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Interval Valued Intuitionistic Fuzzy β-Filters on β-Algebras

Kaliyaperumal Palanivel[1], Prakasam Muralikrishna[2], Perumal Hemavathi[3], Ronnason Chinram[4], Pattarawan Singavananda[5,*]

[1] *Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore-632014, India*

[2] *PG and Research Department of Mathematics, Muthurangam Government Arts College (Autonomus), Vellore-632002, India*

[3] *Saveetha School of Engineering, SIMATS, Thandalam-602025, India*

[4] *Division of Computational Science, Faculty of Science, Prince of Songkla University, Hat Yai, Songkhla 90110, Thailand*

[5] *Program in Mathematics, Faculty of Science and Technology, Songkhla Rajabhat University, Songkhla 90000, Thailand*

* *Corresponding author:* pattarawan.pe@skru.ac.th

Abstract. This study establishes the concept of interval valued intuitionistic fuzzy (InVInF) β-filters on β-algebras and a few of its related properties are investigated. Some compelling results of interval valued fuzzy β-filters have been examined. Further, the notions of products and strong β-filters are also introduced. In addition that, the level set and homomorphism of interval valued intuitionistic fuzzy β-filters are too discussed. Furthermore, we enacted that the intersection between two interval valued intuitionistic fuzzy $\beta-$filters is again an interval valued intuitionistic fuzzy β-filter.

1. Introduction

In 2002, Neggers et al. [12] proposed the idea of a β-algebra which is an algebraic structure with two operations. The concepts of fuzzy positive implicative and fuzzy associative filters of lattice implication algebras have been initiated in [13, 14]. Further, the authors in [13, 14] have demonstrated that every fuzzy associative filter is a fuzzy associative filter and that every fuzzy positive implicative

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filter is a fuzzy implicative filter. The equivalent conditions for both fuzzy positive implicative filters and fuzzy associative filters were also provided. Xu et al. [20] established the thought of intuitionistic fuzzy implicative filters in lattice implication algebras. Jun et al. [8] proposed the concept of fuzzy BCI-subalgebras with interval valued membership functions. In 2011, Ghorbani [4] proposed the notion of intuitioistic fuzzy filters of residuated lattices. They have illustrated that a residual lattice's collection of all intuitionistic fuzzy filters is a complete lattice and identified its distributive sublattices. Zadeh [21] developed an interval valued fuzzy set was used to extend a fuzzy set (ie. a fuzzy set with an interval valued membership function). An i-v fuzzy set is an interval valued fuzzy set that can be used in various algebraic structures. Biswas et al. [2] created fuzzy subgroups with interval membership values in 1994. Hoo [9] applied the concepts of filters and ideals in BCI-algebras in 1991. In 2015, Hemavathi et al. [5–7] discussed interval valued fuzzy β-subalgebras and also applied the concept to intuitionistic fuzzy sets. Jun et al. [15] introduced foldness of bipolar fuzzy sets and its application in BCK/BCI-algebras. Takallo et al. [19] discussed the concept of multipolar fuzzy p-ideals of BCI-algebras. The concept of multipolar intuitionistic fuzzy hyper BCK-ideals in hyper BCK-algebras has been developed by Seo et al. [16]. Borzooei et al. [3] focused on multipolar intuitionistic fuzzy B-algebras. A new perception of cubic multi-polar structures on BCK/BCI-algebras was approached by Al-Masarwah et al. [1]. In [10], the authors invented a mathematical model for nonlinear optimization which attempts membership functions to address the uncertainties. Muhiuddin et al. [11] applied the theory of linear Diophantine fuzzy set into BCK/BCI-algebras. Sujatha et al. [17, 18] introduced fuzzy filters on β-algebras and also developed the concept of intuitiointic fuzzy filters on β-algebras. With all of this in mind, this paper establishes the idea of interval valued fuzzy β-filters on β-algebras and demonstrate few of its intriguing aspects.

2. Preliminares

This section outlines some of the most important definitions and examples relevant to the study.

Definition 2.1. A β-algebra is a non-empty set Γ with two binary operations $+$ and $-$ and a constant 0 fulfills the following axioms:

(1) $\varrho\delta - 0 = \varrho\delta$
(2) $(0 - \varrho\delta) + \varrho\delta = 0$
(3) $(\varrho\delta - \grave{y}\delta) - \grave{z}\delta = \varrho\delta - (\grave{z}\delta + \grave{y}\delta)$

for all $\varrho\delta, \grave{y}\delta, \grave{z}\delta \in \Gamma$.

Definition 2.2. Let f be a mapping from a β- algebra Γ to a β- algebra Υ, then f is referred as homomorphism, if

(1) $f(\varrho\delta + \grave{y}\delta) = f(\varrho) + f(\grave{y}\delta)$
(2) $f(\varrho\delta - \grave{y}\delta) = f(\varrho\delta) - f(\grave{y}\delta)$

for all $\varrho\delta, \grave{y}\delta \in \mathit{\Gamma}$.

Definition 2.3. A β-subalgebra Ξ on a β-algebra $\mathit{\Gamma}$ is referred as β-filter, if

(1) $\varrho\delta \triangle \grave{y}\delta = \varrho\delta + (\varrho\delta + \grave{y}\delta) \in \Xi$
(2) $\varrho\delta \triangledown \grave{y}\delta = \varrho\delta - (\varrho\delta - \grave{y}\delta) \in \Xi$

for all $\varrho\delta, \grave{y}\delta \in \Xi$.

