International Journal of Analysis and Applications
Volume 17, Number 4 (2019), 530-547
URL: https://doi.org/10.28924/2291-8639
DOI: 10.28924/2291-8639-17-2019-530

MODELING THE EFFECTS OF OUTLIERS ON THE ESTIMATION OF LINEAR STOCHASTIC TIME SERIES MODEL

EMMANUEL ALPHONSUS AKPAN[1,*] AND IMOH UDO MOFFAT[2]

[1]*Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi, Nigeria*

[2]*Department of Mathematics and Statistics, University of Uyo, Nigeria*

**Corresponding author:* eubong44@gmail.com

ABSTRACT. This study investigates the effects of outliers on the estimates of ARIMA model parameters with particular attention given to the performance of two outlier detection and modeling methods targeted at achieving more accurate estimates of the parameters. The two methods considered are: an iterative outlier detection aimed at obtaining the joint estimates of model parameters and outlier effects, and an iterative outlier detection with the effects of outliers removed to obtain an outlier free series, after which a successful ARIMA model is entertained. We explored the daily closing share price returns of Fidelity bank, Union bank of Nigeria, and Unity bank from 03/01/2006 to 24/11/2016, with each series consisting of 2690 observations from the Nigerian Stock Exchange. ARIMA (1, 1, 0) models were selected based on the minimum values of Akaike information criteria which fitted well to the outlier contaminated series of the respective banks. Our findings revealed that ARIMA (1, 1, 0) models which fitted adequately to the outlier free series outperformed those of the parameter-outlier effects joint- estimated model. Furthermore, we discovered that outliers biased the estimates of the model parameters by reducing the estimated values of the parameters. The implication is that, in order to achieve more accurate estimates of ARIMA model parameters, it is needful to account for the presence of significant outliers and preference should be given to the approach of cleaning the series of outliers before subsequent entertainment of adequate linear time series models.

Received 2019-05-01; accepted 2019-06-10; published 2019-07-01.

2010 Mathematics Subject Classification. 82B31.

Key words and phrases. ARIMA; outliers; return series; time series.

1. INTRODUCTION

Outliers are common characterizations of every time series. In general, outliers are extreme observations that deviate from the overall pattern of the sample. Statistically, outliers are those observations whose standard deviations are greater than 3 in absolute value, which is the value of kurtosis occupied by the normal distribution. However, the effects of outliers on the linear time series models cannot be overemphasized; such effects range from false inference, introduction of biases in the model parameters, model misspecification and misleading confidence interval ([1], [2], [3], [4]).

By efficiency, we mean the goodness of an estimator of a model which can be measured by variance, that is, a model with the smallest variance is considered to be superior as regarding efficiency. To reiterate the need for efficiency of the estimates of model parameters by considering the presence of outliers, this study applied two outlier identification and modeling methods. The first is the modified iterative method proposed by [5], which involves the joint estimation of the model parameters and the magnitude of outlier effects. The second is the modified iterative method proposed by [6], which involves identification of outliers sequentially by searching for most relevant anomaly, estimating its effect and removing it from the data. The estimation of the model parameters is again done on the outlier corrected series, and further iteration of the process is carried out until no significant perturbation is found.

Actually, the motivation for this study is derived from the fact that previous studies such as [7], [8], [9], [10] failed to consider outliers while modeling returns series in Nigeria. Thus, this gap in knowledge is fully addressed in our work.

This work is further organized as follows: section 2 takes care of materials and methods; section 3 handles the results and discussion while section 4 treats the conclusion.

2. MATERIALS AND METHODS

2.1 Return Series

The returns series (R_t) can be obtained given that P_t is the price of a unit shares at time t and P_{t-1} is the price of shares at time $t-1$. Thus

$$R_t = \nabla lnP_t = (1-B)lnP_t = ln\,P_t - ln\,P_{t-1} \qquad (1)$$

In equation (1), R_t is regarded as a transformed series of the price (P_t) of shares meant to attain stationarity such that both the mean and the variance of the series are stable [11] while B is the backshift operator.

2.2 Autoregressive Integrated Moving Average (ARIMA) Model

[3] considered the extension of ARMA model to deal with homogenous non-stationary time series in which X_t, is non-stationary but its d^{th}difference is a stationary ARMA model. Denoting the d^{th} difference of X_t by

$$\varphi(B) = \phi(B)\nabla^d X_t = \theta(B)\varepsilon_t \qquad (2)$$

where $\varphi(B)$ is the nonstationary autoregressive operator such that d of the roots of $\varphi(B) = 0$ are unity and the remainder lie outside the unit circle while $\phi(B)$ is a stationary autoregressive operator. It should be noted that in equation (2), the presence of outliers is not taken into consideration.

2.3 Joint Model of ARIMA and Outlier-effects

$$R_t = \sum_{j=1}^{p} \varphi_j R_{t-j} + \sum_{i=1}^{q} \theta_i a_{t-i} + a_t + \sum_{j=1}^{k} \omega_j V_j(B) I_t^{(T)}, \qquad (3)$$

where $V_j(\mathrm{B}) = 1$ for an AO, and $V_j(\mathrm{B}) = \frac{\theta(B)}{\varphi(B)}$ for an IO at $t = T_j$, $V_j(\mathrm{B}) = (1 - B)^{-1}$ for a LS, $V_j(\mathrm{B}) = (1 - \delta B)^{-1}$ for an TC, and ω is the size of the outlier. For more details on the types of outliers and estimation of their effects, see [1], [12], [3], [4], [5], [13].

