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ROTATIONAL SURFACES GENERATED BY PLANAR CURVES IN E^3 WITH DENSITY

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Abstract. In this paper, we obtain the parametric expressions of curves which have zero weighted curvature in a plane with density e^{ax+by} and create the Smarandache curves of the obtaining curves. Also, we construct the rotational surfaces which are generated by planar curves with vanishing weighted curvature and give some characterizations for them.

1. Introduction

Differential geometers have studied the curves in a plane for a long time and the differential geometry of curves has been interested widely in different spaces such as Euclidean, Minkowski, Galilean, pseudo Galilean

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and etc. The Frenet vectors and curvature of a curve $\alpha(u) = (x(u), y(u), 0)$ in the plane are (see [15] and [17])

$$\begin{aligned} T &= \frac{1}{\sqrt{x'(t)^2 + y'(t)^2}}(x'(u), y'(u), 0), \\ N &= \mp\frac{1}{\sqrt{x'(u)^2 + y'(u)^2}}(-y'(u), x'(u), 0), \\ B &= \mp(0, 0, 1), \\ \kappa &= \mp\frac{x'(u)y''(u) - x''(u)y'(u)}{\left((x'(u))^2 + (y'(u))^2\right)^{3/2}}. \end{aligned} \tag{1.1}$$

Here we must note that, the sign of "$\mp$" which is stated in the above formulas of Frenet vectors and curvature will be taken as "+" throughout this study.

Recently, new curves have been obtained by using the Frenet frames of curves. One of these new curves is Smarandache curve and if we denote TN-Smarandache curve as $\beta(u)$, TB-Smarandache curve as $\delta(u)$ and NB-Smarandache curve as $\gamma(u)$, then they are given by

$$\beta(u) = \frac{T(u) + N(u)}{\| T(u) + N(u) \|},\ \delta(u) = \frac{T(u) + B(u)}{\| T(u) + B(u) \|}\ \textit{and}\ \gamma(u) = \frac{N(u) + B(u)}{\| N(u) + B(u) \|},$$

respectively. Since the Smarandache curves play an important role in Smarandache geometry, many geometers have been interested in these curves. More information about Smarandache curves can be found in [1], [2], [18], [19].

Also, lots of studies have been done about rotational surface in Euclidean space E^3 and Lorentz-Minkowski space E_1^3 in recent years (see [5], [6], [7], [12] and etc). A rotational surface in Euclidean space is generated by rotating of an arbitrary curve about an arbitrary axis. In this sense, let $\alpha(u) = (x(u), y(u), 0)$ be a regular plane curve in the $xy-$plane which is defined on a open invertal $I \subset \mathbb{R}$. A rotational surface M in E^3 is defined by the following three cases:

If the axes of revolution are x-axis, y-axis and z-axis, then the rotational surfaces are given by

$$\begin{aligned} X(u,v) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos v & -\sin v \\ 0 & \sin v & \cos v \end{pmatrix} \begin{pmatrix} x(u) \\ y(u) \\ 0 \end{pmatrix} \\ &= (x(u), y(u)\cos v, y(u)\sin v), \end{aligned} \tag{1.2}$$

$$X(u,v)=\begin{pmatrix}\cos v & 0 & \sin v\\ 0 & 1 & 0\\ -\sin v & 0 & \cos v\end{pmatrix}\begin{pmatrix}x(u)\\ y(u)\\ 0\end{pmatrix}$$

$$=(x(u)\cos v, y(u), -x(u)\sin v) \tag{1.3}$$

and

$$X(u,v)=\begin{pmatrix}\cos v & -\sin v & 0\\ \sin v & \cos v & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}x(u)\\ y(u)\\ 0\end{pmatrix}$$

$$=(x(u)\cos v - y(u)\sin v, x(u)\sin v + y(u)\cos v, 0)\,, \tag{1.4}$$

respectively.

On the other hand, after Morgan's article [14], manifolds with density have been a very popular topic in many areas for geometers, physicists, economicists. For example, in physics, an object may have differing internal densities so in order to determine the object's mass it is necessary to integrate volume weighted with density. In differential geometry, let we consider a bounded curve on the closed Euclidean half plane (boundary on the x-axis) and the surfaces of revolution formed by rotating that curve about x-axis. Areas and arc lengths on that surface correspond to areas and arc lengths on the half plane with a weighting of $2\pi y$. In government and economics, it is often necessary to consider aggregate properties of groups and subgroups of people. For large groups, these aggregate properties can be determined by integrating over the members of the group, the differing individual properties (much like different densities).

In 2003, Gromov [8] firstly has described κ_φ weighted curvature of curves and H_φ weighted mean curvature of $n-$dimensional hypersurfaces on a manifold with density e^φ as

$$\kappa_\varphi=\kappa-\frac{d\varphi}{dN}$$

and

$$H_\varphi=H-\frac{1}{n-1}\frac{d\varphi}{d\eta},$$

respectively. Here, N is the normal vector, κ is the curvature of the curve; η is the normal vector field, H is the mean curvature of the hypersurfaces and $\langle N\ ,\nabla\varphi\rangle=\frac{d\varphi}{dN}$, $\langle \eta\ ,\nabla\varphi\rangle=\frac{d\varphi}{d\eta}$. Furthermore, Corwin [4] has defined weighted Gaussian curvature of a surface in a manifold with density by

$$G_\varphi=G-\Delta\varphi,$$

where G is Gaussian curvature of the surface and Δ is the Laplacian operator. For another characterizations of manifolds with density, we refer to [3], [9], [10], [11], [13], [16], [20] and etc.

2. Curves with Vanishing Weighted Curvature in a Plane with Density e^{ax+by}

In this section, firstly we construct the weighted curvature of a curve in a plane with positive density e^{ax+by}. After that, we investigate the curves whose weighted curvatures are equal to zero according to the cases of a and b. Followingly, we create Smarandache curves of these curves as examples.

