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FUZZY STABILITY OF GENERALIZED SQUARE ROOT FUNCTIONAL EQUATION IN SEVERAL VARIABLES: A FIXED POINT APPROACH

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Abstract. In this paper, we investigate the generalized Hyers-Ulam stability of the generalized square root functional equation in several variables in fuzzy Banach spaces, by applying the fixed point method.

1. INTRODUCTION

In the last forty years, fuzzy theory has gained paramount importance and validty on the mathematical scenario by facilitating to focus an ardent attention on multifarious avenues of development in the theory of fuzzy sets to explore the fuzzy analogues of the classical set theory. In the effulgent light of the authentic investigations executed in this branch, the fuzzy sets are being tapped to augment a wide range of applications in science and Engineering with platonic dimensions.

Various mathematical visions, viewed in different perspectives, have triggered scores of scholars to come out with different definitions of fuzzy norms on a vector space. For Example, A.K. Katsaras [26] had accomplished a detailed survey to define a fuzzy norm on a vector space to help to construct a fuzzy vector topological structure. In 1991, R. Biswas [6] defined and studied fuzzy inner product spaces in linear space. In 1992, C. Felbin [18] introduced an alternative definition of a fuzzy norm on a linear topological structures of a fuzzy normed linear spaces. Similarly, T. Bag and S.K. Samanta [4], gliding along the mathematical track of S.C. Cheng and J.M. Mordeson [12], proved that the corresponding fuzzy metric of a fuzzy norm would be the same as that of the metric executed by I. Kramosil and J. Michalek [28]. They had initiated a decomposition theorem of a fuzzy norm into a family of crisp norms by undertaking an analytical investigation of some of the properties of fuzzy normed spaces.

An inquisitive question that was given a serious thought by S.M. Ulam [45] concerning the stability of group homomorphisms gave rise to the stability problem of functional equations. The laborious intellectual strivings of D.H. Hyers [23] did not go in vain because he was the first to come out with a partial answer to solve the question posed by Ulam on Banach spaces. In course of time, the theorem formulated by Hyers was generalized by T. Aoki [2] for additive mappings and by Th.M. Rassias [43] for linear mappings by taking into consideration an unbounded

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Cauchy difference.

The findings of Th.M. Rassias have exercised a delectable influence on the development of what is addressed as the generalized Hyers-Ulam stability of Hyers-Ulam-Rassias stability of functional equations. A generalized and modified form of the theorem evolved by Th.M. Rassias was advocated by P. Gavruta [21] who replaced the unbounded Cauchy difference by driving into study a general control function within the viable approach designed by Th.M. Rassias. A further research materialized by F. Skof [44] found solution to Hyers-Ulam-Rassias stability problem for quadratic functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y) \tag{1}$$

for a class of functions $f : A \to B$, where A is a normed space and B is a Banach space. The stability problems of several functional equations have been extensively investigated by a number of scholars, possed with creative thinking and critial dissent who have arrived at interesting results (see [3], [10], [11], [17], [22], [24], [27], [42]).

In 1996, G. Isac and Th.M. Rassias [25] were the first to provide applications of stability theory of functional equations for the proof of new fixed point theorems with applications. By using fixed point methods, the stability problems of several functional equations have been extensively investigated by a number of authors (see [8], [9], [35], [36], [39]).

Functional equations find a lot of application in information theory, information science, measure of information, coding theory, fuzzy system models, economics, social sciences and physics.

The paper presenters have made use of some basic concept concerning fuzzy normed spaces and some fundamental results in fixed point theory.

