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A NEW CHAOTIC ATTRACTOR WITH QUADRATIC EXPONENTIAL NONLINEAR TERM FROM CHEN'S ATTRACTOR

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Abstract. In this paper a new three-dimensional chaotic system is proposed, which relies on a nonlinear exponential term and a nonlinear quadratic cross term necessary for folding trajectories. Basic dynamical characteristics of the new system are analyzed. Compared with the Chen system, the equilibrium points of the new system does not contain the origin, and has a greater positive Lyapunov index, can produce more complex shaped chaotic attractor.

1. Introduction

Since Lorenz found the first chaotic attractor in a three first order autonomous ordinary differential equations (ODEs) when he studied the atmospheric convection in 1963 [1], many new three dimension (3D) chaotic attractors have been proposed in the last three decades, such as the Rossler system [2], the Chen system [3], the Lü system [4], the Liu system [5], and the generalized Lorenz system family [6]. New chaotic system can also be achieved by adding or changing the linear/nonlinear term of existing chaotic system. The nonlinear term of system is normally the product of variables at different state. when the system contains nonlinear terms of the exponential function Whether there will be chaos phenomenon, yet need research.

Wei and Yang [7] revealed a 3D autonomous chaotic attractor with a nonlinear term in the form of exponential function at the right-hand side in ODEs as $\dot{x} = ay - ax$, $\dot{y} = -by + mxz$, $\dot{z} = n - e^{xy}$, where the existence of singularly degenerate heteroclinic cycles for a suitable choice of the parameters was investigated. Recently, Liang and Zhonglin [8] discussed the basic dynamic characteristics of the new chaotic system containing a nonlinear term of exponential function instead of the nonlinear term in Lü system.
In this paper, a new chaotic system containing a nonlinear term of exponential function instead of the nonlinear term in Chen's system is proposed. The dynamic characteristics and simulation show clearly that proposed system is chaotic same as Lorenz chaotic attractor and others, but its topological structure is different from all existing chaotic attractors.

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This paper is organized as follows. Section 2 presents the design of a new chaotic system. Section 3 outlines the basic properties of the new system. Finally, conclusions are given in section 4.

2. Design of a new chaotic system

Chen's chaotic system is given by

$$\begin{cases} \dot{x} = a(y-x) \\ \dot{y} = (c-a)x + cy - xz \\ \dot{z} = xy - bz \end{cases} \tag{1}$$

When $a = 35$, $b = 3$, $c = 28$,the Lyapunov exponents of system (1) are found to be LE1=1.997, LE2=0.0002 , LE3=-11.9943. The fractal dimension of system (1) is D1=2.1665 . The chaotic attractor is shown in figure 1. System (I) is still in a state of chaos. In system (I), we use e^{xy} instead of xy in the third equation, and get a new system (2) :

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = (c-a)x + cy - xz, \\ \dot{z} = e^{xy} - bz, \end{cases} \tag{2}$$

When $a = 35$, $b = 3$, $c = 28$ the Lyapunov exponents of system (2) are found to be LEI =3.8065, LE2=0.010904 , LE3=-13.817. There is a positive Lyapunov exponent, which is much larger than that of the system (1). The Lyapunov dimension of system (2) is:

$$D_L = j + \frac{1}{|LE_{j+1}|}\sum_{i=1}^{j} LE_i = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2 + \frac{3.806 + 0.0109}{|-13.817|} = 2.2762$$

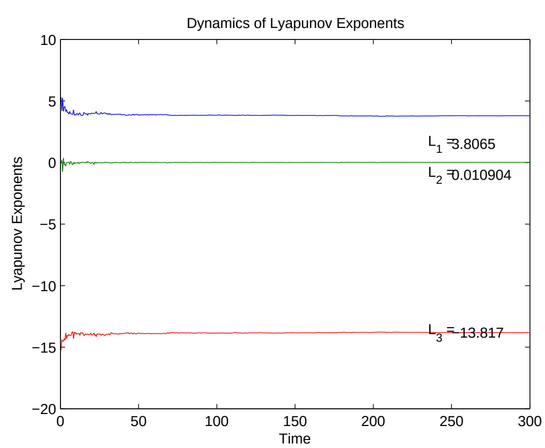

Figure 1. Dynamics of Lyapunov exponents of system (2)

The Lyapunov dimensions of the system are fractional. The chaotic attractor is shown in Fig. 3. System (2) is also at chaos status.

