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On Hybrid Pure Hyperideals in Ordered Hypersemigroups

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Abstract. In this paper, the concepts of hybrid pure hyperideals in ordered hypersemigroups are introduced and some algebraic properties of hybrid pure hyperideals are studied. We characterize weakly regular ordered hypersemigroups in terms of hybrid pure hyperideals. Finally, we introduce the concepts of hybrid weakly pure hyperideals and prove that the hybrid hyperideals are hybrid weakly pure hyperideals if such hybrid hyperideals satisfy the idempotent property.

1. Introduction

The theory of fuzzy sets is the most appropriate theory for dealing with uncertainty and was introduced by Zadeh [25] in 1965. After the introduction of the concept of fuzzy sets by Zadeh, several researchers researched the generalizations of the notions of fuzzy sets with huge applications in computer science, artificial intelligence, control engineering, robotics, automata theory, decision theory, finite state machine, graph theory, logic, operation research and many branches of pure and applied mathematics. For example, Xie et al. [24] applied the fuzzy set theory to the switching method.

Molodtsov [21] introduced the concept of the soft set as a new mathematical tool for dealing with uncertainties being free from the difficulties that have troubled the usual theoretical approaches. The soft sets have many applications in several branches of both pure and applied sciences (see [12], [22], [23]).

As a parallel circuit of fuzzy sets and soft sets, Jun, Song, and Muhiuddin [18] introduced the notion of hybrid structures in a set of parameters over an initial universe set. The hybrid structures can be

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applied in many areas including mathematics, statistics, computer science, electrical instruments, industrial operations, business, engineering, social decisions, etc. (see [2], [13], [14]).

The algebraic hyperstructure theory was first introduced in 1934 by Marty [20]. Hyperstructures have many applications in several branches of both pure and applied sciences (see [8–11, 15]). Recently, Heidari and Davvaz applied the hyperstructure theory to ordered semigroups and introduced the concept of ordered semihypergroups (or hypersemigroups)(see [16]), which is a generalization of the concept of ordered semigroups. Furthermore, the ordered semihypergroup theory was enriched by the work of many researchers, for example, [3, 4, 11, 15]. In particular, the hyperideal theory on semihypergroups and ordered hypersemigroups can be seen in [3–5, 17].

In 1989, Ahsan and Takahashi [1] introduced the notions of pure ideals and purely prime ideals of semigroups. Later, Changphas and Sanborisoot [7] defined the notions of left pure, right pure, left weakly pure, and right weakly pure ideals in ordered semigroups and gave some of their characterizations. In 2020, Changphas and Davvaz [6] studied the purity of hyperideals in ordered hyperstructures. They introduced the notions of pure hyperideals and weakly pure hyperideals in an ordered semihypergroup. We now apply hybrid structures to ordered hyperstructures. In this present paper, the concepts of hybrid pure hyperideals in ordered hypersemigroups are introduced and some algebraic properties of such hyperideals are studied. We characterize weakly regular ordered hypersemigroups in terms of hybrid pure hyperideals. Finally, we introduce the concepts of hybrid weakly pure hyperideals and prove that the hybrid hyperideals are hybrid weakly pure hyperideals if such hybrid hyperideals satisfy the idempotent property.

2. Preliminaries

In this section, we will recall the basic terms and definitions from the ordered hypersemigroup theory and the hybrid structure theory that we will use later in this paper. Throughout this paper, we will use the concepts of ordered hypersemigroups introduced by Kehayopulu [19] and hybrid structures introduced by Anis [2].

Definition 2.1. *A hypergroupoid is a nonempty set H with a hyperoperation*

$$\circ : H \times H \to \mathcal{P}^*(H) \mid (a, b) \mapsto a \circ b$$

on H and an operation

$$* : \mathcal{P}^*(H) \times \mathcal{P}^*(H) \to \mathcal{P}^*(H) \mid (A, B) \mapsto A * B$$

on $\mathcal{P}^*(H)$ (induced by the hyperoperation $\circ$) defined by

$$A * B = \bigcup_{a \in A, b \in B} (a \circ b)$$

($\mathcal{P}^*(H)$ is the set of all nonempty subsets of H.)

We have $\{x\} * \{y\} = x \circ y$. Further more $A \subseteq B$ implies $A * C \subseteq B * C$ and $C * A \subseteq C * B$ for any nonempty subsets A, B and C of H.

Definition 2.2. *A hypergroupoid $(H; \circ)$ is called a hypersemigroup if*

$$\{x\} * (y \circ z) = (x \circ y) * \{z\}$$

for every x, y, $z \in H$.

For convenient, the previous equation could be identified as

$$x * (y \circ z) = (x \circ y) * z.$$

Let $(H; \leq)$ be a partial order set. We define a relation $\preceq$ on $\mathcal{P}^*(H)$ as follows: For two nonempty subsets A and B of H,

$$A \preceq B := \{(x, y) \in A \times B \mid x \leq y, \forall x \in A, \exists y \in B\}.$$

Definition 2.3. *The structure $(H; \circ, \leq)$ is called an ordered hypersemigroup if the following conditions are satisfied:*

(1) *$(H; \circ)$ is a hypersemigroup.*
(2) *$(H; \leq)$ is a partial order set.*
(3) *For $a, b, c \in H$, if $a \leq b$ then $a \circ c \preceq b \circ c$ and $c \circ a \preceq c \circ b$.*

For simplicity, we denoted an ordered hypersemigroup $(H; \circ, \leq)$ by its carrier set as a bold letter $\mathbf{H}$.