Let X be a real linear space. A function $N : X \times \mathbb{R} \to [0,1]$ is said to be a fuzzy norm on X if for all $x, y \in X$ and all $u, v \in \mathbb{R}$:

(N_1) $N(x,u) = 0$ for $u \leq 0$

(N_2) $x = 0$ if and only if $N(x,u) = 1$ for all $u > 0$

(N_3) $N(ux,v) = N\left(x, \frac{v}{|u|}\right)$ if $u \neq 0$

(N_4) $N(x+y,u+v) \geq min\{N(x,u), N(y,v)\}$

(N_5) $N(x,.)$ is non-decreasing function on $\mathbb{R}$ and $\lim_{t\to\infty} N(x,u) = 1$

(N_6) For $x \neq 0$, $N(x,.)$ is (uppersemi) continuous on $\mathbb{R}$.

The pair (X,N) is called a fuzzy normed linear space. One may regard $N(x,u)$ as the truth value of the statement the norm of x is less than or equal to the real number u.

Definition 1.1. Let (X,N) be a fuzzy normed linear space. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is said to be convergent if there exists $x \in X$ such that $\lim_{n\to\infty} N(x_n - x, u) = 1$ for all $u > 0$. In this case, x is called the limit of the sequence $\{x_n\}$ and we denote it by $N\text{-}\lim_{n\to\infty} x_n = x$.

Definition 1.2. A sequence $\{x_n\}$ in X is called Cauchy if for each $\epsilon > 0$ and each $u > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ and all $p > 0$, we have $N(x_{n+p} - x_n, u) > 1 - \epsilon$.

It is known that every convergent sequence in fuzzy normed space is Cauchy. If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.

We say that a mapping $f : X \to Y$ between fuzzy normed linear spaces X and Y is continuous at a point $x_0 \in X$ if for each sequence $\{x_n\}$ converging to x_0 in X, then the sequence $\{f(x_n)\}$ converges to $f(x_0)$. If $f : X \to Y$ is continuous at each $x \in X$, then $f : X \to Y$ is said to be continuous on X.

Let X be a set. A function $d : X \times X \to [0, \infty]$ is called a generalized metric on X if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;
(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(3) $d(x, z) \le d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Theorem 1.3. *Let (X, d) be a complete generalized metric space and let $\sigma : X \to X$ be a strictly contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(\sigma^n x, \sigma^{n+1} x) = \infty$$

for all nonnegative integers n or there exists a positive integer n_0 such that

(1) *$d(\sigma^n x, \sigma^{n+1} x) < \infty$ for all $n \ge n_0$;*
(2) *the sequence $\{\sigma^n x\}$ converges to a fixed point y^* of σ;*
(3) *y^* is the unique fixed point of σ in the set*
$$Y = \{y \in X / d(\sigma^{n_0} x, y) < \infty\};$$
(4) *$d(y, y^*) \le \frac{1}{1-L} d(y, \sigma y)$ for all $y \in Y$.*

K. Ravi and B.V. Senthil Kumar[41] introduced the generalized square root functional equation (or GSRF equation) in several variables of the form

$$s\left(\sum_{i=1}^{p} \rho_i x_i + 2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \sqrt{\rho_i \rho_j x_i x_j}\right) = \sum_{i=1}^{p} \sqrt{\rho_i}\, s(x_i) \tag{2}$$

for arbitrary but fixed real numbers $(\rho_i, \rho_2, \dots, \rho_p) \neq (0, 0, \dots, 0)$, so that $0 < \rho = \sqrt{\rho_1} + \sqrt{\rho_2} + \cdots + \sqrt{\rho_p} = \sum_{i=1}^{p} \sqrt{\rho_i} \neq 1$ and $s : X \to \mathbb{R}$ with X as space of non-negative real numbers and investigated generalized Hyers-Ulam stability of equation (2). It is easy to verify that the function $f : X \to \mathbb{R}$ such that $f(x) = \sqrt{x}$ is a solution of the functional equation (2).

In this paper, we will show the generalized Hyers-Ulam stability of the equation (2) on fuzzy normed spaces using fixed point method.

Throughout this paper, let us assume that X be space of non-negative real numbers and Y be a fuzzy normed linear space.

