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Fuzzy Ideals and Fuzzy Filters on Topologies Generated by Fuzzy Relations

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Abstract. Recently, Mishra and Srivastava have introduced and studied the notion of fuzzy topology generated by fuzzy relation and several properties were proved. In this paper, we mainly investigate the lattice structure of fuzzy open sets in this topology, and show its various properties and characteristics. Additionally, we extend to this lattice the notions of fuzzy ideal and fuzzy filter. For each of these notions, we fully characterize them in terms of this lattice meet and join operations.

1. Introduction

The notions of ideals and filters have studied in many algebraic structures (e.g., semi-groups, rings, MV-algebras, lattices, et cetera) and used as tools to investigate properties, representations and characterizations of these algebraic structures [16], [7], [13], [14]. In addition to their theoretical roles, they have used in some areas of applied mathematics, especially, in topology and its analysis approaches. Ideals and filters are appeared to provide very general contexts to unify the various notions of sequences convergence and limit in arbitrary topological spaces, and to express completeness and compactness in metric spaces [3], [17].

In the fuzzy setting and its extensions, several authors introduced and investigated the concepts of fuzzy ideals and fuzzy filters in different structures. The first approach considered fuzzy ideal and fuzzy filter as fuzzy sets on crisp structures, like on lattices or on residuated lattices [5], [9], [15]. The

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second approach proposed similar notions on (intuitionistic) fuzzy structures [2], [10], [12]. The third approach considered neutrosophic ideal and neutrosophic filter as neutrosophic sets [1], [20].

The present study is motivated by the work of Mishra and Srivastava [11] that have considered the notion of fuzzy topology generated by a fuzzy relation. More specifically, we deepen the study of a lattice structure of fuzzy open sets on this topology, and providing its various characteristics and properties. We pay particular attention to the notion of fuzzy ideal (resp. fuzzy filter) on this topology generated by a fuzzy relation. Furthermore, we provide a characterization of these notion of fuzzy ideal (resp. fuzzy filter) based on the meet and the join operations of the introduced lattice.

This paper is organized as follows. In Section 2, we recall some basic concepts related to fuzzy sets, fuzzy relations and fuzzy topology. In Section 3, we provide the lattice structure of fuzzy open sets in a fuzzy topology generated by a fuzzy relation, and we show its various properties and characteristics. In Section 4, we introduce the notions of fuzzy ideal (resp. fuzzy filter) on the lattice of fuzzy open sets, and some basic properties are given. Finally, some conclusions and future research in Section 5 are presented.

2. Basic concepts

This section contains the basic definitions and properties of fuzzy sets, fuzzy topology and some related notions that will be needed throughout this paper.

2.1. **Fuzzy sets.** In this subsection, we recall some basic concepts of fuzzy sets.

Let X be a universe, a fuzzy subset $A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}$ of X defined by Zadeh in 1965 [18] is characterized by a membership function $\mu_A : X \to [0, 1]$, where $\mu_A(x)$ is interpreted as the degree of membership of the element x in the fuzzy subset A for each $x \in X$.

For fuzzy sets, several operations are defined. Here we present only those which are related to the present paper.

Let $A = \{\langle x, \mu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x)\rangle \mid x \in X\}$ be two fuzzy subsets on X, then

(i) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$, for any $x \in X$;
(ii) $A = B$ if $\mu_A(x) = \mu_B(x)$, for any $x \in X$;
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x)\rangle \mid x \in X\}$;
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x)\rangle \mid x \in X\}$;
(v) $\overline{A} = \{\langle x, 1 - \mu_A(x)\rangle \mid x \in X\}$;
(vi) $Supp(A) = \{x \in X \mid \mu_A(x) > 0\}$;
(vii) $Ker(A) = \{x \in X \mid \mu_A(x) = 1\}$.

In the sequel, we need the following definition of level set (which is also often called α-cuts) of a fuzzy set.

Definition 2.1. [6] *Let A be a fuzzy set on a nonempty set X. The α-cut of A is the crisp subset*

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\},$$

for any $\alpha \in [0, 1]$.