Definition 2.4. *Let $\mathbf{H}$ be an ordered hypersemigroup. A nonempty subset A of H is called a left (resp., right) hyperideal of $\mathbf{H}$ if*

(1) *$H * A \subseteq A$ (resp., $A * H \subseteq A$).*
(2) *For $a \in H, b \in A$, if $a \leq b$, then $a \in A$.*

A nonempty subset A of H is called a *two-sided hyperideal*, or simply a *hyperideal* of $\mathbf{H}$ if it is both a left and a right hyperideal of $\mathbf{H}$.

Let A be a nonempty subset of H. Define

$$(A] := \{x \in H \mid \ x \leq a \ \text{ for some } a \in A\}.$$

Note that condition (2) in Definition 2.4 is equivalent to $A = (A]$. If A and B are nonempty subsets of H, then we obtain

(1) $A \subseteq (A]$.
(2) $(A] \cup (B] \subseteq (A \cup B]$.
(3) $((A] * (B]] = (A * B]$.
(4) $(A] * (B] \subseteq (A * B]$.

Definition 2.5. *[7] Let* **H** *be an ordered hypersemigroup. A hyperideal* A *of* **H** *is called a left (resp., right) pure hyperideal of* **H** *if* $x \in (A * x]$ *(resp.,* $x \in (x * A]$*) for all* $x \in A$*.*

A hyperideal A of **H** is called a *pure hyperideal* of **H** if it is both a left and a right pure hyperideal of **H**.

Let I be the unit interval, H a set of parameters and $\mathcal{P}(U)$ denote the power set of an initial universe set U. Hybrid structures are defined as follows.

Definition 2.6. *A hybrid structure in* H *over* U *is defined to be a mapping*

$$f := (f^*, f^+) : H \to \mathcal{P}(U) \times I, x \mapsto (f^*(x), f^+(x)),$$

where

$$f^* : H \to \mathcal{P}(U) \text{ and } f^+ : H \to I$$

are mappings.

Let us denote by $Hyb(H)$ the set of all hybrid structures in H over U. We define a binary relation $\ll$ on $Hyb(H)$ as follows: For all $f = (f^*, f^+), g = (g^*, g^+) \in Hyb(H)$,

$$f \ll g \Leftrightarrow f^* \sqsubseteq g^*, f^+ \succeq g^+,$$

where $f^* \sqsubseteq g^*$ means that $f^*(x) \subseteq g^*(x)$ and $f^+ \succeq g^+$ means that $f^+(x) \geq g^+(x)$ for all $x \in H$ and $f = g$ if $f \ll g$ and $g \ll f$.

Definition 2.7. *Let* $f = (f^*, f^+)$ *and* $g = (g^*, g^+)$ *be hybrid structures in* H *over* U*. Then the hybrid intersection of* f *and* g *is denoted by* $f \Cap g$ *and is defined to be a hybrid structure*

$$f \Cap g : H \to \mathcal{P}(U) \times I, x \mapsto ((f^* \cap g^*)(x), (f^+ \vee g^+)(x)),$$

where

$$(f^* \cap g^*)(x) := f^*(x) \cap g^*(x) \text{ and } (f^+ \vee g^+)(x) := \max\{f^+(x), g^+(x)\}.$$

We denote $\widetilde{H} := (H^*, H^+)$ the hybrid structure in H over U and is defined as follows:

$$\widetilde{H} : H \longrightarrow \mathcal{P}(U) \times I : x \mapsto \left(H^*(x), H^+(x)\right),$$

where

$$H^*(x) := U \text{ and } H^+(x) := 0.$$

Let a be an element of H. Then we set

$$\mathbf{H}_a := \{(x, y) \in H \times H \mid a \in (x \circ y]\}.$$

Definition 2.8. *Let $f = (f^*, f^+)$ and $g = (g^*, g^+)$ be hybrid structures in H over U. Then the hybrid products of f and g is denoted by $f \otimes g$ and is defined to be a hybrid structure*

$$f \otimes g : H \to \mathcal{P}(U) \times I, x \mapsto ((f^* \odot g^*)(x), (f^+ \oplus g^+)(x)),$$

where

$$(f^* \odot g^*)(x) := \begin{cases} \bigcup\limits_{(a,b)\in \mathrm{H}_x} (f^*(a) \cap g^*(b)) & \textit{if } \mathbf{H}_x \neq \emptyset \\ \emptyset & \textit{otherwise,} \end{cases}$$

and

$$(f^+ \oplus g^+)(x) := \begin{cases} \bigwedge\limits_{(a,b)\in \mathrm{H}_x} \{\max\{f^+(a), g^+(b)\}\} & \textit{if } \mathbf{H}_x \neq \emptyset \\ 1 & \textit{otherwise.} \end{cases}$$

Let A be a nonempty subset of H. We denote by $\chi_A := (\chi_A^*, \chi_A^+)$ the characteristic hybrid structure of A in H over U which is defined to be a hybrid structure

$$\chi_A : H \to \mathcal{P}(U) \times I, x \mapsto (\chi_A^*(x), \chi_A^+(x)),$$

where

$$\chi_A^*(x) := \begin{cases} U & \text{if } x \in A \\ \emptyset & \text{otherwise,} \end{cases}$$

and

$$\chi_A^+(x) := \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise.} \end{cases}$$

We set $\chi_A := \widetilde{H}$ in the case that $A = H$.

3. Main Results

In this main section, we introduce the concepts of hybrid pure hyperideals in ordered hypersemigroups and study some algebraic properties of such hybrid pure hyperideals. We also characterize weakly regular ordered hypersemigroups in terms of hybrid pure hyperideals. Finally, we introduce the concept of hybrid weakly pure hyperideals and prove that any hybrid hyperideal is a hybrid weakly pure hyperideal whenever it is idempotent.