2.4 ARIMA Model for Outlier-Adjusted Return Series

$$R_t - \sum_{j=1}^{p} \varphi_j R_{t-j} - \sum_{i=1}^{q} \theta_i a_{t-i} - \sum_{j=1}^{k} \omega_j V_j(B) I_t^{(T)} = a_t, \qquad (4)$$

where a_t is the outlier free series. Meanwhile, equations (3) and (4) represent major modifications on equation (2) to account for the presence of outliers.

2.5 Outliers in Time Series

Generally, in time series, four types of outliers are identified and they are as follows: additive outlier, innovative outlier, level shift outlier and temporary outlier [12].

2.5.1 Additive Outlier (AO)

A time series $Y_1, \ldots, Y_T$ affected by the presence of an additive outlier at t = T is given by

$$Y_t = \begin{cases} X_t, & t \neq T \\ X_t + \omega, & t = T \end{cases} = X_t + \omega I_t^{(T)} = \frac{\theta(B)}{\varphi(B)} a_t + \omega I_t^{(T)} \qquad (5)$$

for t = 1, …,T, where $I_t^{(T)} = \begin{cases} 1, & t = T, \\ 0, & t \neq T, \end{cases}$ is the indicator variable representing the presence or absence of an outlier at time T, X_t follows an ARIMA model, ω is an outlier size. Hence, an additive outlier affects only a single observation (see also [1], [12], [3], [4]).

2.5.2 Innovative Outlier (IO)

A time series$Y_1, \ldots, Y_T$ affected by the presence of an innovative outlier at t = T is given by

$$Y_t = X_t + \frac{\theta(B)}{\varphi(B)}\omega I_t^{(T)} = \frac{\theta(B)}{\varphi(B)}(a_t + \omega I_t^{(T)}) \quad (6)$$

hence, an innovative outlier affects all observations $Y_t, Y_{t+1},\ldots,$ beyond time T through the memory of the system described by ψ(B) = $\frac{\theta(B)}{\varphi(B)}$, such that $Y_t = X_t + \psi(B)\omega I_t^{(T)}$.

Meanwhile, according to [12], the innovation of a time series $Y_1, \ldots, Y_T$ is affected by

$$Y_t = e_t + \omega I_t^{(T)} \quad (5)$$

where e_t are the innovations of the uncontaminated series X_t.

2.6.3 Level Shift (LS)

A time series $Y_1, \ldots, Y_T$ affected by the presence of a level shift at t = T is given by

$$Y_t = X_t + \omega S_t^{(T)} \quad (6)$$

where $S_t^{(T)} = (1 - B)^{-1} I_t^{(T)}$. Note that level shift affects all the observation of the series after t = T. Hence, according to [12], level shift serially affects the innovations as follows:

$$a_t = e_t + \pi(B)\omega S_t^{(T)} \quad (7)$$

where $\pi(B) = (1 - \pi_1 B - \pi_2 B^2 - \cdots)$

2.7.4 Temporary Change (TC)

A time series $Y_1, \ldots, Y_T$ affected by the presence of a temporary change at t = T is given by

$$Y_t = X_t + \frac{1}{1-\delta B}\omega I_t^{(T)} \quad (8)$$

where δ is an exponential decay parameter such that $0 < \delta < 1$. If δ tends to 0, the temporary change reduces to an additive outlier, whereas if δ tends to 1, the temporary change reduces to a level shift. The temporary change affects the innovations as follows:

$$a_t = e_t + \frac{\pi(B)}{1-\delta B}\omega I_t^{(T)} \quad (9)$$

If $\pi(B)$ is close to $1 - \delta B$, the effect of temporary change on the innovations is very close to the effect of an innovative outlier. Otherwise, the temporary change can affect several observations with a decreasing effect after t = T [12].

3. RESULTS AND DISCUSSION

3.1 Time Plots

Inspecting the plots in Figures 1-3, it is obvious that they are characterized by upward and downward movements away from the common mean, which clearly indicates the existence of nonstationarity.

