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A Class of Tests for Testing Better Failure Rate at Specific Age Distribution With Randomly Right Censored Data

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ABSTRACT: A device has a better failure rate at specific age t_0 property, denoted by $BFR - t_0$ if its failure rate $r(t)$ increases for $t \leq t_0$ and for $t > t_0$, $r(t)$ is not less than its value at t_0. A test statistic is proposed to test exponentiality versus $BFR - t_0$ based on a randomly right censored sample of size n. Kaplan-Meier estimator is used to estimate the empirical life distribution. Properties of the test are measured by power estimates, estimated risks, and test of normality. The efficiency loss due to censoring is investigated by using tests for censored sample data.

1. INTRODUCTION

The concept of ageing for engineering devices, biological organs or their corresponding systems has been characterized by various life distribution classes. The increasing failure rate (*IFR*) class of life distributions is the most used and have all other notions of ageing in reliability literature.

Among these notions are the increasing failure rate average (*IFRA*), new better than used (*NBU*), new better than used failure rate (*NBUFR*), new better than used in average failure rate (*NBAFR*), decreasing mean remaining life (*DMRL*), new better than used in expectation (*NBUE*) and harmonic new better than used in expectation (*HNBUE*). See ([14], [23], [21], [6]) for definitions, properties and interrelationships of these classes of life distributions.

In many practical situations, it is familiar that properties of life distributions may not be completely observed after a specific time. This arises in data collection in companies for their commodities with guarantee for some fixed time t_0, say.

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In study of [15] for the new better than used at specific age t_0 $(NBU - t_0)$ have considered the hypothetical cancer patients problem. It is interesting to investigate the problem of testing whether new diagnosed cancer patient has smaller chance of survival than a patient with similar initial diagnosis and survived, on treatment for a certain year. The decreasing mean remaining life at a specific age t_0 $(DMRL - t_0)$ is defined in similar line by [10].

In a study by [2] introduced the class of better failure rate at a specific age t_0 $(BFR - t_0)$ and its dual class $(WFR - t_0)$. Its closure properties under some reliability operations are studied. Test statistic for testing exponentiality against $BFR - t_0$ or its dual class $WFR - t_0$ is also proposed for the complete sample.

Different classes of life distributions based on a random censored samples, have been studied in references such as ([26], [27]) for testing *NBU* and *IFR* classes of life distributions, [20] for testing *IFRA* class of life distributions and ([3], [4]) for testing *NBRU* and *NBRUE* classes of life distributions and their dual classes.

In research for [12] defined classes of life distributions *IFRA*-t_o and *NBU*-t_0. The properties of these classes are studied, and a nonparametric test is proposed which is designed to test the hypothesis whether *NBU*-t_0 element is strictly new better than used after time t_0.

A paper for [19] about NBU class made by [15] to investigate the testing of new better than used at specified age (*NBU*-t_0) based on a U-statistic whose kernel depends on sub-sample minima. A member of the class of tests proposed by [17] for this problem belongs to the class of tests are covered and distribution properties of the class of test statistics are studied. The performances of a few members of the proposed class of tests are studied in terms of Pitman asymptotic relative efficiency.

In a paper for [9] he introduced some properties of the new better than used in convex ordering at age t_0 (*NBUC*– t_0) and new better than used of second order (2) at age t_0 (*NBU (2)*– t_0) classes of life distributions, where the survival probability at age 0 is greater than or equal to the conditional survival probability at specified age $t_0 > 0$. Preservation properties of the two classes under various reliability operations and shock model are arriving according to homogeneous Poisson process are established.

Researchers [13] have defined two classes of life distributions, namely new better than used in expectation at specific age t_0 $(NBUE - t_0)$ and harmonic new better than used in expectation at specific age t_0 $(HNBUE - t_0)$ and their dual classes $(NWUE - t_0)$ and $(HNWUE - t_0)$. The closure properties under various reliability operations such as convolution, mixture, mixing and the homogeneous Poisson shock model of these classes are studied. Nonparametric tests are proposed to test exponentiality versus the *NBUE-t_0* and *HNBUE-t_0* classes. The critical values and the powers of these tests are calculated to assess the performance of the tests. They show that the proposed tests have high efficiencies for some commonly used distributions in reliability.

