

Bipolar Fuzzy Magnified Translations in Groups

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Abstract. In this paper, we define a bipolar fuzzy magnified translation (BFMT) of a bipolar fuzzy subgroup (BFSG) of a group. Based on this concept we have also developed some important results and theorems on bipolar fuzzy groups.

1. Introduction

The notion of a fuzzy set (FS) was introduced in 1965 by Zadeh [16]. The FS theory has various expansions, such as intuitionistic fuzzy sets (IFS), interval-valued fuzzy sets (IVFS), vague sets (VS), and so on. With the traditional FS representation it is not easy to explicitly express the difference of the irrelevant elements from the contrary elements. Based on these observations, in 2000, Lee [1] introduced a bipolar valued fuzzy set (BVFS), an extension of FSs whose range of the membership degree (MSD) is enlarged from $[0, 1]$ to $[-1, 1]$. Kalyani and Eswarlal [6–10] have introduced and studied the bipolar vague cosets, normal groups, bipolar fuzzy sublattices, ideals and also gave the application of TOPSIS and ELECTRE1 method on bipolar vague sets. The idea of fuzzy magnified translation (FMT) has been coined by Majumder and Sardar [15] in 2008. Jun [14] explored the bipolar fuzzy translations (BFT) in BCK/BCI-algebras in 2009. Kumar [13] popularized bipolar valued fuzzy translations (BVFT) in semigroups in 2012. The notion of an intuitionistic fuzzy magnified

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translation (IMFT) in groups has been discussed by Sarma [12] in 2012. The notion of translation (T), multiplication (M), and extension (E) applied in distinct aspects on different structures in algebra. In 2019, Iampan [4] studied translation (T) and density (D) of a bipolar valued fuzzy set (BFS) in UP-algebras. Anggraeni [5] has given BFT(bipolar fuzzy translation), BFE(bipolar fuzzy extension), and BFM(bipolar fuzzy multiplication) on bipolar anti-fuzzy ideals of K-algebras in 2019. In 2021, Alshehri [2] studied fuzzy translation (FT) and multiplication (FM) in BRK algebras, Khamrot [3] studied on a right weakly regular semigroup (RWRSG) of generalized a bipolar fuzzy subsemigroup (BFSSG).

Here in this paper, we introduce a Bipolar fuzzy magnified translation in groups and obtained some interesting results.

Throughout the paper FS stands for a fuzzy set, FG stands for a fuzzy subgroup, BFS stands for a bipolar fuzzy subset, BFSG stands for a bipolar fuzzy subgroup, BFNSG stands for a bipolar fuzzy normal subgroup, BFMT stands for a bipolar fuzzy magnified translation, G is always a group and \mathbb{D} is universe of discourse.

2. Preliminaries

Here, we will review a few standard definitions that are relevant to this work.

Definition 2.1. [16] A mapping $\delta : \mathbb{Z} \rightarrow [0, 1]$ is represented to as a fuzzy subset (FS) of a nonempty set \mathbb{Z} .

Definition 2.2. [1] A BFS \mathbb{B} in \mathbb{D} is an object having the form $\mathbb{B} = \{ \langle \mathcal{T}, \mathbb{B}^N(\mathcal{T}), \mathbb{B}^P(\mathcal{T}) \rangle : \mathcal{T} \in \mathbb{D} \}$, where $\mathbb{B}^P : \mathbb{D} \rightarrow [0, 1]$ and $\mathbb{B}^N : \mathbb{D} \rightarrow [-1, 0]$. The positive membership degree (+ve MSD) $\mathbb{B}^P(\mathcal{T})$ denotes the satisfaction degree of an element \mathcal{T} to the property corresponding to \mathbb{B} and the negative membership degree (-ve MSD) $\mathbb{B}^N(\mathcal{T})$ denotes the satisfaction degree of \mathcal{T} to some implicit counter property of \mathbb{B} . For the sake of simplicity, we shall use the symbol $\mathbb{B} = \langle \mathbb{B}^N, \mathbb{B}^P \rangle$ for the BFS $\mathbb{B} = \{ \langle \mathcal{T}, \mathbb{B}^N(\mathcal{T}), \mathbb{B}^P(\mathcal{T}) \rangle : \mathcal{T} \in \mathbb{D} \}$.

