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Nigh-Open Sets in Topological Space

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Abstract. In this paper, we aim to introduce a new class of open sets namely nigh-open set. Accordingly, we define a topological space called a nigh-topological space. This consequently leads us to outline several new operations in connection with the sets in nigh-topological space coupled with deriving several their properties and relations.

1. Introduction

A topology on a non-empty set $\daleth$ defined as a collection $\aleph$ of subset of $\daleth$ called open sets. These sets commonly satisfy three main axioms; the empty set $\daleth$ itself belong to $\aleph$, any arbitrary (finite or infinite) union of member of $\aleph$ belongs to $\aleph$, and finally that the intersection of any finite number of member of $\aleph$ belongs to $\aleph$ [1].

In regard with the literature about this topic, we point out that all results reported in this work are not addressed at all. However, we state some other related works for completeness. For instance, R. A. Hosny et al. provided a definition of r-neighborhood (open set) related to the usual topology in [2]. In [3], A. S. Salama et al. used a novel type of open sets called rough open set, which was used to define the so-called rough continuous functions. In [4, 5], J. Oudetallah studied other type of open sets called D-open sets, which were employed to define the D-metacompact spaces. Some other related works can be found in [6–9].

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In this research paper, we intend to define a new topological space namely a nigh-topological space such that the member of the collection $\aleph_{(n)}$ is a nigh-open set. Such a set is defined as a nigh-open set ς if there exist two open sets ϱ and ϑ such that $\vartheta \subseteq \varsigma \subseteq Ext(\varrho)$ and $\varrho \cap \varsigma = \phi$. Consequently, we define the complement of this nigh-open set; the nigh-closed set. Furthermore, based on some basic topological meanings, we define a nigh-closure of a set nigh-derive sets nigh-interior, night-exterior and nigh-boundary set. Several results and theorems related to the properties and relations between these sets are studied and derived in a well-defined topological space.

2. Basic definitions

In this section, we aim to pave the way to our main results by recalling two significant definitions. These definitions are connected with the concept of the topological space and the concept of regular open/closed/semi set.

Definition 2.1. *[10] A topological space is a pair $(\beth, \aleph)$ consisting of a set $\beth$ and a family $\aleph$ of subsets of $\beth$ satisfying the following conditions:*

1. *$\phi \in \aleph$ and $\beth \in \aleph$.*
2. *The union of any numbers of members in $\aleph$ is a member in $\aleph$.*
3. *The intersection of any two members in $\aleph$ is a member in $\aleph$.*

Definition 2.2. *[11, 12] Suppose $A \subseteq \beth$ and $(\beth, \tau)$ is a topological space. We say that:*

1. *A is a regular open set in $\beth$ if $A = \overline{A}^{o}$.*
2. *A is regular closed set in $\beth$ if and only if $A = \overline{A^{o}}$.*
3. *A is a semi open set in $\beth$ if and only if there exists an open set U such that $U \subseteq A \subseteq \bar{U}$.*

3. Formulations

This part aims to lay the groundwork for understanding and theorizing the nigh-open set and the nigh-topology. As a result, some results are reported for further investigation in this study.

Definition 3.1. *Let $(\beth, \aleph)$ be a topological space and ς be a subset of $\beth$. Then ς is said to be a nigh-open set if there exist two open sets ϱ and ϑ such that $\vartheta \subseteq \varsigma \subseteq Ext(\varrho)$ and $\varrho \cap \varsigma = \phi$. The complement of a nigh-open set is called nigh-closed set.*

Remark 3.1. *From the above definition, one might assert:*

1. *$\varrho \cap \vartheta = \phi$.*
2. *ϑ is called the first open set and ϱ is called the second open set.*

Based on what we previously established, we state and prove the next results.

Theorem 3.1. *Every open set in any topological space is a nigh-open set.*

Proof. To prove this result, we first let ς be an open set in the topological space $(\beth, \aleph)$. Consequently, $\varsigma \subseteq \varsigma \subseteq Ext(\phi)$, $\varsigma \cap = \phi = \phi$. Thus ς is a nigh-open set. □

Remark 3.2. *The converse of the above theorem need not be true. For example, if we take the set* $[1, 2]$ *in the usual topology defined on* $\mathbb{R}$, *we find that this set represents a nigh-open set. This is because there exist two open sets* $(0, 1)$ *and* $(4, 5)$ *such that* $(0, 1) \subseteq (1, 2] \subseteq Ext(4, 5)$. *But it is clearly the set* $(1, 2]$ *is not open set in the usual topology.*