Definition 3.1. *Let $\mathbf{H}$ be an ordered hypersemigroup. A hybrid structure $f = (f^*, f^+)$ in H over U is called a hybrid left (resp., right) hyperideal in $\mathbf{H}$ over U if for every $x, y \in H$:*

(1) $\bigcap\limits_{a\in x\circ y} f^*(a) \supseteq f^*(y)$ *(resp.,* $\bigcap\limits_{a\in x\circ y} f^*(a) \supseteq f^*(x)$*);*
(2) $\bigvee\limits_{a\in x\circ y} f^+(a) \leq f^+(y)$ *(resp.,* $\bigvee\limits_{a\in x\circ y} f^+(a) \leq f^+(x)$*);*
(3) $x \leq y$ *implies* $f^*(x) \supseteq f^*(y)$ *and* $f^+(x) \leq f^+(y)$*.*

A hybrid structure f is called a *hybrid hyperideal* in $\mathbf{H}$ over U if it is both a hybrid left and a hybrid right hyperideal in $\mathbf{H}$ over U.

Definition 3.2. *Let* **H** *be an ordered hypersemigroup. A hybrid hyperideal f in* **H** *over U is*

(1) *left pure if $f \Cap g = f \otimes g$ for every hybrid left hyperideal g in* **H** *over U.*
(2) *right pure $g \Cap f = g \otimes f$ for every hybrid right hyperideal g in* **H** *over U.*

The following remark is a useful tool in calculating the purity of hybrid hyperideals.

Remark 3.1. *Let $f = (f^*, f^+)$ be a hybrid left (resp., right) pure hyperideal in* **H** *over U. Then for any hybrid left (resp., right) hyperideal $g = (g^*, g^+)$ in* **H** *over U, we have*

(1) $f^* \cap g^* = f^* \odot g^*$ *(resp., $g^* \cap f^* = g^* \odot f^*$);*
(2) $f^+ \vee g^+ = f^+ \oplus g^+$ *(resp., $g^+ \vee f^+ = g^+ \oplus f^+$).*

A hybrid hyperideal in **H** over U is called a *hybrid pure hyperideal* in **H** over U if it is both a hybrid right pure and a hybrid left pure hyperideal in **H** over U.

The following lemmas are important in illustrating our first theorem.

Lemma 3.1. *Let* **H** *be an ordered hypersemigroup and A a nonempty subset of H. Then the following conditions are equivalent:*

(1) *A is a right (resp., left) hyperideal of* **H**.
(2) *$\chi_A = (\chi_A^*, \chi_A^+)$ is a hybrid right (resp., left) hyperideal in* **H** *over U.*

Proof. (1)⇒(2). Let A be a right hyperideal of an ordered hypersemigroup **H**. First, let $x, y \in H$. If $x \in A$, then $x \circ y \subseteq A$ and we obtain

$$\bigcap_{a \in x \circ y} \chi_A^*(a) = U \supseteq \chi_A^*(x) \quad \text{and} \quad \bigvee_{a \in x \circ y} \chi_A^+(a) = 0 \leq \chi_A^+(x).$$

If $x \notin A$, we obtain

$$\bigcap_{a \in x \circ y} \chi_A^*(a) \supseteq \emptyset = \chi_A^*(x), \quad \text{and} \quad \bigvee_{a \in x \circ y} \chi_A^+(a) \leq 1 = \chi_A^+(x).$$

Secondly, let $x, y \in H$ be such that $x \leq y$. If $y \in A$, then $x \in A$ and then

$$\chi_A^*(x) = U \supseteq \chi_A^*(y), \quad \text{and} \quad \chi_A^+(x) = 0 \leq \chi_A^*(y).$$

If $y \notin A$, we obtain

$$\chi_A^*(x) \supseteq \emptyset = \chi_A^*(y), \quad \text{and} \quad \chi_A^+(x) \leq 1 = \chi_A^+(y).$$

Altogether, it is complete to prove that χ_A is a hybrid right hyperideal in **H** over U.

(2)⇒(1). Let χ_A be a hybrid right hyperideal in **H** over U. Firstly, let $x \in A$ and $y \in H$. We obtain $U \supseteq \bigcap_{a \in x \circ y} \chi_A^*(a) \supseteq \chi_A^*(x) = U$, which implies that $\bigcap_{a \in x \circ y} \chi_A^*(a) = U$ and since $U = \bigcap_{a \in x \circ y} \chi_A^*(x) \subseteq \chi_A^*(a) \subseteq U$, we obtain $\chi_A^*(a) = U$. Similarly, since $0 \leq \bigvee_{a \in x \circ y} \chi_A^+(a) \leq \chi_A^+(x) = 0$, which implies that $\bigvee_{a \in x \circ y} \chi_A^+(a) = 0$, and since $0 = \bigvee_{a \in x \circ y} \chi^+(a) \geq \chi_A^+(a) \geq 0$, we obtain that $\chi_A^+(a) = 0$. Altogether, we have $a \in A$ true.

Secondly, let $x, y \in H$ be such that $x \leq y$ and $y \in A$. We obtain $U \supseteq \chi^*_A(x) \supseteq \chi^*_A(y) = U$, which implies that $\chi^*_A(x) = U$. It means that $x \in A$. For χ^+_A need not to consider. Therefore A is a right hyperideal of $\mathbf{H}$. Similarly, we can show that A is a left hyperideal if and only if χ_A is a hybrid left hyperideal. □

As a consequence of the above lemma, we have that A is a hyperideal of $\mathbf{H}$ if and only if χ_A is a hybrid hyperideal in $\mathbf{H}$ over U. The following lemma provides some good characterzations relating to two sets and their characteristic hybrid structures.