Definition 2.4. A β-subalgebra Ξ on a β-algebra $\mathit{\Gamma}$ is referred as fuzzy β-filter, if

(1) $\epsilon_{\Xi}(\varrho\delta \triangle \grave{y}\delta) \geq min\{\epsilon_{\Xi}(\varrho\delta), \epsilon_{\Xi}(\varrho\delta + \grave{y}\delta)\}$ and
$\epsilon_{\Xi}(\varrho\delta \triangledown \grave{y}\delta) \geq min\{\epsilon_{\Xi}(\varrho\delta), \epsilon_{\Xi}(\varrho\delta - \grave{y}\delta)\}$
(2) $\epsilon_{\Xi}(\grave{y}\delta) \geq \epsilon_{\Xi}(\varrho\delta)$ if $\varrho\delta \leq \grave{y}\delta$

for all $\varrho\delta, \grave{y}\delta \in \Xi$.

3. Interval Valued Fuzzy β-Filters

The concept of an Interval valued fuzzy (InVF) β-filter on a β-subalgebra is introduced in this section.

Definition 3.1. An InVF β-subalgebra Ξ on a β-algebra $\mathit{\Gamma}$ is referred as an InVF fuzzy β-filter, it satisfies

(1) $\bar{\epsilon}_{\Xi}(\varrho\delta \triangle \grave{y}\delta) \geq rmin\{\bar{\epsilon}_{\Xi}(\varrho\delta), \bar{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\}$ and
$\bar{\epsilon}_{\Xi}(\varrho\delta \triangledown \grave{y}\delta) \geq rmin\{\bar{\epsilon}_{\Xi}(\varrho\delta), \bar{\epsilon}_{\Xi}(\varrho\delta - \grave{y}\delta)\}$
(2) $\bar{\epsilon}_{\Xi}(\grave{y}\delta) \geq \bar{\epsilon}_{\Xi}(\varrho\delta)\ if\ \varrho\delta \leq \grave{y}\delta$

for all $\varrho\delta, \grave{y}\delta \in \Xi$.

Example 3.1. Consider a β-algebra $\mathit{\Gamma} = \{0, \gamma_1, \gamma_2, \gamma_3\}$ with two binary operations $+$ and $-$ and a constant 0 defined on $\mathit{\Gamma}$ with the Cayley's table:

$+$	0	γ_1	γ_2	γ_3
0	0	0	0	0
γ_1	γ_1	γ_1	γ_1	γ_1
γ_2	0	0	γ_2	γ_3
γ_3	γ_3	γ_3	γ_3	γ_3

$-$	0	γ_1	γ_2	γ_3
0	0	0	0	0
γ_1	γ_1	γ_1	γ_1	γ_1
γ_2	γ_2	γ_2	γ_2	γ_2
γ_3	γ_3	γ_3	γ_3	γ_3

Then $(\mathit{\Gamma}, +, -, 0)$ is a β-algebra. Thus $\Xi = \{\gamma_1, \gamma_3\}$ is a β-filter on $\mathit{\Gamma}$. We have Ξ is an InVF β-subalgebra, with interval membership function

$$\bar{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.3, 0.5] : & \varrho\delta = \gamma_1 \\ [0.4, 0.6] : & \varrho\delta = \gamma_3 \end{cases}.$$

Then it is observed that, Ξ is an InVF β-filter on $\mathit{\Gamma}$.

Example 3.2. From the Example 3.1, Ξ is an InVF β-subalgebra, define by the membership function

$$\bar{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.2, 0.6] : & \varrho\delta = \gamma_1 \\ [0.1, 0.4] : & \varrho\delta = \gamma_2 \\ [0.3, 0.5] : & \varrho\delta = \gamma_3 \end{cases}.$$

Then it is observed that, Ξ is not an InVF β-filter on Γ because $\bar{\epsilon}_{\Xi}(\gamma_2) \geq \bar{\epsilon}_{\Xi}(\gamma_1) \Rightarrow [0.1, 0.4] \not\geq [0.2, 0.6]$.

Lemma 3.1. *If Ξ_1 and Ξ_2 are two InVF β-filters on Γ, then so is $\Xi_1 \cap \Xi_2$.*

Proof. For $\varrho\delta, \grave{y}\delta \in \Gamma$,

$$\begin{aligned} \bar{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta \triangle \grave{y}\delta) &= rmin\{\bar{\epsilon}_{\Xi_1}(\varrho\delta \triangle \grave{y}\delta), \bar{\epsilon}_{\Xi_2}(\varrho\delta \triangle \grave{y}\delta)\} \\ &\geq rmin\{rmin\{\bar{\epsilon}_{\Xi_1}(\varrho\delta), \bar{\epsilon}_{\Xi_1}(\varrho\delta + \grave{y}\delta)\}, rmin\{\bar{\epsilon}_{\Xi_2}(\varrho\delta), \bar{\epsilon}_{\Xi_2}(\varrho\delta + \grave{y}\delta)\}\} \\ &\geq rmin\{rmin\{\bar{\epsilon}_{\Xi_1}(\varrho\delta), \bar{\epsilon}_{\Xi_2}(\varrho\delta)\}, rmin\{\bar{\epsilon}_{\Xi_1}(\varrho\delta + \grave{y}\delta), \bar{\epsilon}_{\Xi_2}(\varrho\delta + \grave{y}\delta)\}\} \\ &= rmin\{\bar{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta), \bar{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta + \grave{y}\delta)\}. \end{aligned}$$

Similarly, $\bar{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta \bigtriangledown \grave{y}\delta) \geq rmin\{\bar{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta), \bar{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta - \grave{y}\delta)\}$. Hence $\Xi_1 \cap \Xi_2$ is an InVF β-filter of Γ. □

