Int. J. Anal. Appl. (2023), 21:29

International Journal of Analysis and Applications

Analytic Solution of Black-Scholes-Merton European Power Put Option Model on Dividend Yield with Modified-Log-Power Payoff Function

S.E. Fadugba[1,2,3], A.A. Adeniji[4], M.C. Kekana[4], J.T. Okunlola[5], O. Faweya[6]

[1] *Department of Mathematics, Ekiti State University, Ado Ekiti, 360001, Nigeria*

[2] *Department of Physical Sciences, Mathematics Programme, Landmark University, Omu-Aran, Nigeria*

[3] *Landmark University SDG 4 (Quality Education Research Group), Omu-Aran, Nigeria*

[4] *Department of Mathematics, Tshwane University of Technology, Pretoria, South Africa*

[5] *Department of Mathematical and Physical Sciences, Afe Babalola University, Ado Ekiti, Nigeria*

[6] *Department of Statistics, Ekiti State University, Ado Ekiti, 360001, Nigeria*

* *Corresponding author:* sunday.fadugba@eksu.edu.ng, fadugba.sunday@lmu.edu.ng

Abstract. This paper proposes a framework based on the celebrated transform of Mellin type (MT) for the analytic solution of the Black-Scholes-Merton European Power Put Option Model (BSMEPPOM) on Dividend Yield (DY) with Modified-Log-Power Payoff Function (MLPPF) under the geometric Brownian motion. The MT has the capability of tackling complex functions by means of its fundamental properties and it is closely related to other well-known transforms such as Laplace and Fourier types. The main goal of this paper is to use MT to obtain a valuation formula for the European Power Put Option (EPPO) which pays a DY with MLPPF. By means of MT and its inversion formula, the price of EPPO on DY was expressed in terms of integral equation. Moreover, the valuation formula of EPPO was obtained with the help of the convolution property of MT and final time condition. The MT was tested on an illustrative example in order to measure its performance, effectiveness and suitability. The MLPPF was compared with other existing payoff functions. Hence, the effect of DY on the pricing of EPPO with MLPPF was also investigated.

1. Introduction

The popularity of option pricing in financial mathematics has been displayed as one of the key major areas in derivative security. In other words, option valuation has contributed greatly to the financial

Received: Jan. 9, 2023.

2010 *Mathematics Subject Classification.* 91B24, 91G20.

Key words and phrases. convolution property; final time condition; integral representation; Mellin transform.

https://doi.org/10.28924/2291-8639-21-2023-29

ISSN: 2291-8639

markets. There is a massive growth in trading activities on derivatives globally from the inception of the Black-Scholes pricing formula [1,2]. It is noteworthy to say that the Black-Scholes model for linear payoff has been used by many researchers and as well as become one of the utmost areas in financial markets over the last few decades. Immediately after the huge success recorded by the Black-Scholes model for vanilla option flavours, several other valuation formula were developed for options pricing with different payoff functions such as Mellin transform, binomial model, finite difference method, Monte Carlo method, e.t.c; see [3] – [6]. For mathematical framework, some implementations of transform methods of different types in financial markets; see [7]– [15]. Ghevariya [16] solved the classical Black-Scholes European put option model for modified-log payoff function with the help of the MT. Fadugba *et al*. [17] obtained a direct solution of the Black-Scholes-Merton European put option model on dividend yield with modified-log payoff function via a framework based on MT. In this paper, an analytic solution of BSMEPPOM via the celebrated transform of Mellin type is proposed in the sense of DY and MLPPF. The remaining part of the paper is listed as follows; Section 2 captures the brief concepts of MT. The governing model for EPPO on a DY with MLPPF is presented in Section 3. Section 4 captures the solution of BSMEPPOM with DY and MLPPF. An illustrative example on the application of MT to EPPO is captured in Section 5. Section 6 is the concluding part of the paper.

2. Mellin Transform

This section captures some definitions of terms based on the framework of the Mellin transform and its inversion formula [18].

Definition 2.1. *Let $f(x)$ be a locally Lebesgue integrable function. The Mellin transform of $f(x)$ is defined as*

$$\mathcal{M}[f(x),\omega] := \tilde{f}(\omega) = \int_0^{\infty} f(x)x^{\omega-1}dx \tag{2.1}$$

The Mellin transform variable ω is a complex number, $\omega = \Re(.) + i\Im(.)$, where $\Re(.)$ is the real part, i is the imaginary unit and $\Im(.)$ is the imaginary part..