Definition 2.3. [11] A BFS \mathbb{B} of G is said to be a BFSG of G if the following conditions are satisfied:

- (i) $\mathbb{B}^P(\xi\eta) \geq \min\{\mathbb{B}^P(\xi), \mathbb{B}^P(\eta)\}$,
- (ii) $\mathbb{B}^P(\xi^{-1}) \geq \mathbb{B}^P(\xi)$,
- (iii) $\mathbb{B}^N(\xi\eta) \leq \max\{\mathbb{B}^N(\xi), \mathbb{B}^N(\eta)\}$, and
- (iv) $\mathbb{B}^N(\xi^{-1}) \leq \mathbb{B}^N(\xi)$ for all ξ, η in G .

Definition 2.4. [11] A BFSG \mathbb{B} of G is said to be a BFNSG of G if the following conditions are satisfied:

- (i) $\mathbb{B}^P(\xi\eta) = \mathbb{B}^P(\eta\xi)$ and
- (ii) $\mathbb{B}^N(\xi\eta) = \mathbb{B}^N(\eta\xi)$ for all ξ, η in G .

Definition 2.5. [11] Let η be a mapping from a group \mathbb{K} to a group \mathbb{K}' and let $\mathbb{A} = (\mathbb{K}; \mathbb{A}^N, \mathbb{A}^P)$ BFS in \mathbb{K} and $\mathbb{B} = (\mathbb{K}'; \mathbb{B}^N, \mathbb{B}^P)$ BFS in $\eta(g) = \mathbb{K}'$ defined by $\mathbb{B}^P(h) = \sup\{\mathbb{A}^P(g)\}$ and $\mathbb{B}^N(h) = \inf\{\mathbb{A}^N(g)\}$,

where $g \in \eta^{-1}(h)$ for all $g \in \mathbb{K}$ and $h \in \mathbb{K}'$. \mathbb{A} is called the preimage of \mathbb{B} under η and is denoted by $\eta^{-1}(\mathbb{B})$ and is defined by for $g \in \mathbb{K}$, $(\eta^{-1}(\mathbb{B}^P(g))) = \mathbb{B}^P(\eta(g))$ and $(\eta^{-1}(\mathbb{B}^N(g))) = \mathbb{B}^N(\eta(g))$.

Remark 2.1. [4] For any BFS $\mathbb{B} = \langle \mathbb{B}^N, \mathbb{B}^P \rangle$ in \mathbb{D} , we denote $\nabla = -1 - \inf\{\mathbb{B}^N(\mathcal{T}) : \mathcal{T} \in \mathbb{D}\}$ and $\Delta = 1 - \sup\{\mathbb{B}^P(\mathcal{T}) : \mathcal{T} \in \mathbb{D}\}$. Let $\mathbb{B} = \langle \mathbb{B}^N, \mathbb{B}^P \rangle$ be a BFS in \mathbb{D} and $(\theta, \vartheta) \in [\nabla, 0] \times [0, \Delta]$. By a bipolar fuzzy (θ, ϑ) -translation of $\mathbb{B} = \langle \mathbb{B}^N, \mathbb{B}^P \rangle$, we mean a BFS $\mathbb{B}_{(\theta, \vartheta)}^T = \langle \mathbb{B}_{(\theta, T)}^N, \mathbb{B}_{(\vartheta, T)}^P \rangle$, where $\mathbb{B}_{(\theta, T)}^N : \mathbb{D} \rightarrow [-1, 0]$ defined by $\mathbb{B}_{(\theta, T)}^N(\mathcal{T}) = \mathbb{B}^N(\mathcal{T}) + \theta$ and $\mathbb{B}_{(\vartheta, T)}^P : \mathbb{D} \rightarrow [0, 1]$ defined by $\mathbb{B}_{(\vartheta, T)}^P(\mathcal{T}) = \mathbb{B}^P(\mathcal{T}) + \vartheta$ for all $\mathcal{T} \in \mathbb{D}$.