Theorem 3.1. *Every β-filter in InVF is also a β-subalgebra in InVF.*

Proof. This proof is self-evident, as it follows clearly from the definition of the InVF β-filter. Every InVF β-subalgebra, on the other hand, does not have to be an InVF β-filter. □

Theorem 3.2. *For an InVF β-filter $\bar{\epsilon}$ of Γ, we have $\bar{\epsilon}_{\Xi}(\varrho\delta \triangle \grave{y}\delta) \geq \bar{\epsilon}_{\Xi}(\varrho\delta)$ and $\bar{\epsilon}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) \geq \bar{\epsilon}_{\Xi}(\varrho\delta)$ where $\varrho\delta \leq \grave{y}\delta$.*

Proof. Assume that $\bar{\epsilon}_{\Xi}$ is an InVF β-filter of Γ. Let $\varrho\delta, \grave{y}\delta \in \Gamma$. Then

$$\begin{aligned} \bar{\epsilon}_{\Xi}(\varrho\delta \triangle \grave{y}\delta) &= \bar{\epsilon}_{\Xi}(\varrho\delta + (\varrho\delta + \grave{y}\delta)) \\ &\geq rmin\{\bar{\epsilon}_{\Xi}(\varrho\delta), \bar{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\} \\ &= rmin\{\bar{\epsilon}_{\Xi}(\varrho\delta), rmin\{\bar{\epsilon}_{\Xi}(\varrho\delta), \bar{\epsilon}_{\Xi}(\grave{y}\delta)\}\} \\ &\text{(because InVF } \beta\text{-filter is an InVF } \beta\text{-subalgebra)} \\ &= rmin\{\bar{\epsilon}_{\Xi}(\varrho\delta), \bar{\epsilon}_{\Xi}(\varrho\delta)\} \\ &\text{(because } \varrho\delta \leq \grave{y}\delta \Rightarrow \bar{\epsilon}_{\Xi}(\grave{y}\delta) \geq \bar{\epsilon}_{\Xi}(\varrho\delta)) \\ &= \bar{\epsilon}_{\Xi}(\varrho\delta). \end{aligned}$$

Similarly, $\bar{\epsilon}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) \geq \bar{\epsilon}_{\Xi}(\varrho\delta)$. □

Definition 3.2. Consider an InVF β-filter $\overline{\epsilon}_{\Xi}$ of a β-subalgebra Γ. For $[s_1, s_2] \in D[0,1]$, the set $\overline{\epsilon}_{\Xi} = \{\varrho\delta \in \Gamma : \overline{\epsilon}_{\Xi}(\varrho)\delta \geq [s_1, s_2]\}$ is referred to as a level set of an InVF β-filter $\overline{\epsilon}_{\Xi}$ of Γ.

Theorem 3.3. *An InVF subset Ξ of a β-algebra Γ is an InVF β-filter if and only if for any $\overline{t} \in D[0,1]$ the $\overline{t}-$InVF level subset $\Xi_{\overline{t}} = \{\varrho\delta \in \Gamma : \Xi(\varrho\delta) \geq \overline{t}\}$ is either a β-filter or $\Xi_{\overline{t}} \neq \emptyset$.*

Proof. For an InVF level subset of Ξ in Γ, $\Xi_{\overline{t}} \neq \emptyset$. Then $\varrho\delta, \grave{y}\delta \in \Xi_{\overline{t}}, \Xi(\varrho\delta) \geq \overline{t}$. Now,

$$\begin{aligned}\Xi(\varrho\delta \bigtriangleup \grave{y}) &= \Xi(\varrho\delta + (\varrho\delta + \grave{y}\delta))\\ &\geq rmin\{\Xi(\varrho\delta), \Xi(\varrho\delta + \grave{y}\delta)\}\\ &= rmin\{\Xi(\varrho\delta), rmin\{\Xi(\varrho\delta), \Xi(\grave{y}\delta)\}\}\\ &= rmin\{\overline{t}, rmin\{\overline{t}, \overline{t}\}\}\\ &= \overline{t}.\end{aligned}$$

This implies that $\varrho\delta \bigtriangleup \grave{y}\delta \in \Xi_{\overline{t}}$. Similarly, $\varrho\delta \bigtriangledown \grave{y}\delta \in \Xi_{\overline{t}}$. Then $\Xi_{\overline{t}}$ is a β-filter of Γ. Suppose that $\Xi_{\overline{t}}$ is a β-filter of Γ, on the other hand. For $\varrho\delta, \grave{y}\delta \in \Gamma, \varrho\delta \bigtriangleup \grave{y}\delta$ *and* $\varrho\delta \bigtriangledown \grave{y}\delta \in \Xi_{\overline{t}}$, this implies that $\Xi(\varrho\delta \bigtriangleup \grave{y}\delta) \geq \overline{t}$ and $\Xi(\varrho\delta \bigtriangledown \grave{y}\delta) \geq \overline{t}$ $\Xi(\varrho\delta \bigtriangleup \grave{y}\delta) = \Xi(\varrho\delta + (\varrho\delta + \grave{y}\delta)) \geq \overline{t} = rmin\{\Xi(\varrho\delta), \Xi(\varrho\delta + \grave{y}\delta)\}$. Similarly, $\Xi(\varrho\delta \bigtriangledown \grave{y}\delta) \geq \overline{t}$. This proved Ξ is an InVF β-filter. □

Theorem 3.4. *Consider an onto β-algebra homomorphism f from Γ to Υ. If Ξ_2 is a InVF β-filter of Y, hence its inverse image $f^{-1}(\Xi_2)$ is again an InVF $\beta-$filter on Γ.*