Definition 2.2. *If $f(x)$ is an integrable function with fundamental strips (a, b), then if c is such that $a < c < b$ and $\{\tilde{f}(\omega) : \omega = c + it\}$ is integrable, the inverse Mellin transform is defined as*

$$\mathcal{M}^{-1}[\tilde{f}(\omega)] = f(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \tilde{f}(\omega)x^{-\omega}d\omega \tag{2.2}$$

Remark 2.1. *It is clearly seen that the Mellin transform $\mathcal{M}[f(x),\omega]$ and the inverse Mellin transform $\mathcal{M}^{-1}[\tilde{f}(\omega)]$ are linear integral operators.*

Remark 2.2. *For more details on the condition that ensures the existence of MT; see [18, 19].*

Remark 2.3. *The fundamental operational properties of the Mellin transforms such as scaling, shifting, derivatives, integrals, convolution, multiplicative convolution and Parseval's formula are well detailed in [9, 12, 18, 20].*

3. The BSMEPPOM on a DY with MLPPF

The BSMEPPOM on a DY with MLPPF is given by

$$\begin{aligned}\frac{\partial P_\rho(S_t^\rho,t)}{\partial t}+\rho\left(r-q+\frac{(\rho-1)\sigma^2}{2}\right)S_t^\rho\frac{\partial P_\rho(S_t^\rho,t)}{\partial S_t^\rho}\\ +\frac{(\sigma S_t^\rho)^2}{2}\frac{\partial^2 P_\rho(S_t^\rho,t)}{\partial (S_t^\rho)^2}=rP_\rho(S_t^\rho,t)\end{aligned}\tag{3.1}$$

subject to the boundary conditions

$$\lim_{S_t^\rho\to\infty}P_\rho(S_t^\rho,t)=0 \qquad \text{on } [0,T)\tag{3.2}$$

$$\lim_{S_t^\rho\to 0}P_\rho(S_t^\rho,t)=\frac{K}{e^{r(T-t)}} \qquad \text{on } [0,T)\tag{3.3}$$

and MLPPF

$$P_\rho(S_T^\rho,T)=\left[S_T^\rho\ln\left(\frac{K}{S_T^\rho}\right)\right]^+ \qquad \text{on } [0,\infty)\tag{3.4}$$

where $P_\rho(S_t^\rho,t)$, ρ, t, T, S_t^ρ, K, σ, r and q are the price of EPPO, power of the option, current time, time to expiry, underlying asset price, strike price, volatility, risk-free interest rate and DY, respectively.

4. Solution of the Black-Scholes-Merton European Put Option Model with MLPPF

Ghevariya derived BSM formula on non-dividend yield for ML-payoff function [16]. In this section, analytic solution of BSMEPPOM with dividend yield for ML-power payoff function is obtained via the MT as follows.

Taking the MT of (3.1) and using its linearity, independence of time derivatives and shifting properties and rearranging terms, one obtains

$$\frac{\partial \tilde{P}_\rho(\omega,t)}{\partial t}=-\frac{\sigma^2\rho^2}{2}(\omega^2+\omega(1-b_1)-b_2)\tilde{P}_\rho(\omega,t)\tag{4.1}$$

where

$$\begin{aligned}b_1&=\frac{\rho-1}{\rho}+\frac{2(r-q)}{\rho\sigma^2},\\ b_2&=\frac{2r}{\rho^2\sigma^2}\end{aligned}\tag{4.2}$$

Solving (3.1), yields

$$\tilde{P}_\rho(\omega,t)=l(\omega)e^{-\frac{1}{2}\rho^2\sigma^2(\omega^2+\omega(1-b_1)-b_2)t}\tag{4.3}$$

But

$$l(\omega)=\mathcal{M}(P_\rho(S_T,T),\omega)e^{\frac{1}{2}\rho^2\sigma^2(\omega^2+(1-b_1)\omega-b_2)T}\tag{4.4}$$

which is equivalent to

$$I(\omega) = \tilde{f}(\omega) e^{\frac{1}{2}\rho^2\sigma^2(\omega^2+(1-b_1)\omega-b_2)T} \tag{4.5}$$

Substituting (4.5) into (4.3), yields

$$\tilde{P}_\rho(\omega, t) = \tilde{f}(\omega) e^{\frac{1}{2}\rho^2\sigma^2(\omega^2+(1-b_1)\omega-b_2)\tau} \tag{4.6}$$

where $\tau = T - t$. By means of (2.2), (4.6) yields

$$P_\rho(S_t^\rho, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \tilde{f}(\omega) e^{\frac{1}{2}\rho^2\sigma^2(\omega^2+(1-b_1)\omega-b_2)\tau} (S_t^\rho)^{-\omega} d\omega \tag{4.7}$$

which is the integral equation for governing equation (3.1). Let

$$\xi(S_t^\rho) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\frac{\rho^2\sigma^2}{2}(\omega^2+\omega(1-b_1)-b_2)} (S_t^\rho)^{-\omega} d\omega \tag{4.8}$$

Using the fact that

$$e^{\frac{1}{2}\rho^2\sigma^2(\omega^2+(1-b_1)\omega-b_2)\tau} = e^{-\alpha_1(\alpha_2^2+b_2)+\alpha_1(\omega+\alpha_2)^2} \tag{4.9}$$

where

$$\alpha_1 = \frac{\rho^2\sigma^2\tau}{2}, \alpha_2 = \frac{1-b_1}{2} \tag{4.10}$$