Lemma 3.2. *Let* $\mathbf{H}$ *be an ordered hypersemigroup and* A, B *nonempty subsets of* H. *Then the following conditions hold:*

(1) $A \subseteq B$ *if and only if* $\chi_A \ll \chi_B$*;*
(2) $\chi_A \Cap \chi_B = \chi_{A\cap B}$*;*
(3) $\chi_A \otimes \chi_B = \chi_{(A*B]}$.

Proof. We will give a proof of (3) only. Let $x \in (A * B]$. Then there exists $c \in a \circ b$ for some $a \in A$ and $b \in B$ such that $x \leq c$. Then $\mathbf{H}_x \neq \emptyset$, and we obtain

$$\begin{aligned} U &\supseteq (\chi^*_A \odot \chi^*_B)(x) \\ &= \bigcup_{(y,z)\in \mathrm{H}_x} [\chi^*_A(y) \cap \chi^*_B(z)] \\ &\supseteq \chi^*_A(a) \cap \chi^*_B(b) \\ &= U. \end{aligned}$$

This implies that $(\chi^*_A \odot \chi^*_B)(x) = U = \chi^*_{(A*B]}(x)$ and

$$\begin{aligned} 0 &\leq (\chi^+_A \oplus \chi^+_B)(x) \\ &= \bigwedge_{(y,z)\in \mathrm{H}_x} \{\max\{\chi^+_A(y), \chi^+_B(z)\}\} \\ &\leq \max\{\chi^+_A(a), \chi^+_B(b)\} \\ &= 0. \end{aligned}$$

This implies that $(\chi^+_A \oplus \chi^+_B)(x) = 0 = \chi^+_{(A*B]}(x)$. Therefore $\chi_A \otimes \chi_B = \chi_{(A*B]}$. □

In 2020, Changphas [6] characterized right (resp., left) pure hyperideals as the following lemmas.

Lemma 3.3. *Let* $\mathbf{H}$ *be an ordered hypersemigroup and* A *a hyperideal of* $\mathbf{H}$. *Then the following conditions are equivalent:*

(1) *A is a right pure hyperideal of* $\mathbf{H}$.
(2) $B \cap A = (B * A]$ *for every right hyperideal* B *of* $\mathbf{H}$.

Lemma 3.4. *Let* **H** *be an ordered hypersemigroup and* A *a hyperideal of* **H**. *Then the following conditions are equivalent:*

(1) *A is a left pure hyperideal of* **H**.
(2) $A \cap B = (A * B]$ *for every left hyperideal* B *of* **H**.

The following theorem provides a characterization of right (resp., left) pure hyperideals in ordered hypersemigroups using hybrid right (resp., left) pure hyperideals.

Theorem 3.1. *Let A be a hyperideal of an ordered hypersemigroup* **H**. *Then the following conditions are equivalent:*

(1) *A is a right (resp., left) pure hyperideal of* **H**.
(2) $\chi_A = (\chi_A^*, \chi_A^+)$ *is a hybrid right (resp., left) pure hyperideal in* **H** *over* U.

Proof. (1)⇒(2). Assume that A is a right pure hyperideal of **H**. By Lemma 3.1, $\chi_A = (\chi_A^*, \chi_A^+)$ is a hybrid hyperideal in **H** over U. Let $f = (f^*, f^+)$ be a hybrid right hyperideal in **H** over U and $a \in H$. Suppose that $a \notin A$. We consider two cases as follows: If $\mathbf{H}_a = \emptyset$, then we have

$$\begin{aligned}(f^* \odot \chi_A^*)(a) &= \emptyset \\ &= f^*(a) \cap \chi_A^*(a) \\ &= (f^* \cap \chi_A^*)(a),\end{aligned}$$

and

$$\begin{aligned}(f^+ \oplus \chi_A^+)(a) &= 1 \\ &= \max\{f^+(a), \chi_A^+(a)\} \\ &= (f^+ \vee \chi_A^+)(a).\end{aligned}$$

If $\mathbf{H}_a \neq \emptyset$, then, by a right purity of A, we have that $v \notin A$ for all $(u, v) \in \mathbf{H}_a$. Then

$$\begin{aligned}(f^* \odot (\chi_A^*)(a) &= \bigcup_{(x,y)\in \mathrm{H}_a} [f^*(x) \cap \chi_A^*(y)] \\ &= \emptyset \\ &= f^*(a) \cap \chi_A^*(a) \\ &= (f^* \cap \chi_A^*)(a),\end{aligned}$$

and

$$\begin{aligned}(f^+ \oplus \chi^+)_A)(a) &= \bigwedge_{(x,y)\in \mathrm{H}_a} \{\max\{f^+(x), \chi_A^+(y)\}\} \\ &= 1 \\ &= \max\{f^+(a), \chi_A^+(a)\} \\ &= (f^+ \vee \chi_A^+)(a),\end{aligned}$$

Now, we assume that $a \in A$. By the right purity of A, we have $\mathbf{H}_a \neq \emptyset$. More precisely, there exists $(a, x) \in \mathbf{H}_a$ such that $x \in A$. Then, by the hybrid right ideality of f and the hybrid left ideality of χ_A, we have that

$$\begin{aligned}
(f^* \cap \chi_A^*)(a) &= f^*(a) \cap \chi_A^*(a) \\
&= f^*(a) \\
&= f^*(a) \cap \chi_A^*(x) \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [f^*(u) \cap \chi_A^*(v)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [f^*(uv) \cap \chi_A^*(uv)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [f^*(a) \cap \chi_A^*(a)] \\
&= f^*(a) \cap \chi_A^*(a) \\
&= (f^* \cap \chi_A^*)(a).
\end{aligned}$$