Proof. Let Ξ_2 be an InVF β-filter of Y. For any $\varrho\delta, \grave{y}\delta \in \Gamma$,

$$\begin{aligned}f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta \bigtriangleup \grave{y}\delta)) &= f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta + (\varrho\delta + \grave{y}\delta)))\\ &= \overline{\epsilon}_{\Xi_2}(f(\varrho\delta + (\varrho\delta + \grave{y}\delta)))\\ &= \overline{\epsilon}_{\Xi_2}(f(\varrho\delta)\delta + f(\varrho\delta + \grave{y}\delta))\\ &\geq rmin\{\overline{\epsilon}_{\Xi_2}(f(\varrho\delta)), \overline{\epsilon}_{\Xi_2}(f(\varrho\delta + \grave{y}\delta))\}\\ &= rmin\{f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta)), f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta + \grave{y}\delta))\}.\end{aligned}$$

Similarly, $f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta \bigtriangledown \grave{y}\delta)) \geq rmin\{f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta)), f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta - \grave{y}\delta))\}$. Let $\varrho\delta, \grave{y}\delta \in \Gamma$, so that $\varrho\delta \geq \grave{y}\delta$. Subsequently, Ξ_2 is an InVF β-filter, $\overline{\epsilon}_{\Xi_2}(f(\grave{y}\delta)) \geq \overline{\epsilon}_{\Xi_2}(f(\varrho\delta)) = f^{-1}(\overline{\epsilon}_B(\varrho\delta))$ such that $f^{-1}(\overline{\epsilon}_{\Xi_2}(\grave{y}\delta)) \geq f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta))$. □

3.1. **Products on InVF β-Filters on β-Algebras.** The basic concepts and examples of product on InVF β-filters settings are covered in this section.

Theorem 3.5. *An InVF β-filter is the Cartesian product of any two InVF β-filters.*

Proof. Take $\varrho\delta = (\varrho\delta_1, \varrho\delta_2)$ & $\grave{y}\delta = (\grave{y}\delta_1, \grave{y}\delta_2) \in \Gamma \times Y$ & $\overline{\sigma} = (\overline{\epsilon}_{\Xi_1} \times \overline{\epsilon}_{\Xi_2})$. So

$$\begin{aligned}
\overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta \triangle \grave{y}\delta) &= \overline{\epsilon}_{\Xi_1}((\varrho\delta_1, \varrho\delta_2) \triangle (\grave{y}\delta_1, \grave{y}\delta_2)) \\
&= (\overline{\epsilon}_{\Xi_1} \times \overline{\epsilon}_{\Xi_2})\{((\varrho\delta_1, \varrho\delta_2) + ((\varrho\delta_1, \varrho\delta_2) + (\grave{y}\delta_1, \grave{y}\delta_2)))\} \\
&\geq rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta_1 + (\varrho\delta_1 + \grave{y}\delta_1)), \overline{\epsilon}_{\Xi_2}(\varrho\delta_2 + (\varrho\delta_2 + \grave{y}\delta_2))\} \\
&= rmin\{rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta_1), \overline{\epsilon}_{\Xi_1}(\varrho\delta_1 + \grave{y}\delta_1)\}, rmin\{\overline{\epsilon}_{\Xi_2}(\varrho\delta_2), \overline{\epsilon}_{\Xi_2}(\varrho\delta_2 + \grave{y}\delta_2)\}\} \\
&= rmin\{rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta_1), \overline{\epsilon}_{\Xi_2}(\varrho\delta_2)\}, rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta_1 + \grave{y}\delta_1), \overline{\epsilon}_{\Xi_2}(\varrho\delta_2 + \grave{y}\delta_2)\}\} \\
&= rmin\{\overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta_1, \varrho\delta_2), \overline{\sigma}_{\Xi_1\times\Xi_2}((\varrho\delta_1, \varrho\delta_2) + (\grave{y}\delta_1, \grave{y}\delta_2))\} \\
&= rmin\{\overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta), \overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta + \grave{y}\delta)\}.
\end{aligned}$$

Similarly, $\overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta \triangledown \grave{y}\delta) \geq rmin\{\overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta), \overline{\sigma}_{\Xi_1\times\Xi_2}(\varrho\delta - \grave{y}\delta)\}$. This proved that $\Xi_1 \times \Xi_2$ is also an InVF β-filter. □

Theorem 3.6. *Let Γ and Υ be two β-algebras. Let $\Xi_{1\overline{t}}$ and $\Xi_{2\overline{s}}$ be InVF β-filters on $\Gamma \times Y$. Then($\Xi_{1\overline{t}} \times \Xi_{2\overline{s}}$) is also a β-filter, if $\overline{t} \geq \overline{s}$.*

Proof. Take $\varrho\delta = (\varrho\delta_1, \varrho\delta_2)$ & $\grave{y}\delta = (\grave{y}\delta_1, \grave{y}\delta_2) \in \Gamma \times Y$ & $\overline{\sigma} = (\overline{\epsilon}_{\Xi_1} \times \overline{\epsilon}_{\Xi_2})$ *if* $\overline{t} \geq \overline{s}$. Using above theorem, $\overline{\sigma}(\Xi_{1\overline{t}} \times \Xi_{2\overline{s}})(\varrho\delta \triangle \grave{y}\delta) \geq \overline{s}$. Similarly, $\overline{\sigma}(\Xi_{1\overline{t}} \times \Xi_{2\overline{s}})(\varrho\delta \triangledown \grave{y}\delta) \geq \overline{s}$. □

3.2. **InVF Strong β-Filters.** Beginning with a description and some examples, this section introduces the notion of an InVF strong β-filter on a β-subalgebra.