Thus

$$\xi(S_t^\rho) = \frac{e^{-\alpha_1(\alpha_2^2+b_2)}}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\alpha_1(\omega+\alpha_2)^2} (S_t^\rho)^{-\omega} d\omega \tag{4.11}$$

By means of the transformation given by [21], one obtains

$$e^{\phi\omega^2} = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{1}{\sqrt{\phi}} \exp\left(\frac{-(\ln S_t^\rho)^2}{4\phi}\right) (S_t^\rho)^{\omega-1} dS_t^\rho, \ \ \Re(\phi) \geq 0 \tag{4.12}$$

Therefore,

$$\xi(S_t^\rho) = e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(S_t^\rho)^{\alpha_2}}{\rho\sigma\sqrt{2\pi\tau}} e^{-\frac{1}{2}\left(\frac{\ln(S_t^\rho)}{\rho\sigma\sqrt{\tau}}\right)^2} \tag{4.13}$$

Similarly,

$$\xi\left(\frac{S_t^\rho}{v}\right) = e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(\frac{S_t^\rho}{v})^{\alpha_2}}{\rho\sigma\sqrt{2\pi\tau}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} \tag{4.14}$$

Using (3.4), then

$$h(S_t^\rho) = \mathcal{M}^{-1}(\tilde{f}(\omega)) = \left[S_T^\rho \ln\left(\frac{K}{S_T^\rho}\right)\right]^+ \tag{4.15}$$

Thus,

$$h(v) = \left[v \ln\left(\frac{K}{v}\right)\right]^+ \tag{4.16}$$

With the help of the convolution property of MT, (4.7) becomes

$$P_\rho(S_t^\rho, t) = \int_0^\infty h(v)\xi\left(\frac{S_t^\rho}{v}\right)\frac{1}{v} dv \tag{4.17}$$

Using (4.14) and (4.16), (4.17) becomes

$$P_\rho(S_t^\rho, t) = \int_0^\infty \left[v \ln\left(\frac{K}{v}\right)\right]^+ e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(\frac{S_t^\rho}{v})^{\alpha_2}}{\rho\sigma\sqrt{2\pi\tau}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} \frac{1}{v} dv \tag{4.18}$$

$$P_\rho(S_t^\rho, t) = e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(S_t^\rho)^{\alpha_2}}{\rho\sigma\sqrt{2\pi\tau}} \int_0^K \left[v^{1-\alpha_2} \ln\left(\frac{K}{v}\right)\right] e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} \frac{1}{v} dv \tag{4.19}$$

Simplifying further, yields

$$\begin{aligned} P_\rho(S_t^\rho, t) = {} & e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(S_t^\rho)^{\alpha_2}}{\rho\sigma\sqrt{2\pi\tau}} \int_0^K \ln(K) \frac{1}{v^{\alpha_2}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} dv \\ & - e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(S_t^\rho)^{\alpha_2}}{\rho\sigma\sqrt{2\pi\tau}} \int_0^K \ln(v) \frac{1}{v^{\alpha_2}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} dv \end{aligned} \tag{4.20}$$

$$P_\rho(S_t^\rho, t) = e^{-\alpha_1(\alpha_2^2+b_2)} \frac{(S_t^\rho)^{\alpha_2}}{\rho\sigma\sqrt{\tau}} [\ln(K)H_1 - H_2] \tag{4.21}$$

where

$$H_1 = \frac{1}{\sqrt{2\pi}} \int_0^K \frac{1}{v^{\alpha_2}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} dv \tag{4.22}$$

$$H_2 = \frac{1}{\sqrt{2\pi}} \int_0^K \frac{\ln(v)}{v^{\alpha_2}} e^{-\frac{1}{2}\left(\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}\right)^2} dv \tag{4.23}$$

Let

$$x = \frac{\ln\left(\frac{S^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}} \tag{4.24}$$

Thus

$$H_2 = \rho\sigma\sqrt{\tau}(S_t^\rho)^{-\alpha_2+1} e^{\alpha_1(\alpha_2-1)^2} [\rho\sigma\sqrt{\tau}G_1 - \ln(S_t^\rho)G_2] \tag{4.25}$$

where

$$G_1 = \frac{1}{\sqrt{2\pi}} \int_\infty^{\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}} x e^{-\frac{1}{2}(x-\rho\sigma\sqrt{\tau}(\alpha_2-1))^2} dx \tag{4.26}$$

$$G_2 = \frac{1}{\sqrt{2\pi}} \int_\infty^{\frac{\ln\left(\frac{S_t^\rho}{v}\right)}{\rho\sigma\sqrt{\tau}}} e^{-\frac{1}{2}(x-\rho\sigma\sqrt{\tau}(\alpha_2-1))^2} dx \tag{4.27}$$

