Int. J. Anal. Appl. (2022), 20:49

International Journal of Analysis and Applications

On Tripolar Fuzzy Pure Ideals in Ordered Semigroups

Nuttapong Wattanasiripong[1], **Jirapong Mekwian**[2], **Hataikhan Sanpan**[2], **Somsak Lekkoksung**[2,*]

[1] *Division of Applied Mathematics, Faculty of Science and Technology, Valaya Alongkorn Rajabhat University under the Royal Patronage, Pathum Thani 13180, Thailand*

[2] *Division of Mathematics, Faculty of Engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Khon Kaen 40000, Thailand*

[*] *Corresponding author:* lekkoksung_somsak@hotmail.com

Abstract. Tripolar fuzzy sets are a concept that deals with tripolar information. This idea is a generalization of bipolar and intuitionistic fuzzy sets. In this paper, the notions of tripolar fuzzy pure ideals in ordered semigroups are introduced, and some algebraic properties of tripolar fuzzy pure ideals are studied. We obtain some characterizations of weakly regular ordered semigroups in terms of tripolar fuzzy pure ideals. Finally, we introduce the concepts of tripolar weakly pure ideals and prove that the tripolar fuzzy ideals are tripolar weakly pure ideals if such tripolar fuzzy ideals satisfy the idempotent property.

1. Introduction

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty was introduced by Zadeh [14] in 1965. After the introduction of the concept of fuzzy sets by Zadeh, several researchers researched the generalizations of the notions of fuzzy sets with huge applications in computer science, artificial intelligence, control engineering, robotics, automata theory, decision theory, finite state machine, graph theory, logic, operations research and many branches of pure and applied mathematics. For example, Xie et al. applied fuzzy set theory to switching method [13]. There are many extensions of fuzzy sets, for example, intuitionistic fuzzy sets, hesitant fuzzy sets, interval-valued fuzzy sets, vague sets, type-2 fuzzy sets, fuzzy multi-sets, bipolar fuzzy sets, and cubic sets. The

Received: Aug. 10, 2022.

2010 *Mathematics Subject Classification.* 06F05, 08A72.

Key words and phrases. ordered semigroup; weakly regular ordered semigroup; tripolar fuzzy pure ideal; tripolar fuzzy weakly pure ideal.

https://doi.org/10.28924/2291-8639-20-2022-49
ISSN: 2291-8639

fuzzification of the algebraic structure was introduced by Rosenfeld [11] and he introduced the notion of fuzzy subgroups in 1971.

A bipolar fuzzy set is an extension of a fuzzy set whose membership degree range is $[-1, 1]$. In 1994, Zhang [15] initiated the concept of the bipolar fuzzy set as a generalization of the fuzzy set. The notion of an intuitionistic fuzzy set was introduced by Atanassov [2] as a generalization of the notion of a fuzzy set.

The tripolar fuzzy set is a generalization of fuzzy set, bipolar fuzzy set, and intuitionistic fuzzy set. Rao [7] introduced the notion of tripolar fuzzy set as a generalization of fuzzy set, bipolar fuzzy set, and intuitionistic fuzzy set, and studied tripolar fuzzy interior ideals of Γ-semigroup. Rao and Venkateswarlu [8, 9] investigated tripolar fuzzy interior ideal, tripolar fuzzy soft ideal, and tripolar fuzzy soft interior ideal of Γ-semigroup and Γ-semiring. Rao [10] introduced the notion of tripolar fuzzy interior ideal, tripolar fuzzy soft ideal, and tripolar fuzzy soft interior ideal over semigroup and studied some of their algebraic properties.

Ahsan and Takahashi [1] introduced the notions of pure ideals and purely prime ideals in semigroups without order. Bashir et al. [3] defined the concepts of pure ideals, weakly purely ideals, and purely prime ideals in ternary semigroups. In [4] Changphas and Sanborisoot introduced the concepts of pure ideals, weakly pure ideals, and purely prime ideals in ordered semigroups. Siribute and Sanborisoot [12] applied fuzzy theory to semigroup theory. They introduced the concepts of pure fuzzy and weakly pure fuzzy ideals in ordered semigroups and characterized weakly regular ordered semigroups by pure fuzzy ideals. Linesawat et al. [6] introduced the concepts of anti-hybrid pure ideals in ordered semigroups and studied some algebraic properties of anti-hybrid pure ideals. They characterize weakly regular ordered semigroups in terms of anti-hybrid pure ideals. Finally, they also gave the concept of anti-hybrid weakly pure ideals. They prove that the anti-hybrid ideals are anti-hybrid weakly pure ideals if such anti-hybrid ideals satisfy the idempotent property.

Based on the concept of the purity of fuzzy ideals considered by Siribute and Sanborisoot [12], we apply the concept of purity to tripolar fuzzy ideals in ordered semigroups. In this present paper, the notions of tripolar fuzzy pure ideals in ordered semigroups are introduced, and some algebraic properties of tripolar fuzzy pure ideals are studied. We obtain some characterizations of weakly regular ordered semigroups in terms of tripolar fuzzy pure ideals. Finally, we introduce the concepts of tripolar weakly pure ideals and prove that the tripolar fuzzy ideals are tripolar weakly pure ideals if such tripolar fuzzy ideals satisfy the idempotent property.

2. Preliminaries

In this section, we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

An ordered semigroup is a structure $(S; \cdot, \leq)$ such that

(1) $(S;\cdot)$ is a semigroup.

(2) $(S;\leq)$ is a partially ordered set, and

(3) $x\leq y$ implies $u\cdot x\leq v\cdot y$ and $x\cdot u\leq y\cdot v$ for all $u,v,x,y\in S$.

For simplicity, we will be written xy instead of $x\cdot y$, and an ordered semigroup $(S;\cdot,\leq)$, will be written in its universe set as a bold letter $\mathbf{S}$.

For $K\subseteq S$, we denote

$$(K] := \{a\in S \mid a\leq k \text{ for some } k\in K\}.$$

Let A and B be two nonempty subsets of S. Then we define

$$AB := \{ab \mid a\in A \text{ and } b\in B\}.$$

Definition 2.1. *[5] Let $\mathbf{S}$ be an ordered semigroup. A nonempty subset A of S is called a left (resp., right) ideal of $\mathbf{S}$ if*

(1) *$SA\subseteq A$ (resp., $AS\subseteq A$).*

(2) *For $a\in S, b\in A$, if $a\leq b$, then $a\in A$.*

A nonempty subset I of S is called a *two-side ideal (ideal)* of $\mathbf{S}$ if it is both a left and a right ideal of $\mathbf{S}$.