Definition 3.3. An InVF β-subalgebra Ξ of a β-algebra is referred as an InVF strong β-filter, if

(1) $\overline{\epsilon}_{\Xi}(\varrho\delta \triangle \grave{y}\delta) = \overline{\epsilon}_{\Xi}(\varrho\delta \triangledown \grave{y}\delta)$
(2) $\overline{\epsilon}_{\Xi}(\grave{y}\delta) \geq \overline{\epsilon}_{\Xi}(\varrho\delta)$ *if* $\varrho\delta \leq \grave{y}\delta$

for all $\varrho\delta, \grave{y}\delta \in \Xi$.

Example 3.3. For a β-algebra $\Gamma = \{0, \eta_1, \eta_2, \eta_3\}$ be a with two binary operations $+$ and $-$ constant 0 and defined on Γ, we have a Cayley's table

$+$	0	η_1	η_1	η_3
0	0	0	0	0
η_1	η_1	η_1	η_1	η_1
η_1	0	0	η_1	η_3
η_3	η_3	η_3	η_3	η_3

$-$	0	η_1	η_1	η_3
0	0	0	0	0
η_1	η_1	η_1	η_1	η_1
η_1	η_1	η_1	η_1	η_1
η_3	η_3	η_3	η_3	η_3

Then $(\Gamma, +, -, 0)$ is a β-algebra. Thus $\Xi = \{\eta_1, \eta_3\}$ is a β-filter on Γ. Defining the membership function for an InVF β-subalgebra Ξ as

$$\overline{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.2, 0.5] : & \varrho\delta = \eta_1 \\ [0.3, 0.6] : & \varrho\delta = \eta_3 \end{cases}.$$

Then it is observed that, Ξ is an InVF strong β-filter on Γ.

Theorem 3.7. *Every InVF strong β-filter is also an InVF β-subalgebra.*

It is not necessary for the converse part of the theorem to be correct.

Example 3.4. For a β-algebra $\Gamma = \{0, \eta_1, \eta_2, \eta_3\}$ with two binary operations $+$ and $-$ and constant 0 defined on Γ, we have a Cayley's table

$+$	0	η_1	η_1	η_3
0	0	0	0	0
η_1	η_1	η_1	η_3	0
η_1	η_1	0	η_1	η_3
η_3	η_3	η_1	η_3	η_3

$-$	0	η_1	η_1	η_3
0	0	0	0	0
η_1	η_1	η_1	η_1	η_1
η_1	η_1	η_1	η_1	η_1
η_3	η_3	η_3	η_3	η_3

Then $(\Gamma, +, -, 0)$ is a β-algebra.

Then $\Xi = \{\eta_1, \eta_3\}$ is β-filter on Γ. Ξ is an InVF β-subalgebra, define by the membership function

$$\overline{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.4, 0.6] : & \varrho\delta = \eta_3 \\ [0.3, 0.5] : & \varrho\delta = \eta_1 \end{cases}.$$

Then it is observed that, Ξ is not an InVF strong β-filter on Γ since $\overline{\epsilon}_{\Xi}(\eta_1 \bigtriangleup \eta_3) \neq \overline{\epsilon}_{\Xi}(\eta_1 \bigtriangledown \eta_3)$.

Theorem 3.8. *If $\overline{\sigma}$ is an InVF strong β-filter of Γ, then $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) \geq \overline{\epsilon}_{\Xi}(\grave{y}\delta)$ where $\grave{y}\delta \leq \varrho\delta$.*

Theorem 3.9. *Consider be an onto β-algebra homomorphism f from Γ to Υ. If Ξ_2 is a InVF strong β-filter of Υ,then its inverse image $f^{-1}(\Xi_2)$ is again an InVF strong β-filter on Γ.*

4. InVInF β-Filters on β-Algebras

The concept of Interval valued intuitionstic fuzzy (InVInF) β-filters on a β-subalgebra is introduced in this section, which starts with the definition.

Definition 4.1. An InVInF β-subalgebra of a β-algebra Γ is called as an InVInF β-filter on Γ, if

(1) $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) \geq rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\}$ and
$\overline{\phi}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) \leq rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\varrho\delta + \grave{y}\delta)\}$
(2) $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) \geq rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\varrho\delta - \grave{y}\delta)\}$ and
$\overline{\phi}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) \leq rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\varrho\delta - \grave{y}\delta)\}$
(3) $\overline{\epsilon}_{\Xi}(\grave{y}) \geq \overline{\epsilon}_{\Xi}(\varrho\delta)$ and $\overline{\phi}_{\Xi}(\grave{y}\delta) \leq \overline{\phi}_{\Xi}(\varrho\delta)$ if $\varrho\delta \leq \grave{y}\delta$

for all $\varrho\delta, \grave{y}\delta \in \Gamma$.