Once again, by variable transformation

$$y = x - \rho\sigma\sqrt{\tau}(\alpha_2 - 1),$$
$$d_\rho = \frac{\ln\left(\frac{S_t^\rho}{K}\right) - \rho^2\sigma^2\tau(\alpha_2 - 1)}{\rho\sigma\sqrt{\tau}} = \frac{\ln\left(\frac{S_t^\rho}{K}\right) + \left(r - q + \left(\rho - \frac{1}{2}\right)\sigma^2\right)\tau}{\rho\sigma\sqrt{\tau}} \tag{4.28}$$

Equations (4.26) and (4.27) become

$$G_1 = -[\eta(d_\rho) + \rho\sigma\sqrt{\tau}(\alpha_2 - 1)\mathcal{N}(-d_\rho)] \tag{4.29}$$

and

$$G_2 = -\mathcal{N}(-d_\rho) \tag{4.30}$$

respectively, with

$$\eta(\kappa) = \frac{1}{\sqrt{2\pi}}e^{-\frac{\kappa^2}{2}}, \mathcal{N}(\kappa) = \int_{-\infty}^{\kappa}\eta(\kappa)d\kappa \tag{4.31}$$

Substituting (4.29) and (4.30) into (4.25), yields

$$\begin{aligned} H_2 = -\rho\sigma\sqrt{\tau}(S_t^\rho)^{-\alpha_2+1}e^{\alpha_1(\alpha_2-1)^2}[&\rho\sigma\sqrt{\tau}\eta(d_\rho) \\ &+(\rho^2\sigma^2\tau(\alpha_2 - 1) - \ln(S_t^\rho))\mathcal{N}(-d_\rho)] \end{aligned} \tag{4.32}$$

Similarly,

$$H_1 = \rho\sigma\sqrt{\tau}(S_t^\rho)^{-\alpha_2+1}e^{\alpha_1(\alpha_2-1)^2}[\mathcal{N}(-d_\rho)] \tag{4.33}$$

Using (4.32) and (4.33) and the values of b_2, α_1 and α_2, (4.21) becomes

$$P_\rho(S_t^\rho, t) = S_t^\rho e^{\left[r(\rho-1)-\rho q+\frac{1}{2}\rho(\rho-1)\sigma^2\right]\tau}\left[\rho\sigma\sqrt{\tau}\eta(d_\rho) - (Q_1 + Q_2\tau)\mathcal{N}(-d_\rho)\right] \tag{4.34}$$

with

$$Q_1 = \ln\left(\frac{S_t^\rho}{K}\right), \tag{4.35}$$

$$Q_2 = \rho\left(r - q + \left(\rho - \frac{1}{2}\right)\sigma^2\right), \tag{4.36}$$

$$d_\rho = \frac{Q_1 + Q_2\tau}{\rho\sigma\sqrt{\tau}}, \tag{4.37}$$

$$\tau = T - t. \tag{4.38}$$

Remark 4.1. *Setting $q = 0$ in (4.34), yields the fundamental valuation formula for BSMEPPOM on non DY with MLPPF*

$$P_\rho(S_t^\rho, t) = S_t^\rho e^{\left[r(\rho-1)+\frac{1}{2}\rho(\rho-1)\sigma^2\right]\tau}\left[\rho\sigma\sqrt{\tau}\eta(d_\rho) - (R_1 + R_2\tau)\mathcal{N}(-d_\rho)\right] \tag{4.39}$$

with

$$R_1 = \ln\left(\frac{S_t^\rho}{K}\right), \tag{4.40}$$

$$R_2 = \rho \left(r + \left(\rho - \frac{1}{2} \right) \sigma^2 \right), \tag{4.41}$$

$$d_\rho = \frac{R_1 + R_2\tau}{\rho\sigma\sqrt{\tau}}, \tag{4.42}$$

$$\tau = T - t. \tag{4.43}$$

Remark 4.2. *Setting $\rho = 1$ in (4.34), yields the fundamental valuation formula for plain EPO on DY*

$$P_1(S_t, t) = S_t e^{-q\tau} \left[\sigma\sqrt{\tau}\eta(d_1) - (D_1 + D_2\tau)\mathcal{N}(-d_1) \right] \tag{4.44}$$

with

$$D_1 = \ln\left(\frac{S_t}{K} \right), \tag{4.45}$$

$$D_2 = \left(r - q + \frac{\sigma^2}{2} \right), \tag{4.46}$$

$$d_1 = \frac{D_1 + D_2\tau}{\sigma\sqrt{\tau}}, \tag{4.47}$$

$$\tau = T - t. \tag{4.48}$$

Remark 4.3. *Setting $\rho = 1$ and $q = 0$ in (4.34), yields the fundamental valuation formula for plain EPO on non DY*

$$P_1(S_t, t) = S_t \left[\sigma\sqrt{\tau}\eta(d_1) - (B_1 + B_2\tau)\mathcal{N}(-d_1) \right] \tag{4.49}$$

with

$$B_1 = \ln\left(\frac{S_t}{K} \right), \tag{4.50}$$

$$B_2 = \left(r + \frac{\sigma^2}{2} \right), \tag{4.51}$$

$$d_1 = \frac{B_1 + B_2\tau}{\sigma\sqrt{\tau}}, \tag{4.52}$$

$$\tau = T - t. \tag{4.53}$$

5. Numerical Example

Consider the valuation of the EPPO on a DY with MLPPF via the MT using the following parameters in Table 1. The results obtained are displayed in Figures 1 - 10.