3. Bipolar fuzzy magnified translations in groups

Definition 3.1. Let $\mathbb{B} = \langle \mathbb{B}^N, \mathbb{B}^P \rangle$ be a BFS in \mathbb{D} and $(\alpha, \beta) \in [0, 1]$, $(\theta, \vartheta) \in [\nabla, 0] \times [0, \Delta]$. By a BFMT of $\mathbb{B} = \langle \mathbb{B}^N, \mathbb{B}^P \rangle$, we mean a BFS $M = \{ \langle r, \mathbb{B}_{(\alpha, \theta)}^N(r), \mathbb{B}_{(\beta, \vartheta)}^P(r) \rangle : r \in \mathbb{D} \}$ or simply as $M = \{ \langle r, \mathbb{B}_M^N(r), \mathbb{B}_M^P(r) \rangle : r \in \mathbb{D} \}$, where $\mathbb{B}_M^N = \mathbb{B}_{(\alpha, \theta)}^N : \mathbb{D} \rightarrow [-1, 0]$ and $\mathbb{B}_M^P(r) = \mathbb{B}_{(\beta, \vartheta)}^P : \mathbb{D} \rightarrow [0, 1]$ defined by $\mathbb{B}_M^N(r) = \mathbb{B}_{(\alpha, \theta)}^N(r) = \alpha \mathbb{B}^N(r) + \theta$ and $\mathbb{B}_M^P(r) = \mathbb{B}_{(\beta, \vartheta)}^P(r) = \beta \mathbb{B}^P(r) + \vartheta$ for all $r \in \mathbb{D}$.

Example 3.1. Let $\mathbb{D} = \{1, \omega, \omega^2\}$ and let $\mathbb{B} = \{ \langle 1, -0.2, 0.3 \rangle, \langle \omega, -0.3, 0.4 \rangle, \langle \omega^2, -0.1, 0.5 \rangle \}$. Then $\theta \in [-0.9, 0]$ and $\vartheta \in [0, 0.5]$. Let $\alpha = 0.1, \beta = 0.2, \theta = -0.8, \vartheta = 0.2$. Hence, a BFMT $M = \{ \langle 1, -0.8, 0.26 \rangle, \langle \omega, -0.83, 0.36 \rangle, \langle \omega^2, -0.81, 0.3 \rangle \}$.

Theorem 3.1. Let M be a BFMT of a BFSG \mathbb{B} of G .

Then (i) $\mathbb{B}_T^P(r^{-1}) = \mathbb{B}_T^P(r)$ and $\mathbb{B}_T^N(r^{-1}) = \mathbb{B}_T^N(r)$,

(ii) $\mathbb{B}_T^P(r) \leq \mathbb{B}_T^P(e)$ and $\mathbb{B}_T^N(r) \geq \mathbb{B}_T^N(e)$ for all $r, e \in G$.

Proof. (i) $\mathbb{B}_T^P(r^{-1}) = \mathbb{B}_{(\alpha, \theta)}^P(r^{-1}) = \alpha \mathbb{B}^P(r^{-1}) + \theta = \alpha \mathbb{B}^P(r) + \theta = \mathbb{B}_{(\alpha, \theta)}^P(r)$. Similarly, $\mathbb{B}_T^N(r^{-1}) = \mathbb{B}_T^N(r)$.

(ii) $\mathbb{B}_T^P(e) = \mathbb{B}_{(\alpha, \theta)}^P(e) = \alpha \mathbb{B}^P(e) + \theta \geq \alpha \mathbb{B}^P(r) + \theta = \mathbb{B}_{(\alpha, \theta)}^P(r)$. Similarly, $\mathbb{B}_T^N(r) \geq \mathbb{B}_T^N(e)$. □

Theorem 3.2. Let M be a BFMT of a BFSG \mathbb{B} of G . Then

(i) $\mathbb{B}_T^P(ry^{-1}) = \mathbb{B}_T^P(e) \Rightarrow \mathbb{B}_T^P(r) = \mathbb{B}_T^P(y)$,

(ii) $\mathbb{B}_T^N(ry^{-1}) = \mathbb{B}_T^N(e) \Rightarrow \mathbb{B}_T^N(r) = \mathbb{B}_T^N(y)$ for all r, y, e in G .