Theorem 3.2. *If* ς *is a nigh-open set in the topological space* $(\beth, \aleph)$, *then* $(\overline{\varrho}) \subset Ext(\varsigma) \subseteq Ext(\vartheta)$, *where* ϑ *and* ϱ *are the first and second open sets reported in Remark 3.1, respectively.*

Proof. Let ς be a nigh-open set in $\beth$, then there exist two open sets ϑ and ϱ such that $\vartheta \subseteq \varsigma \subseteq Ext(\varrho)$ and $\varrho \cap \vartheta = \phi$. Thus, we have:

$$\overline{\varsigma} \subseteq \overline{Ext(\varrho)} = \bigcap_{\substack{F \text{ closed} \\ Ext(\varrho) \subseteq F}} F.$$

Consequently, we obtain:

$$\bigcup_{\substack{F^c \text{ open set} \\ F^c \subset (Ext(\varrho))^c}} F^c = \overline{\varrho} \subset \overline{\varsigma}^c = Ext(\varsigma).$$

Now, putting $w = F^c$ leads to assert that w is an open set and accordingly we get:

$$\bigcup_{\substack{w \text{ open} \\ w \subset \overline{\varrho}}} w \subset Ext(\varsigma).$$

But we have:

$$\bigcup_{\substack{w \text{ open} \\ w \subset \overline{\varrho}}} w = Int(\varsigma),$$

and since $\vartheta \subseteq \varsigma$, then $Ext(\varsigma) \subseteq Ext(\varrho)$. This gives $Int(\overline{\varrho} \subseteq Ext(\varsigma) \subseteq Ext(\vartheta)$, which finishes the proof of this result. □

In what follow, based on the main axioms founded for the traditional topology, we establish one of the most targets of this work; the nigh-topological space. This would help us to derive further results in the upcoming section.

Definition 3.2. *Let* $\beth$ *be a non-empty set and* $\aleph \subseteq p(\beth)$. *Then an* $\aleph$ *is said to be a nigh-topology on* $\beth$ *if the following statements are hold:*

(1) $\phi, \beth \in \aleph$.
(2) *The intersection of any two nigh-open sets is a nigh-open set.*
(3) *The union of any family of nigh-open sets is a nigh-open set.*

4. Results

In this section, several novel relations and properties in respect to the nigh-open sets and the nigh-topological space are stated and derived well.

Theorem 4.1. *Every topological space is a nigh-topological space.*

Proof. In order to prove this result, we let $(\beth, \aleph)$ be a topological space. To show that $(\beth, \aleph)$ is a nigh-topological space, we should note the following points:

(1) By Definition 2.1, it should be noticed that $\beth, \phi \in \aleph$.

(2) If one lets ς, ϖ be two nigh-open sets, then there exist the open sets $\vartheta_1, \vartheta_2, \varrho_1$ and ϱ_2 such that:

$$\vartheta_1 \subseteq \varsigma \subseteq Ext(\varrho_1, \vartheta_2 \subseteq B \subseteq Ext(\varrho_2).$$

This consequently implies:

$$\vartheta_1 \cap \vartheta_2 \subseteq \varsigma \cap \varpi \subseteq Ext(\varrho_1) \cap Ext(\varrho_2) = Ext(\varrho_1 \cap \varrho_2).$$

Putting $\vartheta_1 \cap \vartheta_2 = \vartheta$ and $\varrho_1 \cap \varrho_2 = \varrho$ yields to assert that ϑ and ϱ are two open sets. This means that $\vartheta \subseteq \varsigma \cap \varpi \subseteq Ext(\varrho)$ and so $\varsigma \cap \varpi$ is a nigh-open set.

(3) If one lets $\varsigma = \{\varsigma_\alpha : \alpha \in \Lambda\}$ be a nigh-open set, then to show that $\cup\varsigma_\alpha$ is a nigh-open set for every $\alpha \in \Lambda, \varsigma_\alpha$, we should note that there exist two open sets ϑ_α and ϱ_α such that $\vartheta_\alpha \subseteq \varsigma_\alpha \subseteq Ext(\varrho_\alpha)$ for every $\alpha \in \Lambda$. Also, we have:

$$\bigcup_{\alpha\in\Lambda} \vartheta_\alpha \subseteq \bigcup_{\alpha\in\Lambda} \varsigma_\alpha \subseteq \bigcup_{\alpha\in\Lambda} Ext(\varrho_\alpha) \subseteq Ext(\phi).$$

Thus, we deduce that $\bigcup_{\alpha\in\Lambda} \varsigma_\alpha$ is a nigh-open set.