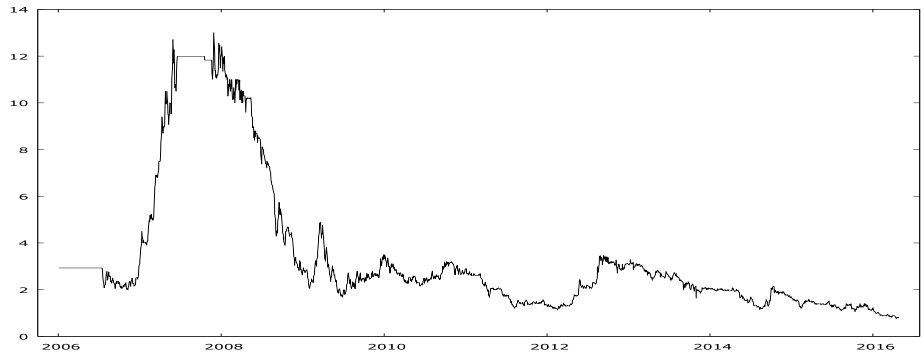

Figure 1: Price Series of Fidelity Bank shares

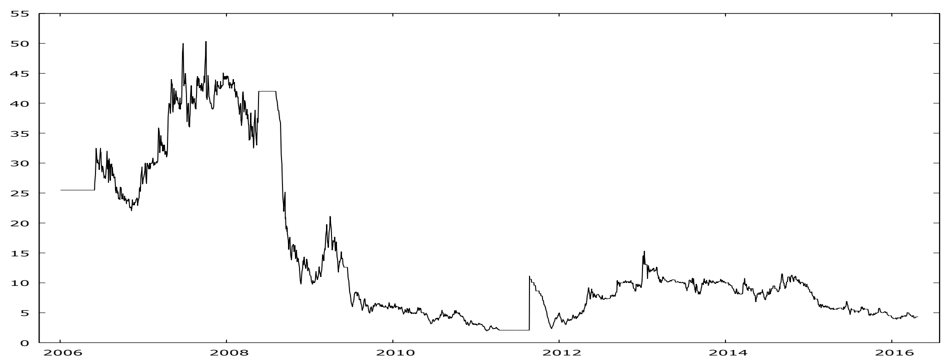

Figure 2: Price Series of Union Bank shares

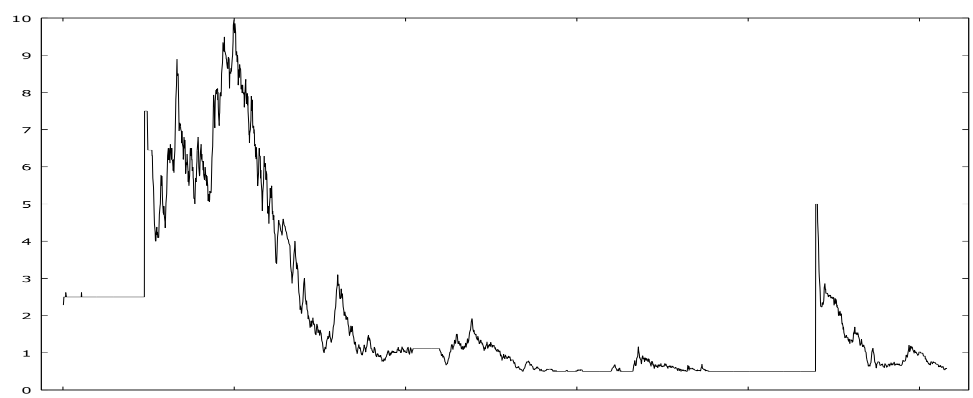

Figure 3: Price Series of Unity Bank shares

Also, the plots in Figures 4 - 6 indicate that the returns series cluster around the mean which implies that the series are stationary.

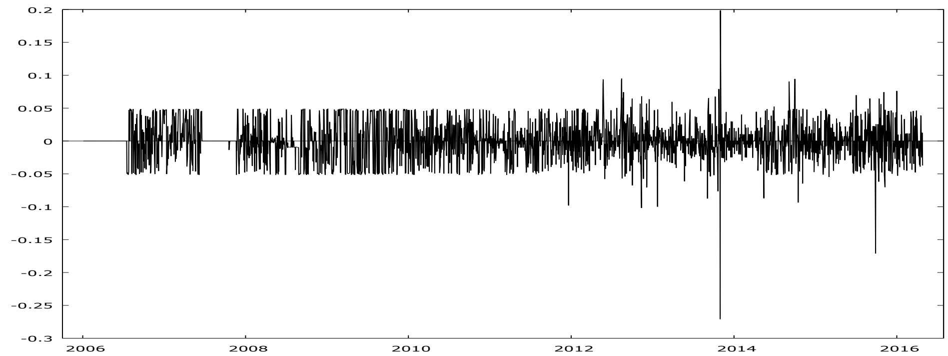

Figure 4: Return Series of Fidelity Bank

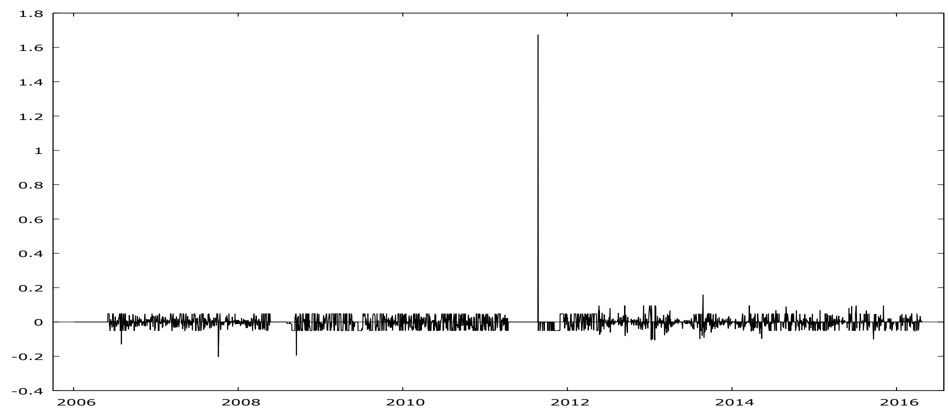

Figure 5: Return Series of Union Bank

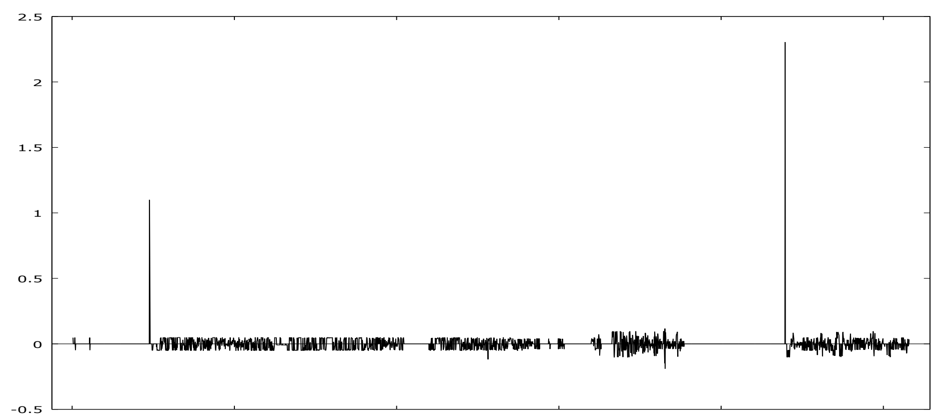

Figure 6: Return Series of Unity Bank

3.2 Linear Time Series Modeling Return Series of Fidelity Bank

From Figures 7 and 8, both ACF and PACF indicate that mixed model is possible. The following models, ARIMA(1, 1, 0), ARIMA(0, 1, 1), ARIMA(1, 1, 1), ARIMA(1, 1, 2) and ARIMA (2, 1, 1) are entertained tentatively.