Definition 2.2. *[4] Let $\mathbf{S}$ be an ordered semigroup. An ideal I of $\mathbf{S}$ is called a right (resp., left) pure ideal of $\mathbf{S}$ if for each $a\in I$ there exist $x\in I$ such that $a\leq ax$ (resp., $a\leq xa$).*

An ideal I of $\mathbf{S}$ is called a *pure ideal* of $\mathbf{S}$ if it is both a left and a right pure ideal of $\mathbf{S}$.

A fuzzy subset of a nonempty set X (or fuzzy set in a nonempty subset X) is a mapping $f: X\to[0,1]$ from X to a unit closed interval, (see [14]).

Definition 2.3. *[7] A fuzzy set f of a universe set X is said to be a tripolar fuzzy set, if*

$$f := \{(x; f^{+}(x), f^{*}(x), f^{-}(x)) \mid x\in X \text{ and } 0\leq f^{+}(x)+f^{*}(x)\leq 1\},$$

where $f^{+}: X\to[0,1], f^{*}: X\to[0,1]$ and $f^{-}: X\to[-1,0]$. The membership degree $f^{+}(x)$ characterizes the extent that the element X satisfies the property corresponding to tripolar fuzzy set f, $f^{*}(x)$ characterizes the extent that the element X satisfies the not property (irrelevant) corresponding to tripolar fuzzy set f, and $f^{-}(x)$ characterizes the extent that the element X satisfies the implicit counter property corresponding to tripolar fuzzy set f. For simplicity $f := (f^{+}, f^{*}, f^{-})$ has been used for $f := \{(x; f^{+}(x), f^{*}(x), f^{-}(x)) \mid x\in X$ and $0\leq f^{+}(x)+f^{*}(x)\leq 1\}$.

Let $a\in S$. Then, we set $\mathbf{S}_a := \{(x,y)\in S\times S \mid a\leq xy\}$. We denote $Tri(S)$ the set of all tripolar fuzzy subsets of S and define an operation on such set as follows: Let $f := (f^{+}, f^{*}, f^{-}), g :=$

(g^+, g^*, g^-) be elements in $Tri(S)$. Then the product $f \circ g$ of f and g as the tripolar fuzzy set, denote by $f \circ g := (f^+ \circ g^+, f^* \circ g^*, f^- \circ g^-)$, of S defined as follows: For each $x \in S$.

$$(f^+ \circ g^+)(x) = \begin{cases} \bigvee_{(a,b)\in \mathrm{S}_x} \{\min\{f^+(a), g^+(b)\}\} & \text{if } \mathbf{S}_x \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

$$(f^* \circ g^*)(x) = \begin{cases} \bigwedge_{(a,b)\in \mathrm{S}_x} \{\max\{f^*(a), g^*(b)\}\} & \text{if } \mathbf{S}_x \neq \emptyset \\ 1 & \text{otherwise,} \end{cases}$$

and

$$(f^- \circ g^-)(x) = \begin{cases} \bigwedge_{(a,b)\in \mathrm{S}_x} \{\max\{f^-(a), g^-(b)\}\} & \text{if } \mathbf{S}_x \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

It is easy to verify that the structure $(Tri(S); \circ)$ is a semigroup. In the set of all tripolar fuzzy subsets of S we define the order relation as follows: $f \subseteq g$ if and only if $f^+(x) \leq g^+(x)$, $f^*(x) \geq g^*(x)$ and $f^-(x) \geq g^-(x)$ for all $x \in S$ and then such structure $(Tri(S); \circ, \subseteq)$ is an ordered semigroup. Finally for tripolar fuzzy subsets f and g of S we define the operation $f \cap g$ as the tripolar fuzzy subset of S defined by:

$$f \cap g := (f^+ \cap g^+, f^* \cup g^*, f^- \cup g^-),$$

where $(f^+ \cap g^+)(x) := \min\{f^+(x), g^+(x)\}$, $(f^* \cup g^*)(x) := \max\{f^*(x), g^*(x)\}$, and $(f^- \cup g^-)(x) := \max\{f^-(x), g^-(x)\}$ for all $x \in S$.

The tripolar fuzzy subset $\mathbf{1} := (1^+, 0, 1^-)$ of S defined by $1^+(x) := 1$, $0(x) := 0$ and $1^-(x) = -1$ for all $x \in S$.

Let $A \subseteq S$. We denoted by $\chi_A := (\chi_A^+, \chi_A^*, \chi_A^-)$ the characteristic tripolar fuzzy subset of A in S and it is defined as follows:

$$(\chi_A^+)(x) := \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise,} \end{cases}$$

$$(\chi_A^*)(x) := \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise,} \end{cases}$$

and

$$(\chi_A^-)(x) := \begin{cases} -1 & \text{if } x \in A \\ 0 & \text{otherwise.} \end{cases}$$

In the case of $A = S$, we define $\chi_A = \mathbf{1}$.

Definition 2.4. *Let* $\mathbf{S}$ *be an ordered semigroup. A tripolar fuzzy subset* $f = (f^+, f^*, f^-)$ *of* S *is called a tripolar fuzzy right (resp., left) ideal of* $\mathbf{S}$ *if for every* $x, y \in S$,

(1) $f^+(xy) \geq f^+(x)$ *(resp.,* $f^+(xy) \geq f^+(y)$*),*
(2) $f^*(xy) \leq f^*(x)$ *(resp.,* $f^*(xy) \leq f^*(y)$*),*
(3) $f^-(xy) \leq f^-(x)$ *(resp.,* $f^-(xy) \leq f^-(y)$*),*

(4) *if* $x \leq y$, *then* $f^{+}(x) \geq f^{+}(y)$, $f^{*}(x) \leq f^{*}(y)$, $f^{-}(x) \leq f^{-}(y)$.

f is called a *tripolar fuzzy two-side ideal* (*tripolar fuzzy ideal*) of $\mathbf{S}$ if f is both a tripolar fuzzy right and a tripolar fuzzy left ideal of $\mathbf{S}$.