This implies that $(f^* \cap \chi_A^*)(a) = \bigcup_{(u,v)\in \mathrm{H}_a} [f^*(u) \cap \chi_A^*(v)] = (f^* \odot \chi_A^*)(a)$, and

$$\begin{aligned}
(f^+ \vee \chi_A^+)(a) &= \max\{f^+(a), \chi_A^+(a)\} \\
&= f^+(a) \\
&= \max\{f^+(a), \chi_A^+(x)\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{f^+(u), \chi_A^+(v)\}\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{f^+(uv), \chi_A^+(uv)\}\} \\
&\geq \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{f^+(a), \chi_A^+(a)\}\} \\
&= \max\{f^+(a), \chi_A^+(a)\} \\
&= (f^+ \vee \chi_A^+)(a).
\end{aligned}$$

This implies that $(f^+ \vee \chi_A^+)(a) = \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{f^+(u), \chi_A^+(v)\}\} = (f^+ \oplus \chi_A^+)(a)$. Altogether, we have that χ_A is a hybrid right pure hyperideal in $\mathbf{H}$ over U.

(2)⇒(1). Assume that χ_A is a hybrid right pure hyperideal in $\mathbf{H}$ over U. Let B be a right hyperideal of $\mathbf{H}$. By Lemma 3.1, χ_B is a hybrid right hyperideal in $\mathbf{H}$ over U. By assumption, we obtain

$$\chi_{B\cap A} = \chi_B \Cap \chi_A = \chi_B \otimes \chi_A = \chi_{(B*A]}.$$

By Lemma 3.2 (1), we have $B \cap A = (B * A]$ and, by Lemma 3.3, we obtain that A is a right pure hyperideal of $\mathbf{H}$. Similarly, we can prove that A is a left pure hyperideal of $\mathbf{H}$ if and only if χ_A is a hybrid left pure hyperideal in $\mathbf{H}$ over U. □

By the above theorem, we obtain the following consequence.

Corollary 3.1. *Let A be a hyperideal of an ordered hypersemigroup $\mathbf{H}$. Then the following conditions are equivalent:*

(1) *A is a pure hyperideal of $\mathbf{H}$.*
(2) *$\chi_A = (\chi_A^*, \chi_A^+)$ is a hybrid pure hyperideal in $\mathbf{H}$ over U.*

The following results illustrate some properties of hybrid right (resp., left) pure hyperideals in an ordered hypersemigroup.

Theorem 3.2. *Let $f = (f^*, f^+)$ and $g = (g^*, g^+)$ be hybrid right pure hyperideals in $\mathbf{H}$ over U. Then $f \Cap g$ is a hybrid right pure hyperideal in $\mathbf{H}$ over U.*

Proof. Let h be a hybrid right hyperideal in $\mathbf{H}$ over U and $a \in H$. We consider two cases as follows. If $\mathbf{H}_a = \emptyset$, we obtain

$$\begin{aligned}
(h^* \odot (f^* \cap g^*))(a) &= \emptyset \\
&= (h^* \odot f^*)(a) \cap (h^* \odot g^*)(a) \\
&= (h^* \cap f^*)(a) \cap (h^* \cap g^*)(a) \\
&= [h^*(a) \cap f^*(a)] \cap [h^*(a) \cap g^*(a)] \\
&= h^*(a) \cap [f^*(a) \cap g^*(a)] \\
&= h^*(a) \cap (f^* \cap g^*)(a) \\
&= [h^* \cap (f^* \cap g^*)](a),
\end{aligned}$$

and

$$\begin{aligned}
(h^+ \oplus (f^+ \vee g^+))(a) &= 1 \\
&= \max\{(h^+ \oplus f^+)(a), (h^+ \oplus g^+)(a)\} \\
&= \max\{(h^+ \vee f^+)(a), (h^+ \vee g^+)(a)\} \\
&= \max\{\max\{h^+(a), f^+(a)\}, \max\{h^+(a), g^+(a)\}\} \\
&= \max\{h^+(a), \max\{f^+(a), g^+(a)\}\} \\
&= \max\{h^+(a), (f^+ \vee g^+)(a)\} \\
&= [h^+ \vee (f^+ \vee g^+)](a).
\end{aligned}$$

It is complete to prove that $f \Cap g$ is a hybrid right pure hyperideal in $\mathbf{H}$ over U. □

By a similar method to Theorem 3.2, we have the following theorem.

Theorem 3.3. *Let $f = (f^*, f^+)$ and $g = (g^*, g^+)$ be hybrid left pure hyperideals in* $\mathbf{H}$ *over* U*. Then* $f \Cap g$ *is a hybrid left pure hyperideal in* $\mathbf{H}$ *over* U*.*

Combining Theorem 3.2 and 3.3, we obtain the following result.

Corollary 3.2. *Let* f *and* g *be hybrid pure hyperideals in* $\mathbf{H}$ *over* U*. Then* $f \Cap g$ *is a hybrid pure hyperideal in* $\mathbf{H}$ *over* U*.*

Let $f = (f^*, f^+)$ and $g = (g^*, g^+)$ be hybrid structures in H over U. The hybrid union of f and g denoted by $f \Cup g$ and is defined to be a hybrid structure

$$f \Cup g := (f^* \cup g^*, f^+ \wedge g^+) : H \to \mathcal{P}^*(U) \times I \mid x \mapsto ((f^* \cup g^*)(x), (f^+ \wedge g^+)(x)),$$

where

$$(f^* \cup g^*)(x) := f^*(x) \cup g^*(x) \text{ and } (f^+ \wedge g^+)(x) := \min\{f^+(x), g^+(x)\}.$$

