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Pairwise Semiregular Properties on Generalized Pairwise Lindelöf Spaces

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Abstract. Let (X, τ_1, τ_2) be a bitopological space and $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ its pairwise semiregularization. Then a bitopological property $\mathcal{P}$ is called pairwise semiregular provided that (X, τ_1, τ_2) has the property $\mathcal{P}$ if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ has the same property. In this work we study pairwise semiregular property of (i, j)-nearly Lindelöf, pairwise nearly Lindelöf, (i, j)-almost Lindelöf, pairwise almost Lindelöf, (i, j)-weakly Lindelöf and pairwise weakly Lindelöf spaces. We prove that (i, j)-almost Lindelöf, pairwise almost Lindelöf, (i, j)-weakly Lindelöf and pairwise weakly Lindelöf are pairwise semiregular properties, on the contrary of each type of pairwise Lindelöf space which are not pairwise semiregular properties.

1. Introduction

Semiregular properties in topological spaces have been studied by many topologist. Some of them related to this research studied by Mrsevic et al. [14, 15] and Fawakhreh and Kılıçman [3]. But in bitopological space, the study of this topic is still open for investigation. The purpose of this paper is to study pairwise semiregular properties on generalized pairwise Lindelöf spaces, that we have studied in [9, 13, 16, 17], namely, (i, j)-nearly Lindelöf, pairwise nearly Lindelöf, (i, j)-almost Lindelöf, pairwise almost Lindelöf, (i, j)-weakly Lindelöf and pairwise weakly Lindelöf spaces.

In 2010, Salleh and Kılıçman [19] studied the pairwise semiregular properties of (i, j)-almost regular-Lindelöf, pairwise almost regular-Lindelöf, (i, j)-weakly regular-Lindelöf and pairwise weakly regular-Lindelöf spaces. They also show that the (i, j)-nearly regular-Lindelöf and pairwise nearly-regular-Lindelöf spaces are pairwise semiregular invariant properties.

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The main results is that the Lindelöf, B-Lindelöf, s-Lindelöf and p-Lindelöf spaces are not pairwise semiregular properties. While (i,j)-almost Lindelöf, pairwise almost Lindelöf, (i,j)-weakly Lindelöf and pairwise weakly Lindelöf spaces are pairwise semiregular properties. We also show that (i,j)-nearly Lindelöf and pairwise nearly Lindelöf spaces are satisfying pairwise semiregular invariant properties.

2. Preliminaries

Throughout this paper, all spaces (X,τ) and (X,τ_1,τ_2) (or simply X) are always mean topological spaces and bitopological spaces, respectively unless explicitly stated. If $\mathcal{P}$ is a topological property, then (τ_i,τ_j)-$\mathcal{P}$ denotes an analogue of this property for τ_i has property $\mathcal{P}$ with respect to τ_j, and p-$\mathcal{P}$ denotes the conjunction (τ_1,τ_2)-$\mathcal{P}\wedge(\tau_2,\tau_1)$-$\mathcal{P}$, i.e., p-$\mathcal{P}$ denotes an absolute bitopological analogue of $\mathcal{P}$.

Also note that (X,τ_i) has a property $\mathcal{P} \iff (X,\tau_1,\tau_2)$ has a property τ_i-$\mathcal{P}$. Sometimes the prefixes (τ_i,τ_j)- or τ_i- will be replaced by (i,j)- or i- respectively, if there is no chance for confusion. By i-open cover of X, we mean that the cover of X by i-open sets in X; similar for the (i,j)-regular open cover of X etc. By i-int(A) and i-cl(A), we shall mean the interior and the closure of a subset A of X with respect to topology τ_i, respectively. In this paper always $i,j\in\{1,2\}$ and $i\neq j$. The reader may consult [2] for the detail notations.

The following are some basic concepts.