Example 2.1. *Let* $S = \{a, b, c\}$. *Define the binary operation* $*$ *on* S *by the followsing table:*

$*$	a	b	c
a	a	a	a
b	a	a	a
c	a	b	c

and define an order on S *as follows:*

$$\leq := \{(a, b)\} \cup \Delta_S,$$

where Δ_S *is an equality relation on* S. *That is,* $\Delta_S := \{(x, x) \in S \times S \mid x \in S\}$. *Then,* $\mathbf{S} := (S; *, \leq)$ *is an ordered semigroup. We define a tripolar fuzzy subset* $f = (f^{+}, f^{*}, f^{-})$ *of* S *by:*

S	$f^{+}(x)$	$f^{*}(x)$	$f^{-}(x)$
a	0.7	0.1	-0.6
b	0.5	0.2	-0.5
c	0.7	0.1	-0.6

Then f *is a tripolar fuzzy left ideal of* $\mathbf{S}$ *but,* f *is not a tripolar fuzzy right ideal of* $\mathbf{S}$ *since* $f^{+}(c * b) = f^{+}(b) = 0.5 < 0.7 = f^{+}(c)$.

Example 2.2. *Let* $S = \{a, b, c\}$. *Define the binary operation* $\circ$ *on* S *by the following table:*

$\circ$	a	b	c
a	a	a	a
b	a	a	a
c	a	c	c

and define an order on S *as follows:*

$$\leq := \{(a, b), (a, c)\} \cup \Delta_S,$$

where Δ_S *is an equality relation on* S. *That is,* $\Delta_S := \{(x, x) \in S \times S \mid x \in S\}$. *Then,* $\mathbf{S} := (S; \circ, \leq)$ *is an ordered semigroup. We define a tripolar fuzzy subset* $f = (f^{+}, f^{*}, f^{-})$ *of* S *by:*

S	$f^{+}(x)$	$f^{*}(x)$	$f^{-}(x)$
a	0.8	0.1	-0.8
b	0.7	0.2	-0.7
c	0.7	0.2	-0.7

Then f *is a tripolar fuzzy ideal of* $\mathbf{S}$.

3. Main Results

In this main section, we introduce the concepts of tripolar fuzzy pure ideals in ordered semigroups and study some algebraic properties of such tripolar fuzzy pure ideals. We also characterize weakly regular ordered semigroups in terms of tripolar fuzzy pure ideals. Finally, we introduce the concept of tripolar weakly pure ideals and prove that any tripolar fuzzy ideal is a tripolar fuzzy weakly pure ideal whenever it is idempotent.

Definition 3.1. *Let* **S** *be an ordered semigroup. A tripolar fuzzy ideal* f *of* **S** *is*

(1) *left pure if* $f \cap g = f \circ g$ *for every tripolar fuzzy left ideal* g *of* **S**.
(2) *right pure* $g \cap f = g \circ f$ *for every tripolar fuzzy right ideal* g *of* **S**.

A tripolar fuzzy ideal of **S** is called a *tripolar fuzzy pure ideal* of **S** it is both a tripolar fuzzy right pure and a tripolar fuzzy left pure ideal of **S**.

The following lemmas are important in illustrating our first theorem. Therefore, we give useful tools.

Lemma 3.1. *Let* **S** *be an ordered semigroup and* A *a nonempty subset of* S*. Then the following conditions are equivalent:*

(1) A *is a right (resp., left) ideal of* **S**.
(2) χ_A *is a tripolar fuzzy right (resp., left) ideal of* **S**.

Proof. (1)⇒(2). Let A be a right ideal of an ordered semigroup **S**. First, let $x, y \in S$ and $x \in A$. Then $xy \in A$ and, we obtain

$$\chi_A^+(xy) = 1 = \chi_A^+(x), \quad \chi_A^*(xy) = 0 = \chi_A^*(x), \quad \text{and} \quad \chi_A^-(xy) = -1 = \chi_A^-(x).$$

If $x \notin A$, we obtain

$$\chi_A^+(xy) \geq 0 = \chi_A^+(x), \quad \chi_A^*(xy) \leq 1 = \chi_A^*(x), \quad \text{and} \quad \chi_A^-(xy) \leq 0 = \chi_A^-(x).$$

Secondly, let $x, y \in S$ be such that $x \leq y$. If $y \in A$, then $x \in A$ and then

$$\chi_A^+(x) = 1 = \chi_A^+(y), \quad \chi_A^*(x) = 0 = \chi_A^*(y), \quad \text{and} \quad \chi_A^-(x) = -1 = \chi_A^-(y).$$

If $y \notin A$, we obtain

$$\chi_A^+(x) \geq 0 = \chi_A^+(y), \quad \chi_A^*(x) \leq 1 = \chi_A^*(y), \quad \text{and} \quad \chi_A^-(x) \leq 0 = \chi_A^-(y).$$

Altogether, it is completed to prove that χ_A is a tripolar fuzzy right ideal of **S**.

(2)⇒(1). Let χ_A be a tripolar fuzzy right ideal of **S**. First, let $x, y \in S$ and $x \in A$. We obtain $1 \geq \chi_A^+(xy) \geq \chi_A^+(x) = 1$, which implies that $\chi_A^+(xy) = 1$ and then $xy \in A$, $0 \leq \chi_A^*(xy) \leq \chi_A^*(x) = 0$, which implies that $\chi_A^*(xy) = 0$ and then $xy \in A$, similarly, $-1 \leq \chi_A^-(xy) \leq \chi_A^-(x) = -1$, which implies that $\chi_A^-(xy) = -1$ and then $xy \in A$. Secondly, let $x, y \in S$ be such that $x \leq y$ and $y \in A$. We obtain

$1 \geq \chi_A^+(x) \geq \chi_A^+(y) = 1$, which implies that $\chi_A^+(x) = 1$ and then $x \in A$, $0 \leq \chi_A^*(x) \leq \chi_A^*(y) = 0$, which implies that $\chi_A^*(x) = 0$ and then $x \in A$, similarly, $-1 \leq \chi_A^-(x) \leq \chi_A^-(y) = -1$, which implies that $\chi_A^-(x) = -1$ and then $x \in A$. Altogether, we obtain A is a right ideal of $\mathbf{S}$. Similarly, we can show that A is left ideal if and only if χ_A is tripolar fuzzy left ideal. □

As a consequence of the above lemma, we have that A is an ideal of $\mathbf{S}$ if and only if χ_A is a tripolar fuzzy ideal of $\mathbf{S}$.