A test statistic has been built by [7] for two classes of life distributions defined earlier by [1] namely new better than used renewal failure rate (*NBURFR*) and new better than average renewal failure rate (*NBARFR*). These two classes include many other classes of life distributions. Test statistics for testing of exponentiality as a null hypothesis against these two renewal ageing criteria, and their duals are derived in the case of randomly censored samples. Percentiles tables, power estimates, estimated risks are calculated. The normality of their test statistics is also studied.

2. THE $\boldsymbol{BFR-t_0}$ AND $\boldsymbol{WFR-t_0}$ CLASSES

Let T be a life length of a device with continuous distribution *F*, survival function $\bar{F}(t)=1-F(t)$ and failure rate $r(t)$, then it is called better failure rate at time t_0 $BFR-t_0$ $(WFR-t_0)$ if

$$r(s)\leq(\geq)r(t) \quad \forall \quad s<t<t_0 \qquad \& \qquad t\in[0,t_0], \qquad (2.1)$$

and

$$r(t_0)\leq(\geq)r(t) \ \ \forall \ \ t\geq t_0 \qquad (2.2)$$

This means that any device of age t_0 or less has smaller failure rate than an older device, whereas a device of age t_0 or more cannot have a failure rate less than $r(t_0)$.

3. TESTING EXPONENTIALITY VERSUS $\boldsymbol{BFR-t_0}$ AND $\boldsymbol{WFR-t_0}$ CLASSES

In this section we consider the problem of testing:

H_0: $F(t)=1-e^{-\lambda t_0}$, *i.e.* $r(t)=r(t+x)\forall x\geq 0, 0\leq t\leq t_0$, versus

H_1: F is $BFR-t_0$ *i.e.* $r(t)$ is increasing for $t\leq t_0$ and $r(t)\geq r(t_0)$ for $t\geq t_0$.

This test is based on randomly right-censored data by using Kaplan-Meier estimator [11] for the empirical survival function $(Z_{(i)}, \delta_{(i)}), 1\leq i\leq n$.

Let $\{T_i\} i=1,2,\ldots,n$ be independent and identically distributed non-negative continuous random variables having a common distribution F and survival function $\bar{F}(t)=1-F(t)$.

Let $\{Y_i\} i=1,2,\ldots,n$ be independent and identically distributed random variables according to a continuous censoring distribution H. $\{T_i\}$ and $\{Y_i\}$ are independent of each other. The censoring distribution H is usually, but not necessary, unknown and is considered as nuisance parameter. The pairs $(T_1, Y_1), \ldots\ldots, (T_n, Y_n)$ are defined on a common probability space.

In the censored situations of sample size *n,* the $T_1, \ldots\ldots\ldots T_n$ are not completely observed but the pairs $(Z_1, \delta_1), \ldots\ldots, (Z_n, \delta_n)$ are observed data, where

$$Z_i = min(T_i, Y_i), \forall i = 1,2, \ldots, n \qquad \text{and} \qquad \delta_i = \begin{cases} 1 & \text{if } T_i \le Y_i \\ 0 & \text{if } T_i > Y_i \end{cases}$$

4. TEST STATISTICS FOR $\boldsymbol{BFR - t_0}$

In the $BFR - t_0$ the required test can be based on the estimation of the parameter

$$\mathrm{M(F)} = \iint_{0<z<t\le t_0} \bar{F}(s)\bar{F}(t)\{r(s)\text{-}r(t)\}dF(s)\, dF(t) + \int_{t_0}^{\infty} \bar{F}(t_0)\bar{F}(t)\{r(t_0)\text{-}r(t)\}dF(t) \qquad (4.1)$$

The statistic $M(F) = 0$ under H_0 whereas $M(F) < 0$ under H_1 where strict inequality is due to the continuity of the underlying distribution *F.* Also the statistic $M(F)$ gives a measure of deviation from the exponentiality towards the $BFR - t_0$ property.

