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CONVERGENCE THEOREMS OF AN IMPLICIT ITERATES WITH ERRORS FOR TOTAL ASYMPTOTICALLY PSEUDO-CONTRACTIVE MAPPINGS

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Abstract. The goal of this paper is to establish weak and strong convergence theorems of an implicit iteration process with errors to converge to common fixed points for a finite family of uniformly L-Lipschitzian total asymptotically pseudo-contractive mappings in the framework of Banach spaces. Our results extend the corresponding result of [2, 5, 8, 10] and many others.

1. **Introduction and Preliminaries**

In recent years, the implicit iteration scheme for approximating fixed point of nonlinear mappings has been introduced and studied by various authors (see, e.g., [1, 4, 7, 10, 11]).

In 2001, Xu and Ori [11] have introduced an implicit iteration process for a finite family of nonexpansive mappings in a Hilbert space H. Let C be a nonempty subset of H. Let $T_1, T_2, \dots, T_N$ be self-mappings of C and suppose that $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$, the set of common fixed points of $T_i, i = 1, 2, \dots, N$. An implicit iteration process for a finite family of nonexpansive mappings is defined as follows, with $\{t_n\}$ a real sequence in $(0, 1)$, $x_0 \in C$:

$$\begin{aligned} x_1 &= t_1 x_0 + (1 - t_1) T_1 x_1, \\ x_2 &= t_2 x_1 + (1 - t_2) T_2 x_2, \\ &\vdots \\ x_N &= t_N x_{N-1} + (1 - t_N) T_N x_N, \\ x_{N+1} &= t_{N+1} x_N + (1 - t_{N+1}) T_1 x_{N+1}, \\ &\vdots \end{aligned}$$

which can be written in the following compact form:

$$x_n = t_n x_{n-1} + (1 - t_n) T_n x_n, \quad n \geq 1 \tag{1.1}$$

where $T_k = T_{k \bmod N}$. (Here the mod N function takes values in $\{1, 2, \dots, N\}$). And they proved the weak convergence of the process (1.1).

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In 2002, Zhou and Chang [12] introduced the following implicit iteration scheme for common fixed points of a finite family of asymptotically nonexpansive mappings $\{T_i\}_{i=1}^N$ in Banach space:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T^n_{n\ (mod\ N)} x_n, \quad n \geq 1 \tag{1.2}$$

By this implicit iteration scheme, Zhou and Chang proved some weak and strong convergence theorems in Banach spaces for a finite family of asymptotically nonexpansive mappings.

In 2003, Sun [10] modified the implicit iteration process of Xu and Ori [11] and applied the modified averaging iteration process for the approximation of fixed points of asymptotically quasi-nonexpansive mappings. Sun introduced the following implicit iteration process for common fixed points of a finite family of asymptotically quasi-nonexpansive mappings $\{T_i\}_{i=1}^N$ in Banach spaces:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_i^k x_n, \quad n \geq 1, \tag{1.3}$$

where $n = (k-1)N + i$, $i \in I = \{1, 2, \ldots, N\}$.

In this paper, we propose the following implicit iteration process with errors for a finite family of total asymptotically pseudo-contractive mappings $\{T_i\}_{i=1}^N$ and prove some strong convergence theorems for said mappings and iteration scheme in Banach spaces. The results presented in this paper extend the corresponding results of Chang [2], Miao et al. [5], Sun [10], Osilike and Akuchu [8] and many others. The proposed implicit iteration scheme is as follows: $x_1 \in C$ and

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n^n x_n + u_n, \quad \forall n \geq 1, \tag{1.4}$$

where C is a closed convex subset of a Banach space E with $C+C \subset C$, $T_n^n = T^n_{n\ (mod\ N)}$ and $\{u_n\}$ is a bounded sequence in C.

Definition 1.1. ([5]) A mapping $T\colon C \to C$ is said to be total asymptotically pseudo contractive if there exists a nonnegative real sequence $\{\mu_n\}$, $n \geq 1$, with $\mu_n \to 0$ as $n \to \infty$ and there exists a strictly function $\phi\colon R^+ \to R^+$ with $\phi(0) = 0$ such that for all $x, y \in C$,

$$\langle T^n x - T^n y, j(x-y)\rangle \leq \left\|x-y\right\|^2 + \mu_n \phi(\left\|x-y\right\|). \tag{1.5}$$

Remark 1.2. If $\phi(\lambda) = \lambda^2$, then (1.5) reduces to

$$\langle T^n x - T^n y, j(x-y)\rangle \leq (1+\mu_n)\left\|x-y\right\|^2. \tag{1.6}$$

The total asymptotically pseudo contractive mappings coincide with asymptotically pseudo contractive mappings. If $\mu_n = 0$ for all $n \geq 1$, we obtain from (1.5) the class of mappings that includes the class of pseudo contractive mappings.