Proof. (i)

$$\begin{aligned} \mathbb{B}_T^P(r) &= \mathbb{B}_{(\alpha, \theta)}^P(r) \\ &= \alpha \mathbb{B}^P(r) + \theta \\ &= \alpha \mathbb{B}^P(ry^{-1}y) + \theta \\ &\geq \alpha \mathbb{B}^P(r) + \theta \\ &\geq \alpha \{ \min\{\mathbb{B}^P(ry^{-1}), \mathbb{B}^P(y)\} \} + \theta \\ &\geq \min\{ \alpha(\mathbb{B}^P(ry^{-1}) + \theta), \alpha(\mathbb{B}^P(y) + \theta) \} \end{aligned}$$

$$\begin{aligned}
&= \min\{\mathbb{B}_{(\alpha,\theta)}^P(ry^{-1}), \mathbb{B}_{(\alpha,\theta)}^P(y)\} \\
&= \min\{\mathbb{B}_{(\alpha,\theta)}^P(e), \mathbb{B}_{(\alpha,\theta)}^P(y)\} \\
&= \mathbb{B}_{(\alpha,\theta)}^P(y).
\end{aligned}$$

Similarly, we can prove (ii). □

Theorem 3.3. *Let M be a BFMT of a BFGS \mathbb{B} of G . Then M is a BFGS of G .*

Proof. Let $x, y \in G$. Then we have

$$\begin{aligned}
\mathbb{B}_T^P(ry^{-1}) &= \mathbb{B}_{(\beta,\vartheta)}^P(ry^{-1}) \\
&= \beta\mathbb{B}^P(ry^{-1}) + \vartheta \\
&\geq \beta\min\{\mathbb{B}^P(r), \mathbb{B}^P(y^{-1})\} + \vartheta \\
&= \beta\min\{\mathbb{B}^P(r), \mathbb{B}^P(y)\} + \vartheta \\
&= \min\{(\beta\mathbb{B}^P(r) + \vartheta), (\beta\mathbb{B}^P(y) + \vartheta)\} \\
&= \min\{\mathbb{B}_{(\beta,\vartheta)}^P(r), \mathbb{B}_{(\beta,\vartheta)}^P(y)\} \\
&= \min\{\mathbb{B}_T^P(r), \mathbb{B}_T^P(y)\}.
\end{aligned}$$

Therefore, $\mathbb{B}_T^P(ry^{-1}) \geq \min\{\mathbb{B}_T^P(r), \mathbb{B}_T^P(y)\}$.

Similarly, $\mathbb{B}_T^N(ry^{-1}) \leq \max\{\mathbb{B}_T^N(r), \mathbb{B}_T^N(y)\}$.

Hence, M is a BFGS of G . □

Theorem 3.4. *Let M be a BFMT of a BFGS \mathbb{B} of G . Then $H = \{r \in G : \mathbb{B}_T^P(r) = \mathbb{B}_T^P(e) \text{ and } \mathbb{B}_T^N(r) = \mathbb{B}_T^N(e)\}$ is a subgroup of G .*

Proof. Let $r, y \in H$.

By Theorem 3.1, we have

$$\mathbb{B}_T^P(r^{-1}) = \mathbb{B}_T^P(r) = \mathbb{B}_T^P(e).$$

Similarly,

$$\mathbb{B}_T^N(r^{-1}) = \mathbb{B}_T^N(r) = \mathbb{B}_T^N(e).$$

Thus $\mathbb{B}_T^P(r^{-1}) = \mathbb{B}_T^P(e)$ and $\mathbb{B}_T^N(r^{-1}) = \mathbb{B}_T^N(e)$. So $r^{-1} \in H$.