Lemma 3.2. *Let $\mathbf{S}$ be an ordered semigroup and A, B nonempty subsets of a set S. Then the following conditions are equivalent:*

(1) *$A \subseteq B$ if and only if $\chi_A \subseteq \chi_B$;*
(2) *$\chi_A \cap \chi_B = \chi_{A\cap B}$;*
(3) *$\chi_A \circ \chi_B = \chi_{(AB]}$.*

Proof. We will give proof only (3). Let $x \in (AB]$. Then $x \leq ab$ for some $a \in A$ and $b \in B$ and then $\mathbf{S}_x \neq \emptyset$, we obtain

$$1 \geq (\chi_A^+ \circ \chi_B^+)(x) = \bigvee_{(y,z)\in \mathrm{S}_x} \{\min\{\chi_A^+(y), \chi_B^+(z)\}\} \geq \min\{\chi_A^+(a), \chi_B^+(b)\} = 1.$$

This implies that $(\chi_A^+ \circ \chi_B^+)(x) = 1 = \chi_{(AB]}^+(x)$,

$$0 \leq (\chi_A^* \circ \chi_B^*)(x) = \bigwedge_{(y,z)\in \mathrm{S}_x} \{\max\{\chi_A^*(y), \chi_B^*(z)\}\} \leq \max\{\chi_A^*(a), \chi_B^*(b)\} = 0.$$

This implies that $(\chi_A^* \circ \chi_B^*)(x) = 0 = \chi_{(AB]}^*(x)$ and

$$-1 \leq (\chi_A^- \circ \chi_B^-)(x) = \bigwedge_{(y,z)\in \mathrm{S}_x} \{\max\{\chi_A^-(y), \chi_B^-(z)\}\} \leq \max\{\chi_A^-(a), \chi_B^-(b)\} = -1.$$

This implies that $(\chi_A^- \circ \chi_B^-)(x) = -1 = \chi_{(AB]}^-(x)$. Therefore $\chi_A \circ \chi_B = \chi_{(AB]}$. □

Lemma 3.3. *[4] Let $\mathbf{S}$ be an ordered semigroup and A an ideal of $\mathbf{S}$. Then the following conditions are equivalent:*

(1) *A is a right pure ideal of $\mathbf{S}$.*
(2) *$B \cap A = (BA]$ for every right ideal B of $\mathbf{S}$.*

Lemma 3.4. *[4] Let $\mathbf{S}$ be an ordered semigroup and A an ideal of $\mathbf{S}$. Then the following conditions are equivalent:*

(1) *A is a left pure ideal of $\mathbf{S}$.*
(2) *$A \cap B = (AB]$ for every left ideal B of $\mathbf{S}$.*

The following theorem provides a characterization of right (resp., left) pure ideals in ordered semigroups using tripolar fuzzy right (resp., left) pure ideals.

Theorem 3.1. *Let A be an ideal of an ordered semigroup $\mathbf{S}$. Then the following conditions are equivalent:*

(1) *A is a right (resp., left) pure ideal of $\mathbf{S}$.*
(2) *χ_A is a tripolar fuzzy right (resp., left) pure ideal of $\mathbf{S}$.*

Proof. (1)⇒(2). Assume that A is a right pure ideal of $\mathbf{S}$. Let f be a tripolar fuzzy right ideal of $\mathbf{S}$ and $a \in S$. Suppose that $a \notin A$, we consider two cases as follows: If $\mathbf{S}_a = \emptyset$, then we have

$$(f^+ \circ \chi_A^+)(a) = 0 = \chi_A^+(a) = \min\{f^+(a), \chi_A^+(a)\} = (f^+ \cap \chi_A^+)(a),$$

$$(f^* \circ \chi_A^*)(a) = 1 = \chi_A^*(a) = \max\{f^*(a), \chi_A^*(a)\} = (f^* \cup \chi_A^*)(a), \text{ and}$$

$$(f^- \circ \chi_A^-)(a) = 0 = \chi_A^-(a) = \max\{f^-(a), \chi_A^-(a)\} = (f^- \cup \chi_A^-)(a).$$

If $\mathbf{S}_a \neq \emptyset$, then, by a right purity of A, we have that $v \notin A$ for all $(u, v) \in \mathbf{S}_a$. Then

$$\begin{aligned}
(f^+ \circ \chi_A^+)(a) &= \bigvee_{(x,y)\in \mathrm{S}_a} \{\min\{f^+(x), \chi_A^+(y)\}\} \\
&= 0 \\
&= \min f^+(a), \chi_A^+(a) \\
&= (f^+ \cap \chi_A^+)(a),
\end{aligned}$$

$$\begin{aligned}
(f^* \circ \chi_A^*)(a) &= \bigwedge_{(x,y)\in \mathrm{S}_a} \{\max\{f^*(x), \chi_A^*(y)\}\} \\
&= 1 \\
&= \max\{f^*(a), \chi_A^*(a)\} \\
&= (f^* \cup \chi_A^*)(a),
\end{aligned}$$

and

$$\begin{aligned}
(f^- \circ \chi_A^-)(a) &= \bigwedge_{(x,y)\in \mathrm{S}_a} \{\max\{f^-(x), \chi_A^-(y)\}\} \\
&= 0 \\
&= \max\{f^-(a), \chi_A^-(a)\} \\
&= (f^- \cup \chi_A^-)(a).
\end{aligned}$$

Now, we assume that $a \in A$. By the right purity of A, we have $\mathbf{S}_a \notin \emptyset$. More precisely, there exists $(a, x) \in \mathbf{S}_a$ such that $x \in A$. Then, by the tripolar fuzzy right ideality of f and the tripolar fuzzy left ideality of χ_A, we have that

$$\begin{aligned}(f^+\cap\chi_A^+)(a) &= \min\{f^+(a),\chi_A^+(a)\} = f^+(a) = \min\{f^+(a),\chi_A^+(x)\}\\ &\le \bigvee_{(u,v)\in \mathrm{S_a}}\{\min\{f^+(u),\chi_A^+(v)\}\} \le \bigvee_{(u,v)\in \mathrm{S_a}}\{\min\{f^+(uv),\chi_A^+(uv)\}\}\\ &\le \bigvee_{(u,v)\in \mathrm{S_a}}\{\min\{f^+(a),\chi_A^+(a)\}\} = \min\{f^+(a),\chi_A^+(a) = (f^+\cap\chi_A^+)(a).\end{aligned}$$

This implies that $(f^+\cap\chi_A^+)(a) = \bigvee_{(u,v)\in \mathrm{S_a}}\{\min\{f^+(u),\chi_A^+(v)\}\} = (f^+\circ\chi_A^+)(a)$,

$$\begin{aligned}(f^*\cup\chi_A^*)(a) &= \max\{f^*(a),\chi_A^*(a)\} = f^*(a) = \max\{f^*(a),\chi_A^*(x)\}\\ &\ge \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^*(u),\chi_A^*(v)\}\} \ge \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^*(uv),\chi_A^*(uv)\}\}\\ &\ge \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^*(a),\chi_A^*(a)\}\} = \max\{f^*(a),\chi_A^*(a) = (f^*\cup\chi_A^*)(a).\end{aligned}$$