2.2. **Fuzzy relations.** Zadeh [19] introduced the concept of fuzzy relation as a natural generalization of crisp relation.

Definition 2.2. [19] *A fuzzy binary relation (a fuzzy relation, for short) from a nonempty set X to a nonempty set Y is a fuzzy subset in $X \times Y$, i.e., is an expression R given by*

$$R = \{\langle (x, y), \mu_R(x, y)\rangle \mid (x, y) \in X \times Y\},$$

where

$$\mu_R : X \times Y \to [0, 1]$$

for any $(x, y) \in X \times Y$. The value $\mu_R(x, y)$ is called the degree of relation between x and y under the fuzzy relation R.

Next, we need to recall the following definitions [19].

Let R and P are two fuzzy relations from a nonempty set X to a nonempty set Y. R is said to be contained in P or we say that P contains R, denoted by $R \subseteq P$, if for all $(x, y) \in X \times Y$ it holds that $\mu_R(x, y) \leq \mu_P(x, y)$.

The transpose (inverse) R^t of R is the fuzzy relation from the nonempty set Y to the nonempty set X defined by

$$R^t = \{\langle (x, y), \mu_{R^t}(x, y)\rangle \mid (x, y) \in X \times Y\},$$

where $\mu_{R^t}(x, y) = \mu_R(y, x)$ for any $(x, y) \in X \times Y$.

The intersection of two fuzzy relations R and P from a nonempty set X to a nonempty set Y is defined as

$$R \cap P = \{\langle (x, y), \mu_{R\cap P}(x, y)\rangle\},$$

where $\mu_{R\cap P}(x, y) = \min(\mu_R(x, y), \mu_P(x, y))$ for any $(x, y) \in X \times Y$.

The union of two fuzzy relations R and P from a nonempty set X to a nonempty set Y is defined as

$$R \cup P = \{\langle (x, y), \mu_{R\cup P}(x, y)\rangle\},$$

where $\mu_{R\cup P}(x, y) = \max(\mu_R(x, y), \mu_P(x, y))$ for any $(x, y) \in X \times Y$.

In general, if A is a set of fuzzy relations from a nonempty set X to a nonempty set Y, then

$$\bigcap_{R\in A} R = \{\langle (x, y), \mu_{\cap_{R\in A} R}(x, y)\rangle\},$$

where $\mu_{\cap_{R\in A}R}(x,y) = \inf_{R\in A}\mu_R(x,y)$ for any $(x,y)\in X\times Y$;

$$\bigcup_{R\in A} R = \{\langle (x,y), \mu_{\cup_{R\in A}R}(x,y)\rangle\},$$

where $\mu_{\cup_{R\in A}R}(x,y) = \sup_{R\in A}\mu_R(x,y)$ for any $(x,y)\in X\times Y$.

2.3. **Fuzzy topology.**

Definition 2.3. [4] *[Fuzzy topology] A fuzzy topology (FT, for short) on a nonempty set X is a family τ of fuzzy sets on X which satisfies the following axioms:*

(i) $\emptyset, X \in \tau$;
(ii) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$;
(iii) $\bigcup G_i \in \tau$ *for any* $\{G_i : \ i \in J\} \subseteq \tau$.

In this case, the pair (X,τ) is called a fuzzy topological space (FTS, for short) and any FS in τ is known as a fuzzy open set (FOS, for short) in X. The complement of a fuzzy open set is called a fuzzy closed set (FCS, for short) in X.

Example 2.1. *Let $X = \{x_1, x_2, x_3\}$ and $A_1, A_2, A_3 \in FS(X)$ such that*
$A_1 = \{\langle x_1, 0.4\rangle, \langle x_2, 0.7\rangle, \langle x_3, 0.1\rangle\}$,
$A_2 = \{\langle x_1, 0.3\rangle, \langle x_2, 0.6\rangle, \langle x_3, 0.2\rangle\}$,
$A_3 = \{\langle x_1, 0.2\rangle, \langle x_2, 0.5\rangle, \langle x_3, 0.2\rangle\}$.
Then, $\tau = \{\emptyset, X, A_1, A_2, A_3\}$ is a fuzzy topology on X.