For the present of randomly censored data, we base our test on the Kaplan-Meier estimator of $\bar{F}_n$, see [11], which is given by

$$\bar{F}_n(t) = \prod_{\{i: z_{(i)} \le t\}} \left\{ \frac{n-i}{n-i+1} \right\}^{\delta_{(i)}} \quad 0 \le t < z_{(n)} \qquad (4.2)$$

Where $0 = z_{(0)} \le z_{(1)} \le \ldots\ldots \le z_{(n)}$, denote ordered sample of $z_1, z_2, \ldots\ldots, z_n$ and $\delta_{(1)}, \delta_{(2)}, \ldots\ldots, \delta_{(n)}$ are the corresponding δ_i for $i = 1,2,\ldots,n$ the ordered $z_{(i)}$ for $i = 1,2, \ldots, n$. Note that $\delta_{(n)}$ is taken as one, that is $z_{(n)}$ is treated as uncensored observation whether it is the case or not. The corresponding density function and failure rate of $\bar{\hat{F}}_n(t)$ are given in the following.

$$\hat{f}_n(t) = \frac{\bar{\hat{F}}_{i-1,n}(t) - \bar{\hat{F}}_{i,n}(t)}{z_{(i)} - z_{(i-1)}} \qquad (4.3)$$

where $\bar{\hat{F}}_n(t) = 1 - \hat{F}_n(t)$,

Hence

$$\hat{f}_n(t) = \left(z_{(i)} - z_{(i-1)}\right)^{-1} \left[\prod_{\{i-1: z_{(i)} \le t\}} \left\{\frac{n-i}{n-i+1}\right\}^{\delta_{(i)}} - \prod_{\{i: z_{(i)} \le t\}} \left\{\frac{n-i}{n-i+1}\right\}^{\delta_{(i)}} \right]$$

$$= \left(z_{(i)} - z_{(i-1)}\right)^{-1} \left[\prod_{\substack{j=1 \\ Z_{(i)} \le t}}^{i-1} \left\{\frac{n-j}{n-j+1}\right\}^{\delta_{(j)}} - \prod_{j=1}^{i} \left\{\frac{n-j}{n-j+1}\right\}^{\delta_{(j)}} \right]$$

$$= \left(z_{(i)} - z_{(i-1)}\right)^{-1} \prod_{j=1}^{i-1} \left\{\frac{n-j}{n-j+1}\right\}^{\delta_{(j)}} \left[1 - \left\{\frac{n-i}{n-i+1}\right\}^{\delta_{(i)}} \right]$$

$$= \left(z_{(i)} - z_{(i-1)}\right)^{-1}\left[\left[1 - \left\{\frac{n-i}{n-i+1}\right\}^{\delta_{(i)}}\right]\prod_{j=1}^{i-1}\left\{\frac{n-j}{n-j+1}\right\}^{\delta_{(j)}}\right] \quad (4.4)$$

It is reasonable to base a test of exponentiality versus $BFR - t_0$ property on a consistent estimator$M(F)$, given by (3) and then exploit $M(\bar{\bar{F}}_n)$,w here $\bar{\bar{F}}_n$ the Kaplan-Meier estimator. In fact the Kaplan-Meier estimator has been shown to be weakly convergent by ([5], [16], [22]) and strongly consistent by ([24], [18]).