Theorem 3.4. *Let $f = (f^*, f^+)$ and $g = (g^*, g^+)$ be hybrid right pure hyperideals in* $\mathbf{H}$ *over* U*. Then* $f \Cup g$ *is a hybrid right pure hyperideal in* $\mathbf{H}$ *over* U*.*

Proof. Let $h = (h^*, h^+)$ be a hybrid right hyperideal in $\mathbf{H}$ over U and $a \in H$. If $\mathbf{H}_a = \emptyset$, then we obtain that

$$\begin{aligned}
(h^* \cap (f^* \cup g^*))(a) &= h^*(a) \cap (f^* \cup g^*)(a) \\
&= h^*(a) \cap (f^*(a) \cup g^*(a)) \\
&= (h^*(a) \cap f^*(a)) \cup (h^*(a) \cap g^*(a)) \\
&= (h^* \cap f^*)(a) \cup (h^* \cap g^*)(a) \\
&= (h^* \odot f^*)(a) \cup (h^* \odot g^*)(a) \\
&= \emptyset \\
&= (h^* \odot (f^* \cup g^*))(a),
\end{aligned}$$

and

$$\begin{aligned}
(h^+ \vee (f^+ \wedge g^+))(a) &= \max\{h^+(a), (f^+ \wedge g^+)(a)\} \\
&= \max\{h^+(a), \min\{f^+(a), g^+(a)\}\} \\
&= \min\{\max\{h^+(a), f^+(a)\}, \max\{h^+(a), g^+(a)\}\} \\
&= \min\{(h^+ \vee f^+)(a), (h^+ \vee g^+)(a)\} \\
&= \min\{(h^+ \oplus f^+)(a), (h^+ \oplus g^+)(a)\} \\
&= 1 \\
&= (h^+ \oplus (f^+ \wedge g^+))(a).
\end{aligned}$$

Suppose that $\mathbf{H}_a \neq \emptyset$, and we obtain

$$\begin{aligned}
(h^* \cap (f^* \cup g^*))(a) &= h^*(a) \cap (f^* \cup g^*)(a) \\
&= h^*(a) \cap (f^*(a) \cup g^*(a)) \\
&= (h^*(a) \cap f^*(a)) \cup (h^*(a) \cap g^*(a)) \\
&= (h^* \cap f^*)(a) \cup (h^* \cap g^*)(a) \\
&= (h^* \odot f^*)(a) \cup (h^* \odot g^*)(a) \\
&= \bigcup_{(x,y)\in \mathrm{H}_a} [h^*(x) \cap f^*(y)] \cup \bigcup_{(x,y)\in \mathrm{H}_a} [h^*(x) \cap g^*(y)] \\
&= \bigcup_{(x,y)\in \mathrm{H}_a} [(h^*(x) \cap f^*(y))] \cup [(h^*(x) \cap g^*(y))] \\
&= \bigcup_{(x,y)\in \mathrm{H}_a} [h^*(x) \cap (f^*(y) \cup g^*(y))] \\
&= \bigcup_{(x,y)\in \mathrm{H}_a} [h^*(x) \cap (f^* \cup g^*)(y)] \\
&= (h^* \odot (f^* \cup g^*))(a),
\end{aligned}$$

and

$$\begin{aligned}
(h^+ \vee (f^+ \wedge g^+))(a) &= \max\{h^+(a), (f^+ \wedge g^+)(a)\} \\
&= \max\{h^+(a), \min\{f^+(a), g^+(a)\}\} \\
&= \min\{\max\{h^+(a), f^+(a)\}, \max\{h^+(a), g^+(a)\}\} \\
&= \min\{(h^+ \vee f^+)(a), (h^+ \vee g^+)(a)\} \\
&= \min\{(h^+ \oplus f^+)(a), (h^+ \oplus g^+)(a)\} \\
&= \min\{\bigwedge_{(x,y)\in \mathrm{H}_a} \{\max\{h^+(x), f^+(y)\}\}, \bigwedge_{(x,y)\in \mathrm{H}_a} \{\max\{h^+(x), g^+(y)\}\}\} \\
&= \bigwedge_{(x,y)\in \mathrm{H}_a} \{\max\{h^+(x), \min\{f^+(y), g^+(y)\}\}\} \\
&= \bigwedge_{(x,y)\in \mathrm{H}_a} \{\max\{h^+(x), (f^+ \wedge g^+)(y)\}\} \\
&= (h^+ \oplus (f^+ \wedge g^+))(a).
\end{aligned}$$

Altogether, we obtain that $f \Cup g$ is a hybrid right pure hyperideal in $\mathbf{H}$ over U. □

By a similar method to Theorem 3.4, we have the following theorem.

Theorem 3.5. *Let f and g be hybrid left pure hyperideals in an ordered hypersemigroup $\mathbf{H}$ over U. Then $f \Cup g$ is a hybrid left pure hyperideal in $\mathbf{H}$ over U.*

Combining Theorem 3.4 and 3.5, we obtain the following result.

Corollary 3.3. *Let f and g be hybrid pure hyperideals in* **H** *over U. Then $A \Cup B$ is a hybrid pure hyperideal in* **H** *over U.*

An ordered hypersemigroup **H** is said to be *right (resp., left) weakly regular* [7] if for any $a \in H$ there exist $b, x, y \in H$ such that $a \leq b$ and $b \in (a \circ x) * (a \circ y)$ (resp., $a \leq b$ and $b \in (x \circ a) * (y \circ a)$).

An ordered hypersemigroup **H** is called *weakly regular* if it is both a right and a left weakly regular ordered hypersemigroup.