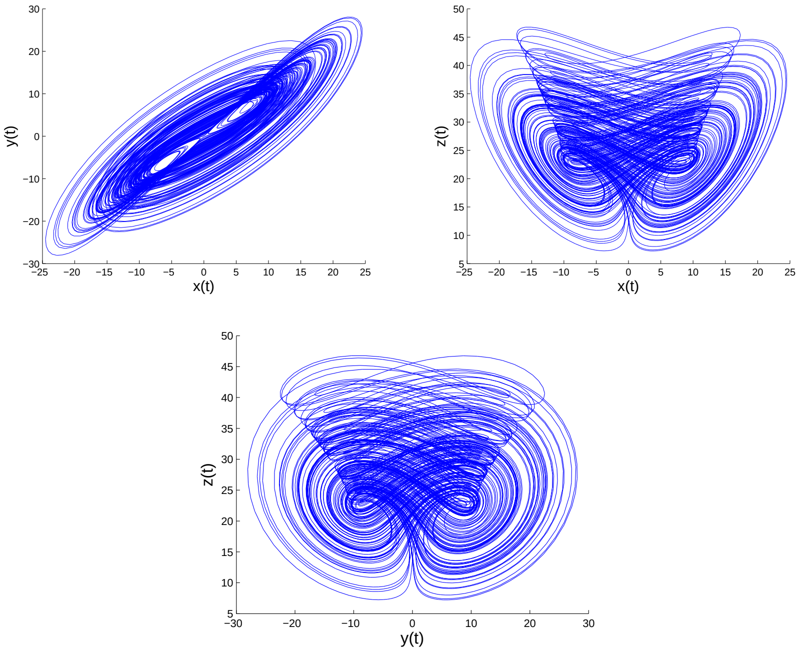

FIGURE 2. Phase portraits of chaotic attractors of system (I)

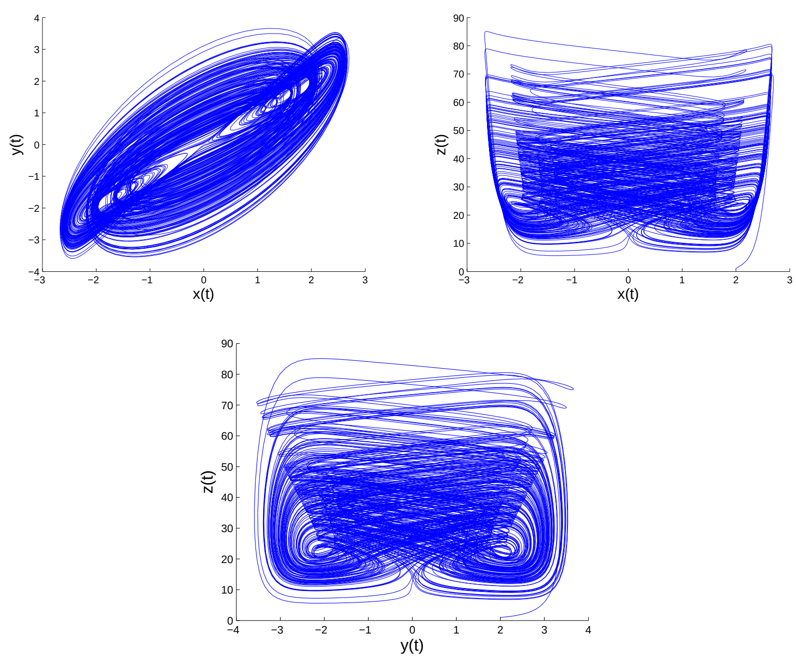

FIGURE 3. Phase portraits of chaotic attractors of system (2)

3. Basic properties of the new system

A. Equilibria

For equilibrium points we take

$$
\begin{cases}
a(y-x)=0, \\
(c-a)x+cy-xz=0, \\
e^{xy}-bz=0,
\end{cases} \tag{3}
$$

When $a=35$, $b=3$, $c=28$, system (2) has three equilibrium points: $E_1(0,0,0.333)$, $E_2(2.035,2.035,21)$, $E_3(-2.035,-2.035,21)$. The equilibrium point of system (2) does not contain the origin. The Jacobian matrix of system (2) is given by

$$
J=\begin{bmatrix} -a & a & 0 \\ c-a-z & c & -x \\ ye^{xy} & xe^{xy} & -b \end{bmatrix} \tag{4}
$$