This implies that $(f^*\cup\chi_A^*)(a) = \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^*(u),\chi_A^*(v)\}\} = (f^*\circ\chi_A^*)(a)$, and

$$\begin{aligned}(f^-\cup\chi_A^-)(a) &= \max\{f^-(a),\chi_A^-(a)\} = f^-(a) = \max\{f^-(a),\chi_A^-(x)\}\\ &\ge \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^-(u),\chi_A^-(v)\}\} \ge \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^-(uv),\chi_A^-(uv)\}\}\\ &\ge \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^-(a),\chi_A^-(a)\}\} = \max\{f^-(a),\chi_A^-(a) = (f^-\cup\chi_A^-)(a).\end{aligned}$$

This implies that $(f^-\cup\chi_A^-)(a) = \bigwedge_{(u,v)\in \mathrm{S_a}}\{\max\{f^-(u),\chi_A^-(v)\}\} = (f^-\circ\chi_A^-)(a)$. Altogether, we have χ_A is a tripolar fuzzy right pure ideal of S.

(2)⇒(1). Assume that χ_A is a tripolar fuzzy right pure ideal of **S**. Let B be a right ideal of **S**. By Lemma 3.1, χ_B is a tripolar fuzzy right ideal of **S**, by assumption, we obtain

$$\chi_{B\cap A} = \chi_B \cup \chi_A = \chi_B \circ \chi_A = \chi_{(BA]}.$$

By Lemma 3.2 (1), we have $B\cap A = (BA]$ and, by Lemma 3.3, we obtain A is a right pure ideal of **S**. Similarly to prove A is a left pure ideal of **S** if and only if χ_A is a tripolar fuzzy left pure ideal of **S**. □

By the above theorem, we obtain the following consequence.

Corollary 3.1. *Let A be an ideal of an ordered semigroup* **S**. *Then the following conditions are equivalent:*

(1) *A is a pure ideal of* **S**.
(2) *χ_A is a tripolar fuzzy pure ideal of* **S**.

The following results illustrate some properties of tripolar fuzzy right (resp., left) pure ideals in an ordered semigroup.

Theorem 3.2. *Let f and g be tripolar fuzzy right pure ideals of an ordered semigroup* **S***. Then $f \cap g$ is a tripolar fuzzy right pure ideal of* **S**

Proof. Let h be a tripolar fuzzy right ideal of **S**. We have

$$\begin{aligned} h \circ (f \cap g) &= (h \circ f) \cap (h \circ g) \\ &= (h \cap f) \cap (h \cap g) \\ &= h \cap (f \cap g). \end{aligned}$$

It is completed to prove that $f \cap g$ is a tripolar fuzzy right pure ideal of **S**. □

By similar method of Theorem 3.2, we have the following theorem.

Theorem 3.3. *Let f and g be tripolar fuzzy left pure ideals of an ordered semigroup* **S***. Then $f \cap g$ is a tripolar fuzzy left pure ideal of* **S**

Combining Theorem 3.2 and 3.3, we obtain the following result.

Corollary 3.2. *Let f and g be tripolar fuzzy pure ideals of an ordered semigroup* **S***. Then $f \cap g$ is a tripolar fuzzy pure ideal of* **S**

Let f and g be tripolar fuzzy subsets of S. Then $f \Cup g := (f^+ \cup g^+, f^* \cap g^*, f^- \cap g^-)$ is a tripolar fuzzy subset of S and is defined as follows:

$$(f^+ \cup g^+)(x) := \max\{f^+(x), g^+(x)\},\ (f^* \cap g^*)(x) := \min\{f^*(x), g^*(x)\}$$

and

$$(f^- \cap g^-)(x) := \min\{f^-(x), g^-(x)\}$$

for all $x \in S$.

Theorem 3.4. *Let f and g be tripolar fuzzy right pure ideals of an ordered semigroup* **S***. Then $f \Cup g$ is a tripolar fuzzy right pure ideal of* **S***.*

Proof. Let h be a tripolar fuzzy right ideal of an ordered semigroup **S** and $a \in S$. If $\mathbf{S}_a = \emptyset$, then we obtain that

$$\begin{aligned} (h^+ \cap (f^+ \cup g^+))(a) &= \min\{h^+(a), (f^+ \cup g^+)(a)\} \\ &= \min\{h^+(a), \max\{f^+(a), g^+(a)\}\} \\ &= \max\{\min\{h^+(a), f^+(a)\}, \min\{h^+(a), g^+(a)\}\} \\ &= \max\{(h^+ \cap f^+)(a), (h^+ \cap g^+)(a)\} \\ &= \max\{(h^+ \circ f^+)(a), (h^+ \circ g^+)(a)\} \\ &= 0 \\ &= (h^+ \circ (f^+ \cup g^+))(a), \end{aligned}$$

$$\begin{aligned}
(h^* \cup (f^* \cap g^*))(a) &= \max\{h^*(a), (f^* \cap g^*)(a)\} \\
&= \max\{h^*(a), \min\{f^*(a), g^*(a)\}\} \\
&= \min\{\max\{h^*(a), f^*(a)\}, \max\{h^*(a), g^*(a)\}\} \\
&= \min\{(h^* \cup f^*)(a), (h^* \cup g^*)(a)\} \\
&= \min\{(h^* \circ f^*)(a), (h^* \circ g^*)(a)\} \\
&= 1 \\
&= (h^* \circ (f^* \cap g^*))(a),
\end{aligned}$$

and

$$\begin{aligned}
(h^- \cup (f^- \cap g^-))(a) &= \max\{h^-(a), (f^- \cap g^-)(a)\} \\
&= \max\{h^-(a), \min\{f^-(a), g^-(a)\}\} \\
&= \min\{\max\{h^-(a), f^-(a)\}, \max\{h^-(a), g^-(a)\}\} \\
&= \min\{(h^- \cup f^-)(a), (h^- \cup g^-)(a)\} \\
&= \min\{(h^- \circ f^-)(a), (h^- \circ g^-)(a)\} \\
&= 0 \\
&= (h^- \circ (f^- \cap g^-))(a).
\end{aligned}$$