2.4. **Fuzzy topology generated by a fuzzy relation.**

The notion of fuzzy topology generated by a fuzzy relation was previously proposed by Mishra and Srivastava [11].

Definition 2.4. [11] *Let X be a nonempty crisp set and $R = \{\langle (x,y), \mu_R(x,y)\rangle \mid x,y\in X\}$ a fuzzy relation on X. Then for any $x\in X$, the fuzzy sets $\mathcal{L}_x$ and $\mathcal{R}_x$ defined by:*
$\mu_{\mathcal{L}_x}(y) = \mu_R(y,x)$, *for any* $y\in X$,
$\mu_{\mathcal{R}_x}(y) = \mu_R(x,y)$, *for any* $y\in X$,
are called respectively the lower and the upper contour of x.

We denote by τ_1, the fuzzy topology generated by the set of all lower contours and τ_2, the fuzzy topology generated by the set of all upper contours. Consequently, we denote by τ_R, the fuzzy topology generated by S the set of all lower and upper contours and it's called the fuzzy topology generated by R.

Definition 2.5. *Let R be a fuzzy relation on the set X and τ_R is the fuzzy topology generated by R and let U_1, U_2 are two fuzzy open sets on τ_R. The U_1 is said to be contained in U_2 (in symbols, $U_1 \subseteq U_2$) if $\mu_{U_1}(x_i) \leq \mu_{U_2}(x_i)$ for any $x_i \in X$.*
In this case, we also say that U_1 is smaller than U_2.

Example 2.2. *Let $X = \{x, y\}$ and R be a fuzzy relation on X given by:*

$\mu_R(.,.)$	x	y
x	0.5	0.7
y	0.4	0.6

Then $\mathcal{L}_x$, $\mathcal{L}_y$, $\mathcal{R}_x$ and $\mathcal{R}_y$ are the fuzzy sets on X given by :

$$\begin{aligned}\mathcal{L}_x &= \{\langle x, 0.5\rangle; \langle y, 0.4\rangle\},\\ \mathcal{L}_y &= \{\langle x, 0.7\rangle; \langle y, 0.6\rangle\},\\ \mathcal{R}_x &= \{\langle x, 0.5\rangle; \langle y, 0.7\rangle\},\\ \mathcal{R}_y &= \{\langle x, 0.4\rangle; \langle y, 0.6\rangle\}.\end{aligned}$$

Notice that $\mathcal{R}_y \subseteq \mathcal{R}_x$ and $\mathcal{R}_y \subseteq \mathcal{L}_y$. The fuzzy topology $\tau_{\mathcal{R}}$ generated by the fuzzy topology generated by R is the fuzzy topology generated by $S = \{\mathcal{L}_x, \mathcal{L}_y\} \cup \{\mathcal{R}_x, \mathcal{R}_y\}$, i.e., $\tau_{\mathcal{R}} = \{\emptyset, X, \mathcal{L}_x, \mathcal{L}_y, \mathcal{R}_x, \mathcal{R}_y, \mathcal{L}_x \cap \mathcal{R}_y, \mathcal{L}_y \cap \mathcal{R}_x, \mathcal{L}_x \cup \mathcal{R}_y, \mathcal{L}_y \cup R_x\}$, where
$\mathcal{L}_x \cap \mathcal{R}_y = \{\langle x, 0.4\rangle; \langle y, 0.4\rangle\}$, $\mathcal{L}_y \cap \mathcal{R}_x = \{\langle x, 0.5\rangle; \langle y, 0.6\rangle\}$,
$\mathcal{L}_x \cup \mathcal{R}_y = \{\langle x, 0.5\rangle; \langle y, 0.6\rangle\}$, and $\mathcal{L}_y \cup \mathcal{R}_x = \{\langle x, 0.7\rangle; \langle y, 0.7\rangle\}$.