Table 1. The parameters

Parameters	**Values**
S in dollars	100
K in dollars	100, 110, 120, 130, 140, 150
r	8%
σ	50%
q	0, 5%, 20%, 60%, 100%
T in years	$\frac{1}{2}$

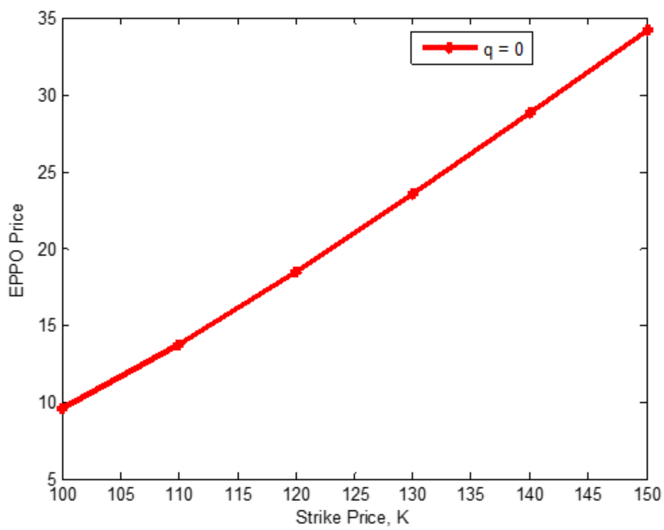

Figure 1. The Effect of DY on the price of EPPO with $\rho = 1$, $q = 0$

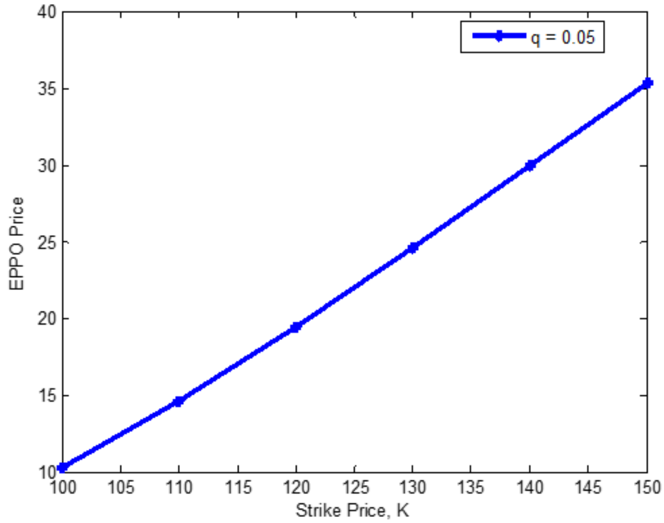

Figure 2. The Effect of DY on the price of EPPO with $\rho = 1$, $q = 0.05$

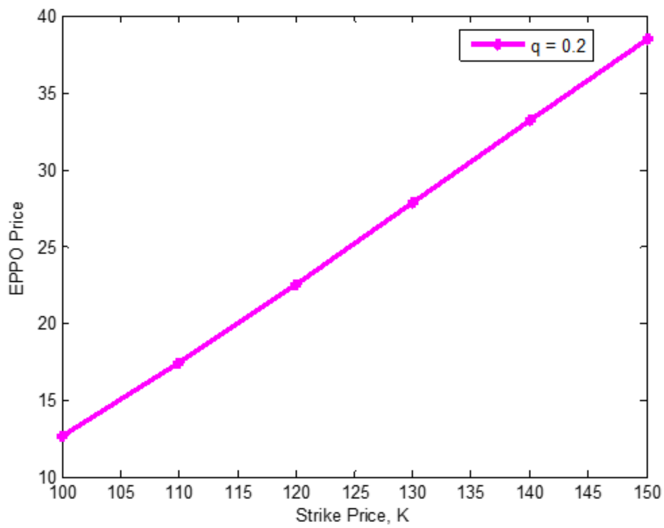