For the sake of convenience, let us define

$$D_\rho s(x_1, x_2, \dots, x_p) = s\left(\sum_{i=1}^{p} \rho_i x_i + 2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \sqrt{\rho_i \rho_j x_i x_j}\right) - \sum_{i=1}^{p} \sqrt{\rho_i}\, s(x_i)$$

for all $x_1, x_2, \dots, x_p \in X$ and $p \in \mathbb{N} - \{1\}$.

2. GENERALIZED HYERS-ULAM STABILITY OF THE FUNCTIONAL EQUTION (2) IN FUZZY NORMED SPACES

Theorem 2.1. *Let $\varphi : X^p \to \mathbb{R}$ be a function such that there exists an $L < 1$ with*

$$\varphi(\rho^2 x_1, \rho^2 x_2, \dots, \rho^2 x_p) \le \rho L \varphi(x_1, x_2, \dots, x_p)$$

for all $x_1, x_2, \ldots, x_p \in X$. *Let* $f : X \to \mathbb{R}$ *be a mapping satisfying*

$$N\left(D_\rho f(x_1, x_2, \ldots, x_p), t\right) \geq \frac{t}{t + \varphi(x_1, x_2, \ldots, x_p)} \tag{1}$$

for all $x_1, x_2, \ldots, x_p \in X$ *and all* $t > 0$, *where* $0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} < 1$. *Then* $s(x) = N\text{-}\lim_{n\to\infty} \rho^{-n} f\left(\rho^{2n} x\right)$ *exists for each* $x \in X$ *and defines a square root mapping* $s : X \to Y$ *such that*

$$N(f(x) - s(x), t) \geq \frac{(1-L)t}{(1-L)t + L\varphi(x, x, \ldots, x)} \tag{2}$$

for all $x \in X$ *and all* $t > 0$.

Proof. Taking x_i as x for $1 \leq i \leq p$ in (1), we get

$$N\left(f(\rho^2 x) - \rho f(x)\right) \geq \frac{t}{t + \varphi(x, x, \ldots, x)} \tag{3}$$

for all $x \in X$.

Consider the set

$$S = \{g : X \to Y / g \text{ is a function}\}$$

and introduce the generalized metric d on S as follows:

$$d(g, h) = \inf\{C \in R_+ : N(g(x) - h(x), Ct) \geq \frac{t}{t + \varphi(x, x, \ldots, x)}, \forall x \in X, \forall t > 0\},$$

where, as usual, $\inf \phi = +\infty$. It is easy to show that (S, d) is complete. (See the proof of Lemma 2.1 of [30]).

Define a mapping $\sigma : S \to S$ by

$$\sigma h(x) = \frac{1}{\rho} h\left(\rho^2 x\right) (x \in X)$$

Let $g, h \in S$ be given such that $d(g, h) = \epsilon$. Then

$$N\left(g(x) - h(x), \epsilon t\right) \geq \frac{t}{t + \varphi(x, x, \ldots, x)}$$

for all $x \in X$ and all $t > 0$. Hence

$$\begin{aligned}
N(\sigma g(x) - \sigma h(x), L\epsilon t) &= N\left(\frac{1}{\rho} g(\rho^2 x) - \frac{1}{\rho} h(\rho^2 x), L\epsilon t\right) \\
&= N\left(g(\rho^2 x) - h(\rho^2 x), \rho L\epsilon t\right) \\
&\geq \frac{\rho L t}{\rho L t + \varphi(\rho^2 x, \rho^2 x, \ldots, \rho^2 x)} \\
&\geq \frac{\rho L t}{\rho L t + \rho L \varphi(x, x, \ldots, x)} \\
&\geq \frac{t}{t + \varphi(x, x, \ldots, x)}
\end{aligned}$$

for all $x \in X$ and all $t > 0$. So $d(g,h) = \epsilon$ implies that $d(\sigma g, \sigma h) \leq L\epsilon$. This means that

$$d(\sigma g, \sigma h) \leq L d(g,h)$$

for all $g, h \in S$.