Suppose that $\mathbf{S}_a \neq \emptyset$, and we obtain

$$\begin{aligned}
(h^+ \cap (f^+ \cup g^+))(a) &= \min\{h^+(a), (f^+ \cup g^+)(a)\} \\
&= \min\{h^+(a), \max\{f^+(a), g^+(a)\}\} \\
&= \max\{\min\{h^+(a), f^+(a)\}, \min\{h^+(a), g^+(a)\}\} \\
&= \max\{(h^+ \cap f^+)(a), (h^+ \cap g^+)(a)\} \\
&= \max\{(h^+ \circ f^+)(a), (h^+ \circ g^+)(a)\} \\
&= \max\{\bigvee_{(x,y)\in S_a} \{\min\{h^+(x), f^+(y)\}, \bigvee_{(x,y)\in S_a} \{\min\{h^+(x), g^+(y)\}\}\} \\
&= \bigvee_{(x,y)\in S_a} \{\min\{h^+(x), \max\{f^+(y), g^+(y)\}\}\} \\
&= \bigvee_{(x,y)\in S_a} \{\min\{h^+(x), (f^+ \cup g^+)(y)\}\} \\
&= (h^+ \circ (f^+ \cup g^+))(a),
\end{aligned}$$

$$\begin{aligned}
(h^* \cup (f^* \cap g^*))(a) &= \max\{h^*(a), (f^* \cap g^*)(a)\} \\
&= \max\{h^*(a), \min\{f^*(a), g^*(a)\}\} \\
&= \min\{\max\{h^*(a), f^*(a)\}, \max\{h^*(a), g^*(a)\}\} \\
&= \min\{(h^* \cup f^*)(a), (h^* \cup g^*)(a)\} \\
&= \min\{(h^* \circ f^*)(a), (h^* \circ g^*)(a)\} \\
&= \min\{\bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^*(x), f^*(y)\}, \bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^*(x), g^*(y)\}\}\} \\
&= \bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^*(x), \min\{f^*(y), g^*(y)\}\}\} \\
&= \bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^*(x), (f^* \cap g^*)(y)\}\} \\
&= (h^* \circ (f^* \cap g^*))(a),
\end{aligned}$$

and

$$\begin{aligned}
(h^- \cup (f^- \cap g^-))(a) &= \max\{h^-(a), (f^- \cap g^-)(a)\} \\
&= \max\{h^-(a), \min\{f^-(a), g^-(a)\}\} \\
&= \min\{\max\{h^-(a), f^-(a)\}, \max\{h^-(a), g^-(a)\}\} \\
&= \min\{(h^- \cup f^-)(a), (h^- \cup g^-)(a)\} \\
&= \min\{(h^- \circ f^-)(a), (h^- \circ g^-)(a)\} \\
&= \min\{\bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^-(x), f^-(y)\}, \bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^-(x), g^-(y)\}\}\} \\
&= \bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^-(x), \min\{f^-(y), g^-(y)\}\}\} \\
&= \bigwedge_{(x,y)\in \mathrm{S_a}} \{\max\{h^-(x), (f^- \cap g^-)(y)\}\} \\
&= (h^- \circ (f^- \cap g^-))(a).
\end{aligned}$$

For any two cases, we obtain $f \Cup g$ is a tripolar fuzzy right pure ideal of $\mathbf{S}$. $\square$

By similar method of Theorem 3.4, we have the following theorem.

Theorem 3.5. *Let f and g be tripolar fuzzy left pure ideals of an ordered semigroup $\mathbf{S}$. Then $f \Cup g$ is a tripolar fuzzy left pure ideal of $\mathbf{S}$.*

Combining Theorem 3.4 and 3.5, we obtain the following result.

Corollary 3.3. *Let f and g be tripolar fuzzy pure ideals of an ordered semigroup $\mathbf{S}$. Then $A \Cup B$ is a tripolar fuzzy pure ideal of $\mathbf{S}$.*

An ordered semigroup **S** is said to be *right (resp., left) weakly regular* [4] if for any $a \in S$ there exist $x, y \in S$ such that $a \leq axay$ (resp., $a \leq xaya$).

An ordered semigroup **S** is called *weakly regular* if it is both a right and a left weakly regular ordered semigroup.

Lemma 3.5. *[4] Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) **S** *is right (resp., left) weakly regular.*
(2) *Every ideal of* **S** *is a right (resp., left) pure ideal of* **S**.

Lemma 3.6. *[4] Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) **S** *is weakly regular.*
(2) *Every ideal of* **S** *is a pure ideal of* **S**.

We characterize right weakly regular ordered semigroups in terms of tripolar fuzzy right pure ideals as follows:

Theorem 3.6. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) **S** *is right weakly regular.*
(2) *Every tripolar fuzzy ideal of* **S** *is right pure.*

Proof. (1)⇒(2). Let f be a tripolar fuzzy ideal of **S** and g a tripolar fuzzy right ideal of **S**. Let $a \in S$. Since **S** is right weakly regular, there exist $x, y \in S$ such that $a \leq axay = (ax)(ay)$. This implies that $\mathbf{S}_a \neq \emptyset$ and then

$$\begin{aligned}
(g^+ \circ f^+)(a) &= \bigvee_{(u,v)\in \mathrm{S_a}} \{\min\{g^+(u), f^+(v)\}\} \\
&\leq \bigvee_{(u,v)\in \mathrm{S_a}} \{\min\{g^+(uv), f^+(uv)\}\} \\
&\leq \bigvee_{(u,v)\in \mathrm{S_a}} \{\min\{g^+(a), f^+(a)\}\} \\
&= \min\{g^+(a), f^+(a)\} \\
&= (g^+ \cap f^+)(a).
\end{aligned}$$

On other inclusion, we have

$$\begin{aligned}
(g^+ \circ f^+)(a) &= \bigvee_{(u,v)\in \mathrm{S_a}} \{\min\{g^+(u), f^+(v)\}\} \\
&\geq \min\{g^+(ax), f^+(ay)\} \\
&\geq \min\{g^+(a), f^+(a)\} \\
&= (g^+ \cap f^+)(a).
\end{aligned}$$

Therefore $(g^+ \circ f^+)(a) = (g^+ \cap f^+)(a)$,

$$
\begin{aligned}
(g^* \circ f^*)(a) &= \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^*(u), f^*(v)\}\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^*(uv), f^*(uv)\}\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^*(a), f^*(a)\}\} \\
&= \max\{g^*(a), f^*(a)\} \\
&= (g^* \cup f^*)(a).
\end{aligned}
$$

On other inclusion, we have

$$
\begin{aligned}
(g^* \circ f^*)(a) &= \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^*(u), f^*(v)\}\} \\
&\leq \max\{g^*(ax), f^*(ay)\} \\
&\leq \max\{g^*(a), f^*(a)\} \\
&= (g^* \cup f^*)(a).
\end{aligned}
$$