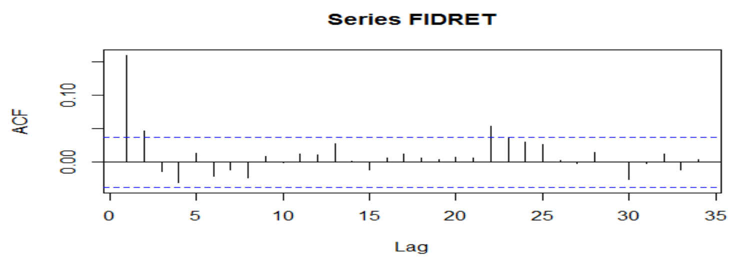

Figure 7: ACF of Return Series of Fidelity Bank

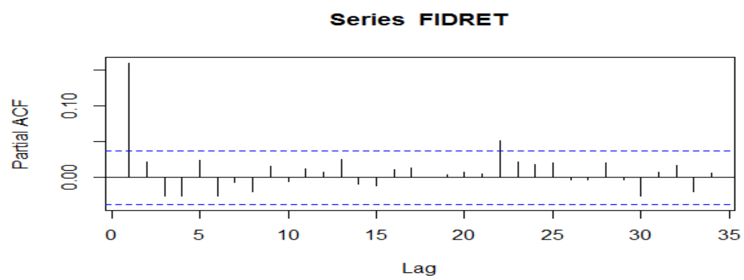

Figure 8: PACF of Return Series of Fidelity Bank

From Table I, ARIMA(1, 1, 1) model has the smallest AIC but one of its parameters is not significant. Secondly, ARIMA(1, 1, 2) model has the second smallest AIC yet its parameters are not significant. Hence, ARIMA(1, 1, 0) model is selected based on the ground that its parameter is significant and has the nearest minimum AIC.

Table I: ARIMA Models for Return Series of Fidelity Bank

Model	Parameter				Akaike Information Criteria (AIC)	Log likelihood
	φ_1	φ_2	θ_1	θ_2		
ARIMA(1,1,0)	0.1606***				−11562.17	5783.09
ARIMA(0,1,1)			0.1494***		−11559.28	5780.64
ARIMA(1,1,1)	0.2569***		− 0.0986		−11563.16	5783.58
ARIMA(1,1,2)	−0.0498		0.2071	0.0628	−11562.88	5784.44
ARIMA(2,1,1)	−0.0721	0.0619	0.2288		−11561.98	5783.99

***** significance at 5% level**

Furthermore, evidence from Ljung-Box Q-statistics shows that ARIMA(1, 1, 0) model is adequate at 5% level of significance given the Q-statistics at lags 1, 4, 8, and 24 given by Q(1) = 0.0376, Q(4) = 5.4261, Q(8) = 9.8001, and Q(24) = 23.379 with the respective p-values of 0.8462, 0.2463, 0.2793, and 0.4975.

3.3 Identification of Outliers in the Residual Series of ARIMA(1, 1, 0) Model fitted to the Return Series of Fidelity Bank

Considering the critical value, C = 4, and based on the condition that $n \geq 450$, we identified sixteen (16) different outliers that have contaminated the residual series of ARIMA(1, 1, 0) model, as indicated in Table II. They are: two (2) innovation outliers (IO), five (5) additive outliers, and nine (9) temporary change outliers.

Table II: Types of Outliers Identified in the Residual Series of ARIMA(1, 1, 0) Model fitted to the Return Series of Fidelity Bank

Type	Observation index	Location	Estimate	T-statistic
IO	1555	26/04/2012	-0.09798041	-4.198390
AO	1789	08/04/2013	-0.10865950	-4.715609
AO	1841	21/06/2013	-0.10673597	-4.632131
AO	2539	15/04/2016	-0.17301613	-7.508560
AO	2042	11/04/2014	-0.30477209	-13.226510
TC	827	18/05/2009	0.07540548	4.049288
TC	847	16/06/2009	-0.07692527	-4.130901
TC	859	02/07/2009	-0.07537282	-4.047534
TC	1665	04/10/2012	0.08360953	4.489847
TC	1724	01/02/2013	0.07510564	4.033187
TC	2263	05/03/2015	0.07816849	4.197662
TC	2280	30/03/2015	0.09555288	5.131207
IO	2292	17/04/2015	-0.09220965	-4.046644
AO	2043	14/04/2014	0.24193892	10.598998
TC	691	27/10/2008	-0.06641433	-4.025161
TC	950	11/11/2009	0.06557061	4.004060

To account for the effect of outliers, the method of joint estimation of the parameter of ARIMA (1, 1, 0) model with outliers identified in Table II is performed as indicated in Table III. Comparing the values of AIC = −11922.67 and log likelihood = 5979.34 of the joint model of ARIMA(1, 1, 0) with outliers effects with that of ARIMA (1, 1, 0) model having AIC = −11562.17 and log likelihood = 5783.09, it is obvious that the joint model of ARIMA (1, 1, 0) with outliers effects has a lower AIC and a higher log likelihood value, thus making it a better model than the ARIMA (1, 1, 0) model where the influence of outliers is not taken into consideration.