It follows from (3) that

$$N\left(\frac{1}{\rho} f(\rho^2 x) - f(x), \frac{Lt}{\rho}\right) \geq \frac{t}{t + \varphi(x, x, \ldots, x)}$$

for all $x \in X$ and all $t > 0$. So $d(\sigma f, f) \leq \frac{L}{\rho}$.

By Theorem 1.3, there exists a mapping $s : X \to Y$ satisfying the following:

(1) s is a fixed point of σ, ie.,

$$s(\rho^2 x) = \rho s(x) \tag{4}$$

for all $x \in X$. The mapping s is a unique fixed point of σ in the set

$$\mu = \{g \in S : d(f,g) < \infty\}.$$

This implies that s is a unique mapping satisfying (4) such that there exists a $C \in (0, \infty)$ satisfying

$$N(f(x) - s(x), Ct) \geq \frac{t}{t + \varphi(x, x, \ldots, x)}$$

for all $x \in X$.

(2) $d(\sigma^n f, s) \to 0$ as $n \to \infty$. This implies the equality

$$N\text{-}\lim_{n\to\infty} \frac{1}{\rho^n} f(\rho^{2n} x) = s(x)$$

for all $x \in X$.

(3) $d(f,s) \leq \frac{1}{1-L} d(\sigma f, f)$, which implies the inequality

$$d(f,s) \leq \frac{L}{\rho - \rho L}.$$

This implies that the inequality (2) holds.

By (1),

$$N\left(\frac{1}{\rho^n} D_\rho f(\rho^{2n} x_1, \rho^{2n} x_2, \ldots, \rho^{2n} x_p), \frac{t}{\rho^n}\right) \geq \frac{t}{t + \varphi(\rho^{2n} x_1, \rho^{2n} x_2, \ldots, \rho^{2n} x_p)}$$

for all $x_1, x_2, \ldots, x_p \in X$, all $t > 0$ and all $n \in \mathbb{N}$. So

$$N\left(\frac{1}{\rho^n} D_\rho f(\rho^{2n} x_1, \rho^{2n} x_2, \ldots, \rho^{2n} x_p), t\right) \geq \frac{\rho^n t}{\rho^n t + L^n \rho^n \varphi(x_1, x_2, \ldots, x_p)}$$

for all $x_1, x_2, \ldots, x_p \in X$, all $t > 0$ and all $n \in \mathbb{N}$. Since

$$\lim_{n\to\infty} \frac{\rho^n t}{\rho^n t + L^n \rho^n \varphi(x_1, x_2, \ldots, x_p)} = 1$$

for all $x_1, x_2, \ldots, x_p \in X$, all $t > 0$,

$$N\left(D_\rho s(x_1, x_2, \ldots, x_p), t\right) = 1$$

for all $x_1, x_2, \ldots, x_p \in X$, all $t > 0$. Thus the mapping $s : X^p \to Y$ is square root as desired. $\square$

Theorem 2.2. *Let $\varphi : X^p \to Y$ be a function such that there exists an $L < 1$ with*

$$\varphi\left(\frac{x_1}{\rho^2}, \frac{x_2}{\rho^2}, \ldots, \frac{x_p}{\rho^2}\right) \leq \frac{L}{\rho}\varphi(x_1, x_2, \ldots, x_p)$$

for all $x_1, x_2, \ldots, x_p \in X$. Let $f : X \to Y$ be a mapping satisfying

$$N\left(D_\rho f(x_1, x_2, \ldots, x_p), t\right) \geq \frac{t}{t + \varphi(x_1, x_2, \ldots, x_p)} \tag{5}$$

for all $x_1, x_2, \ldots, x_p \in X$ and all $t > 0$, where $0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} > 1$. Then $s(x) = N\text{-}\lim_{n\to\infty} \rho^n f\left(\rho^{-2n}x\right)$ exists for each $x \in X$ and defines a square root mapping $s : X \to Y$ such that

$$N(f(x) - s(x), t) \geq \frac{(1-L)t}{(1-L)t + L\varphi(x, x, \ldots, x)} \tag{6}$$

for all $x \in X$ and all $t > 0$.