The test statistic in (3) can by simplified in the following. For easy calculation we write $M(F) = A + B$, where

$$A = \iint_{0<s<t<t_0} \bar{F}(s)\bar{F}(t)\{r(s) - r(t)\}dF(s)dF(t) \quad (4.5)$$

and $$B = \int_{t_0}^{\infty} \bar{F}(t_0)\bar{F}(t)\{r((t_0) - r(t))\}dF(t) \quad (4.6)$$

Now

$$\begin{aligned} A &= \int_0^{t_0}\int_0^{t} \bar{F}(s)\bar{F}(t)r(s)dF(s)dF(t) - \int_0^{t_0}\int_0^{t} \bar{F}(s)\bar{F}(t)r(t)dF(s)dF(t) \\ &= \int_0^{t}\int_0^{t} \bar{F}(s)\bar{F}(t)\frac{f(s)}{\bar{F}(s)} f(s)ds\, dF(t) - \int_0^{t_0}\int_0^{t} \bar{F}(s)\bar{F}(t)\frac{f(t)}{\bar{F}(t)} f(t)dF(s)dt \\ &= \int_0^{t_0}\left[\int_0^{t}\{f(s)\}^2 ds\right]\bar{F}(t)dF(t) - \int_0^{t_0}\left[\bar{F}(s)dF(s)\right]\{f(t)\}^2 dt \\ &= \int_0^{t_0}\left[\int_0^{t}\{f(s)\}^2 ds\right]\bar{F}(t)dF(t) - \int_0^{t_0}\left\{\frac{\left(\bar{F}(s)\right)^2}{2}\right\}_0^{t}\{f(t)\}^2 dt \\ &= \int_0^{t_0}\left[\int_0^{t}\{f(s)\}^2 ds\right]\bar{F}(t)dF(t) - \frac{1}{2}\int_0^{t_0}\left[\{\bar{F}(t)\}^2 - 1\right]\{f(t)\}^2 dt \end{aligned} \quad (4.7)$$

and

$$\begin{aligned} B &= \int_{t_0}^{\infty} \bar{F}(t_0)\bar{F}(t)r(t_0)dF(t) - \int_{t_0}^{\infty} \bar{F}(t_0)\bar{F}(t)r(t)dF(t) \\ &= \int_{t_0}^{\infty} \bar{F}(t_0)\bar{F}(t)\frac{f(t_0)}{\bar{F}(t_0)}dF(t) - \int_{t_0}^{\infty} \bar{F}(t_0)\bar{F}(t)\frac{f(t)}{\bar{F}(t)}dF(t) \\ &= f(t_0)\int_{t_0}^{\infty} \bar{F}(t)dF(t) - \bar{F}(t_0)\int_{t_0}^{\infty}\{f(t)\}^2 dt \quad (4.8) \\ &= f(t_0)\frac{1}{2}\left\{\left(\bar{F}(t)\right)^2\right\}_{t_0}^{\infty} - \bar{F}(t_0)\int_{t_0}^{\infty}\{f(t)\}^2 dt \\ &= -\frac{1}{2} f(t_0)\left(\bar{F}(t_0)\right)^2 - \bar{F}(t_0)\int_{t_0}^{\infty}\{f(t)\}^2 dt \end{aligned}$$

Hence

$$M(F)=\int_0^{t_0}\left[\int_0^t\{f(s)\}^2ds\right]\bar{F}(t)dF(t)-\frac{1}{2}\int_0^{t_0}[\{\bar{F}(t)\}^2-1]\{f(t)\}^2dt$$
$$-\tfrac{1}{2}f(t_0)\left(\bar{F}(t_0)\right)^2-\bar{F}(t_0)\int_{t_0}^{\infty}\{f(t)\}^2dt \tag{4.9}$$

or

$$M(F)=\int_0^{t_0}\left[\int_0^t\{f(s)\}^2ds\right]\bar{F}(t)f(t)dt-\frac{1}{2}\int_0^{t_0}\{\bar{F}(t)\}^2\{f(t)\}^2dt+\frac{1}{2}\int_0^{t_0}\{f(t)\}^2dt$$
$$-\bar{F}(t_0)\int_{t_0}^{\infty}\{f(t)\}^2dt-\tfrac{1}{2}f(t_0)\left(\bar{F}(t_0)\right)^2 \tag{4.10}$$