Weighted curvature κ_φ of a curve $\alpha(u) = (x(u), y(u), 0)$ in a plane with density e^{ax+by} is obtained as

$$\kappa_\varphi = \kappa - \frac{d\varphi}{dN}$$

$$= \frac{x'(u)y''(u) - x''(u)y'(u) - (x'(u)^2 + y'(u)^2)(-ay'(u) + bx'(u))}{(x'(u)^2 + y'(u)^2)^{3/2}}. \tag{2.1}$$

So,

Corollary 2.1. *If the curve $\alpha(u) = (x(u), y(u), 0)$ is a unit speed curve, then the weighted curvature κ_φ of the curve $\alpha(u)$ in the plane with density e^{ax+by} is*

$$\kappa_\varphi = x'(u)y''(u) - x''(u)y'(u) + ay'(u) - bx'(u).$$

Proposition 2.1. *Weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in the plane with density e^{ax+by} vanishes if and only if*

$$x'(u)y''(u) + ax'(u)^2y'(u) + ay'(u)^3 = x''(u)y'(u) + bx'(u)y'(u)^2 + bx'(u)^3.$$

Now, we will examine the weighted curvature of a curve in a plane with density e^{ax+by} for different values of constants a and b.

First, let's assume that $b = 0$; i.e. the density of the plane is e^{ax}.
Then from (2.1), the weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in a plane with density e^{ax} is obtained as

$$\kappa_\varphi = \frac{x'(u)y''(u) - x''(u)y'(u) + ay'(u)\left(x'(u)^2 + y'(u)^2\right)}{(x'(u)^2 + y'(u)^2)^{3/2}}.$$

Hence, we have

Proposition 2.2. *Weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in the plane with density e^{ax} is zero if and only if*

$$x'(u)y''(u) + ay'(u)\left(x'(u)^2 + y'(u)^2\right) = x''(u)y'(u). \tag{2.2}$$

From (2.2), we have

$$y(u) = c_2 \mp \frac{\arctan\left(\sqrt{c_1 e^{2ax(u)} - 1}\right)}{a}.$$

So, taking the sign of "$\mp$" which has been stated in the last equation as "$+$", we have

Theorem 2.1. *Let $\alpha(u)$ be a curve with vanishing weighted curvature in a plane with density e^{ax}. Then, it can be parametrized by*

$$\alpha(u) = \left(x(u), \frac{\arctan\left(\sqrt{c_1 e^{2ax(u)} - 1}\right)}{a} + c_2, 0 \right), \tag{2.3}$$

where $c_1, c_2 \in \Re$.

In Figure 1, one can see the graph of curve (2.3) for $x(u) = sin(u),\ c_1 = 3,\ c_2 = 5$ and some different values of a.

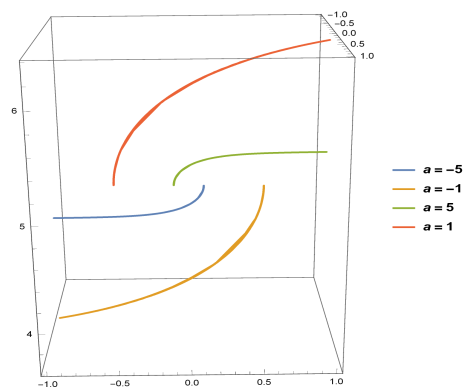

FIGURE 1. Curve (2.3) with vanishing weighted curvature in a plane with density e^{ax}

Now, we construct Smarandache curve of curve (2.3), obtain the Frenet vectors and curvature of this Smarandache curve and draw it.

Example 2.1. *From (1.1), the tangent, normal and binormal vectors of the curve (2.3) are obtained as*

$$T(u) = \frac{1}{\sqrt{c_1 e^{2ax(u)}}} \left(\sqrt{-1 + e^{2ax(u)} c_1}, 1, 0 \right),$$
$$N(u) = \frac{1}{\sqrt{c_1 e^{2ax(u)}}} \left(-1, \sqrt{-1 + c_1 e^{2ax(u)}}, 0 \right),$$
$$B(u) = (0, 0, 1),$$

respectively. Then, the TB-Smarandache curve $\delta(u)$ which is defined with the aid of the Frenet vectors of curve (2.3) can be written as

$$\delta(u) = \frac{T(u) + B(u)}{\|T(u) + B(u)\|} = \left(\frac{\sqrt{-1 + c_1 e^{2ax(u)}}}{\sqrt{2c_1 e^{2ax(u)}}}, \frac{1}{\sqrt{2c_1 e^{2ax(u)}}}, \frac{1}{\sqrt{2}} \right). \tag{2.4}$$

Also, the Frenet vectors and the curvature of the TB-Smarandache curve $\delta(u)$ are obtained as

$$T_\delta(u) = \frac{1}{\sqrt{c_1 e^{2ax(u)}}}\left(1, -\sqrt{-1+c_1 e^{2ax(u)}}, 0\right),$$
$$N_\delta(u) = \frac{1}{\sqrt{c_1 e^{2ax(u)}}}\left(\sqrt{-1+c_1 e^{2ax(u)}}, 1, 0\right),$$
$$B_\delta(u) = (0,0,1),$$
$$\kappa_\delta(u) = -\sqrt{2},$$

respectively.

Note that, the TN-Smarandache curve $\beta(u)$ and the NB-Smarandache curve $\gamma(u)$ of the curve (2.3) can be obtained with the similar procedure of above.

Now, taking $x(u) = \sin u$, $c_1 = 3$ in (2.4), we get

$$\delta(u) = \left(\frac{\sqrt{-1+3e^{2a\sin(u)}}}{\sqrt{6e^{2a\sin(u)}}}, \frac{1}{\sqrt{6e^{2a\sin(u)}}}, \frac{1}{\sqrt{2}}\right). \tag{2.5}$$

Figure 2 shows the graph of this curve for some different values of a.