Note. The idea of Definition 1.1 is to unify various definitions of classes of mappings associated with the class of asymptotically pseudo contractive mappings and which are extensions of pseudo contractive mappings.

Observe that if C is a nonempty closed convex subset of a real Banach space E with $C + C \subset C$ and $\{T_i\}_{i=1}^N\colon C \to C$ be N uniformly L_i-Lipschitzian total asymptotically pseudo-contractive mappings. If $(1-\alpha_n)L < 1$, where $L = \max\{L_i : i =$

$1, 2, \ldots, N\}$, then for given $x_n \in C$, the mapping $W_n : C \to C$ defined by

$$W_n(x) = \alpha_n x_{n-1} + (1-\alpha_n)T_n^n x + u_n, \ \forall n \geq 1, \tag{1.7}$$

is a contraction mapping. In fact, the following are observed

$$\begin{aligned} \|W_n x - W_n y\| =& \|\alpha_n x_{n-1} + (1-\alpha_n)T_n^n x \\ &+ u_n - (\alpha_n x_{n-1} + (1-\alpha_n)T_n^n y + u_n)\| \\ =& (1-\alpha_n)\left\|T_n^n x - T_n^n y\right\| \\ \leq& (1-\alpha_n)L\left\|x - y\right\|, \ \forall x, \ y \in C. \end{aligned} \tag{1.8}$$

Since $(1-\alpha_n)L < 1$ for all $n \geq 1$, hence $W_n : C \to C$ is a contraction mapping. By Banach contraction mapping principle, there exists a unique fixed point $x_n \in C$ such that

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n^n x_n + u_n, \ \forall n \geq 1. \tag{1.9}$$

Therefore, if $(1-\alpha_n)L < 1$ for all $n \geq 1$, then the iterative sequence (1.4) can be employed for the approximation of common fixed points for a finite family of uniformly L_i-Lipschitzian total asymptotically pseudo-contractive mappings.

Recall that a Banach space E satisfies the *Opial's condition* [6] if for each sequence $\{x_n\}$ in E weakly convergent to a point x and for all $y \neq x$

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \left\|x_n - y\right\|.$$

The examples of Banach spaces which satisfy the Opial's condition are Hilbert spaces and all $L^p[0, 2\pi]$ with $1 < p \neq 2$ fail to satisfy Opial's condition [6].

Let C be a nonempty closed convex subset of a Banach space E. Then $I - T$ is demiclosed at zero if, for any sequence $\{x_n\}$ in C, condition $x_n \to x$ weakly and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ implies $(I - T)x = 0$.

Recall that a family $\{T_i\}_{i=1}^N : C \to C$ with $\mathcal{F} = \cap_{i=1}^N F(T_i) \neq \emptyset$ is said to satisfy *Condition* (B) [3] on C if there is a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for all $r \in (0, \infty)$ such that for all $x \in C$

$$\max_{1\leq i\leq N} \left\{ \|x - T_i x\| \right\} \geq f(d(x, \mathcal{F})).$$

In the sequel we need the following lemma to prove our main results.

Lemma 1.3. *(see* [9]*) Let $\{a_n\}_{n=1}^\infty$, $\{b_n\}_{n=1}^\infty$ and $\{c_n\}_{n=1}^\infty$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} \leq (1 + b_n)a_n + c_n, \quad n \geq 1.$$

If $\sum_{n=1}^\infty b_n < \infty$ and $\sum_{n=1}^\infty c_n < \infty$, then $\lim_{n\to\infty} a_n$ exists. If in addition $\{a_n\}_{n=1}^\infty$ has a subsequence which converges strongly to zero, then $\lim_{n\to\infty} a_n = 0$.