Definition 2.1. *[6,20] A subset S of a bitopological space (X,τ_1,τ_2) is said to be (i,j)-regular open (resp. (i,j)-regular closed) if i-int$(j$-cl$(S))=S$ (resp. i-cl$(j$-int$(S))=S$), where $i,j\in\{1,2\}, i\neq j$. S is called pairwise regular open (resp. pairwise regular closed) if it is both $(1,2)$-regular open and $(2,1)$-regular open (resp. $(1,2)$-regular closed and $(2,1)$-regular closed).*

Definition 2.2. *[6,21] A bitopological space (X,τ_1,τ_2) is said to be (i,j)-almost regular if for each point $x\in X$ and for each (i,j)-regular open set V containing x, there exists an (i,j)-regular open set U such that $x\in U\subseteq j$-cl$(U)\subseteq V$. X is called pairwise almost regular if it is both $(1,2)$-almost regular and $(2,1)$-almost regular.*

In any bitopological space (X,τ_1,τ_2), the family of all (i,j)-regular open sets is closed under finite intersections. Thus the family of (i,j)-regular open sets in any bitopological space (X,τ_1,τ_2) forms a base for a coarser topology called (i,j)-semiregularization of (X,τ_1,τ_2), which is defined as follows.

Definition 2.3. *[16] The topology generated by the (i,j)-regular open subsets of (X,τ_1,τ_2) is denoted by $\tau^s_{(i,j)}$ and it is called (i,j)-semiregularization of X. The topologies is pairwise semiregularization of X if the first topology is $(1,2)$-semiregularization of X and the second topology is $(2,1)$-semiregularization of X. If $\tau_i\equiv\tau^s_{(i,j)}$, then X is said to be (i,j)-semiregular. (X,τ_1,τ_2) is called pairwise semiregular if it is both $(1,2)$-semiregular and $(2,1)$-semiregular, that is, whenever*

$\tau_i \equiv \tau^s_{(i,j)}$ for each $i,j \in \{1,2\}$ and $i \neq j$. In other words, (X, τ_1, τ_2) is (i,j)-semiregular if the family of (i,j)-regular open sets form a base for the topology τ_i.

It is very clear that $\tau^s_{(i,j)} \subseteq \tau_i$, but it is not necessary $\tau_i \subseteq \tau^s_{(i,j)}$. Thus with every given bitopological space (X, τ_1, τ_2) there is associated another bitopological space $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ in the manner described above (see [20]). We provide the following example in order to understand the concept of pairwise semiregular spaces clearly.

Example 2.1. *For the set of all real numbers $\mathbb{R}$, let τ_u denotes the usual topology and τ_s denote the Sorgenfrey topology, i.e., topology generated by right half-open intervals (see [22]). Then $(\mathbb{R}, \tau_u, \tau_s)$ is (τ_u, τ_s)-semiregular since $\tau_u = \tau^s_{(\tau_u,\tau_s)}$, i.e., τ_u generated by (τ_u, τ_s)-regular open subsets of $\mathbb{R}$. $(\mathbb{R}, \tau_u, \tau_s)$ is also (τ_s, τ_u)-semiregular since $\tau_s = \tau^s_{(\tau_s,\tau_u)}$ because any set $E \in \tau_s$ is the union of a collection of (τ_s, τ_u)-regular open sets in $\mathbb{R}$. Thus $(\mathbb{R}, \tau_u, \tau_s)$ is pairwise semiregular.*

Khedr and Alshibani [6] defined the equivalent definition of (i,j)-semiregular spaces as follows.

Definition 2.4. *A bitopological space X is said to be (i,j)-semiregular if for each $x \in X$ and for each i-open subset V of X containing x, there is an i-open set U such that $x \in U \subseteq i\text{-int}\,(j\text{-cl}\,(U)) \subseteq V$. X is called pairwise semiregular if it is both $(1,2)$-semiregular and $(2,1)$-semiregular.*

Definition 2.5. *[1] A bitopological space (X, τ_1, τ_2) is said to be (i,j)-extremally disconnected if the i-closure of every j-open set is j-open. X is called pairwise extremally disconnected if it is both $(1,2)$-extremally disconnected and $(2,1)$-extremally disconnected.*

Recall that a property $\mathcal{P}$ will be called bitopological property (resp. p-topological property) if whenever (X, τ_1, τ_2) has property $\mathcal{P}$, then every space homeomorphic (resp. p-homeomorphic) to (X, τ_1, τ_2) also has property $\mathcal{P}$ (see [8]). If a bitopological space X has bitopological (or p-topological) property $\mathcal{P}$, one may ask, does the pairwise semiregularization of X satisfies the property $\mathcal{P}$ also? Now we arrive to the concept of pairwise semiregular property.