Example 2.3. *Let $X = \{x, y, z\}$ and R be a fuzzy relation on X given by:*

$\mu_R(.,.)$	x	y	z
x	1	0.5	0
y	0	1	0.8
z	0.7	0	1

$\mathcal{L}_x$, $\mathcal{L}_y$, $\mathcal{L}_z$, $\mathcal{R}_x$, $\mathcal{R}_y$ and $\mathcal{R}_z$ are the fuzzy sets on X given by :

$$\begin{aligned}\mathcal{L}_x &= \{\langle x, 1\rangle; \langle y, 0\rangle; \langle z, 0.7\rangle\}\ ;\\ \mathcal{L}_y &= \{\langle x, 0.5\rangle; \langle y, 1\rangle; \langle z, 0\rangle\}\ ;\\ \mathcal{L}_z &= \{\langle x, 0\rangle; \langle y, 0.8\rangle; \langle z, 1\rangle\}\ ;\\ \mathcal{R}_x &= \{\langle x, 1\rangle; \langle y, 0.5\rangle; \langle z, 0\rangle\}\ ;\\ \mathcal{R}_y &= \{\langle x, 0\rangle; \langle y, 1\rangle; \langle z, 0.8\rangle\}\ ;\\ \mathcal{R}_z &= \{\langle x, 0.7\rangle; \langle y, 0\rangle; \langle z, 1\rangle\}.\end{aligned}$$

The fuzzy topology $\tau_{\mathcal{R}}$ is generated by $S = \{\mathcal{L}_x, \mathcal{L}_y, \mathcal{L}_z\} \cup \{\mathcal{R}_x, \mathcal{R}_y, \mathcal{R}_z\}$. Thus,

$$\tau_{\mathcal{R}} = \Big\{\emptyset, X, \mathcal{L}_x, \mathcal{L}_y, \mathcal{L}_z, \mathcal{R}_x, \mathcal{R}_y, \mathcal{R}_z, \{\langle x, 0.5\rangle; \langle y, 0\rangle; \langle z, 0\rangle\}, \{\langle x, 0\rangle; \langle y, 0\rangle; \langle z, 0.7\rangle\},$$

$$\{\langle x, 1\rangle; \langle y, 0\rangle; \langle z, 0\rangle\}, \{\langle x, 0.7\rangle; \langle y, 0\rangle; \langle z, 0.7\rangle\}, \{\langle x, 0\rangle; \langle y, 0.8\rangle; \langle z, 0\rangle\}, \{\langle x, 0.5\rangle; \langle y, 0.5\rangle; \langle z, 0\rangle\},$$

$\{\langle x,0\rangle;\langle y,1\rangle;\langle z,0\rangle\},\{\langle x,0\rangle;\langle y,0.8\rangle;\langle z,0.8\rangle\},\{\langle x,0\rangle;\langle y,0\rangle;\langle z,1\rangle\},\{\langle x,0\rangle;\langle y,0.5\rangle;\langle z,0\rangle\},$
$\{\langle x,0\rangle;\langle y,0\rangle;\langle z,0.8\rangle\},\{\langle x,0.7\rangle;\langle y,0\rangle;\langle z,0\rangle\},\{\langle x,1\rangle;\langle y,1\rangle;\langle z,0.7\rangle\},\{\langle x,1\rangle;\langle y,0.8\rangle;\langle z,1\rangle\},$
$\{\langle x,1\rangle;\langle y,0.5\rangle;\langle z,0.7\rangle\},\{\langle x,1\rangle;\langle y,1\rangle;\langle z,0.8\rangle\},\{\langle x,1\rangle;\langle y,0\rangle;\langle z,1\rangle\},\{\langle x,0.5\rangle;\langle y,1\rangle;\langle z,1\rangle\},$
$\{\langle x,1\rangle;\langle y,1\rangle;\langle z,0\rangle\},\{\langle x,0.5\rangle;\langle y,1\rangle;\langle z,0.8\rangle\},\{\langle x,0.7\rangle;\langle y,1\rangle;\langle z,1\rangle\},\{\langle x,0\rangle;\langle y,1\rangle;\langle z,1\rangle\},$
$\{\langle x,0.7\rangle;\langle y,0.8\rangle;\langle z,1\rangle\},\{\langle x,1\rangle;\langle y,1\rangle;\langle z,0.8\rangle\},\{\langle x,1\rangle;\langle y,0.5\rangle;\langle z,1\rangle\}\big\}.$

3. The lattice of fuzzy open sets on fuzzy topology generated by a fuzzy relation

In this section, we mainly investigate the lattice of all fuzzy open sets on a given fuzzy topology generated by a fuzzy relation.