Therefore $(g^* \circ f^*)(a) = (g^* \cup f^*)(a)$, and

$$
\begin{aligned}
(g^- \circ f^-)(a) &= \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^-(u), f^-(v)\}\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^-(uv), f^-(uv)\}\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^-(a), f^-(a)\}\} \\
&= \max\{g^-(a), f^-(a)\} \\
&= (g^- \cup f^-)(a).
\end{aligned}
$$

On other inclusion, we have

$$
\begin{aligned}
(g^- \circ f^-)(a) &= \bigwedge_{(u,v)\in \mathrm{S_a}} \{\max\{g^-(u), f^-(v)\}\} \\
&\leq \max\{g^-(ax), f^-(ay)\} \\
&\leq \max\{g^-(a), f^-(a)\} \\
&= (g^- \cup f^-)(a).
\end{aligned}
$$

Therefore $(g^- \circ f^-)(a) = (f^- \cup g^-)(a)$, and then $g \circ f = g \cap f$. Hence f is a tripolar fuzzy right pure ideal of $\mathbf{S}$.

(2)⇒(1). Let A and B be an ideal of **S** and a right ideal of **S**, respectively. By Lemma 3.1, we obtain χ_A and χ_B is a tripolar fuzzy ideal of **S** and a tripolar fuzzy right ideal of **S**, respectively. By our assumption, χ_B is a tripolar fuzzy right pure ideal of **S**. Then

$$\begin{aligned}\chi_{(BA]} &= \chi_B \circ \chi_A \\ &= \chi_B \cap \chi_A \\ &= \chi_{B\cap A}.\end{aligned}$$

By Lemma 3.2 (1), we obtain $B \cap A = (BA]$. This means that A is a right pure ideal of **S**. Therefore, by Lemma 3.5, **S** is right weakly regular. □

By similar method of Theorem 3.6, we have the following theorem.

Theorem 3.7. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) **S** *is left weakly regular.*
(2) *Every tripolar fuzzy ideal of* **S** *is left pure.*

Combining Theorem 3.6 and 3.7, we obtain the following result.

Corollary 3.4. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) **S** *is weakly regular.*
(2) *Every tripolar fuzzy ideal of* **S** *is pure.*

Now, we present the concepts of right weakly purity and left weakly purity of tripolar fuzzy ideals. In our last main result, the coincidence of these two concepts is provided.

Definition 3.2. *A tripolar fuzzy ideal f of* **S** *is called a tripolar fuzzy right (resp., left) weakly pure ideal if $g \circ f = g \cap f$ (resp., $f \circ g = f \cap g$) for every tripolar fuzzy ideal g of* **S**.

A tripolar fuzzy ideal is called a *tripolar weakly pure ideal* of **S** if it is both a tripolar fuzzy right and a tripolar fuzzy left ideal of **S**. A tripolar fuzzy subset f of S is idempotent with respect to $\circ$ if $f \circ f = f$.

Lemma 3.7. *Let* **S** *be an ordered semigroup and f, g are tripolar fuzzy right ideals of* **S**. *Then $f \cap g$ is a tripolar fuzzy right ideal of* **S**.

Proof. Let f, g be tripolar fuzzy right ideals of **S** and $x, y \in S$. Then we obtain

$$(f^+ \cap g^+)(xy) = f^+(xy) \cap g^+(xy) \geq f^+(x) \cap g^+(x) = (f^+ \cap g^+)(x),$$

$$(f^* \cup g^*)(xy) = f^*(xy) \cup g^*(xy) \leq f^*(x) \cup g^*(x) = (f^* \cup g^*)(x),$$

and

$$(f^- \cup g^-)(xy) = f^-(xy) \cup g^-(xy) \leq f^-(x) \cup g^-(x) = (f^- \cup g^-)(x).$$

Let $x, y \in S$ be such that $x \leq y$. Then

$$(f^{+} \cap g^{+})(x) = f^{+}(x) \cap g^{+}(x) \geq f^{+}(y) \cap g^{+}(y) = (f^{+} \cap g^{+})(y),$$

$$(f^{*} \cup g^{*})(x) = f^{*}(x) \cup g^{*}(x) \leq f^{*}(y) \cup g^{*}(y) = (f^{*} \cup g^{*})(y),$$

and

$$(f^{-} \cup g^{-})(x) = f^{-}(x) \cup g^{-}(x) \leq f^{-}(y) \cup g^{-}(y) = (f^{-} \cup g^{-})(y).$$

Therefore $f \cap g$ is a tripolar fuzzy right ideal of **S**. □

By similar method of Lemma 3.7, we have the following lemma.

Lemma 3.8. *Let **S** be an ordered semigroup and f, g are tripolar fuzzy left ideals of S. Then $f \cap g$ is a tripolar fuzzy left ideal of **S**.*

Combining Lemma 3.7 and 3.8, we obtain the following result.

Corollary 3.5. *Let **S** be an ordered semigroup and f, g are tripolar fuzzy ideals of **S**. Then $f \cap g$ is a tripolar fuzzy ideal of **S**.*

Our last result illustrates that the concepts of right weakly purity and left weakly purity of tripolar fuzzy ideals coincide.

Theorem 3.8. *Let **S** be an ordered semigroup and f a tripolar fuzzy ideal of **S**. Then the following statements are equivalent.*

(1) *f is tripolar fuzzy right weakly pure ideal.*
(2) *f is idempotent with respect to $\circ$.*
(3) *f is tripolar fuzzy left weakly pure ideal.*

Proof. (1)⇒(2). Let f be a tripolar fuzzy pure ideal of **S**. Then, we obtain

$$f \circ f = f \cap f = f.$$

Therefore f is idempotent with respect to $\circ$.