Figure 3. The Effect of DY on the price of EPPO with $\rho = 1$, $q = 0.2$

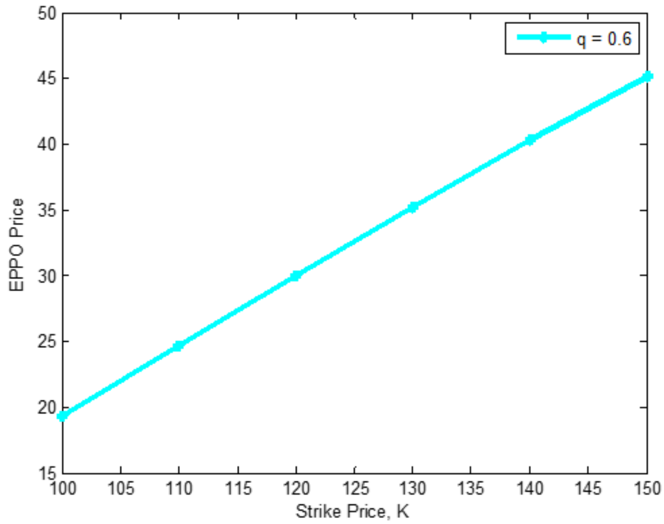

Figure 4. The Effect of DY on the price of EPPO with $\rho = 1$, $q = 0.6$

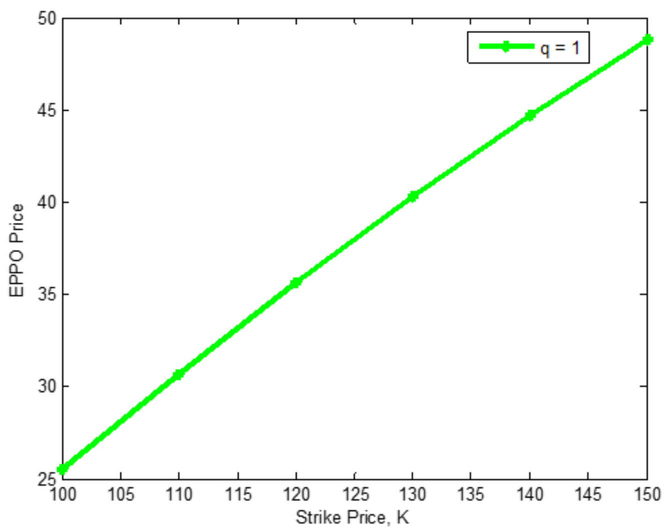

Figure 5. The Effect of DY on the price of EPPO with $\rho = 1$, $q = 1.0$

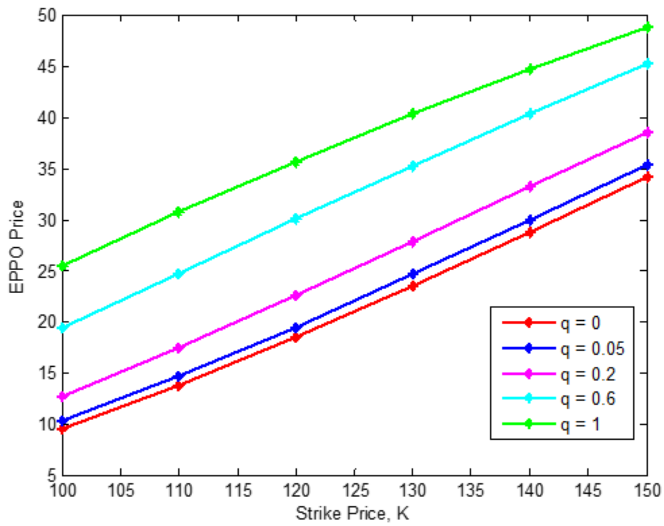

Figure 6. The Comparative Study of EPPO Price with different DY.

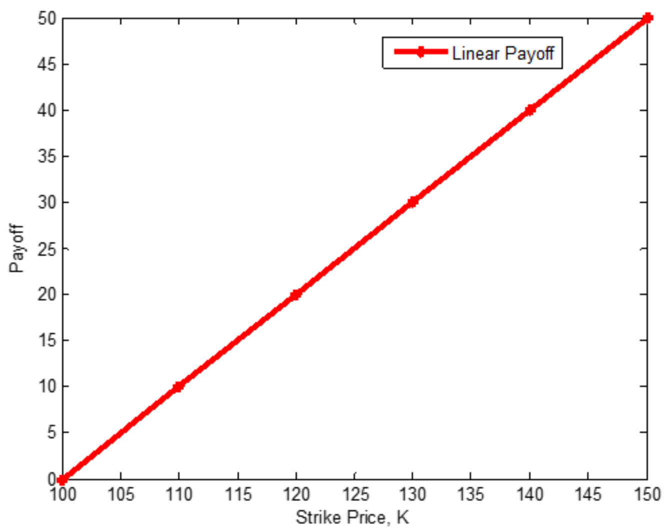

Figure 7. The Plots of Linear Payoff.