Definition 2.6. *Let (X, τ_1, τ_2) be a bitopological space and let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ its pairwise semiregularization. A bitopological property $\mathcal{P}$ is called pairwise semiregular provided that (X, τ_1, τ_2) has the property $\mathcal{P}$ if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ has the property $\mathcal{P}$.*

Lemma 2.1. *[16] Let (X, τ_1, τ_2) be a bitopological space and let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ its pairwise semiregularization. Then*

(a) τ_i-int $(C) = \tau^s_{(i,j)}$-int (C) for every τ_j-closed set C;
(b) τ_i-cl $(A) = \tau^s_{(i,j)}$-cl (A) for every $A \in \tau_j$;
(c) the family of (τ_i, τ_j)-regular open sets of (X, τ_1, τ_2) are the same as the family of $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-regular open sets of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$;

(d) the family of (τ_i, τ_j)-regular closed sets of (X, τ_1, τ_2) are the same as the family of $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-regular closed sets of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$;

(e) $\left(\tau^s_{(i,j)}\right)^s_{(i,j)} = \tau^s_{(i,j)}$.

3. Pairwise Semiregularization of Pairwise Lindelöf Spaces

Definition 3.1. *[5, 7]. A bitopological space (X, τ_1, τ_2) is said to be i-Lindelöf if the topological space (X, τ_i) is Lindelöf. X is called Lindelöf if it is i-Lindelöf for each $i = 1, 2$. In other words, (X, τ_1, τ_2) is called Lindelöf if the topological space (X, τ_1) and (X, τ_2) are both Lindelöf.*

Note that i-Lindelöf property as well as Lindelöf property is not a pairwise semiregular property by the following example.

Example 3.1. *Let X be a set with cardinality 2^c, where $c = \text{card}(\mathbb{R})$. Let τ_1 be a co-c topology on X consisting of $\emptyset$ and all subsets of X whose complements have cardinality at most c and let τ_2 be a cofinite topology on X. Then (X, τ_1, τ_2) is τ_2-Lindelöf but not τ_1-Lindelöf and hence not Lindelöf. Observe that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(1,2)}$-Lindelöf and $\tau^s_{(2,1)}$-Lindelöf since $\tau^s_{(1,2)}$ and $\tau^s_{(2,1)}$ are indiscrete topologies. Hence $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf.*

Definition 3.2. *A bitopological space (X, τ_1, τ_2) is called (i, j)-Lindelöf [5, 7] if for every i-open cover of X there is a countable j-open subcover. X is called B-Lindelöf [5] or p_1-Lindelöf [7] if it is both $(1, 2)$-Lindelöf and $(2, 1)$-Lindelöf.*

An (i, j)-Lindelöf property as well as B-Lindelöf property is not pairwise semiregular property by the following example.

Example 3.2. *Let (X, τ_1, τ_2) be a bitopological space as in Example 3.1. Then (X, τ_1, τ_2) is not (τ_1, τ_2)-Lindelöf but it is (τ_2, τ_1)-Lindelöf and hence not B-Lindelöf. Observe that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$-Lindelöf and $\left(\tau^s_{(2,1)}, \tau^s_{(1,2)}\right)$-Lindelöf since $\tau^s_{(1,2)}$ and $\tau^s_{(2,1)}$ are indiscrete topologies. Hence $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is B-Lindelöf.*

Definition 3.3. *A cover U of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-open [23] if $U \subseteq \tau_1 \cup \tau_2$. If, in addition, U contains at least one nonempty member of τ_1 and at least one nonempty member of τ_2, it is called p-open [4].*

Definition 3.4. *[5] A bitopological space (X, τ_1, τ_2) is called s-Lindelöf (resp. p-Lindelöf) if every $\tau_1\tau_2$-open (resp. p-open) cover of X has a countable subcover.*

A p-Lindelöf property is not pairwise semiregular property by the following example. Thus the s-Lindelöf property is also not pairwise semiregular property.

Example 3.3. *Let (X, τ_1, τ_2) be a bitopological space as in Example 3.1. Then (X, τ_1, τ_2) is not p-Lindelöf and hence not s-Lindelöf. Observe that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is p-Lindelöf and s-Lindelöf since $\tau^s_{(1,2)}$ and $\tau^s_{(2,1)}$ are indiscrete topologies.*