The following theorem provides the lattice structure of fuzzy open sets on a fuzzy topology generated by a fuzzy relation.

Theorem 3.1. *Let X be a nonempty set, R a fuzzy relation on X and τ_R be a fuzzy topology generated R. Then the family*

$$\mathfrak{L}=\{U_i \mid U_i \text{ is a fuzzy opn set on } \tau_R\},$$

is a lattice.

Proof. The fact that τ_R is a fuzzy topology guarantees that the union and intersection of two fuzzy open sets are also fuzzy open sets. Hence, $(\mathfrak{L},\subseteq)$ is a lattice. □

Remark 3.1. *One can easily see that*

(i) $(\mathfrak{L},\cap,\cup,\emptyset,X)$ *is a Boolean algebra.*
(ii) *If X is a finite set, then the lattice $(\mathfrak{L},\subseteq)$ is complete.*

4. Fuzzy ideals and filters on the lattice of fuzzy open sets

In this section, we introduce the notions of fuzzy ideal (resp. fuzzy filter) on the lattice of fuzzy open sets, and a characterization in terms of this lattice meet and join operations is given.

Throughout this section, $\mathfrak{L}$ denotes the lattice of fuzzy open sets of τ_R a fuzzy topology generated by a fuzzy relation R on a nonempty set X.

4.1. **Definitions.**

Definition 4.1. *A fuzzy set I on $\mathfrak{L}$ is called a fuzzy ideal if for all $A,B\in\mathfrak{L}$ the following conditions hold:*

(i) $\mu_I(A\cup B)\geq\mu_I(A)\wedge\mu_I(B)$,
(ii) $\mu_I(A\cap B)\geq\mu_I(A)\vee\mu_I(B)$.

Definition 4.2. *A fuzzy set F on $\mathfrak{L}$ is called a fuzzy filter if for all $A,B\in\mathfrak{L}$ the following conditions hold:*

(i) $\mu_F(A \cup B) \geq \mu_F(A) \vee \mu_F(B)$,

(ii) $\mu_F(A \cap B) \geq \mu_F(A) \wedge \mu_F(B)$.

The following proposition expresses the relationship between a fuzzy ideal and a fuzzy filter on a lattice of fuzzy open sets. Its proof is straightforward.

Proposition 4.1. *Let $\mathfrak{L}^d$ be the order-dual lattice of $\mathfrak{L}$ and A be fuzzy set on $(\mathfrak{L})$. Then it holds that A is a fuzzy ideal on $\mathfrak{L}$ if and only if A is a fuzzy filter on $\mathfrak{L}^d$, and conversely.*

We need also the following result.

Proposition 4.2. *Let A and B are two fuzzy sets on $\mathfrak{L}$, then it holds that*

(i) *If A and B are two fuzzy ideals on $\mathfrak{L}$, then $A \cap B$ is a fuzzy ideal on $\mathfrak{L}$;*

(ii) *If A and B are two fuzzy filters on $\mathfrak{L}$, then $A \cap B$ is a fuzzy filter on $\mathfrak{L}$.*

4.2. **Basic characterization of fuzzy ideals and filters on a lattice of fuzzy open sets.** In this subsection, we provide interesting characterization of fuzzy ideals (resp. filters) on the lattice of fuzzy open sets in terms of its meet and its join operations.