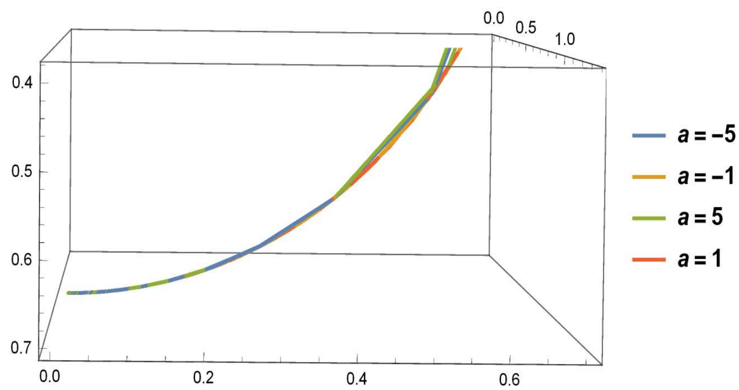

FIGURE 2. TB-Smarandache curve (2.5) of curve (2.3)

Similarly, from (2.2) we have

$$x(u) = c_2 - \frac{\ln(\cos(c_1 + ay(u)))}{a}.$$

So,

Theorem 2.2. *The curve $\alpha(u)$ with vanishing weighted curvature in a plane with density e^{ax} can be parametrized by*

$$\alpha(u) = \left(c_2 - \frac{\ln(\cos(c_1 + ay(u)))}{a}, y(u), 0\right), \tag{2.6}$$

where $c_1, c_2 \in \Re$.

Figure 3 shows the graph of curve (2.6) for $y(u) = arccos(e^u)$, $c_1 = 0$, $c_2 = 5$ and some different values of a.

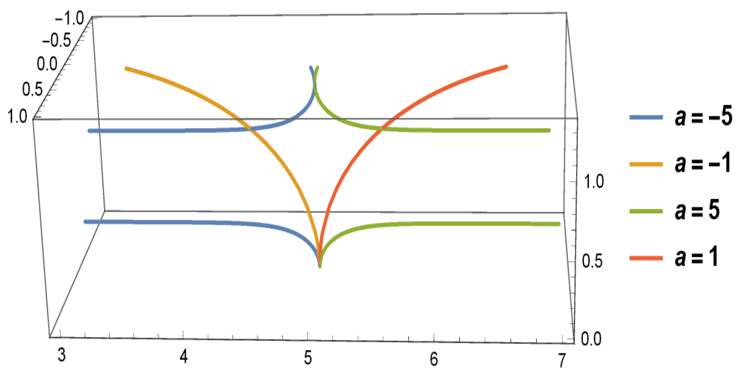

Figure 3. Curve (2.6) with vanishing weighted curvature in a plane with density e^{ax}

Now, let us construct Smarandache curve of curve (2.6), obtain the Frenet vectors and curvature of this Smarandache curve and draw it.

Example 2.2. *Let $\beta(u)$ be the TN-Smarandache curve which is defined with the aid of the Frenet vectors of curve (2.6). Then, the TN-Smarandache curve $\beta(u)$, its tangent, normal and binormal vectors and the curvature are obtained as*

$$\beta(u) = \frac{T(u) + N(u)}{\| T(u) + N(u) \|} = \frac{1}{\sqrt{2}} \left(\sin(c_1 + ay(u)) - \cos(c_1 + ay(u)), \cos(c_1 + ay(u)) + \sin(c_1 + ay(u)), 0 \right), \tag{2.7}$$

$$T_\beta(u) = \frac{1}{\sqrt{2}} (\sin(c_1 + ay(u)) + \cos(c_1 + ay(u)), -\sin(c_1 + ay(u)) + \cos(c_1 + ay(u)), 0),$$

$$N_\beta(u) = \frac{1}{\sqrt{2}} (\sin(c_1 + ay(u)) - \cos(c_1 + ay(u)), \sin(c_1 + ay(u)) + \cos(c_1 + ay(u)), 0),$$

$$B_\beta(u) = (0, 0, 1),$$

$$\kappa_\beta(u) = -1,$$

respectively. Here the Frenet vectors of curve (2.6) are

$$T(u) = (\sin(c_1 + ay(u)), \cos(c_1 + ay(u)), 0),$$

$$N(u) = (-\cos(c_1 + ay(u)), \sin(c_1 + ay(u)), 0),$$

$$B(u) = (0, 0, 1).$$

TN-Smarandache curve of curve (2.6) for conditions $y(u) = arccos(e^u)$, $c_1 = 3$ is

$$\beta(u) = \frac{1}{\sqrt{2}}(\sin(3 + a\arccos(e^u)) - \cos(3 + a\arccos(e^u)),$$

$$\cos(3 + a\arccos(e^u)) + \sin(3 + a\arccos(e^u)), 0) \tag{2.8}$$

and the graph of this curve for some different values of a can be seen in Figure 4 .

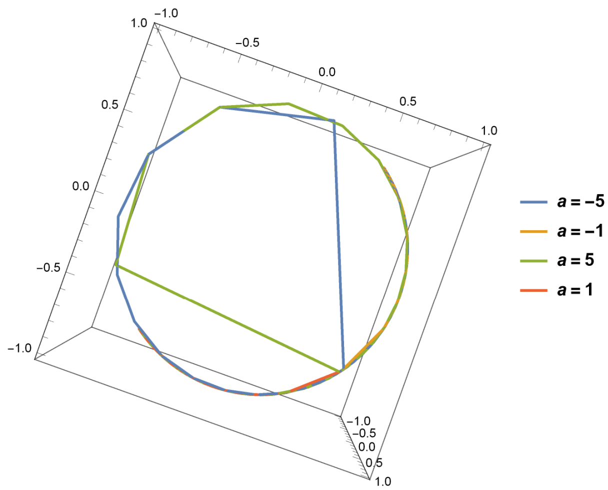

FIGURE 4. TN-Smarandache curve (2.8) of curve (2.6)

Here, the TB-Smarandache curve $\delta(u)$ and the NB-Smarandache curve $\gamma(u)$ of the curve (2.6) can be obtained with the similar procedure of above.

Now, let's assume that $a = 0$; i.e. the density of the plane is e^{by}.
Then from (2.1), the weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in the plane with density e^{by} is obtained as

$$\kappa_\varphi = \frac{x'(u)y''(u) - x''(u)y'(u) - bx'(u)(x'(u)^2 + y'(u)^2)}{(x'(u)^2 + y'(u)^2)^{3/2}}.$$

So, we get

Proposition 2.3. *Weighted curvature κ_φ of the curve $\alpha(u) = (x(u), y(u), 0)$ in the plane with density e^{by} is zero if and only if*

$$x'(u)y''(u) = x''(u)y'(u) + bx'(u)(x'(u)^2 + y'(u)^2). \tag{2.9}$$

From (2.9), the results are similar to the curves with vanishing weighted curvature in the plane with density e^{ax}.