Now substituting the empirical values of $\bar{\hat{F}}(t)$ and $\hat{f}_n(t)$in (12), then the statistics $M_n(F)$ in the case of randomly censored data defined earlier will be

$$M_n(F)=\sum_{i=1}^{m}\left[\sum_{j=1}^{i}\{\hat{f}_n(Z_{(j)})\}^2(Z_{(j)}-Z_{(j-1)})\right]\bar{F}_n(Z_{(i)})\hat{f}_n(Z_{(i)})(Z_{(i)}-Z_{(i-1)})$$
$$+\frac{1}{2}\sum_{i=1}^{m}\{\hat{f}_n(Z_{(i)})\}^2(Z_{(i)}-Z_{(i-1)})-\frac{1}{2}\sum_{i=1}^{m}\{\bar{F}_n(Z_{(i)})\}^2\{\hat{f}_n(Z_{(i)})\}^2(Z_{(i)}-Z_{(i-1)})$$
$$-\vec{F}_n(Z_{(m)})\sum_{i=m}^{n}\{\hat{f}_n(Z_{(i)})\}^2(Z_{(i)}-Z_{(i-1)})-\tfrac{1}{2}\hat{f}_n(Z_{(m_0)})\{\bar{F}_n(Z_{(m_0)})\}^2 \tag{4.11}$$

$\overline{F_n}(Z_{(i)})$ is the Kaplan-Meier, product limit estimator of $\overline{F}_n$ and given by [11] and defined as

$$\overline{F}_n(Z_{(j)})=1-F_n(Z_{(j)})=\prod_{k=1}^{j}\left\{\frac{n-k}{n-k+1}\right\}^{\delta_{(k)}} \tag{4.12}$$

Also, $Z_{(i)}$ and $\delta_{(i)}$ are defined earlier, and $\hat{d}_i=Z_{(i)}-Z_{(i-1)}$,

Here $Z_{(n)}$ is considered as an actual observation, whether or not it is censored, and in this case ∂_n is taken as one to avoid appearance of indefinite values in the empirical calculation of the statistic $M_n(F)$.

For a specific significance level $\alpha=\alpha_0$ and a sample size $n=n_0$, we reject H_0 in favor of H_1, that F has the $BFR-t_0$ property, for small negative values of $M_n(F)$. In otherwards, we reject H_0 in favor of H_1 if the observed or calculated values of the statistic is less than or equal the tabulated value of the statistic for the same value of α_0 and n_0.

For testing H_0 against H_1 that F has the $WFR-t_0$ property, we reject H_0 for small negative values of $-M_n(F)$, or for large positive values of $M_n(F)$.

The asymptotic normality of the sequence, such as $\sqrt{n}[M_n(F) - M(F)]$, has been dealt with in the literature under the following crucial assumption.

$$Sup\left\{\left[\overline{G}(t)\right]^{1-\varepsilon}\left[\overline{H}(t)\right]^{-1},\ t\in[0,\infty)\right\}<\infty \quad \text{for some} \quad 0<\varepsilon<1 \tag{4.13}$$

where G and H are the distribution functions of the actual values of X and the censored values Y respectively. Here G and H are assumed to have support on $[0,\infty)$.

The above sequence, or stochastic process, converges weakly as $n\to\infty$ to a Gaussian process with zero mean, for this result and the details of the corresponding variance function see [22].

A simulation of a small sample is calculated. In practice, simulated percentiles for small samples are commonly used by applied statisticians and reliability analysts.

Based on test statistics in (4.1), empirical evaluation in (4.11) using Kaplan-Meier, product limit estimator in (4.12), the lower percentiles in the *0.01, 0.05,* and *0.10* regions and upper percentiles in the *0.90, 0.95,* and *0.99* regions for the sample sizes *n=10(2),30(5)* and *50* are presented in table 1 for the $BFR - t_0$ and $WFR - t_0$ test statistics.

5. PROPERTIES OF THE TEST

In the following, we will cover three properties of the test, namely, power estimates, estimated risks, and test of normality.