2. **Main Results**

Theorem 2.1. *Let E be a real Banach space. Let C be a closed convex subset of E with $C + C \subset C$ and $\{T_i\}_{i=1}^N$ be a finite family of uniformly L_i-Lipschitzian total asymptotically pseudo contractive self mappings of C into itself such that $\mathcal{F} = \cap_{i=1}^N F(T_i) \neq \emptyset$ is closed. Let $L = \max\{L_i : i = 1, 2, \ldots, N\}$, $\sum_{n=1}^{\infty} \|u_n\| < \infty$, $\sum_{n=1}^{\infty} \mu_n < \infty$, and suppose that there exist $K_i > 0$ such that $\phi_i(\lambda_i) \leq K_i\lambda_i$, $i = 1, 2, \ldots, N$. Given $x_1 \in C$, let $\{x_n\}_{n=1}^{\infty}$ be the sequence generated by an implicit iteration scheme (1.4). If $\{\alpha_n\}$ is chosen such that $\alpha_n \in (0, 1)$ with $0 < \tau < \alpha_n < 1$, where τ is some constant, then $\{x_n\}$ strongly to a common fixed point of the family $\{T_i\}_{i=1}^N$ if and only if $\liminf_{n\to\infty} d(x_n, \mathcal{F}) = 0$, where $d(x, \mathcal{F})$ denotes the distance between x and the set $\mathcal{F}$.*

Proof. The necessity is obvious and so it is omitted. Now, we prove the sufficiency. For any $p \in \mathcal{F} = \cap_{i=1}^N F(T_i)$, from (1.4) and (1.5), we have

$$\begin{aligned}
\left\|x_n - p\right\|^2 &= \left\|\alpha_n x_{n-1} + (1-\alpha_n)T_n^n x_n + u_n - p\right\|^2 \\
&= \alpha_n\langle x_{n-1} - p, j(x_n - p)\rangle + (1-\alpha_n)\langle T_n^n x_n - p, j(x_n - p)\rangle \\
&\quad + \langle u_n, j(x_n - p)\rangle \\
&\leq \alpha_n \left\|x_{n-1} - p\right\| \left\|x_n - p\right\| + (1-\alpha_n)[\left\|x_n - p\right\|^2 \\
&\quad + \mu_n\phi(\left\|x_n - p\right\|)] + \|u_n\| \left\|x_n - p\right\| \\
&\leq \left\|x_{n-1} - p\right\| \left\|x_n - p\right\| + \frac{(1-\alpha_n)}{\alpha_n}\mu_n\phi(\left\|x_n - p\right\|) \\
&\quad + \frac{1}{\alpha_n}\|u_n\| \left\|x_n - p\right\| \\
&\leq \left\|x_{n-1} - p\right\| \left\|x_n - p\right\| + \frac{(1-\tau)}{\tau}\mu_n\phi(\left\|x_n - p\right\|) \\
&\quad + \frac{1}{\tau}\|u_n\| \left\|x_n - p\right\|. \qquad (2.1)
\end{aligned}$$

Simplify both the sides of above inequality, we get

$$\begin{aligned}
\left\|x_n - p\right\| &\leq \left\|x_{n-1} - p\right\| + \frac{(1-\tau)}{\tau}\mu_n\frac{\phi(\left\|x_n - p\right\|)}{\left\|x_n - p\right\|} + \frac{\|u_n\|}{\tau} \\
&\leq \left\|x_{n-1} - p\right\| + \theta_n, \qquad (2.2)
\end{aligned}$$

where

$$\theta_n = \frac{(1-\tau)}{\tau}\mu_n\frac{\phi(\left\|x_n - p\right\|)}{\left\|x_n - p\right\|} + \frac{\|u_n\|}{\tau}.$$

Since ϕ is an strictly increasing continuous function, by hypothesis, there exists K such that $\frac{\phi(\|x_n - p\|)}{\|x_n - p\|} \leq K$ and by the assumptions of the theorem we know that $\sum_{n=1}^{\infty} \mu_n < \infty$ and $\sum_{n=1}^{\infty} \|u_n\| < \infty$, it follows that $\sum_{n=1}^{\infty} \theta_n < \infty$. Then from (2.2), we have

$$d(x_n, \mathcal{F}) \leq d(x_{n-1}, \mathcal{F}) + \theta_n. \qquad (2.3)$$

By Lemma 1.3, we know that $\lim_{n\to\infty} d(x_n, \mathcal{F})$ exists. Because $\liminf_{n\to\infty} d(x_n, \mathcal{F}) = 0$, then

$$\lim_{n\to\infty} d(x_n, \mathcal{F}) = 0. \tag{2.4}$$

Next we prove that $\{x_n\}$ is a Cauchy sequence in C. It follows from (2.2) that for any $m \geq 1$, for all $n \geq n_0$ and for any $p \in \mathcal{F}$, we have

$$\begin{aligned} \left\|x_{n+m} - p\right\| &\leq \left\|x_{n+m-1} - p\right\| + \theta_{n+m} \\ &\leq \left\|x_{n+m-2} - p\right\| + [\theta_{n+m} + \theta_{n+m-1}] \\ &\leq \dots \\ &\leq \dots \\ &\leq \left\|x_n - p\right\| + \sum_{k=n+1}^{n+m} \theta_k. \end{aligned} \tag{2.5}$$