3. Rotational Surfaces Generated By Planar Curves with Vanishing Weighted Curvature in E^3 With Density

In this section, we obtain the parametric representations of rotational surfaces which are generated by curves with vanishing weighted curvature in a plane with density e^{ax+by} and give their mean and Gaussian curvatures. Also, we draw the graphics of these curves, surfaces, the Gaussian and mean curvatures functions' graphics and the variations of Gaussian and mean curvatures on these surfaces.

3.1. Rotational Surfaces Generated By Curve (2.3).

First, we suppose that the axis of revolution is x-axis. Then, from (1.2) and (2.3), the rotational surface M can be parametrized by

$$\begin{aligned} X(u,v) &= (x(u), (\frac{\arctan(\sqrt{c_1 e^{2ax(u)}-1})}{a} + c_2)\cos(v), \\ &\quad (\frac{\arctan(\sqrt{c_1 e^{2ax(u)}-1})}{a} + c_2)\sin(v)). \end{aligned} \tag{3.1}$$

The Gaussian and mean curvatures of this surface are obtained by

$$G = \frac{a^2 e^{-2ax(u)}\sqrt{c_1 e^{2ax(u)}-1}}{ac_1c_2 + c_1\arctan(\sqrt{c_1 e^{2ax(u)}-1})}$$

and

$$H = \frac{\left(c_1 e^{2ax(u)} - 1 + ac_2\sqrt{c_1 e^{2ax(u)}-1} + \sqrt{c_1 e^{2ax(u)}-1}\arctan(\sqrt{c_1 e^{2ax(u)}-1})\right)a}{2(ac_2 + \arctan(\sqrt{c_1 e^{2ax(u)}-1}))\sqrt{c_1 e^{2ax(u)}(c_1 e^{2ax(u)}-1)}},$$

respectively.

We know that, minimal surfaces are defined as surfaces with zero mean curvature and a surface for which the Gaussian curvature vanishes everywhere is called a flat surface. Also, a point p on a regular surface $M \subset \mathbb{R}^3$ is said to be elliptic, parabolic and hyperbolic, if the Gaussian curvature $G(p) > 0$, $G(p) = 0$ and $G(p) < 0$, respectively.

Hence, we have

Theorem 3.1. *The rotational surface (3.1) which is generated by curve (2.3) with vanishing weighted curvature isn't minimal and flat.*

Theorem 3.2. *i) All of the points of the surface (3.1) are elliptic, if $ac_2 > 0$;*

ii) the points of the surface (3.1) which satisfy the conditions "$x(u) = -\frac{\ln(c_1)}{2a}$" and "$c_2 \neq 0$" are parabolic;

iii) all of the points of the surface (3.1) are hyperbolic, if $ac_2 \leq -\frac{\pi}{2}$.

In Figure 5, one can see the curve (2.3) which generates the rotational surface and the rotational surface (3.1) for $x(u) = \sin u$, $c_1 = 3$, $c_2 = 5$ and $a = 1$.

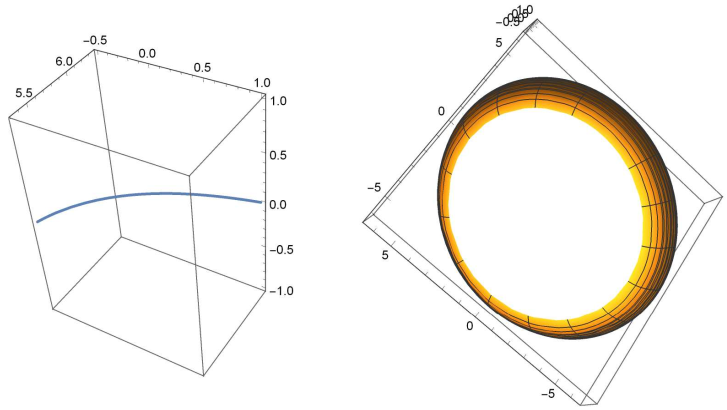

FIGURE 5. The curve (2.3) which generates the rotational surface and the rotational surface (3.1)

Also, in Figure 6, one can see the Gaussian and mean curvatures functions' graphics above and the variations of Gaussian and mean curvatures on rotational surface (3.1) below.

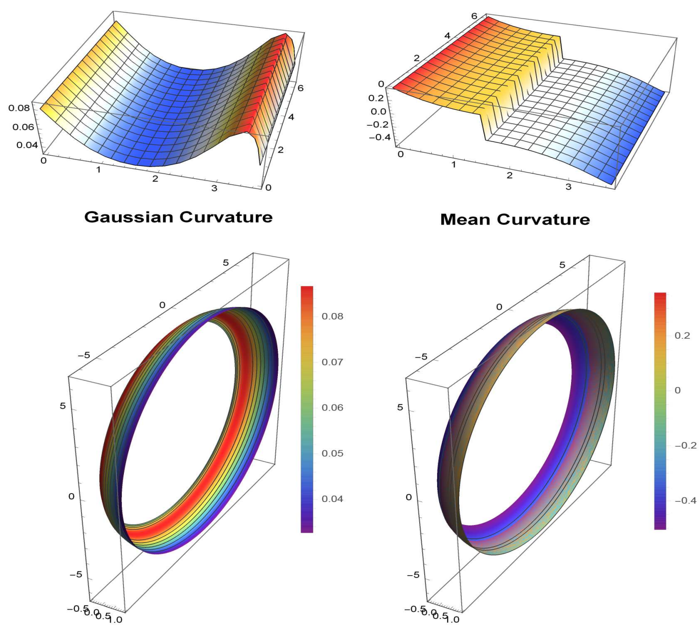

FIGURE 6.