Theorem 4.1. *I is a fuzzy ideal on $\mathfrak{L}$ if and only if the following condition is satisfied:*

$$\mu_I(A \cup B) = \mu_I(A) \wedge \mu_I(B), \text{ for any } A, B \in \mathfrak{L}\,.$$

Proof. Suppose that I is a fuzzy ideal on $\mathfrak{L}$, then for any $A, B \in \mathfrak{L}$ it holds that $\mu_I(A \cup B) \geq \mu_I(A) \wedge \mu_I(B)$. Since $A \sqsubseteq A \cup B$ and $B \sqsubseteq A \cup B$, it follows from Definition 4.1 (ii) that

$$\mu_I(A) = \mu_I(A \cap (A \cup B)) \geq \mu_I(A) \vee \mu_I(A \cup B) \geq \mu_I(A \cup B)\,.$$

In the same manner, $\mu_I(B) \geq \mu_I(A \cup B)$. Hence, $\mu_I(A) \wedge \mu_I(B) \geq \mu_I(A \cup B)$. Thus, $\mu_I(A \cup B) = \mu_I(A) \wedge \mu_I(B)$.

Conversely, suppose that $\mu_I(A \cup B) = \mu_I(A) \wedge \mu_I(B)$ for any $A, B \in \mathfrak{L}$. Then it is easy to see that $\mu_I(A \cup B) \geq \mu_I(A) \wedge \mu_I(B)$ for any $A, B \in \mathfrak{L}$. Next, we will show that $\mu_I(A \cap B) \geq \mu_I(A) \vee \mu_I(B)$ for any $A, B \in \mathfrak{L}$. Let $A, B \in \mathfrak{L}$, since $A \cup (A \cap B) = A$ and $B \cup (A \cap B) = B$ then it holds that $\mu_I(A \cup (A \cap B)) = \mu_I(A)$ and $\mu_I(B \cup (A \cap B)) = \mu_I(B)$. From hypothesis it follows that $\mu_I(A) \wedge \mu_I(A \cap B) = \mu_I(A)$ and $\mu_I(B) \wedge \mu_I(A \cap B) = \mu_I(B)$. Hence, $\mu_I(A \cap B) \geq \mu_I(A)$ and $\mu_I(A \cap B) \geq \mu_I(B)$. Thus, $\mu_I(A \cap B) \geq \mu_I(A) \vee \mu_I(B)$, for any $A, B \in \mathfrak{L}$. Therefore, I is a fuzzy ideal on $\mathfrak{L}$. □

In the same line, the following theorem provides a characterization of fuzzy filters on the lattice of fuzzy open sets in terms of its meet operation.

Theorem 4.2. *F is a fuzzy filter on $\mathfrak{L}$ if and only if the following condition is satisfied:*

$$\mu_F(A \cap B) = \mu_F(A) \wedge \mu_F(B), \text{ for any } A, B \in \mathfrak{L}\,.$$

Proof. The proof is a direct application of Proposition 4.1 and Theorem 4.1. □

As corollaries of the above theorems, we obtain the following interesting properties of fuzzy ideals and fuzzy filters on a lattice of fuzzy open sets.

Corollary 4.1. *Let I be a fuzzy ideal on $\mathfrak{L}$ and $A, B \in \mathfrak{L}$. If $A \sqsubseteq B$, then $\mu_I(A) \geq \mu_I(B)$, (i.e., the mapping μ_I is antitone).*

Corollary 4.2. *Let F be a fuzzy filter on $\mathfrak{L}$ and $A, B \in \mathfrak{L}$. If $A \sqsubseteq B$, then $\mu_F(A) \leq \mu_F(B)$, (i.e., the mapping μ_F is monotone).*

5. Conclusion and Future Work

In this article, we have studied properties of lattices on fuzzy topology generated by fuzzy relation, and provided their various characteristics. We have introduced and studied the notions of fuzzy ideal (resp. fuzzy filter) on the lattice of fuzzy open sets and we have discussed some its basic properties. We anticipate that these notions of fuzzy ideals (resp. fuzzy filters) will facilitate the study and the representations of the different kinds of fuzzy lattices. Due to the usefulness of these notions, we think it makes sense to study some kinds of fuzzy ideals (resp. fuzzy filters) on fuzzy topology generated by fuzzy relation.