For the equilibrium point$E_1=(0,0,0.333)$, system (2) has three eigenvalues: $\lambda_1=-3$, $\lambda_2=23.835$, $\lambda_3=-30.835$. Eigenvalues is not all for the positive or negative, according to the Routh-Hurwitz , $E_1=(0,0,0.333)$ is unstable saddle node. For the equilibrium points $E_2=(2.035,2.035,21)$ and $E_3=(-2.035,-2.035,21)$, system (2) has same eigenvalues: $\lambda_1=-26.053$, $\lambda_{2,3}=8.026\pm 25.203i$. λ_1 is a negative real root and $\lambda_{2,3}$ are a pair of conjugate roots with positive real part. So $E_2=(2.035,2.035,21)$ and $E_3=(-2.035,-2.035,21)$,are unstable saddle-focus points.

B. Symmetry and invariance

It is easy to see the invariance of system under the coordinate transformation $(x,y,z)\to(-x,-y,z)$ i.e., the system has rotation symmetry around the z -axis.

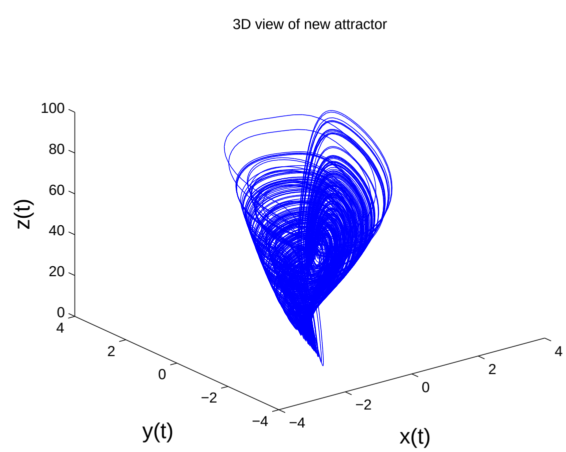

Figure 4. 3D view of new chaotic system (2)

C. Dissipativity

The three Lyapunov exponents and the divergence of the vector field is:

$$\sum_{i=1}^{3} LE_i = \Delta V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -a + c - b = f, \tag{5}$$

where $\sum_{i=1}^{3} LE_i$ denote the three Lyapunov exponents of the system. Note that $f = -a + c - b = -10$ is a negative value, so the system is a dissipative system and an exponential rate is:

$$\frac{dV}{dt} = e^f = e^{-10} \tag{6}$$

From(6), it can be seen that a volume element V_0 is contracted by the flow into a volume element $V_0 e^{-10t}$ in time t . This means that each volume containing the system trajectory shrinks to zero as $t \to \infty$ at an exponential rate of -10 . Therefore, all system orbits are ultimately confined to a specific subset having zero volume and the asymptotic motion settles onto an attractor .

D. Sensitivity to initial conditions Figure 4 shows that the evolution of the chaos trajectories is very sensitive to initial conditions. The initial values of the system are set to $[2, 2, 1]^T$ for the solid line and $[2.01, 2, 1]^T$ for the dashed line.

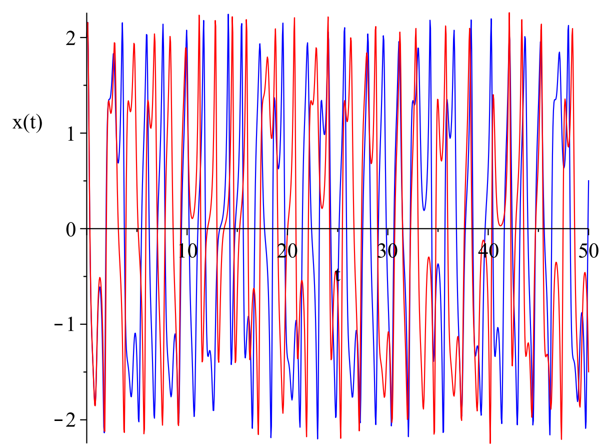

FIGURE 5. Sensitivity of system(2) to initial conditions

4. CONCLUSIONS

A new chaotic system is proposed in this paper, which has exponential term instead of the nonlinear term of the Chen's system. Some basic properties of the system have been investigated. Compare with Chen's system, the new system has greater chaos interval and much larger Lyapunov exponent. Its equilibrium point does not contain the origin. Even though more important analysis of the system like chaos control, boundedness, and synchronization, will take into account in the future work.

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