Table III: Joint Model of ARIMA (1, 1, 0) and Outlier-effects fitted to Return Series Fidelity Bank

	Estimate	Std. Error	z value	Pr(>|z|)
ar1	0.171530	0.019142	8.9607	$< 2.2e^{-16}$ ***
IO1555	-0.084464	0.024209	-3.4890	0.0004849***
AO1789	-0.109211	0.025841	-4.2262	$2.376e^{-05}$ ***
AO1841	-0.107286	0.025841	-4.1518	$3.299e^{-05}$ ***
AO2042	-0.273962	0.026198	-10.4573	$< 2.2e^{-16}$ ***
AO2539	-0.173178	0.025830	-6.7045	$2.022e^{-11}$ ***
TC827	0.075179	0.021072	3.5677	0.0003601***
TC847	-0.076153	0.021068	-3.6147	0.0003007 ***
TC859	-0.074623	0.021069	-3.5418	0.0003974 ***
TC1665	0.083147	0.021082	3.9439	$8.016e^{-05}$ ***
TC1724	0.074547	0.021090	3.5348	0.0004081***
TC2263	0.078614	0.021097	3.7264	0.0001943***
TC2280	0.095246	0.021087	4.5168	6.277e-06***
IO2292	-0.071450	0.024232	-2.9486	0.0031921**
AO2043	0.197831	0.026196	7.5520	4.286e-14***
TC691	-0.070928	0.021078	-3.3651	0.0007653***
TC950	0.071171	0.021068	3.3782	0.0007295***

3.4 Building ARIMA(1, 1, 0) Model for Outlier-Adjusted Return Series of Fidelity Bank

Here, the second method is applied which is the removal of the outliers effects to obtain an outlier-adjusted series. Then, ARIMA(1, 1, 0) model fitted well to the outlier-adjusted series with its parameter significant at 5% level [see Table IV] and is found to be adequate given the

Q-statistics at lags 1, 4, 8, and 24 given by Q(1) = 0.0003, Q(4) = 4.2007, Q(8) = 13.92, and Q(24) = 29.649 with the corresponding p-values of 0.99, 0.38, 0.09, and 0.20.

Table IV: ARIMA (1, 1, 0) Model for Outlier-Adjusted Return Series of Fidelity Bank

Model	Parameter (φ)	Akaike Information Criteria	Log likelihood
ARIMA (1, 1, 0)	0.1715^{***}	−11954.67	5979.34

***** significance at 5% level**

ARIMA (1, 1, 0) model with the least AIC = −11954.67 appears to be better than that of the joint model of ARIMA (1, 1, 0) with outliers effects.

On comparing the estimates of ARIMA(1, 1, 0) model fitted to the outlier contaminated series with the ARIMA(1, 1, 0) model when adjusted for outliers using the two proposed methods, it is found that the estimates of both the joint ARIMA(1, 1, 0) model with outliers effects and the ARIMA(1, 1, 0) model fitted to the outlier adjusted series are the same. However, the later tends to outperform the former on the basis of smallest information criteria. Of paramount interest is the discovery that outliers introduced substantial bias in the estimate of ARIMA (1, 1, 0) model by 0.0109 as shown in Table V. Again, the modified iterative method produced a model with smallest variance as indicated in Table V, hence, adjudged the most efficient method.

Table V: Effect of Outliers on Estimate of ARIMA(1, 1, 0) Model for Return Series of Fidelity Bank

Model	ARIMA (1,1,0) (For outlier-contaminated)	Joint ARIMA (1,1,0) with Outliers Effects	ARIMA (1,1,0) (For outlier-adjusted)	Bias Introduced
Parameter (φ_1)	0.1606	0.1715	0.1715	−0.0109
AIC	−11562.18	-11922.67	−11954.67	
Standard error	0.0190	0.0191	0.0189	
Variance	0.000795	0.000691	0.000687	
Log likelihood	5783.09	5979.34	5979.34	

3.5 Linear Time Series Modeling of Return Series of Union Bank

From Figures 9 and 10, both ACF and PACF indicate that the following mixed model could be entertained tentatively: ARIMA(1, 1, 0), ARIMA(0, 1, 1) and ARIMA(1, 1, 1).

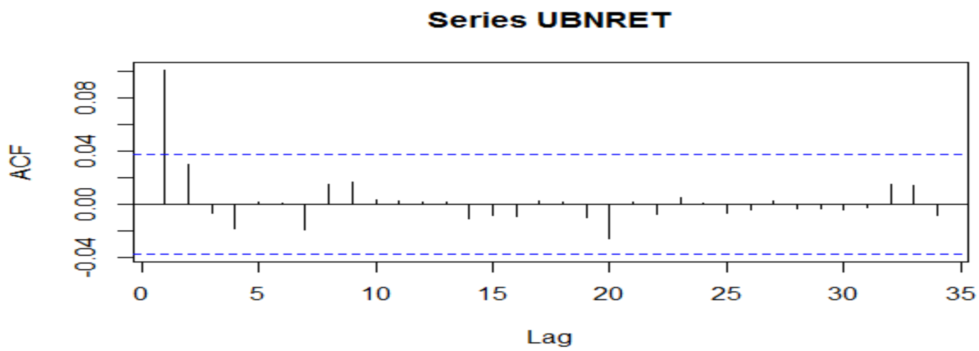

Figure 9: ACF of Return Series of Union Bank

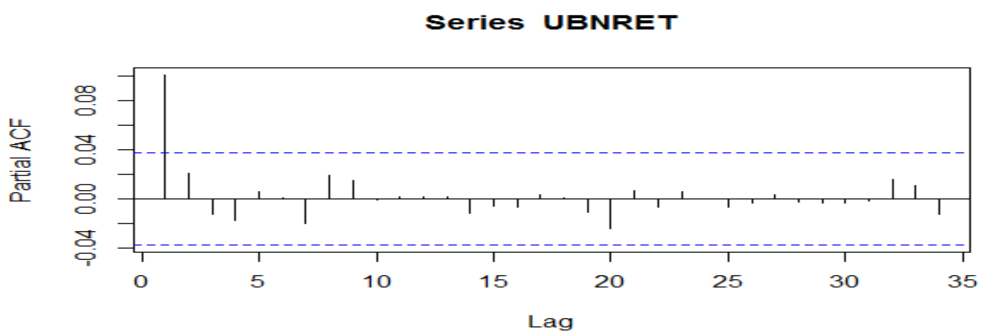

Figure 10: PACF of Return Series of Union Bank

From Table VI, ARIMA (1, 1, 0) model is selected based on the ground that its parameter is significant and has the minimum AIC.