Proof. Taking x_i as $\frac{x}{\rho^2}$ for $1 \leq i \leq p$ in (5) and proceeding further using similar arguments as in Theorem 2.1, the proof is complete. $\square$

Corollary 2.3. *Let $c_1 \geq 0$ and α be real numbers with $\alpha > \frac{1}{2}$ or $\alpha < \frac{1}{2}$. Let $f : X^p \to Y$ be a mapping satisfying*

$$N\left(D_\rho f(x_1, x_2, \ldots, x_p), t\right) \geq \frac{t}{t + c_1\left(\sum_{i=1}^{p} |x_i|^\alpha\right)}$$

for all $x_1, x_2, \ldots, x_p \in X$ and all $t > 0$. Then there exists a unique square mapping $s : X \to Y$ such that

$$N(f(x) - s(x), t) \geq \begin{cases} \frac{(\rho^\alpha - \rho^{\frac{1}{2}})t}{(\rho^\alpha - \rho^{\frac{1}{2}})t + \rho^{\frac{1}{2}} p c_1 |x|^\alpha} & \text{for } \alpha > \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} < 1 \\ \frac{(\rho^{\frac{1}{2}} - \rho^\alpha)t}{(\rho^{\frac{1}{2}} - \rho^\alpha)t + \rho^\alpha p c_1 |x|^\alpha} & \text{for } \alpha < \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} > 1 \end{cases}$$

for all $x \in X$ and all $t > 0$.

Proof. By taking $\varphi(x_1, x_2, \ldots, x_p) = c_1\left(\sum_{i=1}^{p} |x_i|^\alpha\right)$ for all $x_1, x_2, \ldots, x_p \in X$ in Theorem 2.1 and Theorem 2.2, and choosing respectively $L = \rho^{\frac{1}{2}-\alpha}$ and $L = \rho^{\alpha-\frac{1}{2}}$, we get the deisred result. $\square$

Corollary 2.4. *Let $c_2 \geq 0$ and α be real numbers with $\alpha > \frac{1}{2}$ or $\alpha < \frac{1}{2}$. Let $f : X^p \to Y$ be a mapping satisfying*

$$N\left(D_\rho f(x_1, x_2, \dots, x_p), t\right) \geq \frac{t}{t + c_2 \left(\prod_{i=1}^{p} |x_i|^{\frac{\alpha}{p}}\right)} \tag{7}$$

for all $x_1, x_2, \dots, x_p \in X$ and all $t > 0$. Then there exists a unique square root mapping $s : X \to Y$ such that

$$N(f(x) - s(x), t) \geq \begin{cases} \frac{(\rho^{\alpha} - \rho^{\frac{1}{2}})t}{(\rho^{\alpha} - \rho^{\frac{1}{2}})t + \rho^{\frac{1}{2}} c_2 |x|^{\alpha}} & \text{for } \alpha > \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} < 1 \\ \frac{(\rho^{\frac{1}{2}} - \rho^{\alpha})t}{(\rho^{\frac{1}{2}} - \rho^{\alpha})t + \rho^{\alpha} c_2 |x|^{\alpha}} & \text{for } \alpha < \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} > 1 \end{cases}$$

for all $x \in X$ and all $t > 0$.

