdots,z_{n-1}||\Big)\Big]\geq\epsilon\Big\}\in\mathcal{I},\ \text{for all}\ n\in\mathbb{N}\Big\},$$

$\mathcal{I}-c^{\sigma}(A,\mathcal{M},||\cdot,\cdots,\cdot||)=$

$$\Big\{x\in w:\Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}(A(x)-L)}{\rho},z_1,\cdots,z_{n-1}||\Big)\Big]\geq\epsilon\Big\}\in\mathcal{I},\ \text{for all}\ n\in\mathbb{N}\ \&\ \text{for some}\ L\in\mathbb{C}\Big\},$$

$\mathcal{I}-\ell_{\infty}^{\sigma}(A,\mathcal{M},||\cdot,\cdots,\cdot||)=$

$$\Big\{x\in w:\exists\ K>0\ \text{such that}\ \Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}A(x)}{\rho},z_1,\cdots,z_{n-1}||\Big)\Big]\geq K\Big\}\in\mathcal{I},\ \text{for all}\ n\in\mathbb{N}\Big\}.$$

The following inequality will be used throughout the paper. If $0\leq p_k\leq\sup p_k=H$, $D=\max(1,2^{H-1})$ then

$$|a_k+b_k|^{p_k}\leq D\{|a_k|^{p_k}+|b_k|^{p_k}\}\tag{1.1}$$

for all k and $a_k,b_k\in\mathbb{C}$. Also $|a|^{p_k}\leq\max(1,|a|^H)$ for all $a\in\mathbb{C}$.

The main goal of this paper is to introduce the sequence spaces $\mathcal{I}-c_0^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$, $\mathcal{I}-c^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$ and $\mathcal{I}-\ell_{\infty}^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$ defined by a Musielak-Orlicz function $\mathcal{M}=(M_k)$ over n-normed spaces. We also make an effort to study some topological properties and prove some inclusion relation between these spaces.

2. Main Results

Theorem 2.1 *Let $\mathcal{M}=(M_k)$ be a Musielak-Orlicz function, $p=(p_k)$ be a bounded sequence of positive real numbers. Then the spaces $\mathcal{I}-c_0^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$, $\mathcal{I}-c^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$ and $\mathcal{I}-\ell_{\infty}^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$ are linear.*

Proof. Let $x,y\in\mathcal{I}-c_0^{\sigma}(A,\mathcal{M},p,||\cdot,\cdots,\cdot||)$ and let α,β be scalars. Then there exist positive numbers ρ_1 and ρ_2 such that for every $\epsilon>0$

$$D_1=\Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho_1},z_1,\cdots,z_{n-1}||\Big)\Big]^{p_k}\geq\frac{\epsilon}{2D}\Big\}\in\mathcal{I},\tag{2.1}$$

$$D_1=\Big\{k\in\mathbb{N}:\Big[M_k\Big(||\frac{t_{kn}(A(y))}{\rho_2},z_1,\cdots,z_{n-1}||\Big)\Big]^{p_k}\geq\frac{\epsilon}{2D}\Big\}\in\mathcal{I},\tag{2.2}$$

Let $\rho_3 = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Since $\mathcal{M} = (M_k)$ is non-decreasing, convex function and so by using inequality (1.1), we have

$$\begin{aligned}
&\Big[M_k\Big(||\tfrac{t_{kn}(A(\alpha x+\beta y))}{\rho_3}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \\
&\leq \Big[M_k\Big(||\frac{t_{kn}(\alpha A(x))}{\rho_3}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} + \Big[M_k\Big(||\frac{t_{kn}(\beta A(y))}{\rho_3}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \\
&\leq \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} + \Big[M_k\Big(||\frac{t_{kn}(A(y))}{\rho_2}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}
\end{aligned}$$

Now by (2.1) and (2.2), we have

$$\Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(\alpha x+\beta y))}{\rho_3}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} > \epsilon\Big\} \subset D_1 \cup D_2.$$

Therefore $\alpha x + \beta y \in \mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. Hence $\mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$ is a linear space. Similarly we can prove that $\mathcal{I} - c^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$ and $\mathcal{I} - \ell_{\infty}^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$ are linear spaces. □