(2)⇒(1). Let g be a tripolar fuzzy ideal of **S**. By Corollary 3.5, we obtain that $g \cap f$ is a tripolar fuzzy ideal of **S**. By our assumption, we have

$$g \cap f = (g \cap f) \circ (g \cap f) \subseteq g \circ f.$$

On the other hand, let $a \in S$. If $\mathbf{S}_{\mathrm{a}} = \emptyset$, then

$$(g^{+} \circ f^{+})(a) = 0 \leq (g^{-} \cap f^{+})(a),$$

$$(g^{*} \circ f^{*})(a) = 1 \geq (g^{-} \cup f^{*})(a),$$

and

$$(g^{-} \circ f^{-})(a) = 0 \geq (g^{-} \cup f^{-})(a).$$

Suppose that $\mathbf{S}_a \neq \emptyset$. Then,

$$\begin{aligned}(g^+ \circ f^+)(a) &= \bigvee_{(u,v)\in S_a} \{\min\{g^+(u), f^+(v)\}\} \\ &\leq \bigvee_{(u,v)\in S_a} \{\min\{g^+(uv), f^+(uv)\}\} \\ &\leq \bigvee_{(u,v)\in S_a} \{\min\{g^+(a), f^+(a)\}\} \\ &= \min\{g^+(a), f^+(a)\} \\ &= (g^+ \cap f^+)(a),\end{aligned}$$

$$\begin{aligned}(g^* \circ f^*)(a) &= \bigwedge_{(u,v)\in S_a} \{\max\{g^*(u), f^*(v)\}\} \\ &\geq \bigwedge_{(u,v)\in S_a} \{\max\{g^*(uv), f^*(uv)\}\} \\ &\geq \bigwedge_{(u,v)\in S_a} \{\max\{g^*(a), f^*(a)\}\} \\ &= \max\{g^*(a), f^*(a)\} \\ &= (g^* \cup f^*)(a),\end{aligned}$$

and

$$\begin{aligned}(g^- \circ f^-)(a) &= \bigwedge_{(u,v)\in S_a} \{\max\{g^-(u), f^-(v)\}\} \\ &\geq \bigwedge_{(u,v)\in S_a} \{\max\{g^-(uv), f^-(uv)\}\} \\ &\geq \bigwedge_{(u,v)\in S_a} \{\max\{g^-(a), f^-(a)\}\} \\ &= \max\{g^-(a), f^-(a)\} \\ &= (g^- \cup f^-)(a),\end{aligned}$$

Thus $g \circ f \subseteq g \cap f$ and altogether, we have $g \circ f = g \cap f$. This means that f is a tripolar fuzzy right weakly pure ideal of $\mathbf{S}$.

Illustrating (2)⇔(3) can be done similarly. □

By Theorem 3.8, we obtain the following result.

Corollary 3.6. *Let* $\mathbf{S}$ *be an ordered semigroup and* f *a tripolar fuzzy ideal of* $\mathbf{S}$*. Then the following statements are equivalent.*

(1) *f is idempotent with respect to $\circ$.*
(2) *f is tripolar fuzzy weakly pure ideal.*

4. Conclusion

In this present paper, we introduced the concept of tripolar fuzzy pure ideals in ordered semigroups. Some related properties of tripolar pure ideals are studied. We characterized weakly regular ordered semigroups in terms of tripolar fuzzy pure ideals. Finally, we introduced the concept of tripolar weakly pure ideals. We proved that the tripolar fuzzy ideals are tripolar fuzzy weakly pure ideals if such tripolar fuzzy ideals satisfied idempotent property. In our future work, we will apply the notions of tripolar fuzzy ideals and tripolar fuzzy pure ideals to the theory of hyperstructures, ordered hyperstructures, semirings, groups, BCI/BCK algebras, etc.

Conflicts of Interest: The author(s) declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] J. Ahsan, M. Takahashi, Pure Spectrum of a Momoid With Zero, Kobe J. Math. 6 (1989), 163-182. https://cir.nii.ac.jp/crid/1571698601700142976.

[2] K.T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets Syst. 20 (1986), 87-96. https://dl.acm.org/doi/10.5555/1708507.1708520.

[3] N. Rehman, Pure Fuzzy Ideals in Ternary Semigroups, Int. J. Algebra Stat. 1 (2012), 1-7.

[4] T. Changphas, J. Sanborisoot, Pure Ideals in Ordered Semigroups, Kyungpook Math. J. 54 (2014), 123–129. https://doi.org/10.5666/KMJ.2014.54.1.123.

[5] N. Kehayopulu, M. Tsingelis, Fuzzy Sets in Ordered Groupoids, Semigroup Forum. 65 (2001), 128–132. https://doi.org/10.1007/s002330010079.

[6] K. Linesawat, N. Lekkoksung, S. Lekkoksung, Anti-Hybrid Pure Ideals in Ordered Semigroups, Int. J. Innov. Comput. Inform. Control. 18 (2022), 1275-1290. https://doi.org/10.24507/ijicic.18.04.1275.

[7] M.M.K. Rao, Tripolar Fuzzy Interior Ideals of Γ-Semigroup, Ann. Fuzzy Math. Inform. 15 (2018), 199–206. https://doi.org/10.30948/AFMI.2018.15.2.199.

[8] M. Murali Krishna Rao, B. Venkateswarlu, Tripolar Fuzzy Ideals of Γ-Semirings, Asia Pac. J. Math. 5 (2018), 192-207. https://doi.org/10.28924/APJM/5-2-192-207.

[9] M. Murali Krishna Rao, B. Venkateswarlu, Tripolar Fuzzy Interior Ideals and Tripolar Fuzzy Soft Interior Ideals Over Γ-Semirings, Facta Univ. Ser.: Math. Inform. 35 (2020), 29-42. https://doi.org/10.22190/FUMI2001029K.

[10] M. Murali Krishna Rao, Tripolar Fuzzy Interior Ideals and Tripolar Fuzzy Soft Interior Ideals Over Semigroups, Ann. Fuzzy Math. Inform. 20 (2020), 243-256. https://doi.org/10.30948/afmi.2020.20.3.243.

[11] A. Rosenfeld, Fuzzy Groups, J. Math. Anal. Appl. 35 (1971), 338-353.

[12] K. Siribute, J. Sanborisoot, On Pure Fuzzy Ideals in Ordered Semigroups, Int. J. Math. Computer Sci. 14 (2019), 867-877.

[13] Y. Xie, J. Liu, L. Wang, Further Studies on H_∞ Filtering Design for Fuzzy System With Known or Unknown Premise Variables, IEEE Access. 7 (2019), 121975–121981. https://doi.org/10.1109/access.2019.2938797.

[14] L.A. Zadeh, Fuzzy Sets, Inform. Control, 8 (1965), 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X.

[15] Wen-Ran Zhang, Bipolar Fuzzy Sets and Relations: A Computational Framework for Cognitive Modeling and Multiagent Decision Analysis, in: NAFIPS/IFIS/NASA '94. Proceedings of the First International Joint Conference

of The North American Fuzzy Information Processing Society Biannual Conference. IEEE, San Antonio, TX, USA, 1994: pp. 305–309. https://doi.org/10.1109/IJCF.1994.375115.