Example 4.1. Consider a β-algebra $\Gamma = \{0, \rho_1, \rho_2, \rho_3\}$ with two binary operations $+$ and $-$ and a constant 0 defined on Γ with the Cayley's table:

$+$	0	ρ_1	ρ_2	ρ_3
0	0	0	0	0
ρ_1	ρ_1	0	ρ_3	ρ_1
ρ_2	ρ_2	0	ρ_2	ρ_3
ρ_3	ρ_3	ρ_1	ρ_3	ρ_3

$-$	0	ρ_1	ρ_2	ρ_3
0	0	0	0	0
ρ_1	ρ_1	ρ_1	ρ_1	ρ_1
ρ_2	ρ_2	ρ_2	ρ_2	ρ_2
ρ_3	ρ_3	ρ_3	ρ_3	ρ_3

Now, $\Xi = \{\rho_2, \rho_3\}$ is a β-filter on Γ.
Defining the membership and non membership function of an InVInF β-subalgebra Ξ as

$$\bar{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.4, 0.6] : & \varrho\delta = 0, \rho_3 \\ [0.3, 0.5] : & \varrho\delta = \rho_1, \rho_2 \end{cases}.$$

and

$$\overline{\phi}_{\Xi}(\varrho\delta) = \begin{cases} [0.3, 0.4] : & \varrho\delta = 0, \rho_3 \\ [0.4, 0.6] : & \varrho\delta = \rho_1, \rho_2 \end{cases}.$$

Therefore, Ξ is an InVInF β-filter on Γ.

Example 4.2. Consider a β-algebra $\Gamma = \{0, \rho_1, \rho_2, \rho_3\}$ with two binary operations $+$ and $-$ and a constant 0 defined on Γ with the Cayley's table:

$+$	0	ρ_1	ρ_2	ρ_3
0	0	0	0	0
ρ_1	ρ_1	ρ_1	ρ_1	ρ_1
ρ_2	0	ρ_1	ρ_2	ρ_3
ρ_3	ρ_3	ρ_1	ρ_2	ρ_3

$-$	0	ρ_1	ρ_2	ρ_3
0	0	0	0	0
ρ_1	ρ_1	ρ_1	ρ_1	ρ_1
ρ_2	ρ_2	ρ_2	ρ_2	ρ_2
ρ_3	ρ_3	ρ_3	ρ_3	ρ_3

Now, $\Xi = \{\rho_1, \rho_2, \rho_3\}$ is a β-filter on Γ. Defining the membership and non membership function of an InVInF β-subalgebra Ξ as

$$\bar{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.2, 0.6] : & \varrho\delta = 0 \\ [0.4, 0.5] : & \varrho\delta = \rho_1, \rho_3 \\ [0.3, 0.7] : & \varrho\delta = \rho_2 \end{cases}$$

and

$$\overline{\phi}_{\Xi}(\varrho\delta) = \begin{cases} [0.1, 0.6] : & \varrho\delta = 0 \\ [0.2, 0.7] : & \varrho\delta = \rho_1 \\ [0.4, 0.5] : & \varrho\delta = \rho_2, \rho_3 \end{cases}.$$

This shows that, Ξ is not an InVInF β-filter on Γ because $\bar{\epsilon}_{\Xi}(\rho_3) \geq \bar{\epsilon}_{\Xi}(\rho_2) \Rightarrow [0.4, 0.5] \not\geq [0.3, 0.7]$.

Lemma 4.1. *If Ξ_1 and Ξ_2 be any two InVInF β-filters on Γ, then $\Xi_1 \cap \Xi_2$ is also an InVInF β-filter of Γ.*

Proof. For $\varrho\delta, \grave{y}\delta \in \Gamma$

$$\begin{aligned}\overline{\sigma}_{\Xi_1\cap\Xi_2}(\varrho\delta \bigtriangleup \grave{y}\delta) &= rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta \bigtriangleup \grave{y}\delta), \overline{\epsilon}_{\Xi_2}(\varrho\delta \bigtriangleup \grave{y}\delta)\} \\ &\geq rmin\{rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta), \overline{\epsilon}_{\Xi_1}(\varrho\delta + \grave{y}\delta)\}, rmin\{\overline{\epsilon}_{\Xi_2}(\varrho\delta), \overline{\epsilon}_{\Xi_2}(\varrho\delta + \grave{y}\delta)\}\} \\ &\geq rmin\{rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta), \overline{\epsilon}_{\Xi_2}(\varrho\delta)\}, rmin\{\overline{\epsilon}_{\Xi_1}(\varrho\delta + \grave{y}\delta), \overline{\epsilon}_{\Xi_2}(\varrho\delta + \grave{y}\delta)\}\} \\ &= rmin\{\overline{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta), \overline{\epsilon}_{\Xi_1\cap\Xi_2}(\varrho\delta + \grave{y}\delta)\}.\end{aligned}$$

Also, $\overline{\phi}_{\Xi_1\cap\Xi_2}(\varrho\delta \bigtriangleup \grave{y}\delta) \leq rmax\{\overline{\phi}_{\Xi_1\cap\Xi_2}(\varrho\delta), \overline{\phi}_{\Xi_1\cap\Xi_2}(\varrho\delta + \grave{y}\delta)\}$. Hence, $\Xi_1 \cap \Xi_2$ is also an InVInF β-filter of Γ. □

Lemma 4.2. *Every InVInF β-filter is again an InVInF β-subalgebra.*

Proof. The definition of the InVInF β-filter leads to this proof. □

In general, the converse of the preceding lemma does not seems to be true, as shown by the following example (i.e. Every InVInF β-subalgebra need not be an InVInF β-filter).