4. Pairwise Semiregularization of Generalized Pairwise Lindelöf Spaces

Definition 4.1. *[9, 13, 16] A bitopological space X is said to be (i,j)-nearly Lindelöf (resp. (i,j)-almost Lindelöf, (i,j)-weakly Lindelöf) if for every i-open cover $\{U_\alpha : \alpha \in \Delta\}$ of X, there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = \bigcup_{n\in\mathbb{N}} i\text{-int}\left(j\text{-}\mathrm{cl}\left(U_{\alpha_n}\right)\right)$ $\left(\text{resp. } X = \bigcup_{n\in\mathbb{N}} j\text{-}\mathrm{cl}\left(U_{\alpha_n}\right),\ X = j\text{-}\mathrm{cl}\left(\bigcup_{n\in\mathbb{N}} \left(U_{\alpha_n}\right)\right)\right)$. X is called pairwise nearly Lindelöf (resp. pairwise almost Lindelöf, pairwise weakly Lindelöf) if it is both $(1,2)$-nearly Lindelöf (resp. $(1,2)$-almost Lindelöf, $(1,2)$-weakly Lindelöf) and $(2,1)$-nearly Lindelöf (resp. $(2,1)$-almost Lindelöf, $(2,1)$-weakly Lindelöf).*

Our first result is analogue with the result of Mršević et al. [15, Theorem 1].

Theorem 4.1. *A bitopological space (X, τ_1, τ_2) is (τ_i, τ_j)-nearly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.*

Proof. Let (X, τ_1, τ_2) be a (τ_i, τ_j)-nearly Lindelöf and let $\{U_\alpha : \alpha \in \Delta\}$ be a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. For each $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in U_{\alpha_x}$ and since for each $\alpha_x \in \Delta$, $U_{\alpha_x} \in \tau^s_{(i,j)}$, there exists a (τ_i, τ_j)-regular open set V_{α_x} in (X, τ_1, τ_2) such that $x \in V_{\alpha_x} \subseteq U_{\alpha_x}$. So $X = \bigcup_{x\in X} V_{\alpha_x}$ and hence $\{V_{\alpha_x} : x \in X\}$ is a (τ_i, τ_j)-regular open cover of X. Since (X, τ_1, τ_2) is (τ_i, τ_j)-nearly Lindelöf, there exists a countable subset of points $x_1, \ldots, x_n, \ldots$ of X such that $X = \bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}} \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. This shows that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.

Conversely, suppose that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf and let $\{V_\alpha : \alpha \in \Delta\}$ be a (τ_i, τ_j)-regular open cover of (X, τ_1, τ_2). Since $V_\alpha \in \tau^s_{(i,j)}$ for each $\alpha \in \Delta$, $\{V_\alpha : \alpha \in \Delta\}$ is a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf, there exists a countable subcover such that $X = \bigcup_{n\in\mathbb{N}} V_{\alpha_n}$. This implies that (X, τ_1, τ_2) is (τ_i, τ_j)-nearly Lindelöf. □

Corollary 4.1. *A bitopological space (X, τ_1, τ_2) is pairwise nearly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf.*

Proposition 4.1. *A bitopological space $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-nearly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.*

Proof. The sufficient condition is obvious by the definitions. So we need only to prove necessary condition. Suppose that $\{U_\alpha : \alpha \in \Delta\}$ is a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. For each $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in U_{\alpha_x}$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-semiregular, there exists a

$\tau^s_{(i,j)}$-open set V_{α_x} in $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ such that $x \in V_{\alpha_x} \subseteq \tau^s_{(i,j)}\text{-int}\left(\tau^s_{(j,i)}\text{-}\operatorname{cl}\left(V_{\alpha_x}\right)\right) \subseteq U_{\alpha_x}$. Hence $X = \bigcup_{x\in X} V_{\alpha_x}$ and thus the family $\{V_{\alpha_x} : x \in X\}$ forms a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-nearly Lindelöf, there exists a countable subset of points $x_1, \dots, x_n, \dots$ of X such that $X = \bigcup_{n\in\mathbb{N}} \tau^s_{(i,j)}\text{-int}\left(\tau^s_{(j,i)}\text{-}\operatorname{cl}\left(V_{\alpha_{x_n}}\right)\right) \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. This shows that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf. □

Corollary 4.2. *A bitopological space $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is pairwise nearly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf.*

From the Definition 2.6, if the property $\mathcal{P}$ is not bitopological property but it satisfies the condition (X, τ_1, τ_2) has the property $\mathcal{P}$ if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ has the property $\mathcal{P}$, then the property $\mathcal{P}$ will be called pairwise semiregular invariant property. The following theorem prove that (i, j)-nearly Lindelöf as well as pairwise nearly Lindelöf property satisfying the pairwise semiregular invariant property since (i, j)-nearly Lindelöf and pairwise nearly Lindelöf are not i-topological property [8] and bitopological property, respectively. This is because the i-continuity and (i, j)-δ-continuity (resp. continuity and p-δ-continuity) are independent notions (see [12]).