Now, let us assume that the axis of revolution is y-axis. Then, from (1.3) and (2.3), the rotational surface M can be parametrized by

$$X(u,v)=\left(x(u)\cos v,\ \frac{\arctan\left(\sqrt{c_1e^{2ax(u)}-1}\right)}{a}+c_2,\ -x(u)\sin v\right). \tag{3.2}$$

Also, the Gaussian and mean curvatures of this surface are calculated as follows

$$G=-\frac{ae^{-2ax(u)}}{c_1x(u)}$$

and

$$H = \frac{1 - ax(u)}{2x(u)\sqrt{c_1 e^{2ax(u)}}},$$

respectively. Hence,

Theorem 3.3. *The rotational surface (3.2) which is generated by curve (2.3) with vanishing weighted curvature isn't minimal and flat.*

Theorem 3.4. *i) If a and $x(u)$ have different signs, then the points which satisfy this condition on the surface (3.2) are elliptic;*

ii) none of the points of the surface (3.2) are parabolic;

iii) if a and $x(u)$ have same signs, then the points which satisfy this condition on the surface (3.2) are hyperbolic.

Figure 7 shows the curve (2.3) which generates the rotational surface and the rotational surface (3.2), for $x(u) = \sin u,\ c_1 = 3,\ c_2 = 5$ and $a = 1$.

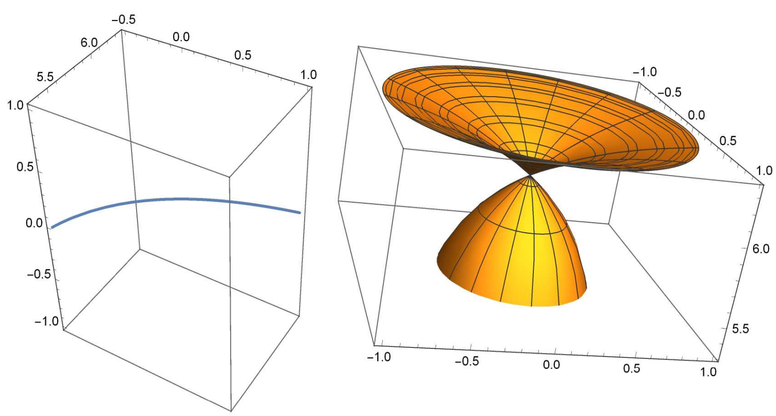

FIGURE 7. The curve (2.3) which generates the rotational surface and the rotational surface (3.2)

In Figure 8, the Gaussian and mean curvatures functions' graphics and the variations of Gaussian and mean curvatures on rotational surface (3.2) can be seen above and below, respectively.

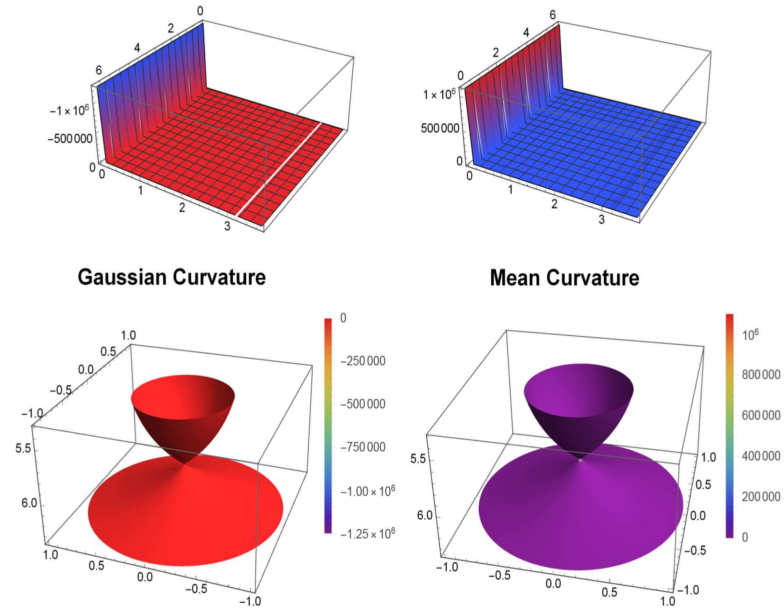

FIGURE 8.

Finally, we suppose that the axis of revolution is z-axis. Then, from (1.4) and (2.3), the rotational surface M can be parametrized by

$$X(u,v) = (x(u)\cos v - (\frac{\arctan(\sqrt{c_1 e^{2ax(u)}-1})}{a} + c_2)\sin v,$$

$$x(u)\sin v + (\frac{\arctan(\sqrt{c_1 e^{2ax(u)}-1})}{a} + c_2)\cos v, 0). \tag{3.3}$$

So, it is clear that the mean and Gaussian curvatures of this surface are zero.

Then, Figure 9 shows the curve (2.3) which generates the rotational surface and the rotational surface (3.3), for $x(u) = \sin u,\ c_1 = 3,\ c_2 = 5$ and $a = 1$.

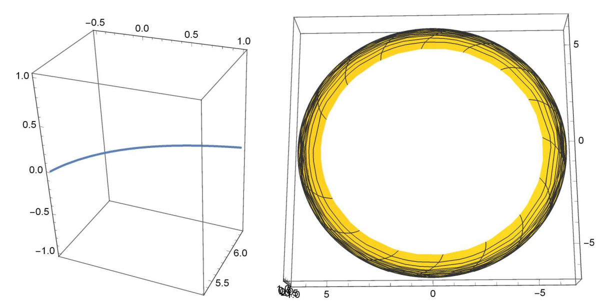

FIGURE 9. The curve (2.3) which generates the rotational surface and the rotational surface (3.3)

In Figure 10, one can see the Gaussian and mean curvatures functions' graphics above and the variations of Gaussian and mean curvatures on rotational surface (3.3) below.