Theorem 2.2 *Let $\mathcal{M} = (M_k)$ be a Musielak-Orlicz function. Then*

$$\mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \subset \mathcal{I} - c^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \subset \mathcal{I} - \ell_{\infty}^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||).$$

Proof. The first inclusion is obvious. For second inclusion, let $x \in \mathcal{I} - c^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. Then there exists $\rho_1 > 0$ such that for every $\epsilon > 0$

$$A_1 = \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x) - L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq \epsilon\Big\} \in \mathcal{I}.$$

Let us define $\rho = 2\rho_1$. Since $\mathcal{M} = (M_k)$ is non-decreasing and convex, we have

$$M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big) \leq M_k\Big(||\frac{t_{kn}(A(x) - L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big) + M_k\Big(||\frac{t_{kn}(L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big).$$

Suppose that $k \notin A_1$. Hence by above inequality and (1.1), we have

$$\begin{aligned}
&\Big[M_k\Big(||\tfrac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \\
&\leq D\Big\{\Big[M_k\Big(||\frac{t_{kn}(A(x) - L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} + \Big[M_k\Big(||\frac{t_{kn}(L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}\Big\} \\
&< D\Big\{\epsilon + \Big[M_k\Big(||\frac{t_{kn}(L)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}\Big\}.
\end{aligned}$$

Because of the fact that $\Big[M_k\Big(||\frac{t_{kn}(L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq \max\Big\{1, \Big[M_k\Big(||\frac{t_{kn}(L)}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{H}\Big\}$, we have

$$\Big[M_k\Big(||\frac{t_{kn}(L)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} < \infty.$$

Put $K = D\Big\{\epsilon + \Big[M_k\Big(||\frac{t_{kn}(L)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}\Big\}$. It follows that

$$\Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} > K\Big\} \in \mathcal{I}$$

which means $x \in \mathcal{I} - \ell_\infty^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. This completes the proof of the theorem. □

Theorem 2.3 *Let $\mathcal{M} = (M_k)$ be a Musielak-Orlicz function, $p = (p_k)$ be a bounded sequence of positive real numbers. Then $\mathcal{I} - \ell_\infty^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$ is a paranormed space with paranorm defined by*

$$g(x) = \inf\Big\{\rho > 0 : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1\Big\}.$$

Proof. It is clear that $g(x) = g(-x)$. Since $M_k(0) = 0$, we get $g(0) = 0$. Let us take $x, y \in \mathcal{I} - c_\infty^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. Let

$$B(x) = \Big\{\rho > 0 : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1\Big\},$$

$$B(y) = \Big\{\rho > 0 : \Big[M_k\Big(||\frac{t_{kn}(A(y))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1\Big\}.$$

Let $\rho_1 \in B(x)$ and $\rho_2 \in B(y)$. If $\rho = \rho_1 + \rho_2$, then we have
$\Big[M_k\Big(||\frac{t_{kn}(A(x+y))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]$

$$\leq \Big(\frac{\rho_1}{\rho_1 + \rho_2}\Big)\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big] + \Big[M_k\Big(||\frac{t_{kn}(A(y))}{\rho_2}, z_1, \cdots, z_{n-1}||\Big)\Big].$$

Thus $\Big[M_k\Big(||\frac{t_{kn}(A(x+y))}{\rho_1+\rho_2}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1$ and

$$\begin{aligned} g(x+y) &\leq \inf\Big\{(\rho_1 + \rho_2) > 0 : \rho_1 \in B(x),\ \rho_2 \in B(y)\Big\} \\ &\leq \inf\Big\{\rho_1 > 0 : \rho_1 \in B(x)\Big\} + \inf\Big\{\rho_2 > 0 : \rho_2 \in B(y)\Big\} \\ &= g(x) + g(y). \end{aligned}$$