Theorem 4.2. *A bitopological space (X, τ_1, τ_2) is (τ_i, τ_j)-nearly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-nearly Lindelöf.*

Proof. It is obvious by Theorem 4.1 and Proposition 4.1. □

Corollary 4.3. *A bitopological space (X, τ_1, τ_2) is pairwise nearly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is pairwise nearly Lindelöf.*

Theorem 4.3. *[20] If (X, τ_1, τ_2) is pairwise semiregular, then $(X, \tau_1, \tau_2) = \left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$.*

The converse of Theorem 4.3 is also true by the definitions.

Proposition 4.2. *Let (X, τ_1, τ_2) be a pairwise semiregular space. Then (X, τ_1, τ_2) is (i, j)-nearly Lindelöf if and only if it is i-Lindelöf.*

Proof. By Theorem 4.3, $(X, \tau_1, \tau_2) = \left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. The result follows immediately by Proposition 4.1. □

Corollary 4.4. *Let (X, τ_1, τ_2) be a pairwise semiregular space. Then (X, τ_1, τ_2) is pairwise nearly Lindelöf if and only if it is Lindelöf.*

Unlike all types of pairwise Lindelöf properties, the (i, j)-almost Lindelöf, pairwise almost Lindelöf, (i, j)-weakly Lindelöf and pairwise weakly Lindelöf properties are pairwise semiregular properties as we prove in the following theorems.

Theorem 4.4. *A bitopological space (X, τ_1, τ_2) is (τ_i, τ_j)-almost Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-almost Lindelöf.*

Proof. Let (X, τ_1, τ_2) be a (τ_i, τ_j)-almost Lindelöf and let $\{U_\alpha : \alpha \in \Delta\}$ be a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Since $\tau^s_{(i,j)} \subseteq \tau_i$, $\{U_\alpha : \alpha \in \Delta\}$ is a τ_i-open cover of the (τ_i, τ_j)-almost Lindelöf space (X, τ_1, τ_2). Then there is a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = \bigcup_{n\in\mathbb{N}} \tau_j$-$\mathrm{cl}\,(U_{\alpha_n})$. By Lemma 2.1, we have $X = \bigcup_{n\in\mathbb{N}} \tau^s_{(j,i)}$-$\mathrm{cl}\,(U_{\alpha_n})$, which implies $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-almost Lindelöf.

Conversely suppose that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-almost Lindelöf and let $\{V_\alpha : \alpha \in \Delta\}$ be a τ_i-open cover of (X, τ_1, τ_2). Since $V_\alpha \subseteq \tau_i$-$\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(V_\alpha))$ and τ_i-$\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(V_\alpha)) \in \tau^s_{(i,j)}$, we have $\{\tau_i\text{-}\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(V_\alpha)) : \alpha \in \Delta\}$ is a $\tau^s_{(i,j)}$-open cover of the $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-almost Lindelöf space $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. So there is a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = \bigcup_{n\in\mathbb{N}} \tau^s_{(j,i)}$-$\mathrm{cl}\,(\tau_i\text{-}\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(V_{\alpha_n})))$. By Lemma 2.1, we have $X = \bigcup_{n\in\mathbb{N}} \tau_j$-$\mathrm{cl}\,(\tau_i\text{-}\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(V_{\alpha_n}))) \subseteq \bigcup_{n\in\mathbb{N}} \tau_j$-$\mathrm{cl}\,(V_{\alpha_n})$. This implies that (X, τ_1, τ_2) is (τ_i, τ_j)-almost Lindelöf. □

Corollary 4.5. *A bitopological space (X, τ_1, τ_2) is pairwise almost Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is pairwise almost Lindelöf.*

Note that, the (i, j)-almost Lindelöf property and the pairwise almost Lindelöf property are both bitopological properties (see [18]). Utilizing this fact, Theorem 4.4 and Corollary 4.5, we easily obtain the following corollary.