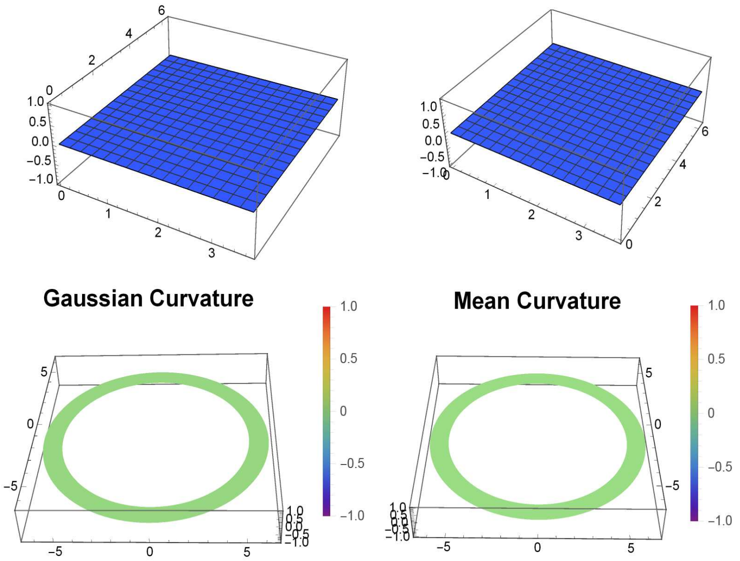

Figure 10.

3.2. **Rotational Surfaces Generated By Curve (2.6).**

First, let the axis of revolution be the x-axis. Then, by following the similar procedure with previous subsection, from (1.2) and (2.6), the rotational surface M is obtained as

$$X(u,v) = \left(c_2 - \frac{\ln\left(\cos(c_1 + ay(u))\right)}{a}, y(u)\cos v, y(u)\sin v\right). \tag{3.4}$$

Hence, the Gaussian and mean curvatures of this surface are

$$G = \frac{a\sin(2(c_1 + ay(u)))}{2y(u)}$$

and

$$H = \frac{\sin(c_1 + ay(u)) + ay(u)\cos(c_1 + ay(u))}{2y(u)},$$

respectively.

Theorem 3.5. *The rotational surface (3.4) which is generated by curve (2.6) with vanishing weighted curvature isn't minimal and flat.*

Theorem 3.6. *For* $2k\pi < c_1 + ay(u) < \frac{\pi}{2} + 2k\pi,\ k \in N,$

i) if a and $y(u)$ have same signs, then the points which satisfy this condition on the surface (3.4) are elliptic;

ii) if a and $y(u)$ have different signs, then the points which satisfy this condition on the surface (3.4) are hyperbolic.

For $\frac{3\pi}{2} + 2k\pi < c_1 + ay(u) < 2\pi + 2k\pi,\ k \in N,$

i) if a and $y(u)$ have different signs, then the points which satisfy this condition on the surface (3.4) are elliptic;

ii) if a and $y(u)$ have same signs, then the points which satisfy this condition on the surface (3.4) are hyperbolic.

Also, the points of the surface (3.4) which satisfy the condition "$c_1 + ay(u) = 2k\pi,\ k \in N$" are parabolic.

Next, taking $y(u) = arccos(e^u)$, $c_1 = 0$, $c_2 = 5$ and $a = 1$, then the curve (2.6) which generates the rotational surface and the rotational surface (3.4) can be seen in Figure 11.

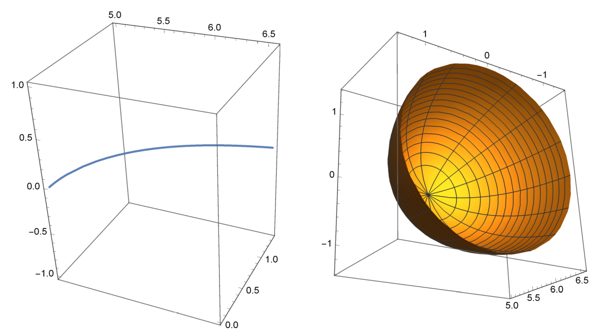

FIGURE 11. The curve (2.6) which generates the rotational surface and the rotational surface (3.4)

In Figure 12, one can see the Gaussian and mean curvatures functions' graphics above and the variations of Gaussian and mean curvatures on rotational surface (3.4) below.

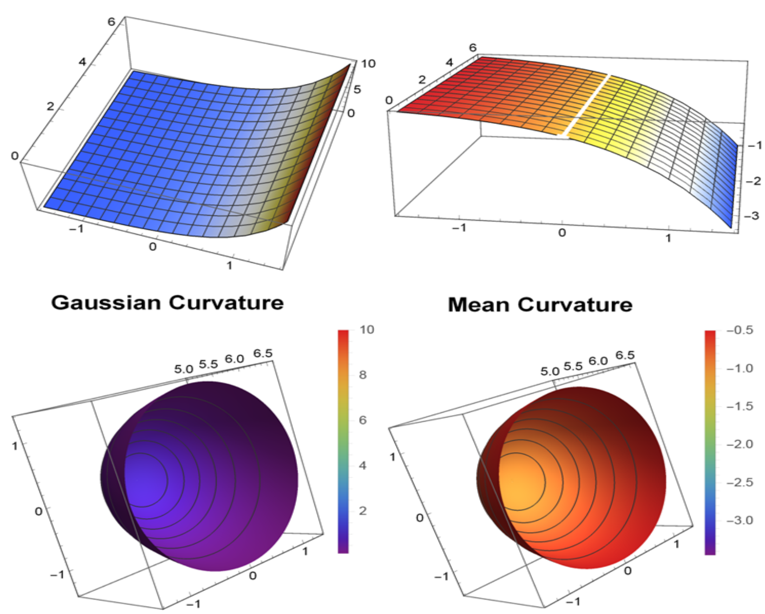

FIGURE 12.

Now, let's assume that the axis of revolution is the y-axis. Then, from (1.3) and (2.6), the rotational surface M can be parametrized by

$$X(u,v)=\left(\left(c_2-\frac{\ln\left(\cos(c_1+ay(u))\right)}{a}\right)\cos v, y(u), -\left(c_2-\frac{\ln\left(\cos(c_1+ay(u))\right)}{a}\right)\sin v\right). \tag{3.5}$$

The Gaussian and mean curvatures of this surface are

$$G=-\frac{a^2\cos(c_1+ay(u))^2}{ac_2-\ln\left(\cos\left(c_1+ay(u)\right)\right)}$$

and

$$H=\frac{\left(1-ac_2+\ln(\cos(c_1+ay(u)))\right)a\cos(c_1+ay(u))}{2\left(\ln(\cos(c_1+ay(u)))-ac_2\right)},$$

respectively.