Proof. By taking $\varphi(x_1, x_2, \dots, x_p) = c_2 \left(\prod_{i=1}^{p} |x_i|^{\frac{\alpha}{p}}\right)$ for all $x_1, x_2, \dots, x_p \in X$ in Theorem 2.1 and Theorem 2.2, and choosing respectively $L = \rho^{\frac{1}{2} - \alpha}$ and $L = \rho^{\alpha - \frac{1}{2}}$, we get the desired result. $\square$

Corollary 2.5. *Let $c_3 \geq 0$ and α be real numbers with $\alpha > \frac{1}{2}$ or $\alpha < \frac{1}{2}$. Let $f : X^p \to Y$ be a mapping satisfying*

$$N\left(D_\rho f(x_1, x_2, \dots, x_p), t\right) \geq \frac{t}{t + c_3 \left[\sum_{i=1}^{p} \left(\prod_{j=1, j \neq i}^{p} |x_j|^{\frac{\alpha}{p-1}}\right)\right]}$$

for all $x_1, x_2, \dots, x_p \in X$ and all $t > 0$. Then there exists a unique square root mapping $s : X \to Y$ such that

$$N(f(x) - s(x), t) \geq \begin{cases} \frac{(\rho^{\alpha} - \rho^{\frac{1}{2}})t}{(\rho^{\alpha} - \rho^{\frac{1}{2}})t + \rho^{\frac{1}{2}} p c_3 |x|^{\alpha}} & \text{for } \alpha > \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} < 1 \\ \frac{(\rho^{\frac{1}{2}} - \rho^{\alpha})t}{(\rho^{\frac{1}{2}} - \rho^{\alpha})t + \rho^{\alpha} p c_3 |x|^{\alpha}} & \text{for } \alpha < \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^{p} \sqrt{\rho_i} > 1 \end{cases}$$

for all $x \in X$ and all $t > 0$.

Proof. By taking $\varphi(x_1, x_2, \dots, x_p) = c_3 \left[\sum_{i=1}^{p} \left(\prod_{j=1, j \neq i}^{p} |x_j|^{\frac{\alpha}{p-1}}\right)\right]$ for all $x_1, x_2, \dots, x_p \in X$ in Theorem 2.1 and Theorem 2.2, and choosing respectively $L = \rho^{\frac{1}{2} - \alpha}$ and $L = \rho^{\alpha - \frac{1}{2}}$, we get the desired result. $\square$

Corollary 2.6. *Let $c_4 \geq 0$ and α be real numbers with $\alpha > \frac{1}{2}$ or $\alpha > \frac{1}{2}$. Let $f : X^p \to Y$ be a mapping satisfying*

$$N\left(D_\rho f(x_1, x_2, \dots, x_p), t\right) \geq \frac{t}{t + c_4 \left[\prod_{i=1}^{p} |x_i|^{\frac{\alpha}{p}} + \left(\sum_{i=1}^{p} |x_i|^{\alpha}\right)\right]}$$

for all $x_1, x_2, \dots, x_p \in X$ *and all* $t > 0$. *Then there exists a unique square root mapping* $s : X \to Y$ *such that*

$$N(f(x)-s(x),t) \geq \begin{cases} \frac{(\rho^\alpha-\rho^{\frac{1}{2}})t}{(\rho^\alpha-\rho^{\frac{1}{2}})t+\rho^{\frac{1}{2}}(p+1)c_4|x|^\alpha} & \text{for } \alpha > \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^p \sqrt{\rho_i} < 1 \\ \frac{(\rho^{\frac{1}{2}}-\rho^\alpha)t}{(\rho^{\frac{1}{2}}-\rho^\alpha)t+\rho^\alpha(p+1)c_4|x|^\alpha} & \text{for } \alpha < \frac{1}{2} \text{ and } 0 < \rho = \sum_{i=1}^p \sqrt{\rho_i} > 1 \end{cases}$$

for all $x \in X$ *and all* $t > 0$.

Proof. By taking $\varphi(x_1, x_2, \dots, x_p) = c_4 \left[\prod_{i=1}^p |x_i|^{\frac{\alpha}{p}} + (\sum_{i=1}^p |x_i|^\alpha)\right]$ for all $x_1, x_2, \dots, x_p \in X$ in Theorem 2.1 and Theorem 2.2, and choosing respectively $L = \rho^{\frac{1}{2}-\alpha}$ and $L = \rho^{\alpha-\frac{1}{2}}$, we get the desired result. □

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