Lemma 3.5. *[7] Let* **H** *be an ordered hypersemigroup. Then the following statements are equivalent:*

(1) **H** *is right (resp., left) weakly regular.*
(2) *Every hyperideal of* **H** *is a right (resp., left) pure hyperideal of* **H**.

Lemma 3.6. *[7] Let* **H** *be an ordered hypersemigroup. Then the following statements are equivalent:*

(1) **H** *is weakly regular.*
(2) *Every hyperideal of* **H** *is a pure hyperideal of* **H**.

We characterize right weakly regular ordered hypersemigroups in terms of hybrid right pure hyperideals as follows:

Theorem 3.6. *Let* **H** *be an ordered hypersemigroup. Then the following statements are equivalent:*

(1) **H** *is right weakly regular.*
(2) *Every hybrid hyperideal in* **H** *over U is right pure.*

Proof. (1)⇒(2). Let $f = (f^*, f^+)$ be a hybrid hyperideal in **H** over U and $g = (g^*, g^+)$ a hybrid right hyperideal in **H** over U. Let $a \in H$. Since **H** is right weakly regular, there exist $b, x, y \in H$ such that $a \leq b$ and $b \in (a \circ x) * (a \circ y)$. This implies that $\mathbf{H}_a \neq \emptyset$ and then

$$\begin{aligned}
(g^* \odot f^*)(a) &= \bigcup_{(u,v)\in \mathrm{H}_a} [g^*(u) \cap f^*(v)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [\bigcap_{c\in u\circ v} g^*(c) \cap \bigcap_{c\in u\circ v} f^*(c)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [g^*(c) \cap f^*(c)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [g^*(a) \cap f^*(a)] \\
&= g^*(a) \cap f^*(a) \\
&= (g^* \cap f^*)(a).
\end{aligned}$$

On the other hand, we obtain

$$\begin{aligned}(g^* \odot f^*)(a) &= \bigcup_{(u,v)\in \mathrm{H}_a} [g^*(u) \cap f^*(v)] \\ &\supseteq \bigcap_{c\in a\circ x} g^*(c) \cap \bigcap_{d\in a\circ y} f^*(d) \\ &\supseteq g^*(a) \cap f^*(a) \\ &= (g^* \cap f^*)(a).\end{aligned}$$

Therefore $(g^* \odot f^*)(a) = (g^* \cap f^*)(a)$, and consider

$$\begin{aligned}(g^+ \oplus f^+)(a) &= \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{g^+(u), f^+(v)\}\} \\ &\geq \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{\bigvee_{c\in u\circ v} g^+(c), \bigvee_{d\in u\circ v} f^+(d)\}\} \\ &\geq \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{g^+(a), f^+(a)\}\} \\ &= \max\{g^+(a), f^+(a)\} \\ &= (g^+ \vee f^+)(a).\end{aligned}$$

On the other hand, we obtain

$$\begin{aligned}(g^+ \oplus f^+)(a) &= \bigwedge_{(u,v)\in \mathrm{H}_a} \{\max\{g^+(u), f^+(v)\}\} \\ &\leq \max\{\bigvee_{c\in a\circ x} g^+(c), \bigvee_{d\in a\circ y} f^+(d)\} \\ &\leq \max\{g^+(a), f^+(a)\} \\ &= (g^+ \vee f^+)(a).\end{aligned}$$

Therefore $(g^+ \oplus f^+)(a) = (g^+ \vee f^+)(a)$ and we obtain that $g \otimes f = g \Cap f$. Hence f is a hybrid right pure hyperideal in **H** over U.

(2)⇒(1). Let A and B be a hyperideal of **H** and a right hyperideal of **H**, respectively. By Lemma 3.1, we obtain χ_A and χ_B is a hybrid hyperideal in **H** over U and a hybrid right hyperideal in **H** over U, respectively. By our assumption, χ_B is a hybrid right pure hyperideal in **H** over U. Then

$$\begin{aligned}\chi_{(B*A]} &= \chi_B \otimes \chi_A \\ &= \chi_B \Cap \chi_A \\ &= \chi_{B\cap A}.\end{aligned}$$

By Lemma 3.2 (1), we obtain $B \cap A = (B * A]$. This means that A is a right pure hyperideal of **H**. Therefore, by Lemma 3.5, **H** is right weakly regular. □

By a similar method to Theorem 3.6, we have the following theorem.

Theorem 3.7. *Let* **H** *be an ordered hypersemigroup. Then the following statements are equivalent:*

(1) **H** *is left weakly regular.*
(2) *Every hybrid hyperideal in* **H** *over* U *is left pure.*

Combining Theorem 3.6 and 3.7, we obtain the following result.

Corollary 3.4. *Let* **H** *be an ordered hypersemigroup. Then the following statements are equivalent:*

(1) **H** *is weakly regular.*
(2) *Every hybrid hyperideal in* **H** *over* U *is pure.*

Now, we present the concepts of right weakly purity and left weakly purity of hybrid hyperideals. In our last main result, the coincidence of these two concepts is provided.

Definition 3.3. *A hybrid hyperideal* f *in* **H** *over* U *is called a hybrid right (resp., left) weakly pure hyperideal if* $g \otimes f = g \Cap f$ *(resp.,* $f \otimes g = f \Cap g$*) for every hybrid hyperideal* g *in* **H** *over* U.

A hybrid hyperideal is called a *hybrid weakly pure hyperideal* in H over U if it is both a hybrid right and a hybrid left weakly pure hyperideal in **H** over U. A hybrid structure f in H over U is idempotent with respect to $\otimes$ if $f \otimes f = f$.

Lemma 3.7. *Let* **H** *be an ordered hypersemigroup and* f, g *are hybrid right hyperideals in* **H** *over* U. *Then* $f \Cap g$ *is a hybrid right hyperideal in* **H** *over* U.