Theorem 3.7. *The rotational surface (3.5) which is generated by curve (2.6) with vanishing weighted curvature isn't minimal and flat.*

Theorem 3.8. *i) If a and c_2 have same signs, then all of the points of the surface (3.5) are hyperbolic;*

ii) none of the points of the surface (3.5) are parabolic;

iii) If a and $x(u)$ have different signs and $|ac_2|>|\ln(\cos(c_1+ay(u)))|$, then the points which satisfy these conditions on the surface (3.5) are elliptic;

if a and $x(u)$ have different signs and $|ac_2|<|\ln(\cos(c_1+ay(u)))|$, then the points which satisfy these conditions on the surface (3.5) are hyperbolic.

Figure 13 shows the curve (2.6) which generates the rotational surface and the rotational surface (3.5), for $y(u)=arccos(e^u)$, $c_1=0$, $c_2=5$ and $a=1$.

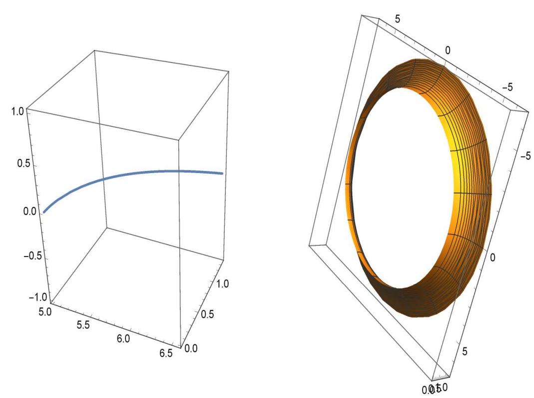

FIGURE 13. The curve (2.6) which generates the rotational surface and the rotational surface (3.5)

In Figure 14, one can see the Gaussian and mean curvatures functions' graphics above and the variations of Gaussian and mean curvatures on rotational surface (3.5) below.

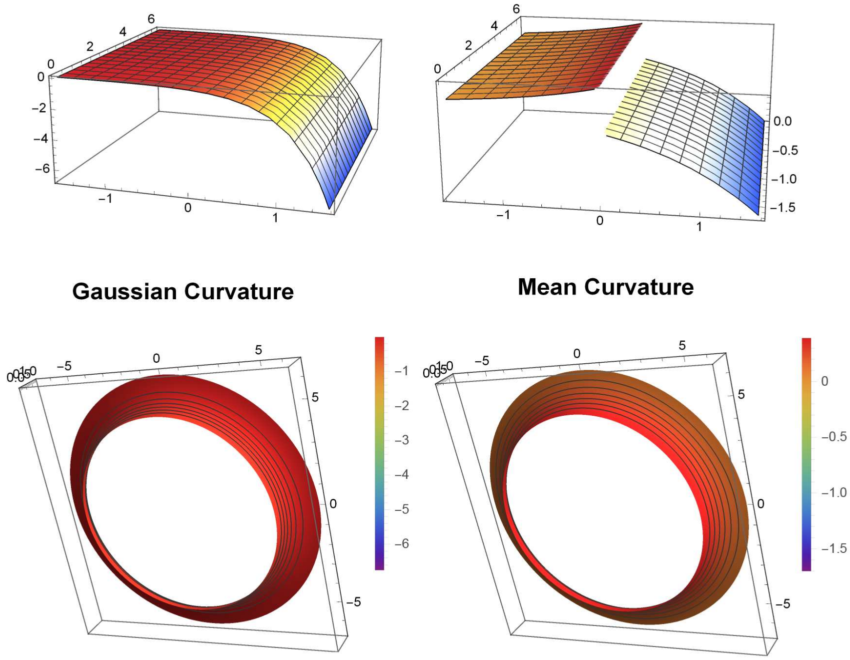

FIGURE 14.

Finally, let we suppose that the axis of revolution is z-axis. Then, from (2.6) and (1.4), the rotational surface M can be parametrized as

$$
\begin{aligned}
X(u,v) \quad = \quad & ((c_2 - \frac{\ln(\cos(c_1 + ay(u)))}{a})\cos(v) - y(u)\sin v, \\
& (c_2 - \frac{\ln(\cos(c_1 + ay(u)))}{a})\sin(v) + y(u)\cos v, 0).
\end{aligned} \tag{3.6}
$$

From above, it is obvious that the mean and Gaussian curvatures of this surface are zero. Figure 15 shows the curve (2.6) which generates the rotational surface and the rotational surface (3.6), for $y(u) = arccos(e^u)$, $c_1 = 0$, $c_2 = 5$ and $a = 1$.

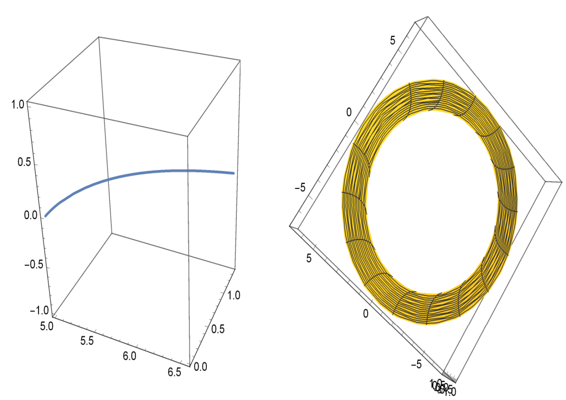

FIGURE 15. The curve (2.6) which generates the rotational surface and the surface of revolution (3.6)

In Figure 16, one can see the Gaussian and mean curvatures functions' graphics above and the variations of Gaussian and mean curvatures on rotational surface (3.6) below.