Corollary 4.6. *The (i, j)-almost Lindelöf property and the pairwise almost Lindelöf property are both pairwise semiregular properties.*

Proposition 4.3. *Let (X, τ_1, τ_2) be a (τ_i, τ_j)-almost regular space. Then (X, τ_1, τ_2) is (τ_i, τ_j)-almost Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.*

Proof. Let (X, τ_1, τ_2) be a (τ_i, τ_j)-almost Lindelöf and let $\{U_\alpha : \alpha \in \Delta\}$ be a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. For each $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in U_{\alpha_x}$ and since $U_{\alpha_x} \in \tau^s_{(i,j)}$, there exists a (τ_i, τ_j)-regular open set V_{α_x} in (X, τ_1, τ_2) such that $x \in V_{\alpha_x} \subseteq U_{\alpha_x}$. Since (X, τ_1, τ_2) is (τ_i, τ_j)-almost regular, there is a (τ_i, τ_j)-regular open set C_{α_x} in (X, τ_1, τ_2) such that $x \in C_{\alpha_x} \subseteq \tau_j$-$\mathrm{cl}\,(C_{\alpha_x}) \subseteq V_{\alpha_x}$. Hence $X = \bigcup_{x\in X} C_{\alpha_x}$ and thus the family $\{C_{\alpha_x} : x \in X\}$ forms a (τ_i, τ_j)-regular open cover of (X, τ_1, τ_2). Since (X, τ_1, τ_2) is (τ_i, τ_j)-almost Lindelöf, there exists a countable subset of points $x_1, \ldots, x_n, \ldots$ of X such that $X = \bigcup_{n\in\mathbb{N}} \tau_j$-$\mathrm{cl}\,(C_{\alpha_{x_n}}) \subseteq \bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}} \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. This shows that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf. Conversely, let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ be a $\tau^s_{(i,j)}$-Lindelöf and let $\{U_\alpha : \alpha \in \Delta\}$ be a τ_i-open cover of (X, τ_1, τ_2). Since $U_\alpha \subseteq \tau_i$-$\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(U_\alpha))$ and τ_i-$\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(U_\alpha)) \in \tau^s_{(i,j)}$, $\{\tau_i\text{-}\mathrm{int}\,(\tau_j\text{-}\mathrm{cl}\,(U_\alpha)) : \alpha \in \Delta\}$ is $\tau^s_{(i,j)}$-open cover of the $\tau^s_{(i,j)}$-Lindelöf space

$\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Then there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = \bigcup_{n\in\mathbb{N}} \tau_i$-int $(\tau_j\text{- cl}(U_{\alpha_n})) \subseteq \bigcup_{n\in\mathbb{N}} \tau_j\text{-cl}(U_{\alpha_n})$. This implies that (X, τ_1, τ_2) is (τ_i, τ_j)-almost Lindelöf. □

Corollary 4.7. *Let (X, τ_1, τ_2) be a pairwise almost regular space. Then (X, τ_1, τ_2) is pairwise almost Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf.*

Proposition 4.4. *Let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ be a $\left(\tau^s_{(j,i)}, \tau^s_{(i,j)}\right)$-extremally disconnected space. Then $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-almost Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.*

Proof. The sufficient condition is obvious by the definitions. So we need only to prove necessary condition. Suppose that $\{U_\alpha : \alpha \in \Delta\}$ is a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. For each $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in U_{\alpha_x}$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-semiregular, there exists a $\tau^s_{(i,j)}$-open set V_{α_x} in $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ such that $x \in V_{\alpha_x} \subseteq \tau^s_{(i,j)}$-int $\left(\tau^s_{(j,i)}\text{- cl}(V_{\alpha_x})\right) \subseteq U_{\alpha_x}$. Hence $X = \bigcup_{x\in X} V_{\alpha_x}$ and thus the family $\{V_{\alpha_x} : x \in X\}$ forms a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-almost Lindelöf and $\left(\tau^s_{(j,i)}, \tau^s_{(i,j)}\right)$-extremally disconnected, there exists a countable subset of points $x_1, \ldots, x_n, \ldots$ of X such that $X = \bigcup_{n\in\mathbb{N}} \tau^s_{(j,i)}\text{-cl}(V_{\alpha_{x_n}}) = \bigcup_{n\in\mathbb{N}} \tau^s_{(i,j)}\text{-int}\left(\tau^s_{(j,i)}\text{- cl}(V_{\alpha_{x_n}})\right) \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. This shows that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf. □

Corollary 4.8. *Let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ be a pairwise extremally disconnected space. Then $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is pairwise almost Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf*

Proposition 4.5. *Let (X, τ_1, τ_2) be a pairwise semiregular and (j, i)-extremally disconnected space. Then (X, τ_1, τ_2) is (i, j)-almost Lindelöf if and only if it is i-Lindelöf.*