Now,

$$\begin{aligned}
\mathbb{B}_T^P(ry) &= \mathbb{B}_{(\beta,\vartheta)}^P(ry) \\
&\geq \min\{\mathbb{B}_{(\beta,\vartheta)}^P(r), \mathbb{B}_{(\beta,\vartheta)}^P(y)\} \\
&= \min\{\mathbb{B}_{(\beta,\vartheta)}^P(e), \mathbb{B}_{(\beta,\vartheta)}^P(e)\}
\end{aligned}$$

$$= \mathbb{B}_{(\beta, \vartheta)}^P(e) = \mathbb{B}_T^P(e).$$

$$\text{Likewise } \mathbb{B}_T^N(ry) \leq \mathbb{B}_T^N(e).$$

Now,

$$\begin{aligned} \mathbb{B}_T^P(e) &= \mathbb{B}_T^P((ry)(ry)^{-1}) \\ &\geq \min\{\mathbb{B}_T^P(ry), \mathbb{B}_T^P(ry)^{-1}\} \\ &= \mathbb{B}_T^P(ry). \end{aligned}$$

Similarly, $\mathbb{B}_T^P(e) \leq \mathbb{B}_T^P(ry)$.

So $ry \in H$.

Hence, H is a subgroup of G . □

The proof of the following two theorems is similar to the proof of Theorem 3.4.

Theorem 3.5. *Let M be a BFMT of a BFSG \mathbb{B} of G . Then $H = \{\langle r, \mathbb{B}_T^P(r) \rangle : \mathbb{B}_T^P(r) = \mathbb{B}_T^P(e)\}$ is a fuzzy subgroup (FSG) of G .*

Theorem 3.6. *Let M be a BFMT of a BFSG \mathbb{B} of G . Then $H = \{\langle r, \mathbb{B}_T^N(r) \rangle : \mathbb{B}_T^N(r) = \mathbb{B}_T^N(e)\}$ is an anti-fuzzy subgroup (AFSG) of G .*

Theorem 3.7. *Let G and G^1 be any two groups. Then the homomorphic image of a BFMT M of a BFSG \mathbb{B} of G is a BFSG of G^1 .*

Proof. Let $\varkappa : G \rightarrow G^1$ be a homomorphism.

Let $\mathbb{V} = \varkappa(M)$, where M is a BFMT of a BFSG \mathbb{B} of G .

We shall show that \mathbb{V} is a BFSG of G^1 .

Now, for $\varkappa(r)$ and $\varkappa(y)$ in G^1 , we have

$$\begin{aligned} \mathbb{V}^P(\varkappa(r)\varkappa(y)^{-1}) &= \mathbb{V}^P(\varkappa(r)\varkappa(y^{-1})) \\ &= \mathbb{V}^P(\varkappa(ry^{-1})) \\ &\geq \mathbb{B}_T^P(ry^{-1}) \\ &= \beta \mathbb{B}^P(ry^{-1}) + \alpha \\ &\geq \beta \min\{\mathbb{B}^P(r), \mathbb{B}^P(y^{-1})\} + \alpha \\ &= \beta \min\{\mathbb{B}^P(r), \mathbb{B}^P(y)\} + \alpha \\ &= \min\{(\beta \mathbb{B}^P(r) + \alpha), (\beta \mathbb{B}^P(y) + \alpha)\} \\ &\geq \min\{\mathbb{V}^P(\varkappa(r)), \mathbb{V}^P(\varkappa(y))\}. \end{aligned}$$

Thus $\mathbb{V}^P(\varkappa(r)\varkappa(y)^{-1}) \geq \min\{\mathbb{V}^P(\varkappa(r)), \mathbb{V}^P(\varkappa(y))\}$.

Now,

$$\begin{aligned}
 \mathbb{V}^N(\varkappa(r)\varkappa(y)^{-1}) &= \mathbb{V}^N(\varkappa(r)\varkappa(y^{-1})) \\
 &= \mathbb{V}^N(\varkappa(ry^{-1})) \\
 &\geq \mathbb{B}_T^N(ry^{-1}) \\
 &= \beta\mathbb{B}^N(ry^{-1}) + \alpha \\
 &\leq \beta \max\{\mathbb{B}^N(r), \mathbb{B}^N(y^{-1})\} + \alpha \\
 &= \beta \max\{\mathbb{B}^N(r), \mathbb{B}^N(y)\} + \alpha \\
 &= \max\{(\beta\mathbb{B}^N(r) + \alpha), (\beta\mathbb{B}^N(y) + \alpha)\} \\
 &\leq \min\{\mathbb{V}^N(\varkappa(r)), \mathbb{V}^N(\varkappa(y))\}.
 \end{aligned}$$

Thus $\mathbb{V}^N(\varkappa(r)\varkappa(y)^{-1}) \leq \max\{\mathbb{V}^N(\varkappa(r)), \mathbb{V}^N(\varkappa(y))\}$.