5.1 Power Estimates

The power estimates of the test statistic $M_n(F)$ are considered for the significance level α*=0.05* and for commonly used distributions in reliability modeling. These distributions are Gamma, Weibull, Pareto, and Rayleigh with the following survival functions and failure rates.

Gamma distribution:

$$\overline{F}(t)=\frac{1}{\Gamma(\lambda)}\int_t^\infty x^{\lambda-1}e^{-x}dx \quad , \quad \lambda>0,\ \ t\ge 0\ , \tag{5.1}$$

$$r^{-1}(t)=\int_0^\infty\left(1+\frac{u}{t}\right)^{\lambda-1}e^{-(u)}du$$

Weibull distribution:

$$\overline{F}(t)=\exp(-t^{\beta}) \quad , \quad \beta>0\ ,\ t\ge 0 \tag{5.2}$$

$$r(t)=\beta t^{\beta-1}.$$

Pareto distribution:

$$\bar{F}(t) = (1+\alpha t)^{-\frac{1}{\alpha}}, \qquad \alpha > 0, \quad t \geq 0, \tag{5.3}$$

$$r(t) = (1+\alpha t)^{-1}, \qquad \alpha > 0, \quad t \geq 0.$$

Rayleigh distribution:

$$\bar{F}(t) = \exp(-\tfrac{1}{2}t^2) \qquad , \ t \geq 0 \tag{5.4}$$

$$r(t) = t \qquad t \geq 0$$

For further references about these distributions, one can refer to [25]. Table 2 contains the power estimates of $M_n(F)$ test statistic with respect to these distributions. The estimates are based on the average of five runs each with *1000* simulated samples of sizes *n=10,20,30,40* and *50* with significance level α*=0.05.*

The power estimates of $BFR - t_0$ statistic for Gamma distribution increases more rapidly as λ increases than the other distributions, namely, Weibull, Pareto, and Rayleigh. This indicates that the power estimate increases as the class tends to be more $BFR - t_0$. The same pattern for the power estimates can be noted when the sample size increases.

This table shows that the percentage of $\delta = 1$ (i.e. the percentage of the uncensored random variables in the sample) is approximately fixed when the sample size increases for the same parameter and decreases as the parameter of the distribution increases for all the distributions, namely, Gamma, Weibull, Pareto and Rayleigh. This study shows that the percentage of censored values is dependent on the parameter values of the distribution more than the sample size. The resulting estimates indicate that the proposed statistic is suitable for different small sample sizes in reliability applications.

5.2 Estimated Risks

The estimated risks (*ER*) of the statistic $M_n(F)$ are given by ;

$$ER\big(M_n(F)\big) = \frac{1}{m}\sum_{k=1}^{m}\big(X_k - \bar{X}_{M(F)}\big)^2 \tag{5.5}$$

where $\bar{X}_{\tilde{D}_A}$ is the mean value, X_i is the statistic value corresponding to the distribution under study, *BFR-*t_0, and m is the sample size. Estimated risks are summarized in Table 3.

It is noted that as the sample size increases, the estimated risks decrease, and the mean value is increasing. More precisely, *ER/n* decreases as *n* increases. For a large sample size *ER/n* approaches to zero. This indicates that our test is a powerful one.

5.3 A Test of Normality

The Kolmogorov-Smirnov (*KS*) test is applied to check how well the underlying statistic $M_n(F)$ tends to normality with unspecified mean and variance. For testing normality let S be the empirical distribution function based on the random sample $X_1, X_2, \ldots\ldots, X_n$. The test statistics *D* is defined as the greatest vertical distance between standardized version of $M_n(F)$, denoted by Φ, and *S*. Symbolically,

$$D = \sup|S - \Phi| \tag{5.6}$$

Here we utilize the modified Kolmogorov-Smirnov test of normality proposed by [8] which accommodates the sample estimated normal parameters.

The *D* values are given in Table 4. By comparing the calculated *KS* value with the tabulated one, one accepts the hypothesis approach those to normality for the considered sample sizes.