□

Definition 4.1. *Let $(\beth, \aleph_{(n)})$ be a nigh-topological space and ς be a subset of $\beth$, then:*

(1) *$\iota \in \beth$ is called a nigh-limit point of a set ς if for any nigh-open set ϱ containing n, we have:* $\begin{cases} \varrho \cap \varsigma_n \neq \phi, & \text{if } \iota \notin \varsigma \\ \varrho \cap \varsigma_n \backslash_{\{\iota\}} \neq \phi, & \text{if } \iota \in \varsigma \end{cases}$.

(2) *The nigh-derived set of ς symbolized by ς'_n is defined by $\varsigma'_{(n)} = \{\varrho \in \beth :$ ι is nigh-limite point of $\varsigma\}$.*

(3) *The nigh-closure set of ς symbolized by $CL_{(n)}(\varsigma)$ is defined by $CL_{(n)}(\varsigma) = \varsigma \cup \varsigma'_{(n)}$.*

(4) *The nigh-exterior set of ς is defined by $Int_{(n)}(\varsigma) = \left(CL_{(n)}(\varsigma^c)\right)^c$.*

(5) *The nigh-exterior set of ς is defined by $Ext_{(n)}(\varsigma)$ and $Ext_{(n)}(\varsigma) = \left(CL_{(n)}(\varsigma)\right)^c$.*

(6) *The nigh-boundary set of ς is defined by $Bd_{(n)}(\varsigma)$ and $Bd_{(n)} = CL_{(n)}(\varsigma) \cap CL_{(n)}(\varsigma^c)$.*

Theorem 4.2. *(Initial nigh-theorem) Let $(\beth, \aleph_{(n)})$ be a nigh-topological space, then:*

(1) $\phi'_{(n)} = \phi$.

(2) $CL_{(n)}(\phi) = \phi$ *and* $CL_{(n)}(\beth) = \beth$.
(3) $Int_{(n)}(\phi) = \phi$ *and* $Int_{(n)}(\beth) = \beth$.
(4) $Ext_{(n)}(\phi) = \beth, Ext_{(n)}(\beth) = \phi$.

Proof. (1) To prove this result, we suppose not, then there is $\iota \in \beth$ such that $\iota \in \phi'_{(n)}$. So $\beth$ is a limit point of $\phi_{(n)}$, and then for all nigh-open sets containing $\beth$, we have $\varrho \cap \phi \neq \phi$, which is contradiction! Hence, the result holds.

(2) It should be noted that

$$CL_{(n)}(\phi) = \phi \cup \phi'_{(n)} = \phi \cup \phi = \phi,$$

and

$$CL_{(n)}(\beth) = \beth \cup \beth'_{(n)} = \beth.$$

(3) Herein, we have:

$$Int_{(n)}(\phi) = \left(CL_{(n)}(\phi^c)\right)^c = \left(CL_{(n)}(\beth)\right)^c = \beth^c = \phi,$$

and

$$Int_{(n)}(\beth) = \left(CL_{(n)}(\beth^c)\right)^c = \left(CL_{(n)}(\phi)\right)^c = \phi^c = \beth.$$

(4) In this part, we note:

$$Ext_{(n)}(\phi) = \left(CL_{(n)}(\phi^c)\right)^c = \phi^c = \beth,$$

and

$$Ext_{(n)}(\beth) = \left(CL_{(n)}(\beth^c)\right)^c = \beth^c = \phi.$$

□

Theorem 4.3. *(Inclusion nigh-Theorem) Let* $(\beth, \aleph)$ *be a nigh-topological space. Suppose* ς *and* ϖ *are two subsets of* $\beth$*, then we have:*

(1) *If* $\varsigma \subseteq \varpi$*, then* $\varsigma'_{(n)} \subseteq \varpi'_{(n)}$.
(2) *If* $\varsigma \subseteq \varpi$*, then* $CL_{(n)}(\varsigma) \subseteq CL_{(n)}(\varpi)$.
(3) *If* $\varsigma \subseteq \varpi$*, then* $Int_{(n)} \subseteq Int_{(n)}(\varpi)$.
(4) *If* $\varsigma \subseteq \varpi$*, then* $Ext_{(n)} \subseteq Ext_{(n)}(\varpi)$.