Proof. Let $f = (f^*, f^+)$ and $g = (g^*, g^+)$ be hybrid right hyperideals in **H** over U and $x, y \in H$. Then we obtain

$$\begin{aligned}\bigcap_{a\in x\circ y}(f^*\cap g^*)(a) &= \bigcap_{a\in x\circ y} f^*(a)\cap\bigcap_{a\in x\circ y} g^*(a)\\ &\supseteq f^*(x)\cap g^*(x)\\ &= (f^*\cap g^*)(x),\end{aligned}$$

and

$$\begin{aligned}\bigvee_{a\in x\circ y}(f^+\vee g^+)(a) &= \max\{\bigvee_{a\in x\circ y} f^+(a), \bigvee_{a\in x\circ y} g^+(a)\}\\ &\leq \max\{f^+(x), g^+(x)\}\\ &= (f^+\vee g^+)(x).\end{aligned}$$

Therefore $f \Cap g$ is a hybrid right hyperideal in **H** over U. □

By a similar method to Lemma 3.7, we have the following lemma.

Lemma 3.8. *Let* $\mathbf{H}$ *be an ordered hypersemigroup and* f, g *are hybrid left hyperideals in* $\mathbf{H}$ *over* U. *Then* $f \Cap g$ *is a hybrid left hyperideal in* $\mathbf{H}$ *over* U.

Combining Lemma 3.7 and 3.8, we obtain the following result.

Corollary 3.5. *Let* $\mathbf{H}$ *be an ordered hypersemigroup and* f, g *are hybrid hyperideals in* $\mathbf{H}$ *over* U. *Then* $f \Cap g$ *is a hybrid hyperideal in* $\mathbf{H}$ *over* U.

Our last result illustrates that the concepts of right weakly purity and left weakly purity of hybrid hyperideals coincide.

Theorem 3.8. *Let* $\mathbf{H}$ *be an ordered hypersemigroup and* $f = (f^*, f^+)$ *a hybrid hyperideal in* $\mathbf{H}$ *over* U. *Then the following statements are equivalent.*

(1) f *is hybrid right weakly pure hyperideal.*
(2) f *is idempotent with respect to* $\otimes$.
(3) f *is hybrid left weakly pure hyperideal.*

Proof. (1)⇒(2). Let f be a hybrid right weakly pure hyperideal in $\mathbf{H}$ over U. Then we obtain

$$f \otimes f = f \Cap f = f.$$

Therefore, f is idempotent with respect to $\otimes$.

(2)⇒(1). Let $g = (g^*, g^+)$ be a hybrid hyperideal in $\mathbf{H}$ over U. By Corollary 3.5, we obtain that $g \Cap f$ is a hybrid hyperideal in $\mathbf{H}$ over U. By our assumption, we have

$$g \Cap f = g \Cap (f \otimes f) = (g \Cap f) \otimes (g \Cap f) \ll g \otimes f.$$

On the other hand, let $a \in H$. If $\mathbf{H}_a = \emptyset$, then

$$(g^* \odot f^*)(a) = \emptyset \subseteq (g^* \cap f^*)(a),$$

and

$$(g^+ \oplus f^+)(a) = 1 \geq (g^+ \vee f^+)(a).$$

Suppose that $\mathbf{H}_a \neq \emptyset$. Then

$$\begin{aligned}
(g^* \odot f^*)(a) &= \bigcup_{(u,v)\in \mathrm{H}_a} [g^*(u) \cap f^*(v)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [\bigcap_{c \in u\circ v} g^*(c) \cap \bigcap_{c \in u\circ v} f^*(c)] \\
&\subseteq \bigcup_{(u,v)\in \mathrm{H}_a} [g^*(a) \cap f^*(a)] \\
&= g^*(a) \cap f^*(a) \\
&= (g^* \cap f^*)(a),
\end{aligned}$$

and

$$\begin{aligned}
(g^+ \oplus f^+)(a) &= \bigwedge_{(u,v)\in H_a} \{\max\{g^+(u), f^+(v)\}\} \\
&\geq \bigwedge_{(u,v)\in H_a} \{\max\{\bigvee_{c\in u\circ v} g^+(c), \bigvee_{c\in u\circ v} f^+(c)\}\} \\
&\geq \bigwedge_{(u,v)\in H_a} \{\max\{g^+(a), f^+(a)\}\} \\
&= \max\{g^+(a), f^+(a)\} \\
&= (g^+ \vee f^+)(a).
\end{aligned}$$

Thus $g \otimes f \ll g \Cap f$ and altogether, we have $g \otimes f = g \Cap f$. This means that f is a hybrid right weakly pure hyperideal in **H** over U.

Illustration of (2)⇔(3) can be done similarly. □

By Theorem 3.8, we obtain the following result.

Corollary 3.6. *Let* **H** *be an ordered hypersemigroup and f a hybrid hyperideal in* **H** *over U. Then the following statements are equivalent.*

(1) *f is idempotent with respect to $\otimes$.*
(2) *f is hybrid weakly pure hyperideal.*

4. Conclusion

We introduced the concept of hybrid pure hyperideals in ordered hypersemigroups. Some related properties of hybrid pure hyperideals are studied. We characterized weakly regular ordered hypersemigroups in terms of hybrid pure hyperideals. Finally, we introduced the concept of hybrid weakly pure hyperideals. We proved that the hybrid hyperideals are hybrid weakly pure hyperideals if such hybrid hyperideals satisfied the idempotent property. In our future work, we will apply the notions of hybrid hyperideals and hybrid pure hyperideals to the theory of hypersemirings, hypergroups, etc.

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