Proof. By Theorem 4.3, $(X, \tau_1, \tau_2) = \left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. The result follows immediately by Proposition 4.4. □

Corollary 4.9. *Let (X, τ_1, τ_2) be a pairwise semiregular and pairwise extremally disconnected space. Then (X, τ_1, τ_2) is pairwise almost Lindelöf if and only if it is Lindelöf.*

Theorem 4.5. *A bitopological space (X, τ_1, τ_2) is (τ_i, τ_j)-weakly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-weakly Lindelöf.*

Proof. The proof is similar to the proof of Theorem 4.4 by using the fact that

$$\begin{aligned}
\tau^s_{(j,i)}\text{- cl}\left(\bigcup_{n\in\mathbb{N}} \tau_i\text{- int}\left(\tau_j\text{- cl}(V_{\alpha_n})\right)\right) &= \tau_j\text{- cl}\left(\bigcup_{n\in\mathbb{N}} \tau_i\text{- int}\left(\tau_j\text{- cl}(V_{\alpha_n})\right)\right) \\
&\subseteq \tau_j\text{- cl}\left(\bigcup_{n\in\mathbb{N}} \tau_j\text{- cl}(V_{\alpha_n})\right) \\
&\subseteq \tau_j\text{- cl}\left(\bigcup_{n\in\mathbb{N}} V_{\alpha_n}\right).
\end{aligned}$$

Thus we choose to omit the details. □

Corollary 4.10. *A bitopological space (X, τ_1, τ_2) is pairwise weakly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is pairwise weakly Lindelöf.*

Note that, the (i,j)-weakly Lindelöf property and the pairwise weakly Lindelöf property are both bitopological properties (see [18]). Utilizing this fact, Theorem 4.5 and Corollary 4.10, we easily obtain the following corollary.

Corollary 4.11. *The (i,j)-weakly Lindelöf property and the pairwise weakly Lindelöf property are both pairwise semiregular properties.*

Recall that, a bitopological space X is called (i,j)-weak P-space [13] if for each countable family $\{U_n : n \in \mathbb{N}\}$ of i-open sets in X, we have $j\text{-cl}\left(\bigcup_{n\in\mathbb{N}} U_n\right) = \bigcup_{n\in\mathbb{N}} j\text{-cl}\,(U_n)$. X is called pairwise weak P-space if it is both $(1,2)$-weak P-space and $(2,1)$-weak P-space.

Proposition 4.6. *Let (X, τ_1, τ_2) be a (τ_i, τ_j)-almost regular and (τ_i, τ_j)-weak P-space. Then (X, τ_1, τ_2) is (τ_i, τ_j)-weakly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.*

Proof. Necessity: Let $\{U_\alpha : \alpha \in \Delta\}$ be a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. For each $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in U_{\alpha_x}$ and since $U_{\alpha_x} \in \tau^s_{(i,j)}$ for each $\alpha_x \in \Delta$, there exists a (τ_i, τ_j)-regular open set V_{α_x} in (X, τ_1, τ_2) such that $x \in V_{\alpha_x} \subseteq U_{\alpha_x}$. Since (X, τ_1, τ_2) is (τ_i, τ_j)-almost regular, there is a (τ_i, τ_j)-regular open set C_{α_x} in (X, τ_1, τ_2) such that $x \in C_{\alpha_x} \subseteq \tau_j\text{-cl}\,(C_{\alpha_x}) \subseteq V_{\alpha_x}$. Hence $X = \bigcup_{x\in X} C_{\alpha_x}$ and thus the family $\{C_{\alpha_x} : x \in X\}$ forms a (τ_i, τ_j)-regular open cover of (X, τ_1, τ_2). Since (X, τ_1, τ_2) is (τ_i, τ_j)-weakly Lindelöf and (τ_i, τ_j)-weak P-space, there exists a countable subset of points $x_1, \dots, x_n, \dots$ of X such that $X = \tau_j\text{-cl}\left(\bigcup_{n\in\mathbb{N}} C_{\alpha_{x_n}}\right) = \bigcup_{n\in\mathbb{N}} \tau_j\text{-cl}\left(C_{\alpha_{x_n}}\right) \subseteq \bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}} \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. This shows that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.