Proof. (1) Let $\iota \in \varsigma'_{(n)}$, so $\beth$ is a nigh-limit point of ς. Therefore, for all nigh-open set ϱ containing n, we have:

$$\begin{cases} \varrho \cap \varsigma_n \neq \phi, & if\ \iota \notin \varsigma \\ \varrho \cap \varsigma_{n)} \backslash_{\{\iota\}} \neq \phi, & if\ \iota \in \varsigma \end{cases}.$$

Now, since $\varsigma \subseteq \varpi$. we have:

$$\begin{cases} \varrho \cap \varpi_n \neq \phi, & if\ \iota \notin \varpi \\ \varrho \cap \varpi_{n)} \backslash_{\{\iota\}} \neq \phi, & if\ \iota \in \varpi \end{cases}.$$

Consequently, $\beth$ is a nigh-limit point of ϖ, and thus $\iota \in \varpi'_{(n)}$.

(2) Since $\varsigma \subseteq \varpi$, then by Theorem 3.1, we have $\varsigma'_{(n)} \subseteq \varpi'_{(n)}$ and $\varsigma \cup \varsigma'_{(n)} \subseteq \varpi \cup \varpi'_{(n)}$. This means $CL_{(n)}(\varsigma) \subseteq CL_{(n)}(\varpi)$.

(3) Since $\varsigma \subseteq \varpi$, then $\varpi^c \subseteq \varsigma^c$. Using Theorem 3.2 yields:

$$CL_{(n)}(\varsigma^c) \subseteq CL_{(n)}(\varsigma^c),$$

and

$$\left(CL_{(n)}(\varsigma^c)\right)^c \subseteq \left(CL_{(n)}(\varpi^c)\right)^c.$$

Consequently, we have $Int_{(n)}(\varsigma) \subseteq Int_{(n)}(\varpi)$.

(4) Since $\varsigma \subseteq \varpi$, then by using theorem 3.2, we have:

$$CL_{(n)}(\varsigma) \subseteq CL_{(n)}(\varpi),$$

and

$$\left(CL_{(n)}(\varpi^c)\right)^c \subseteq \left(CL_{(n)}(\varsigma^c)\right)^c.$$

This means $Ext_{(n)}(\varpi) \subseteq Ext_{(n)}(\varsigma)$, which completes the proof of this result.

□

Theorem 4.4. *(Openness and closeness nigh-theorem) Let $(\beth, \aleph)$ be a nigh-topological space and let $\varsigma \subseteq \beth$, then:*

(1) $CL_{n)}(\varsigma)$ *is a nigh-closed set.*

(2) $Int_{(n)}(\varsigma)$ *is a nigh-open set.*

(3) $Ext_{(n)}(\varsigma)$ *is a nigh-open set.*

(4) $Bd_{(n)}(\varsigma)$ *is a nigh-closed set.*

Proof. (1) To prove this result, we let $\iota \in \left(CL_{(n)}(\varsigma)\right)^c$. Then $\iota \notin CL_{(n)}(\varsigma)$, and so $\iota \notin \varsigma \cup \varsigma'_{(n)}$, $\iota \notin \varsigma$ and $\iota \notin \varsigma'_{(n)}$. Therefore, there exists a nigh-open set ϱ such that $\varrho \cap \varsigma_{(n)} = \phi$ (say *). Consequently, we have:

$$\varrho \cap CL_{(n)(\varsigma)} = \varrho \cap (\varsigma \cap \varsigma'_{(n)}) = (\varrho \cap \varsigma) \cup (\varrho \cap \varsigma'_{(n)} = \phi \cup (\varrho \cap \varsigma'_{(n)}).$$

This means that $\varrho \cap CL_{(n)(\varsigma)} = \varrho \cap \varsigma'_{(n)}$. Now, if $\iota \in (\varrho \cap \varsigma'_{(n)})$, then $\iota \in \varrho$ and $\varrho \cap \varsigma_{(n)} \neq \phi$, which contradicts (*). So, $(\varrho \cap \varsigma'_{(n)}) = \phi$ and hence

$$\varrho \cap CL_{(n)}(\varsigma) = \phi.$$

Thus, we have:

$$\iota \in \varrho_\iota \subseteq \left(CL_{(n)}(\varsigma)\right)^c,$$

which immediately yields:

$$\left(CL_{(n)}(\varsigma)\right)^c = \bigcup_{\substack{\iota \in \varrho_\iota \\ \varrho_\iota \text{is nigh-open set}}} \varrho_\iota,$$

i.e., we have $\left(CL_{(n)}(\varsigma)\right)^c$ is a nigh-open set, and so $CL_{(n)}(\varsigma)$ is a nigh-closed set.

(2) By Theorem 3.1, we have $CL_{(n)}(\varsigma^c)$ is a nigh-closed set. Therefore, $\left(CL_{(n)}(\varsigma)\right)^c$ is a nigh-open set, i.e., $Int_{(n)}(\varsigma)$ is a night-open set.