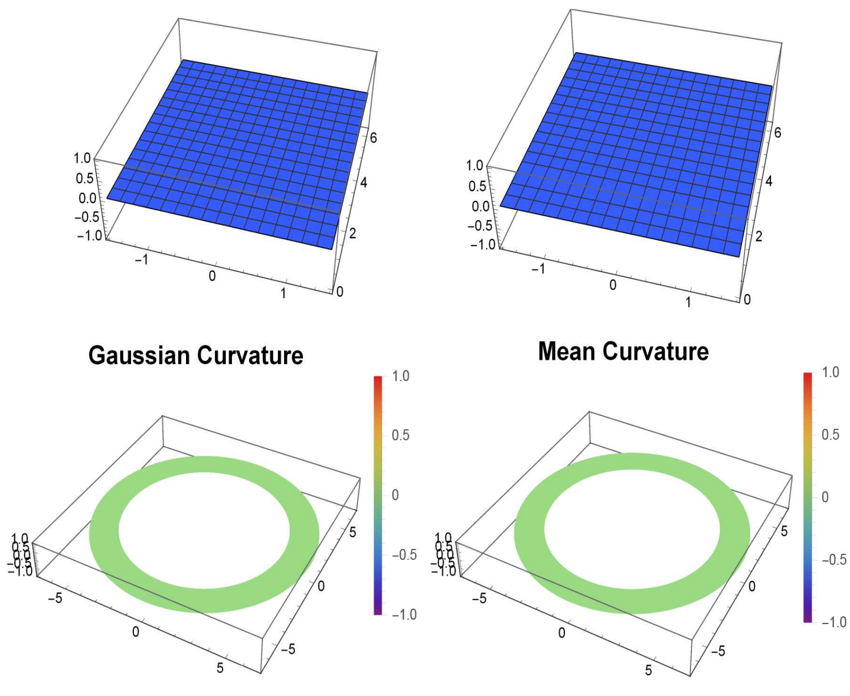

Figure 16.

4. Conclusion and Future Work

In the present paper, we've obtained the weighted curvature of a curve in a plane with density e^{ax+by}, where $a, b \in \mathbb{R}$ not all zero constants and investigated the curves with vanishing weighted curvature according to the cases $a \neq 0,\ b = 0$ and $a = 0,\ b \neq 0$. Later, we've constructed the Smarandache curves of these curves and obtained the tangent, normal and binormal vectors and curvature of them. Also, we've constructed the rotational surfaces which are generated by curves with vanishing weighted curvature in the plane with density e^{ax+by} and give some characterizations by obtaining their mean and Gaussian curvatures. We've drawn these curves and surfaces with the aid of *Mathematica*.

We think that, this study will provide a different perspective to the researchers who're dealing with the weighted spaces. As a problem, these curves and surfaces can be studied in different spaces such as Minkowski space, Galilean space and pseudo Galilean space and etc. Also, these structures can be studied in Euclidean or other spaces with different densities and in the near future, the authors will deal with these open problems in detail.

References

[1] HS. Abdel-Aziz and M.K. Saad, Smarandache Curves Of Some Special Curves In The Galilean 3-Space, Honam Math. J. 37(2) (2015), 253-264.

[2] A.T. Ali, Special Smarandache Curves in the Euclidean Space, Int. J. Math. Comb. 2 (2010), 30-36.

[3] C. Carroll, A. Jacob, C. Quinn and R. Walters, The isoperimetric problem on planes with density, Bull. Aust. Math. Soc. 78(2) (2008), 177-197.

[4] I. Corwin, N. Hoffman, S. Hurder, V. Sesum and Y. Xu, Differential geometry of manifolds with density, Rose-Hulman Und. Math. J. 7(1) (2006), 1-15.

[5] F. Dillen, J. Pas and L. Verstraelen, On the Gauss map of surfaces of revolution, Bull. Inst. Math. Acad. Sinica 18 (1990), 239-246.

[6] A. Ferrandez and P. Lucas, On Surfaces in the 3-dimensional Lorentz-Minkowski space, Pac. J. Math. 152(1) (1992), 93-100.

[7] O.J. Garay, On a certain class of finite type surfaces of revolution, Kodai Math. J. 11(1) (1988), 25-31.

[8] M. Gromov, Isoperimetry of waists and concentration of maps, Geom. Func. Anal. 13 (2003), 178-215.

[9] D.T. Hieu and N.M. Hoang, Ruled Minimal Surfaces In R^3 With Density e^z, Pac. J. Math. 243(2) (2009), 277-285.

[10] D.T. Hieu and T.L. Nam, The classification of constant weighted curvature curves in the plane with a log-linear density, Commun. Pure Appl. Anal. 13(4) (2014), 1641-1652.

[11] A. Kazan and H.B. Karadağ, Weighted Minimal And Weighted Flat Surfaces Of Revolution In Galilean 3-Space With Density, Int. J. Anal. Appl. 16(3) (2018), 414-426.

[12] A. Kazan and H.B. Karadağ, A Classification of Surfaces of Revolution in Lorentz-Minkowski Space, Int. J. Contemp. Math. Sci. 6(39) (2011), 1915-1928.

[13] L. Belarbi and M. Belkhalfa, Surfaces in R^3 with density, i-manager's J. Math. 1(1) (2012), 34-48.

[14] F. Morgan, Manifolds with Density, Notices Amer. Math. Soc. 52(8) (2005), 853-858.

[15] B. O'Neill, Elementary Differential Geometry, Academic Press Inc, 1966.

[16] C. Rosales, A. Canete, V. Bayle and F. Morgan, On the isoperimetric problem in Euclidean space with density, Calc. Var. Partial Differ Equ. 31 (2008), 27-46.

[17] J.W. Rutter, Geometry of Curves, 1st. New York: Chapman and Hall/CRC; 2000, 384.

[18] S. Şenyurt, Y. Altun and C. Cevahir, Smarandache curves for spherical indicatrix of the Bertrand curves pair, Bol. Soc. Paran. Mat. 38(2) (2020) (in press), 27-39.

[19] M. Turgut and S. Yilmaz, Smarandache Curves in Minkowski Space-time, Int. J. Math. Comb. 3 (2008), 51-55.

[20] D.W. Yoon, Weighted Minimal Translation Surfaces in the Galilean Space with Density, Open Math. 15 (2017), 459-466.