Example 4.3. Let $\Gamma = \{0, \omega_1, \omega_2, \omega_3\}$ be a β-algebra with constant 0 and the Cayley's table:

$+$	0	ω_1	ω_2	ω_3
0	0	0	0	0
ω_1	ω_1	ω_1	ω_1	ω_1
ω_2	ω_1	ω_1	ω_2	0
ω_3	ω_3	ω_3	ω_1	ω_1

$-$	0	ω_1	ω_2	ω_3
0	0	0	0	0
ω_1	ω_1	ω_1	ω_1	ω_1
ω_2	ω_2	ω_2	ω_2	ω_2
ω_3	ω_3	ω_3	ω_3	ω_3

Now, $\Xi = \{0, \omega_3\}$ is a β-filter on Γ. Defining the membership and non membership function of an InVInF β-subalgebra Ξ as

$$\overline{\epsilon}_{\Xi}(\varrho\delta) = \begin{cases} [0.3, 0.5]: & \varrho\delta = 0, \omega_2 \\ [0.2, 0.4]: & \varrho\delta = \omega_1, \omega_3 \end{cases}.$$

and

$$\overline{\phi}_{\Xi}(\varrho\delta) = \begin{cases} [0.3, 0.5]: & \varrho\delta = 0, \omega_2 \\ [0.4, 0.6]: & \varrho\delta = \omega_1, \omega_3 \end{cases}.$$

However Ξ is not an InVInF β-filter on Γ because $\overline{\epsilon}_{\Xi}(\omega_3) \geq \overline{\epsilon}_{\Xi}(\omega_1) \Rightarrow [0.2, 0.4] \ngeq [0.3, 0.5]$.

Theorem 4.1. *If Ξ is an InVInF β-filter of Γ, then $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) \geq \overline{\epsilon}_{\Xi}(\varrho\delta)$ and $\overline{\phi}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) \leq \overline{\phi}_{\Xi}(\varrho\delta)$ where $\varrho\delta \leq \grave{y}\delta$.*

Proof. Assume that Ξ is an InVInF β-filter of Γ. Let $\varrho\delta, \grave{y}\delta \in \Gamma$. Then

$$\begin{aligned}\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) &= \overline{\epsilon}_{\Xi}(\varrho\delta + (\varrho\delta + \grave{y}\delta))\\ &\geq rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\}\\ &= rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\grave{y}\delta)\}\}\\ &= rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\varrho\delta)\} \because \varrho\delta \leq \grave{y}\delta \Rightarrow \overline{\epsilon}_{\Xi}(\grave{y}\delta) \leq \overline{\epsilon}_{\Xi}(\varrho\delta)\\ &= \overline{\epsilon}_{\Xi}(\varrho\delta).\end{aligned}$$

Similarly,

$$\begin{aligned}\overline{\phi}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) &= \overline{\phi}_{\Xi}(\varrho\delta - (\varrho\delta - \grave{y}\delta))\\ &\leq rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\varrho\delta - \grave{y}\delta)\}\\ &= rmax\{\overline{\phi}_{\Xi}(\varrho\delta), rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\grave{y}\delta)\}\}\\ &= rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\varrho\delta)\} \because \varrho\delta \leq \grave{y}\delta \Rightarrow \overline{\phi}_{\Xi}(\grave{y}\delta) \leq \overline{\phi}_{\Xi}(\varrho\delta)\\ &= \overline{\phi}_{\Xi}(\varrho\delta).\end{aligned}$$

□

Definition 4.2. Let Ξ be an InVInF β-filter of a β-subalgebra Γ. For $\overline{s}, \overline{t} \in D[0,1]$, the set $\Xi_{\overline{s},\overline{t}} = \{\varrho\delta \in \Gamma : \overline{\epsilon}_{\Xi}(\varrho\delta) \geq \overline{s}\ \&\ \overline{\phi}_{\Xi}(\varrho\delta) \leq \overline{t}\}$ is referred as a level set of InVInF β-filter Ξ of Γ.

Theorem 4.2. *An InVInF subset Ξ of a β-algebra Γ is an InVInF β-filter if and only if for any $\overline{s}, \overline{t} \in D[0,1]$ the $\Xi_{\overline{s}\ \overline{t}}$−InVInF level subset $\Xi_{\overline{s}\ \overline{t}} = \{\varrho\delta \in \Gamma : \overline{\epsilon}_{\Xi}(\varrho\delta) \geq \overline{s}\ \&\ \overline{\phi}_{\Xi}(\varrho\delta) \leq \overline{t}\}$ is either a β-filter or $\Xi_{\overline{s},\overline{t}} \neq \emptyset$.*

Proof. Consider an InVInF level subset of Ξ in Γ, $\Xi_{\overline{s},\overline{t}} \neq \emptyset$. For any $\varrho\delta, \grave{y}\delta \in \Xi_{\overline{s},\overline{t}}$, $\overline{\epsilon}_{\Xi}(\varrho\delta) \geq \overline{s}\ \&\ \overline{\epsilon}_{\Xi}(\varrho\delta) \geq \overline{s}$. Now

$$\begin{aligned}\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}) &= \overline{\epsilon}_{\Xi}(\varrho\delta + (\varrho\delta + \grave{y}\delta))\\ &\geq rmin\{\overline{\epsilon}_{\Xi}(\varrho), \overline{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\}\\ &= rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\grave{y}\delta)\}\}\\ &= rmin\{\overline{s}, rmin\{\overline{s}, \overline{s}\}\}\\ &= \overline{s}.\end{aligned}$$

This implies that $\varrho\delta \bigtriangleup \grave{y}\delta \in \Xi_{\overline{s}\ \overline{t}}$. Similarly, $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta) = rmin\{\overline{\epsilon_{\Xi}}(\varrho\delta), \overline{\epsilon_{\Xi}}(\varrho\delta - \grave{y}\delta)\}$. Analogously,

$$\begin{aligned}\overline{\phi}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}) &= \overline{\phi}_{\Xi}(\varrho\delta + (\varrho\delta + \grave{y}\delta))\\ &\leq rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\varrho\delta + \grave{y}\delta)\}\\ &= rmax\{\overline{\phi}_{\Xi}(\varrho\delta), rmax\{\overline{\phi}_{\Xi}(\varrho\delta), \overline{\phi}_{\Xi}(\grave{y}\delta)\}\}\\ &= rmax\{\overline{t}, rmax\{\overline{t}, \overline{t}\}\}\\ &= \overline{t}.\end{aligned}$$