Let $\eta^s \to \eta$ where $\eta, \eta^s \in \mathbb{C}$ and let $g(x^s - x) \to 0$ as $s \to \infty$. We have to show that $g(\eta^s x^s - \eta x) \to 0$ as $s \to \infty$. Let

$$B(x^s) = \Big\{\rho_s > 0 : \Big[M\Big(||\frac{t_{kn}(A(x^s))}{\rho_s}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1\Big\},$$

$$B(x^s - x) = \Big\{\rho'_s > 0 : \Big[M\Big(||\frac{t_{kn}(A(x^s - x))}{\rho'_s}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1\Big\}.$$

If $\rho_s \in B(x^s)$ and $\rho'_s \in B(x^s - x)$ then we observe that
$\Big[M_k\Big(||\frac{t_{kn}(A(\eta^s x^s - \eta x))}{\rho_s|\eta^s-\eta|+\rho'_s|\eta|}, z_1, \cdots, z_{n-1}||\Big)$

$$\begin{aligned}
&\leq \Big[M_k\Big(||\frac{t_{kn}(A(\eta^s x^s - \eta x^s))}{\rho_s|\eta^s-\eta|+\rho'_s|\eta|} + \frac{|(\eta x^s - \eta x)|}{\rho_s|\eta^s-\eta|+\rho'_s|\eta|}, z_1, \cdots, z_{n-1}||\Big)\Big] \\
&\leq \frac{|\eta^s-\eta|\rho_s}{\rho_s|\eta^s-\eta|+\rho'_s|\eta|}\Big[M_k\Big(||\frac{t_{kn}(A(x^s))}{\rho_s}, z_1, \cdots, z_{n-1}||\Big)\Big] \\
&+ \frac{|\eta|\rho'_s}{\rho_s|\eta^s-\eta|+\rho'_s|\eta|}\Big[M_k\Big(||\frac{t_{kn}(A(x^s - x))}{\rho'_s}, z_1, \cdots, z_{n-1}||\Big).
\end{aligned}$$

From the above inequality, it follows that

$$\Big[M_k\Big(||\frac{t_{kn}(A(\eta^s x^s - \eta x))}{\rho_s|\eta^s-\eta|+\rho'_s|\eta|}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq 1$$

and consequently,

$$\begin{aligned}
g(\eta^s x^s - \eta x) &\leq \inf\Big\{\Big(\rho_s|\eta^s-\eta| + \rho'_s|\eta|\Big) > 0 : \rho_s \in B(x^s), \rho'_s \in B(x^s - x)\Big\} \\
&\leq (|\eta^s - \eta|) > 0 \inf\Big\{\rho > 0 : \rho_s \in B(x^s)\Big\} \\
&+ (|\eta|) > 0 \inf\Big\{(\rho'_s)^{\frac{p_n}{H}} : \rho'_s \in B(x^s - x)\Big\} \\
&\longrightarrow 0 \text{ as } s \longrightarrow \infty.
\end{aligned}$$

This completes the proof of the theorem. □

Theorem 2.4 *Let $\mathcal{M}' = (M'_k)$ and $\mathcal{M}'' = (M''_k)$ are Musielak-Orlicz functions that satisfies the Δ_2-condition. Then*
(i) $\mathcal{I} - c_0^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c_0^\sigma(A, \mathcal{M}' \circ \mathcal{M}'', p, ||\cdot, \cdots, \cdot||)$
(ii) $\mathcal{I} - c^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c^\sigma(A, \mathcal{M}' \circ \mathcal{M}'', p, ||\cdot, \cdots, \cdot||)$
(iii) $\mathcal{I} - l_\infty^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - l_{\infty_\theta}^\sigma(A, \mathcal{M}' \circ \mathcal{M}'', p, ||\cdot, \cdots, \cdot||)$.