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References

[1] A. Bennoui, L. Zedam, S. Milles, Several Types of Single-Valued Neutrosophic Ideals and Filters on a Lattice, TWMS J. App. Eng. Math. In Press.

[2] S. Boudaoud, L. Zedam, S. Milles, Principal Intuitionistic Fuzzy Ideals and Filters on a Lattice, Discuss. Math.: Gen. Algebra Appl. 40 (2020), 75–88. https://doi.org/10.7151/dmgaa.1325.

[3] N. Bourbaki, *Topologie générale*, Springer-Verlag, Berlin Heidelberg, 2007.

[4] C.L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968), 182–190. https://doi.org/10.1016/0022-247x(68)90057-7.

[5] B. Davvaz, O. Kazanci, A New Kind of Fuzzy Sublattice (Ideal, Filter) of a Lattice, Int. J. Fuzzy Syst. 13 (2011), 55–63.

[6] J.A. Goguen, The Logic of Inexact Concepts, Synthese, 19 (1969), 325–373. https://www.jstor.org/stable/20114646.

[7] B.A. Davey, H.A. Priestley, Introduction to Lattices and Order, Second Edition, Cambridge University Press, Cambridge, 2002.

[8] J.L. Kelley, General Topology, Van Nostrand, Princeton, New Jersey, 1955.

[9] Y. Liu, M. Zheng, Characterizations of Fuzzy Ideals in Coresiduated Lattices, Adv. Math. Phys. 2016 (2016), 6423735. https://doi.org/10.1155/2016/6423735.

[10] S. Milles, E. Rak, L. Zedam, Intuitionistic Fuzzy Complete Lattices, in: K.T. Atanassov, O. Castillo, J. Kacprzyk, M. Krawczak, P. Melin, S. Sotirov, E. Sotirova, E. Szmidt, G. De Tré, S. Zadrożny (Eds.), Novel Developments in Uncertainty Representation and Processing, Springer International Publishing, Cham, 2016: pp. 149–160. https://doi.org/10.1007/978-3-319-26211-6_13.

[11] S. Mishra, R. Srivastava, Fuzzy Topologies Generated by Fuzzy Relations, Soft Comput. 22 (2016), 373–385. https://doi.org/10.1007/s00500-016-2458-6.

[12] S. Milles, L. Zedam, E. Rak, Characterizations of Intuitionistic Fuzzy Ideals and Filters Based on Lattice Operations, J. Fuzzy Set Valued Anal. 2017 (2017), 143–159. https://doi.org/10.5899/2017/jfsva-00399.

[13] B.S. Schröder, Ordered Sets, Birkhauser, Boston, USA, 2002.

[14] M.H. Stone, The Theory of Representations of Boolean Algebras, Trans. Amer. Math. Soc. 40 (1936), 37–111. https://doi.org/10.2307/1989664.

[15] M. Tonga, Maximality on Fuzzy Filters of Lattices, Afr. Mat. 22 (2011), 105–114. https://doi.org/10.1007/s13370-011-0009-y.

[16] B. Van Gasse, G. Deschrijver, C. Cornelis, E.E. Kerre, Filters of Residuated Lattices and Triangle Algebras, Inform. Sci. 180 (2010), 3006–3020. https://doi.org/10.1016/j.ins.2010.04.010.

[17] S. Willard, General Topology, Addison-Wesley Publishing Company, Reading, Massachusetts, 1970.

[18] L.A. Zadeh, Fuzzy Sets, Inform. Control. 8 (1965), 331–352.

[19] L.A. Zadeh, Similarity Relations and Fuzzy Orderings, Inform. Sci. 3 (1971), 177–200. https://doi.org/10.1016/s0020-0255(71)80005-1.

[20] L. Zedam, S. Milles, A. Bennoui, Ideals and Filters on a Lattice in Neutrosophic Setting, Appl. Appl. Math. 16 (2021), 1140–1154.