(3) We notice that $Ext_{(n)}(\varsigma) = \left(CL_{(n)}(\varsigma)\right)^c$, which is a nigh-open set

(4) We notice that $Bd_{(n)}(\varsigma) = CL_{(n)}(\varsigma) \cap CL_{(n)}(\varsigma^c)$, which is a nigh-closed set.

□

Theorem 4.5. *$CL_{(n)}(\varsigma) = \varsigma$ if and only if ς is a nigh-closed set.*

Proof. $\Rightarrow$) Trivial.

$\Leftarrow$) Let ς be a nigh-closed set. Clearly, we have $\varsigma \subseteq CL_{(n)}$ (say *). Now, to show that $CL_{(n)} \subseteq \varsigma$, we let $\iota \in \varsigma'$. To show that $\iota \in \varsigma$, we assume not, i.e. $\iota \notin \varsigma$. This gives $\iota \in \varsigma^c$, which is a nigh-open set. Now, since $\iota \in \varsigma'$, then $\varsigma^c \cap \varsigma \neq \phi$, which means that $\phi \neq \phi$. So, there is a contradiction here, and then $\iota \in \varsigma$. This implies $\varsigma' \subseteq \varsigma$ and $\varsigma \subseteq \varsigma$, and so $\varsigma' \cup \varsigma \subseteq \varsigma$, i.e. $CL_{(n)}(\varsigma) \subset \varsigma$. This completes the proof. □

Theorem 4.6. *(Union and intersection nigh theorem) Let $(\beth, \aleph)$ be a nigh-topological space and $\varsigma \subseteq \beth$, then:*

(1) *$CL_{(n)}(\varsigma) = \bigcap\{\varkappa; \varkappa$ is nigh-closed set and $\varsigma \subseteq \varkappa\}$, i.e. $CL_{(n)}(\varsigma)$ is the smallest nigh-closed set containing ς.*

(2) *$Int_{(n)}(\varsigma) = \cup\{T : T$ is nigh-open set and $T \subseteq \varsigma\}$.*

(3) *$Ext_{(n)}(\varsigma) = \cup\{W : W$ is nigh-open set and $W \subseteq \varsigma^c\}$.*

Proof. (1) To show that $\varsigma \subseteq \varkappa$, we note by Theorem 4.5 that $CL_{(n)}(\varsigma) \subseteq CL_{(n)}(\varkappa) = \varkappa$ and $CL_{(n)}(\varsigma)$ is closed. This implies that $CL_{(n)}(\varsigma)$ is one member of $\varkappa'^s$, and so

$$\bigcap_{\substack{\varkappa \text{ is nigh-closed set} \\ \varsigma \subseteq \varkappa}} \{\varkappa\} \subseteq CL_{(n)}(\varsigma).$$

On the other hand, if we want to show that $\varsigma \subseteq \varkappa$, we should note:

$$CL_{(n)}(\varsigma) \subseteq CL_{(n)}(\varkappa),$$

and

$$CL_{(n)}(\varsigma) \subseteq \bigcap_{\substack{\varkappa \text{ is nigh-closed set} \\ \varsigma \subseteq \varkappa}} \{CL_{(n)}(\varkappa)\} = \bigcap_{\substack{\varkappa \text{ is nigh-closed set} \\ \varsigma \subseteq \varkappa}} \{\varkappa\},$$

which gives directly the desired result.

(2) It should be noted here that $Int_{n)}(\varsigma) = \left(CL_{(n)}(\varsigma^c)\right)^c = \cap\varkappa$ such that *$\varkappa$ is nigh-closed set and $\varsigma^c \subseteq \varkappa$*. This means $\left(CL_{(n)}(\varsigma^c)\right)^c = \cup\varkappa^c$ such that *$\varkappa^c$ is nigh-closed set and $\varkappa^c \subseteq \varsigma$*. By putting $\varkappa^c = T$, we obtain $Int_{(n)}(\varsigma) = \cup\{T :$ *T is nigh-open set and $T \subseteq \varsigma$*$\}$, which completes the proof.

(3) By part 2, we can gain:

$$Ext_{(n)}(\varsigma) = Int_{(n)}(\varsigma^c) = \cup\{W : W \text{ is nigh-open set and } W \subseteq \varsigma^c\}.$$

□

5. Conclusions

This work has successfully defined a nigh-open set as well as its corresponding topological space; the nigh-topological space. Several novel results and properties related to these new notions have been consequently generated and derived well.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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