Table VI: ARIMA Models for Return Series of Union Bank

Model	Parameter		Akaike Information Criteria (AIC)	Log likelihood
	φ_1	θ_1		
ARIMA (1,1,0)	0.1014***		−9132.26	4567.13
ARIMA (0,1,1)		0.0963***	−9130.87	4566.43
ARIMA (1,1,1)	0.2455	− 0.1453	−9131.12	4567.56

***** significance at 5% level**

Furthermore, evidence from Ljung-Box Q-statistics shows that ARIMA(1, 1, 0) model is adequate at 5% level of significance given the Q-statistics at lags 1, 4, 8 and 24 given by Q(1) =

0.0133, Q(4) = 2.3753, Q(8) = 4.318 and Q(24) = 7.9309 with the corresponding p-values of 0.9082, 0.6671, 0.8274, and 0.9991.

3.6 Identification of Outliers in the Residual Series of ARIMA(1, 1, 0) Model fitted to the Return Series of Union Bank

Here, we consider the critical value, C = 5 given that C = 4 was not sufficient for computing weights of outliers and about nineteen (19) different outliers are identified to have contaminated the residual series of ARIMA(1, 1, 0) model, four (4) innovation outliers (IO), eight (8) additive outliers and seven (7) temporary change outliers, as shown in Table VII.

Table VII: Types of Outliers identified in the Residual Series of ARIMA(1, 1, 0) Model fitted to the Return Series of Union Bank

Type	Observation index	Location	Estimate	T-statistic
IO	458	16/11/2007	-0.20259320	-9.867965
IO	1472	23/12/2011	-0.22031597	-10.731210
IO	1831	07/06/2013	0.10533493	5.130683
IO	1843	25/06/2013	0.10590627	5.158511
AO	150	15/08/2006	-0.13856541	-6.783874
AO	705	14/11/2008	-0.20086454	-9.833910
AO	1471	22/12/2011	1.67935140	82.217553
AO	1830	06/06/2013	-0.11483241	-5.621956
AO	1842	24/06/2013	-0.10581300	-5.180384
AO	1984	21/01/2014	-0.10648119	-5.213098
AO	1994	04/02/2014	0.16239480	7.950512
TC	691	27/10/2008	-0.08071046	-5.129738
TC	901	31/08/2009	-0.08274861	-5.259278
TC	1470	22/12/2011	0.53378545	33.925958
TC	1523	09/03/2012	-0.08218825	-5.223663
TC	1541	04/04/2012	0.07869209	5.001456
TC	1824	28/05/2013	0.11353246	7.215815
TC	2534	08/04/2016	-0.08059290	-5.122266
AO	1748	06/02/2013	-0.11923771	-5.160464

Again, applying the first method as indicated in Table VIII, it is found that the values of AIC = −11560.27 and log likelihood = 5800.13 for the joint model of ARIMA(1, 1, 0) with outliers effects when compared to that of ARIMA (1, 1, 0) model with AIC = −9132.26 and log likelihood = 4567.13 are respectively smaller and higher, making the former a better model than the later.

Table VIII: Joint Model of ARIMA (1, 1, 0) and Outliers Effects fitted to Return Series of Union Bank

	Estimate	Std. Error	z value	Pr(>|z|)
ar1	0.265411	0.018828	14.0965	< $2.2e^{-16}$ ***
IO458	-0.176690	0.024975	-7.0747	$1.497e^{-12}$ ***
IO1472	-0.045126	0.026449	-1.7061	0.0879825 .
IO1831	0.049676	0.025686	1.9340	0.0531185 .
IO1843	0.049638	0.025664	1.9341	0.0530983 .
AO150	-0.152926	0.027115	-5.6399	$1.701e^{-08}$ ***
AO705	-0.209666	0.027091	-7.7393	$9.999e^{-15}$ ***
AO1471	1.676966	0.029599	56.6554	< $2.2e^{-16}$ ***
AO1830	-0.122852	0.027860	-4.4096	$1.036e^{-05}$ ***
AO1842	-0.094486	0.027815	-3.3969	0.0006816 ***
AO1984	-0.118687	0.027104	-4.3790	$1.192e^{-05}$ ***
AO1994	0.169260	0.027084	6.2495	$4.117e^{-10}$ ***
TC691	-0.072536	0.023951	-3.0285	0.0024576 **
TC901	-0.076155	0.023946	-3.1803	0.0014712 **
TC1470	-0.004099	0.025804	-0.1589	0.8737844
TC1523	-0.075499	0.023947	-3.1528	0.0016173 **
TC1541	0.079253	0.023929	3.3120	0.0009264 ***
TC1824	0.106075	0.024035	4.4134	$1.018e^{-05}$ ***
TC2534	-0.084954	0.023936	-3.5492	0.0003865 ***
AO1748	-0.110790	0.027141	-4.0821	$4.463e^{-05}$ ***

3.7 Building ARIMA (1, 1, 0) Model for Outlier-Adjusted Return Series of Union Bank

Using the second method, which is removing the effects of the outliers and afterward, ARIMA(1, 1, 0) model is fitted to the outlier-adjusted series with its parameter significant at 5% level [Table IX], it is found to be adequate at 5% level of significance given the Q-statistics at lags 1, 14, 18, and 24 having Q(1) = 0.0030, Q(14) = 19.228, Q(18) = 24.611 and Q(24) = 27.717 with the corresponding *p*-values of 0.956, 0.1564, 0.136, and 0.2722.