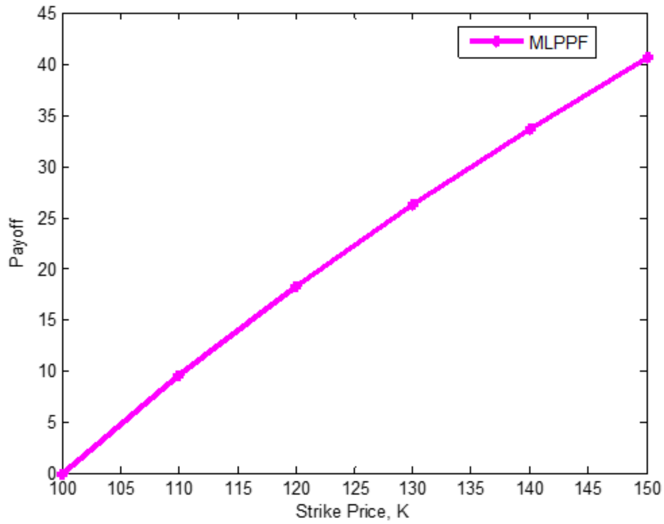

Figure 8. The Plots of MLPPF for $\rho = 1$.

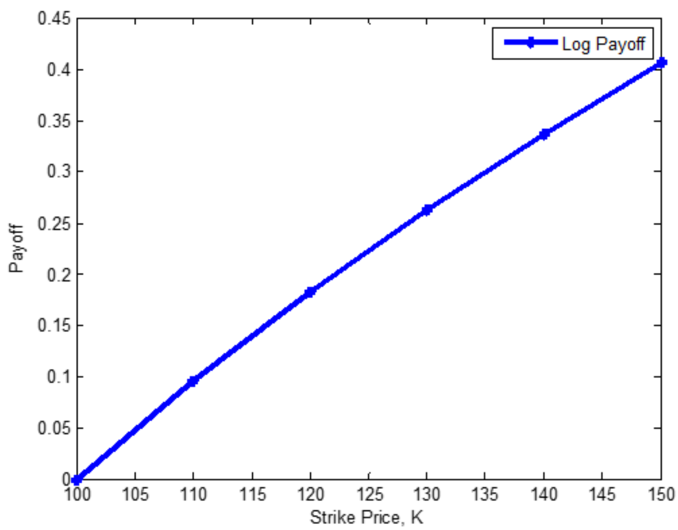

Figure 9. The Plots of Log Payoff.

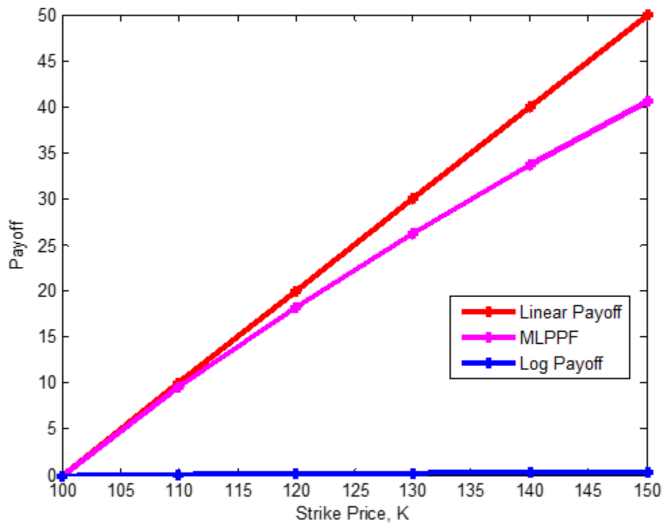

Figure 10. The comparative study of Linear Payoff, MLPPF and Log Payoff.

6. Conclusion

An analytic solution of BSMEPPOM via the celebrated transform of Mellin type in the sense of DY and MLPPF has been proposed in this paper. The integral equation for the representation of the price of EPPO with DY was obtained. The closed form approximation formula for EPPO was also obtained via MT with the help of its convolution property and final time condition. Moreover, the MT was tested on some parameters to show its performance and suitability. The effect of DY is captured in Figures 1 -5. From Figure 6, it is observed that increase in DY leads to increase in the prices of the EPPO with MLPPF. It also is observed from Figures 6, that the holder is more beneficial to enter into a European power put option. In other words, however, the benefits of these cash flows are given to the holder of a put option. The plots of linear payoff, MLPPF and log payoff are displayed in Figures 7, 8 and 9, respectively. From Figure 10, it is clearly seen that the MLPPF used in this present paper performed better than the log payoff used in [22] and also was found to be very close to the linear payoff of plain vanilla [1]. Hence, from the results displayed in Figures 6 and 10, it can be concluded that MT is suitable for the valuation of EPPO with MLPPF and DY due to its capacity power of solving BSMEPPOM directly in terms of market price.

Acknowledgements: The authors wish to thank Tshwane University of Technology for their financial support and the Department of Higher Education and Training, South Africa.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] F. Black, M. Scholes, The Pricing of Options and Corporate Liabilities, J. Political Econ. 81 (1973), 637-654.