Proof. (i) We prove the theorem in two parts. Firstly, let $M'_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big) > \delta$. Since $\mathcal{M}'$ is nondecreasing, convex and satisfies Δ_2-condition, we have
$\Big[M''_k\Big(M'_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big)\Big]^{p_k}$

$$\begin{aligned}
&\leq (K\delta^{-1}M''_2(2)^{p_k})\Big[M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \\
&\leq \max\{1, (K\delta^{-1}M''_k(2)^H)\}^H\Big[M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}\Big)\Big]^{p_k},
\end{aligned}$$

where $K \geq 1$ and $\delta < 1$. From the last inequality, the inclusion
$\Big\{k \in \mathbb{N} : \Big[M''_k\Big(M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big)\Big]^{p_k} \geq \epsilon\Big\}$

$$\begin{aligned} \subseteq & \Big\{k \in \mathbb{N} : \Big[M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}\Big)\Big]^{p_k} \\ \geq & \frac{\epsilon}{\max\{1, (K\delta^{-1}M''_k(2)^H)\}}\Big\} \end{aligned}$$

is obtained. If $x \in \mathcal{I} - c_0^\sigma(\mathcal{M}', A, p, ||\cdot, \cdots, \cdot)$, then the set in the right side of the above inclusion belongs to the ideal and so

$$\Big\{k \in \mathbb{N} : \Big[M''_k\Big(M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1|}|\Big)\Big)\Big]^{p_k} \geq \epsilon\Big\} \in \mathcal{I}.$$

Secondly, suppose that $M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}||\Big) \leq \delta$. Since M''_k is continuous, we have

$$M''_k\Big(M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big) < \epsilon \text{ for all } \epsilon > 0$$

which implies

$$\mathcal{I} - \lim_k \Big[M''_k\Big(M'_k\Big(||\frac{t_{kn}(Ax)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big)\Big]^{p_k} = 0 \quad \text{as}\epsilon \to 0.$$

This completes the proof of (i) part. Similarly, we can prove other parts. □

Theorem 2.5 *Let $\mathcal{M}' = (M'_k)$ and $\mathcal{M}'' = (M''_k)$ are Musielak-Orlicz functions that satisfies the Δ_2-condition. Then*
(i) $\mathcal{I} - c_0^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \cap \mathcal{I} - c_0^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c_0^\sigma(A, \mathcal{M}' + \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$
(ii) $\mathcal{I} - c^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \cap \mathcal{I} - c^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c^\sigma(A, \mathcal{M}' + \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$
(iii) $\mathcal{I} - l_\infty^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \cap \mathcal{I} - l_\infty^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - l_\infty^\sigma(A, \mathcal{M}' + \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$.

Proof. (i) Let $x \in \mathcal{I} - c_0^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \cap \mathcal{I} - c_0^\sigma(A, \mathcal{M}', p, ||\cdot, \cdots, \cdot||)$. Then there exists $K_1 > 0$ and $K_2 > 0$ such that

$$A_1 = \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq K_1\Big\} \in \mathcal{I}$$

and

$$A_2 = \Big\{k \in \mathbb{N} : \Big[M'_k\Big(||\frac{t_{kn}(A(x))}{\rho_1}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq K_2\Big\} \in \mathcal{I}$$

for some $\rho > 0$. Let $k \notin A_1 \cup A_2$. Then we have
$\Big[(M_k + M'_k)\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}$

$$\begin{aligned} \leq & D\Big\{\Big(M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big)^{p_k} \\ + & \Big(M'_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big)^{p_k} \\ < & \{K_1 + K_2\}. \end{aligned}$$

$k \notin B = \Big\{k \in \mathbb{N} : \Big[(M'_k + M_k)\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big)^{p_k} > K\}$. We have $A_1 \cup A_2 \in \mathcal{I}$ and so $B \subset A_1 \cup A_2$ which implies $B \in \mathcal{I}$. This means that $x \in \mathcal{I} - c_0^\sigma(A, \mathcal{M}' + \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. This completes the proof of (i) part of the theorem. Similarly, we can prove (ii) and (iii) part. □

Theorem 2.6 *If* $\sup_k \big[M_k(t)\big]^{p_k} < \infty$ *for all* $t > 0$*, then we have*

$$\mathcal{I} - c^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - \ell_\infty^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||).$$