Similarly, $\overline{\phi}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta)$. Then $\overline{\phi}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) \in \Xi_{\overline{s}\ \overline{t}}$ & $\overline{\phi}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}) \in \Xi_{\overline{s}\ \overline{t}}$. So, $(\varrho\delta \bigtriangleup \grave{y}\delta) \in \Xi_{\overline{s}\ \overline{t}}$ & $(\varrho\delta \bigtriangledown \grave{y}\delta) \in \Xi_{\overline{s}\ \overline{t}}$. Hence $\Xi_{\overline{s}\ \overline{t}}$ is a β-filter of Γ.

On the other hand, assume that $\Xi_{\overline{s}\ \overline{t}}$ is a β-filter of Γ. For all $\varrho\delta, \grave{y}\delta \in X$, $\varrho\delta \bigtriangleup \grave{y}\delta$ and $\varrho\delta \bigtriangledown \grave{y}\delta \in \Xi_{\overline{s}\ \overline{t}}$. Thus $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) \geq \overline{s}$ and $\Xi(\varrho\delta \bigtriangledown \grave{y}\delta) \geq \overline{s}$. Take $\overline{s} = rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\}$ for any $\varrho\delta, \grave{y}\delta \in X$. We have $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangleup \grave{y}\delta) = \overline{\epsilon}_{\Xi}(\varrho\delta + (\varrho\delta + \grave{y}\delta)) \geq \overline{s} = rmin\{\overline{\epsilon}_{\Xi}(\varrho\delta), \overline{\epsilon}_{\Xi}(\varrho\delta + \grave{y}\delta)\}$. Similarly, $\overline{\epsilon}_{\Xi}(\varrho\delta \bigtriangledown \grave{y}\delta)$. Analogously, for the non membership function. This proves Ξ is an InVInF β-filter. □

Theorem 4.3. *Consider an onto β-algebra homomorphism f from Γ to Υ. If Ξ_2 is an InVInF β-filter of Υ, then its inverse image $f^{-1}(\Xi_2)$ is also an InVInF β-filter on Γ.*

Proof. Suppose that Ξ_2 is an InVInF β-filter of Υ. For any $\varrho\delta, \grave{y}\delta \in \Gamma$,

$$\begin{aligned}f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta \bigtriangleup \grave{y}\delta)) &= f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta + (\varrho\delta + \grave{y}\delta)))\\ &= \overline{\epsilon}_{\Xi_2}(f(\varrho\delta + (\varrho\delta + \grave{y}\delta)))\\ &= \overline{\epsilon}_{\Xi_2}(f(\varrho\delta) + f(\varrho\delta + \grave{y}\delta))\\ &\geq rmin\{\overline{\epsilon}_{\Xi_2}(f(\varrho\delta)), \overline{\epsilon}_{\Xi_2}(f(\varrho\delta + \grave{y}\delta))\}\\ &= rmin\{f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta)), f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta + \grave{y}\delta))\}.\end{aligned}$$

Also, $f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta \bigtriangledown \grave{y}\delta)) \geq rmin\{f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta)), f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta - \grave{y}\delta))\}$. Analogously,

$$\begin{aligned}f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta \bigtriangleup \grave{y}\delta)) &= f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta + (\varrho\delta + \grave{y}\delta)))\\ &= \overline{\phi}_{\Xi_2}(f(\varrho\delta + (\varrho\delta + \grave{y}\delta)))\\ &= \overline{\phi}_{\Xi_2}(f(\varrho\delta) + f(\varrho\delta + \grave{y}\delta))\\ &\leq rmax\{\overline{\phi}_{\Xi_2}(f(\varrho\delta)), \overline{\phi}_{\Xi_2}(f(\varrho\delta + \grave{y}\delta))\}\\ &= rmax\{f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta)), f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta + \grave{y}\delta))\}.\end{aligned}$$

Similarly, $f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta \bigtriangledown \grave{y}\delta)) \leq rmax\{f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta)), f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta - \grave{y}\delta))\}$. Let $\varrho\delta, \grave{y}\delta \in \Gamma$, so that $\varrho\delta \geq \grave{y}\delta$. Consequently, Ξ_2 is an InVInF β-filter, $\overline{\epsilon}_{\Xi_2}(f(\grave{y}\delta)) \geq \overline{\epsilon}_{\Xi_2}(f(\varrho\delta)) = f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta))$ such that $f^{-1}(\overline{\epsilon}_{\Xi_2}(\grave{y}\delta) \geq f^{-1}(\overline{\epsilon}_{\Xi_2}(\varrho\delta))$ and $\overline{\phi}_{\Xi_2}(f(\grave{y}\delta)) \leq \overline{\phi}_{\Xi_2}(f(\varrho\delta)) = f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta))$ such that $f^{-1}(\overline{\phi}_{\Xi_2}(\grave{y}\delta) \geq f^{-1}(\overline{\phi}_{\Xi_2}(\varrho\delta))$. This shows that f^{-1} is an InVInF β-filter on Γ. □

5. Conclusion

In this work, we investigated interval valued filters, the product on interval valued fuzzy β- filters on β-algebras, interval valued fuzzy strong β- filters on β-algebras, and interval valued intuitionistic fuzzy β-filters on β-algebras, as well as their associated outcomes. Furthermore, it can be extended to other algebraic structures in the future research works.

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