Hence, \mathbb{V} is a BFSG of G^1 .

□

Theorem 3.8. *Let G and G^1 be any two groups. Then the homomorphic pre-image of a BFMT of a BFSG \mathbb{B} of G^1 is a BFSG of G .*

Proof. Let G and G^1 be any two groups. Let $M = \varkappa(B)$, where M is a BFMT of a BFSG \mathbb{B} of G^1 .

We shall show that \mathbb{B} is a BFSG of G .

Now, for $\varkappa(r)$ and $\varkappa(y)$ in G , we have

$$\begin{aligned}
 \mathbb{B}^P(ry^{-1}) &= B_T^P(\varkappa(ry^{-1})) \\
 &= B_T^P(\varkappa(r)\varkappa(y^{-1})) \\
 &= B_T^P(\varkappa(r)(\varkappa(y))^{-1}) \\
 &= \beta\mathbb{B}^P(\varkappa(r)(\varkappa(y))^{-1}) + \alpha \\
 &\geq \beta \min\{\mathbb{B}^P(\varkappa(r)), \mathbb{B}^P(\varkappa(y))\} + \alpha \\
 &= \min\{\beta\mathbb{B}^P(\varkappa(r)) + \alpha, \beta\mathbb{B}^P(\varkappa(y)) + \alpha\} \\
 &= \min\{B_T^P(\varkappa(r)), B_T^P(\varkappa(y))\} \\
 &= \min\{\mathbb{B}^P(\varkappa(r)), \mathbb{B}^P(\varkappa(y))\}
 \end{aligned}$$

Thus $\mathbb{B}^P(ry^{-1}) \geq \min\{\mathbb{B}^P(\varkappa(r)), \mathbb{B}^P(\varkappa(y))\}$.

Now,

$$\begin{aligned}
 \mathbb{B}^N(ry^{-1}) &= B_T^N(\varkappa(ry^{-1})) \\
 &= B_T^N(\varkappa(r)\varkappa(y^{-1})) \\
 &= B_T^N(\varkappa(r)(\varkappa(y))^{-1}) \\
 &= \beta\mathbb{B}^N(\varkappa(r)(\varkappa(y))^{-1}) + \alpha \\
 &\leq \beta\max\{\mathbb{B}^N(\varkappa(r)), \mathbb{B}^N(\varkappa(y))\} + \alpha \\
 &= \max\{\beta\mathbb{B}^N(\varkappa(r)) + \alpha, \beta\mathbb{B}^N(\varkappa(y)) + \alpha\} \\
 &= \max\{B_T^N(\varkappa(r)), B_T^N(\varkappa(y))\} \\
 &= \max\{\mathbb{B}^N(\varkappa(r)), \mathbb{B}^N(\varkappa(y))\}
 \end{aligned}$$

Thus $\mathbb{B}^N(ry^{-1}) \leq \max\{\mathbb{B}^N(\varkappa(r)), \mathbb{B}^N(\varkappa(y))\}$.

Thus \mathbb{B} is a BFSG of G . Thus the homomorphic pre-image of a BFMT of a BFSG \mathbb{B} of G^1 is a BFSG of G . \square

The proof of the following two theorems is similar to the proof of Theorem 3.7, 3.8.

Theorem 3.9. *Let G and G^1 be any two groups. Then the homomorphic image of a BFMT M of a BFNSG \mathbb{B} of G is a BFNSG of G^1 .*

Theorem 3.10. *Let G and G^1 be any two groups. Then the homomorphic pre-image of a BFMT of a BFNSG \mathbb{B} of G^1 is a BFNSG of G .*

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