Table 1: *Critical values for $M_n(F)$ statistic for testing $BFR - t_0$ and $WFR - t_0$ ageing properties*

n	0.01	0.05	0.1	0.9	0.95	0.99
10	-3.6457	-2.431	-1.9567	0.2251	0.6764	1.7745
12	-3.2025	-2.1203	-1.7422	0.398	0.8059	1.8555
14	-2.9597	-1.9717	-1.617	0.4558	0.9362	2.177
16	-2.5969	-1.8157	-1.4816	0.601	1.1027	2.3588
18	-2.5062	-1.7031	-1.3924	0.6615	1.1451	2.4567
20	-2.2232	-1.6249	-1.3085	0.7522	1.1715	2.328
22	-2.1796	-1.5047	-1.242	0.7844	1.3103	2.4974
24	-1.9614	-1.4286	-1.1464	0.8456	1.3761	2.773
26	-1.8858	-1.317	-1.0815	0.963	1.4602	3.0735
28	-1.7508	-1.261	-1.0356	1.0441	1.6226	2.8913
30	-1.6563	-1.197	-0.9633	1.007	1.4842	2.8249
35	-1.5217	-1.0807	-0.8538	1.1309	1.6884	3.1401
40	-1.3643	-0.926	-0.7355	1.2301	1.7746	3.0854
45	-1.2075	-0.8482	-0.6432	1.307	1.8649	3.0055
50	-1.1091	-0.7636	-0.5834	1.373	1.8875	3.2362

Table 2 : Power estimates for $M_n(F)$ statistic for testing $BFR - t_0$ and $WFR - t_0$ ageing properties.

Distribution	Par.	Sample size				
		10	20	30	40	50
F (Gamma)	2	0.009	0.012	0.014	0.016	0.021
percentage of Delt=1		62	62	63	62	62
F (Gamma)	3	0.162	0.223	0.271	0.272	0.347
percentage of Delt=1		50	50	50	50	50
F (Gamma)	4	0.562	0.613	0.652	0.665	0.691
percentage of Delt=1		40	41	40	40	41
F (Gamma)	5	0.834	0.855	0.862	0.954	1
percentage of Delt=1		32	32	32	32	32
F (Gamma)	6	0.999	1	1	1	1
percentage of Delt=1		26	26	26	26	26
F (Weibull)	0.25	0.08	0.12	0.18	0.3	0.46
percentage of Delt=1		32	35	38	36	37
F (Weibull)	0.5	0	0.02	0.08	0.1	0.18
percentage of Delt=1		40	39	38	38	40
F (Weibull)	2	0.1	0.14	0.2	0.32	0.48
percentage of Delt=1		49	49	51	52	51
F (Weibull)	3	0.11	0.12	0.13	0.14	0.16
percentage of Delt=1		38	42	41	41	40
F (Weibull)	4	0.15	0.19	0.25	0.28	0.35
percentage of Delt=1		35	34	35	35	35
F (Weibull)	5	0.17	0.23	0.47	0.53	0.6
percentage of Delt=1		30	29	29	30	29
F (Weibull)	6	0.26	0.47	0.64	0.72	0.82
percentage of Delt=1		23	23	25	25	25
F (Pareto)	0.25	0.093	0.118	0.094	0.082	0.087
percentage of Delt=1		83	83	83	83	83
F (Pareto)	0.5	0.217	0.238	0.25	0.216	0.228
percentage of Delt=1		81	80	80	80	80
F (Pareto)	2	0.385	0.505	0.569	0.582	0.618
percentage of Delt=1		63	63	63	63	63
F (Pareto)	3	0.325	0.436	0.459	0.506	0.53
percentage of Delt=1		54	55	55	55	54
F (Pareto)	4	0.275	0.321	0.303	0.341	0.371

percentage of Delt=1		*48*	*48*	*48*	*48*	*48*
F (Pareto)	*5*	*0.229*	*0.219*	*0.227*	*0.2*	*0.2*
percentage of Delt=1		*43*	*43*	*43*	*43*	*43*
F (Pareto)	*6*	*0.156*	*0.123*	*0.116*	*0.107*	*0.127*
percentage of Delt=1		*39*	*38*	*38*	*38*	*39*
F (Rayleigh)		*0.013*	*0.121*	*0.147*	*0.153*	*0.16*
percentage of Delt=1		*58*	*58*	*58*	*58*	*58*