Table IX: ARIMA (1,1,0) Model for Outlier Adjusted Return Series of Union Bank

Model	Parameter (φ)	Akaike Information Criteria	Log likelihood
ARIMA (1,1,0)	0.2654***	−11598.27	5800.13

***** significance at 5% level**

ARIMA (1, 1, 0) model fitted to the outlier adjusted series with least AIC = −11598.27 is found to be a better model than that of the joint estimation of ARIMA (1, 1, 0) with outliers effect, and that of ARIMA (1, 1, 0) model without outliers effect.

Again, the effects of outliers on the estimate of ARIMA(1, 1, 0) model fitted to the return series of Union bank is similar to that of the Fidelity bank although the estimate of the model is reduced by 0.164 and the modified iterative method is also adjudged superior in term of efficiency given that it produced a model with minimum variance as shown in Table X.

Table X: Effect of Outliers on Estimate of ARIMA (1, 1, 0) Model for Return Series of Union Bank

Model	ARIMA (1,1,0) (For outlier contaminated)	Joint ARIMA (1,1,0) and Outlier Effect	ARIMA (1,1,0) (For outlier adjusted)	Bias Introduced
Parameter	0.1014	0.2654	0.2654	−0.164
AIC	-9130.26	-11560.27	-11598.27	
Standard error	0.0192	0.0188	0.0186	
Variance	0.001963	0.000785	0.000784	
Log-likelihood	4567.13	5800.13	5800.13	

3.8 Linear Time Series Modeling of Return Series of Unity Bank

Again, using the same procedures as in the first two banks, ARIMA(1, 1, 0) model is found to be adequate for the return series of the Unity bank. However, about thirty three (33) different outliers are identified to have contaminated the residuals series of ARIMA(1,1,0) model, two (2) innovation outliers (IO), six (6) additive outliers, fifteen (15) temporary change and ten (10) level shift at C = 5 as shown in Table XI and the joint estimation of the parameter of ARIMA(1, 1, 0) model and outliers effects is shown in Table XII.

Table XI: Types of Outliers identified in the Residual Series of ARIMA (1, 1, 0) Model fitted to the Return Series of Unity Bank

Type	Observation Index	Location	Estimate	T-statistic
IO	2293	20/04/2015	-0.180979695	-7.444781
AO	248	10/01/2007	1.098612289	45.331893
AO	1906	24/09/2013	-0.200532990	-8.274566
AO	2292	17/04/2015	2.302585093	95.011264
TC	247	09/01/2007	0.365004553	19.903211
TC	1736	18/01/2013	0.107790532	5.877674
TC	1745	01/02/2013	0.112419182	6.130068
TC	1753	13/02/2013	-0.118923561	-6.484743
TC	1762	26/02/2013	0.091895380	5.010932
TC	2291	16/04/2015	0.758297010	41.348923
TC	2298	27/04/2015	-0.142415961	-7.765752
TC	2304	06/04/2015	-0.098876918	-5.391626
TC	2446	30/11/2015	-0.093629262	-5.105479
TC	2458	16/12/2015	0.112419118	6.130064
TC	2460	18/12/2015	0.104980801	5.724463
TC	2467	04/01/2016	-0.106045605	-5.782525
TC	2469	06/01/2016	-0.119002493	-6.489047
IO	1905	23/09/2013	0.127354627	5.132279
AO	1904	20/09/2013	-0.163097022	-6.592937
LS	243	29/12/2006	-0.003141767	-5.772181
LS	251	15/01/2007	-0.002771753	-5.084049
LS	347	11/06/2007	-0.002837928	-5.102009
LS	520	19/06/2008	-0.003114010	-5.387808
LS	598	13/06/2008	-0.003027395	-5.143002
LS	613	04/07/2008	-0.003035068	-5.137530
LS	631	30/07/2008	-0.002988789	-5.037234
LS	635	05/08/2008	-0.003001055	-5.052994
LS	2286	09/04/2015	-0.008202488	-6.130741
TC	1901	17/09/2013	0.097493034	5.208012
TC	2477	18/01/2016	0.096324642	5.145597
LS	607	26/06/2008	0.022239276	28.623733
AO	1336	09/06/2011	-0.128187865	-5.110079
AO	1872	05/08/2013	-0.144642972	-5.766045

Table XII: Joint Model of ARIMA (1, 1, 0) and Outliers Effect fitted to Return Series of Unity Bank