Proof. Let $x \in \mathcal{I} - c^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. By using inequality (1.1), we have

$$\begin{aligned}\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}\Big)\Big]^{p_k} &\leq D\Big\{\Big[M_k\Big(||\frac{t_{kn}(A(x)-L)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \\ &+ \Big[M_k\Big(||\frac{t_{kn}(L)}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}\Big\},\end{aligned}$$

where $\rho = 2\rho_1$. Hence, we have

$$\Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}\Big)\Big]^{p_k} \geq K\Big\} \subseteq \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x)-L)}{\rho_1}, z_1, \cdots, z_{n-1}\Big)\Big]^{p_k} \geq \epsilon\Big\}$$

for all n and some $K > 0$. Since the set in the right side of the above inclusion belongs to the ideal, all of its subsets are in the ideal. Hence

$$\Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}\Big)\Big]^{p_k} \geq K\Big\} \in \mathcal{I}$$

which completes the proof. □

Theorem 2.7 *Let* $0 < p_k \leq q_k < \infty$ *for each* $k \in \mathbb{N}$ *and* $\Big(\frac{q_k}{p_k}\Big)$ *be bounded. Then following inclusions hold*
(i) $\mathcal{I} - c_0^\sigma(A, \mathcal{M}, q, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c_0^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$
(ii) $\mathcal{I} - c^\sigma(A, \mathcal{M}, q, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c^\sigma(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$.

Proof. (i) Let $x \in \mathcal{I} - c_0^\sigma(A, \mathcal{M}, q, ||\cdot, \cdots, \cdot||)$. Write $\alpha_k = \frac{p_k}{q_k}$. By hypothesis, we have $0 < \alpha \leq \alpha_k \leq 1$. If $\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{q_k} \geq 1$, the inequality

$$\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{q_k}$$

holds. This implies the inclusion
$\Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq \epsilon\Big\}$

$$\subseteq \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{q_k} \geq \epsilon\Big\}$$

and so the result is obvious. Conversely, if $\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{q_k} < 1$, we obtain the following inclusion
$\Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq \epsilon\Big\}$

$$\subseteq \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{q_k} \geq \epsilon^{\frac{1}{\alpha}}\Big\}$$

since then the inequality

$$\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \leq \Big(\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{q_k}\Big)^{\alpha}$$

holds. Hence we conclude that $x \in \mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. This completes the proof of (i) part. Similarly, we can prove (ii) part. □

Theorem 2.8 *If* $0 < \inf p_k \leq p_k \leq 1$ *for each* $k \in \mathbb{N}$*. Then the following inclusions hold:*
(i) $\mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, ||\cdot, \cdots, \cdot||)$
(ii) $\mathcal{I} - c^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c^{\sigma}(A, \mathcal{M}, ||\cdot, \cdots, \cdot||)$.

Proof. Let $x \in \mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$. Suppose that $k \notin \Big\{\Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}\Big)\Big]^{p_k} \geq \epsilon\Big\}$ for $0 < \epsilon < 1$. By hypothesis, the inequality

$$M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big) \leq \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k}$$

holds. Then we have $k \notin \Big\{k \in \mathbb{N} : M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big) \geq \epsilon\Big\}$ which implies
$\Big\{k \in \mathbb{N} : M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big) \geq \epsilon\Big\}$

$$\subseteq \Big\{k \in \mathbb{N} : \Big[M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big)\Big]^{p_k} \geq \epsilon\Big\}.$$

Hence $x \in \mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, ||\cdot, \cdots, \cdot||)$ since the set

$$\Big\{k \in \mathbb{N} : M_k\Big(||\frac{t_{kn}(A(x))}{\rho}, z_1, \cdots, z_{n-1}||\Big) \geq \epsilon\Big\} \in \mathcal{I}.$$

This completes the proof of (i) part. Similarly, we can prove (ii) part. □

Corollary 2.9 *If* $0 < \inf p_k \leq p_k \leq 1$ *for each* $k \in \mathbb{N}$*. Then the following inclusions hold:*
(i) $\mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c_0^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$
(ii) $\mathcal{I} - c^{\sigma}(A, \mathcal{M}, ||\cdot, \cdots, \cdot||) \subseteq \mathcal{I} - c^{\sigma}(A, \mathcal{M}, p, ||\cdot, \cdots, \cdot||)$.

Proof. The proof is obvious by Theorem 2.8. □

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