Table 3 : Mean and estimated risks for $M_n(F)$ *statistic for testing* $BFR-t_0$ *and* $WFR-t_0$ *ageing properties.*

n	*Mean*	*ER*	*ER/n*	*n*	*Mean*	*ER*	*ER/n*
10	*-0.3321*	*0.9927*	*0.0993*	*31*	*0.5667*	*0.8982*	*0.0290*
11	*-0.2782*	*0.9567*	*0.0870*	*32*	*0.5036*	*0.7378*	*0.0231*
12	*-0.1888*	*0.9000*	*0.0750*	*33*	*0.5329*	*0.7943*	*0.0241*
13	*-0.2058*	*0.7048*	*0.0542*	*34*	*0.5706*	*0.8068*	*0.0237*
14	*-0.0598*	*0.9599*	*0.0686*	*35*	*0.6364*	*0.8101*	*0.0231*
15	*-0.0485*	*0.7991*	*0.0533*	*36*	*0.6030*	*0.7587*	*0.0211*
16	*0.0075*	*0.9237*	*0.0577*	*37*	*0.6182*	*0.7333*	*0.0198*
17	*0.0579*	*0.8699*	*0.0512*	*38*	*0.6362*	*0.7163*	*0.0188*
18	*0.0768*	*0.7815*	*0.0434*	*39*	*0.6821*	*0.7759*	*0.0199*
19	*0.1144*	*0.8101*	*0.0426*	*40*	*0.6886*	*0.9857*	*0.0246*
20	*0.1872*	*0.9038*	*0.0452*	*41*	*0.6787*	*0.7548*	*0.0184*
21	*0.2494*	*0.8292*	*0.0395*	*42*	*0.7043*	*0.8144*	*0.0194*
22	*0.2207*	*0.7853*	*0.0357*	*43*	*0.7457*	*0.7930*	*0.0184*
23	*0.2370*	*0.7266*	*0.0316*	*44*	*0.7248*	*0.7257*	*0.0165*
24	*0.3127*	*0.8435*	*0.0351*	*45*	*0.7847*	*0.8133*	*0.0181*
25	*0.3467*	*0.8647*	*0.0346*	*46*	*0.7854*	*0.7610*	*0.0165*
26	*0.3367*	*0.7591*	*0.0292*	*47*	*0.8035*	*0.8464*	*0.0180*
27	*0.3529*	*0.7320*	*0.0271*	*48*	*0.8510*	*0.8627*	*0.0180*
28	*0.4842*	*0.9700*	*0.0346*	*49*	*0.8423*	*0.8315*	*0.0170*
29	*0.4765*	*0.9224*	*0.0318*	*50*	*0.8303*	*0.8485*	*0.0170*
30	*0.4503*	*0.8198*	*0.0273*				

Table 4 : Test of normality for $M_n(F)$ *statistic for testing* $BFR - t_0$ *and* $WFR - t_0$ *ageing properties*

n	D	n	D
10	0.073	31	0.093
11	0.0835	32	0.1153
12	0.0853	33	0.1037
13	0.0874	34	0.1158
14	0.0794	35	0.0976
15	0.0882	36	0.1144
16	0.0892	37	0.1209
17	0.0894	38	0.1044
18	0.0803	39	0.0915
19	0.0907	40	0.1147
20	0.0774	41	0.0883
21	0.0913	42	0.0891
22	0.0925	43	0.0925
23	0.0917	44	0.113
24	0.099	45	0.1095
25	0.0951	46	0.1096
26	0.0806	47	0.1116
27	0.1002	48	0.0987
28	0.1153	49	0.1232
29	0.0963	50	0.0809
30	0.0784		

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