	Estimate	Std. Error	z value	Pr(>|z|)
ar1	0.22870458	0.01895846	12.0635	$< 2.2e^{-16}$ ***
IO2293	0.00020692	0.02847462	0.0073	0.9942018
AO248	1.09943444	0.03077300	35.7272	$< 2.2e^{-16}$ ***
AO1906	-0.19318151	0.03242140	-5.9585	$2.546e^{-09}$ ***
AO2292	2.30319017	0.03189738	72.2062	$< 2.2e^{-16}$ ***
TC247	-0.00386092	0.03081394	-0.1253	0.9002878
TC1736	0.09934947	0.02489870	3.9901	$6.603e^{-05}$ ***
TC1745	0.11091559	0.02491948	4.4510	$8.549e^{-06}$ ***
TC1753	-0.12798775	0.02489883	-5.1403	$2.743e^{-07}$ ***
TC1762	0.10385821	0.02488517	4.1735	$3.000e^{-05}$ ***
TC2291	0.00423615	0.02733850	0.1550	0.8768592
TC2298	-0.12316459	0.02511656	-4.9037	$9.404e^{-07}$ ***
TC2304	-0.07679364	0.02506410	-3.0639	0.0021848**
TC2446	-0.08214679	0.02500377	-3.2854	0.0010185**
TC2458	0.09014905	0.02690186	3.3510	0.0008051***
TC2460	0.07420557	0.02694465	2.7540	0.0058872**
TC2467	-0.07462796	0.02693742	-2.7704	0.0055984**
TC2469	-0.08905130	0.02691343	-3.3088	0.0009370***
IO1905	0.08527383	0.03084516	2.7646	0.0056997**
AO1904	-0.19483838	0.03033146	-6.4236	$1.331e^{-10}$***
LS243	0.00112671	0.01580103	0.0713	0.9431541
LS251	0.00098712	0.01609793	0.0613	0.9511048
LS347	-0.00156994	0.00489397	-0.3208	0.7483691
LS520	-0.00603173	0.00523178	-1.1529	0.2489502
LS598	-0.02950821	0.01304279	-2.2624	0.0236717*
LS613	-0.05489807	0.01706044	-3.2179	0.0012915**
LS631	0.00951673	0.01947408	0.4887	0.6250634
LS635	-0.00408405	0.01769983	-0.2307	0.8175170
LS2286	-0.00206496	0.00221715	-0.9314	0.3516679
TC1901	0.11851208	0.02564415	4.6214	$3.811e^{-06}$ ***
TC2477	0.10312584	0.02499954	4.1251	$3.706e^{-05}$ ***
LS607	0.08270945	0.01887366	4.3823	$1.174e^{-05}$ ***
AO1336	-0.12237989	0.02901086	-4.2184	$2.460e^{-05}$ ***
AO1872	-0.12246391	0.02910831	-4.2072	$2.586e^{-05}$ ***

The effects of outliers on the estimate of ARIMA (1, 1, 0) model fitted to the return series of Unity bank is similar to those of the first two banks only that the estimate of the model is reduced by 0.1501, as shown in Table XIII.

Table XIII: Effect of Outliers on Estimate of ARIMA (1, 1, 0) Model for Return Series Unity Bank

Model	ARIMA (1,1,0) (For outlier contaminated)	Joint ARIMA (1,1,0) and Outliers Effects	ARIMA (1,1,0) (For outlier adjusted)	Bias Introduced
Parameter	0.0786	0.2287	0.2287	−0.1501
AIC	−7588.08	−11206.23	−11272.23	
Standard error	0.0192	0.0190	0.0188	
Variance	0.00348	0.000885	0.000884	
Log likelihood	3795.04	5638.12	5638.12	

4. CONCLUSION

In all, it is discovered that outliers introduced substantial biases in the estimates of the ARIMA models of the returns series considered and the two methods employed are sufficient and adequate in handling outliers in such time series. Meanwhile, to ensure efficiency of the estimated parameters of linear models, it is needful and commendable to account for the presence of outliers with preference given to modified iterative method. Furthermore, the fact that volatility clustering exist in the return series calls for entertainment and modeling of heteroscedasticity in future studies.

References

[1] I. U. Moffat and E. A. Akpan, Identification and Modeling of Outliers in a Discrete-Time Stochastic Series, Amer. J. Theor. Appl. Stat. 6 (2017), 191-197.

[2] R. S. Tsay, Analysis of Financial Time Series. (3rd ed.). New York: John Wiley & Sons Inc., (2010), 97-140.

[3] G.E.P. Box, G. M. Jenkins and G.C. Reinsel, Time Series Analysis: Forecasting and Control. (3rd ed.). New Jersey: Wiley and sons, (2008), 5-22.

[4] C. Chen and L. M. Liu, Joint Estimation of Model Parameters and Outlier Effects in Time Series. J. Amer. Stat. Assoc. 8 (1993), 84-297.

[5] W. W. S. Wei, Time Series Analysis Univariate and Multivariate Methods. (2nd ed.). New York: Adison Westley, (2006), 33-59.

[6] F. Battaglia and L. Orfei, Outlier Detection and Estimation in Nonlinear Time Series, J. Time Seri. Anal. 26 (2002), 108-120.

[7] A. B. Abdullahi and H. R. Bakari, Modeling the Nigeria Stock Market (Shares)Evidence from Time Series Analysis, Int. J. Eng. Sci. 3 (2014), 1-12.

[8] E. J. Ekpenyong and U. P. Udoudo, Short-Term Forecasting of Nigeria Inflation Rates using Seasonal ARIMA Model, Sci. J. Appl. Math. Stat. 5 (2016), 101-107.

[9] O. M. Olayiwola, A. A. Amalare and S. O. Adebesin, Prediction of Returns on All-Share Index of Nigeria Stock Exchange, Pac. J. Sci. Technol. 17 (2016), 114-119.

[10] O. S. Ajao, O. S. Obafemi, and F. A. Bolarinwa, Modeling Dollar-Naira Exchange Rate in Nigeria, Nigerian Stat. Soc. 1 (2017), 191-198.

[11] E. A. Akpan and I. U. Moffat, Detection and Modeling of Asymmetric GARCH Effects in a Discrete-Time Series, Int. J. Stat. Probab. 6 (2017), 111-119.

[12] M. J. Sanchez and D. Pena, The Identification of Multiple Outliers in ARIMA Models, Commun. Stat. Theory Methods, 32(6) (2013), 1265-128.

[13] I. Chang, G. C. Tiao and C. Chen, Estimation of Time Series Parameters in the Presence of Outliers, Technometrics, 30 (1988), 193-204.