[2] S.E. Fadugba, C.R. Nwozo, Valuation of European Call Options via the Fast Fourier Transform and the Improved Mellin Transform, J. Math. Finance. 06 (2016), 338–359. https://doi.org/10.4236/jmf.2016.62028.

[3] R.C. Merton, Option Pricing When Underlying Stock Returns Are Discontinuous, J. Financial Econ. 3 (1976), 125–144. https://doi.org/10.1016/0304-405x(76)90022-2.

[4] M.J. Brennan, E.S. Schwartz, Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis, J. Financial Quant. Anal. 13 (1978), 461-474. https://doi.org/10.2307/2330152.

[5] J.C. Cox, S.A. Ross, M. Rubinstein, Option Pricing: A Simplified Approach, J. Financial Econ. 7 (1979), 229–263. https://doi.org/10.1016/0304-405x(79)90015-1.

[6] P. Boyle, M. Broadie, P. Glasserman, Monte Carlo Methods for Security Pricing, J. Econ. Dyn. Control. 21 (1997), 1267–1321. https://doi.org/10.1016/s0165-1889(97)00028-6.

[7] F.S. Emmanuel, The Mellin Transform Method as an Alternative Analytic Solution for the Valuation of Geometric Asian Option, Appl. Comput. Math. 3 (2014), 1-7. https://doi.org/10.11648/j.acm.s.2014030601.11.

[8] D.J. Manuge, P.T. Kim, A Fast Fourier Transform Method for Mellin-Type Option Pricing, arXiv:1403.3756, (2014). https://doi.org/10.48550/arXiv.1403.3756.

[9] C.R. Nwozo, S.E. Fadugba, Mellin Transform Method for the Valuation of Some Vanilla Power Options With Non-Dividend Yield, Int. J. Pure Appl. Math. 96 (2014), 79-104. https://doi.org/10.12732/ijpam.v96i1.7.

[10] C.R. Nwozo, S.E. Fadugba, Performance Measure of Laplace Transforms for Pricing Path Dependent Options, Int. J. Pure Appl. Math. 94 (2014), 175-197. https://doi.org/10.12732/ijpam.v94i2.5.

[11] S.E. Fadugba, C.R. Nwozo, Mellin Transform Method for the Valuation of the American Power Put Option with Non-Dividend and Dividend Yields, J. Math. Finance. 05 (2015), 249–272. https://doi.org/10.4236/jmf.2015.53023.

[12] S.E. Fadugba, Solution of Fractional Order Equations in the Domain of Mellin Transform, J. Nigerian Soc. Phys. Sci. 1 (2019), 138-142.

[13] S.E. Fadugba, C.R. Nwozo, Closed-Form Solution for the Critical Stock Price and the Price of Perpetual American Call Options via the Improved Mellin Transforms, Int. J. Financial Markets Derivatives. 6 (2018), 269-286. https://doi.org/10.1504/ijfmd.2018.097489.

[14] S.E. Fadugba, C.T. Nwozo, Perpetual American Power Put Options With Non-Dividend Yield in the Domain of Mellin Transforms, Palestine J. Math. 9 (2020), 371-385.

[15] S.E. Fadugba, Laplace Transform for the Solution of Fractional Black-Scholes Partial Differential Equation for the American Put Options With Non-Dividend Yield, Int. J. Stat. Econ. 20 (2019), 10-17.

[16] S.J. Ghevariya, BSM European Option Pricing Formula for ML-payoff Function with Mellin Transform, Int. J. Math. Appl. 6 (2018), 34-36.

[17] S.E. Fadugba, A.A. Adeniji, M.C. Kekana, J.T. Okunlola, O. Faweya, Direct Solution of Black-Scholes-Merton European Put Option Model on Dividend Yield With Modified-Log Payoff Function, Int. J. Anal. Appl. 20 (2022), 54. https://doi.org/10.28924/2291-8639-20-2022-54.

[18] P. Flajolet, X. Gourdon, P. Dumas, Mellin Transforms and Asymptotics: Harmonic Sums, Theor. Computer Sci. 144 (1995), 3–58. https://doi.org/10.1016/0304-3975(95)00002-e.

[19] G. Fikioris, Mellin-Transform Method for Integral Evaluation: Introduction and Applications to Electromagnetics, Springer International Publishing, Cham, 2007. https://doi.org/10.1007/978-3-031-01697-4.

[20] A.H. Zemanian, Generalized integral transformation, Dover Publications, New York, 1987.

[21] A. Erdélyi, W. Magnus, F. Oberhettinger, F. Tricomi, Tables of Integral Transforms, Vol. 1-2, First edition, McGraw-Hill, New York, 1954.

[22] P. Wilmott, Paul Wilmott on Quantitative Finance, John Wiley & Sons, Ltd., Second edition, 2006.