Sufficiency: Let $\{U_\alpha : \alpha \in \Delta\}$ be a τ_i-open cover of (X, τ_1, τ_2). Since $U_\alpha \subseteq \tau_i\text{-int}\left(\tau_j\text{-cl}\,(U_\alpha)\right)$ and $\tau_i\text{-int}\left(\tau_j\text{-cl}\,(U_\alpha)\right) \in \tau^s_{(i,j)}$, $\left\{\tau_i\text{-int}\left(\tau_j\text{-cl}\,(U_\alpha)\right) : \alpha \in \Delta\right\}$ is $\tau^s_{(i,j)}$-open cover of the $\tau^s_{(i,j)}$-Lindelöf space $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Then there exists a countable subset $\{\alpha_n : n \in \mathbb{N}\}$ of Δ such that $X = \bigcup_{n\in\mathbb{N}} \tau_i\text{-int}\left(\tau_j\text{-cl}\,(U_{\alpha_n})\right) \subseteq \bigcup_{n\in\mathbb{N}} \tau_j\text{-cl}\,(U_{\alpha_n}) = \tau_j\text{-cl}\left(\bigcup_{n\in\mathbb{N}} U_{\alpha_n}\right)$. This implies that (X, τ_1, τ_2) is (τ_i, τ_j)-weakly Lindelöf. □

Corollary 4.12. *Let (X, τ_1, τ_2) be a pairwise almost regular and pairwise weak P-space. Then (X, τ_1, τ_2) is pairwise weakly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf.*

Proposition 4.7. *Let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ be a $\left(\tau^s_{(j,i)}, \tau^s_{(i,j)}\right)$-extremally disconnected and $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-weak P-space. Then $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-weakly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf.*

Proof. The sufficient condition is obvious by the definitions. So we need only to prove necessary condition. Suppose that $\{U_\alpha : \alpha \in \Delta\}$ is a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. For each $x \in X$, there exists $\alpha_x \in \Delta$ such that $x \in U_{\alpha_x}$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-semiregular, there exists a $\tau^s_{(i,j)}$-open set V_{α_x} in $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ such that $x \in V_{\alpha_x} \subseteq \tau^s_{(i,j)}\text{-int}\left(\tau^s_{(j,i)}\text{- cl}\left(V_{\alpha_x}\right)\right) \subseteq U_{\alpha_x}$. Hence $X = \bigcup_{x\in X} V_{\alpha_x}$ and thus the family $\{V_{\alpha_x} : x \in X\}$ forms a $\tau^s_{(i,j)}$-open cover of $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. Since $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-weakly Lindelöf, $\left(\tau^s_{(j,i)}, \tau^s_{(i,j)}\right)$-extremally disconnected and $\left(\tau^s_{(i,j)}, \tau^s_{(j,i)}\right)$-weak P-space, there exists a countable subset of points $x_1, \dots, x_n, \dots$ of X such that $X = \tau^s_{(j,i)}\text{-cl}\left(\bigcup_{n\in\mathbb{N}} V_{\alpha_{x_n}}\right) = \bigcup_{n\in\mathbb{N}} \tau^s_{(j,i)}\text{-cl}\left(V_{\alpha_{x_n}}\right) = \bigcup_{n\in\mathbb{N}} \tau^s_{(i,j)}\text{-int}\left(\tau^s_{(j,i)}\text{- cl}\left(V_{\alpha_{x_n}}\right)\right) \subseteq \bigcup_{n\in\mathbb{N}} U_{\alpha_{x_n}}$. This shows that $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is $\tau^s_{(i,j)}$-Lindelöf. □

Corollary 4.13. *Let $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ be a pairwise extremally disconnected and pairwise weak P-space. Then $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is pairwise weakly Lindelöf if and only if $\left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$ is Lindelöf.*

Proposition 4.8. *Let (X, τ_1, τ_2) be a pairwise semiregular, (j, i)-extremally disconnected and (i, j)-weak P-space. Then (X, τ_1, τ_2) is (i, j)-weakly Lindelöf if and only if it is i-Lindelöf.*

Proof. By Theorem 4.3, $(X, \tau_1, \tau_2) = \left(X, \tau^s_{(1,2)}, \tau^s_{(2,1)}\right)$. The result follows immediately by Proposition 4.7. □

Corollary 4.14. *Let (X, τ_1, τ_2) be a pairwise semiregular, pairwise extremally disconnected and pairwise weak P-space. Then (X, τ_1, τ_2) is pairwise weakly Lindelöf if and only if it is Lindelöf.*

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