So we have

$$\begin{aligned} \|x_{n+m} - x_n\| &\leq \left\|x_{n+m} - p\right\| + \left\|x_n - p\right\| \\ &\leq 2\left\|x_n - p\right\| + \sum_{k=n+1}^{n+m} \theta_k \\ &\leq 2\left\|x_n - p\right\| + \sum_{k=n}^{\infty} \theta_k \end{aligned} \tag{2.6}$$

Then, we have

$$\|x_{n+m} - x_n\| \leq 2d(x_n, \mathcal{F}) + \sum_{k=n}^{\infty} \theta_k, \quad \forall n \geq n_0. \tag{2.7}$$

For any given $\varepsilon > 0$, there exists a positive integer $n_1 \geq n_0$ such that for any $n \geq n_1$,

$$d(x_n, \mathcal{F}) < \frac{\varepsilon}{4} \quad \text{and} \quad \sum_{k=n}^{\infty} \theta_k < \frac{\varepsilon}{2}. \tag{2.8}$$

Thus, when $n \geq n_1$, we have

$$\|x_{n+m} - x_n\| < 2.\frac{\varepsilon}{4} + \frac{\varepsilon}{2} = \varepsilon. \tag{2.9}$$

This implies that $\{x_n\}$ is a Cauchy sequence in C. Thus, the completeness of E implies that $\{x_n\}$ must be convergent. Assume that $\lim_{n\to\infty} x_n = p^*$. Now, we have to show that p^* is a common fixed point of $\{T_i : i = 1, 2, \dots, N\}$, that is we have to show that $p^* \in \mathcal{F}$. Suppose for contradiction that $p^* \in \mathcal{F}^c$ (where $\mathcal{F}^c$ denotes the complement of $\mathcal{F}$). Since $\mathcal{F}$ is a closed subset of E, we have that $d(p^*, \mathcal{F}) > 0$. But for all $p \in \mathcal{F}$, we have

$$\left\|p^* - p\right\| \leq \left\|p^* - x_n\right\| + \left\|x_n - p\right\|, \tag{2.10}$$

which implies that

$$d(p^*, \mathcal{F}) \leq \left\|x_n - p^*\right\| + d(x_n, \mathcal{F}), \tag{2.11}$$

so that, we obtain $d(p^*, \mathcal{F}) = 0$ as $n \to \infty$, which contradicts $d(p^*, \mathcal{F}) > 0$. Thus, p^* is a common fixed point of the mappings $\{T_i : i = 1, 2, \dots, N\}$. This completes the proof. □

Theorem 2.2. *Let E be a real Banach space. Let C be a closed convex subset of E with $C + C \subset C$ and $\{T_i\}_{i=1}^N$ be a finite family of uniformly L_i-Lipschitzian total asymptotically pseudo contractive self mappings of C into itself such that $\mathcal{F} = \cap_{i=1}^N F(T_i) \neq \emptyset$ is closed. Let $L = \max\{L_i : i = 1, 2, \dots, N\}$, $\sum_{n=1}^{\infty} \|u_n\| < \infty$, $\sum_{n=1}^{\infty} \mu_n < \infty$, and suppose that there exist $K_i > 0$ such that $\phi_i(\lambda_i) \leq K_i\lambda_i$, $i = 1, 2, \dots, N$. Given $x_1 \in C$, let $\{x_n\}_{n=1}^{\infty}$ be the sequence generated by an implicit iteration scheme (1.4). If $\{\alpha_n\}$ is chosen such that $\alpha_n \in (0, 1)$ with $0 < \tau < \alpha_n < 1$, where τ is some constant, then $\{x_n\}$ converges strongly to a common fixed point p^* of the family of mappings $\{T_i\}_{i=1}^N$ if and only if there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ which converges to p^*.*

Proof. The proof of Theorem 2.2 follows from Lemma 1.3 and Theorem 2.1. This completes the proof. □

Theorem 2.3. *Let E be a real Banach space. Let C be a closed convex subset of E with $C + C \subset C$ and $\{T_i\}_{i=1}^N$ be a finite family of uniformly L_i-Lipschitzian total asymptotically pseudo contractive self mappings of C into itself such that $\mathcal{F} = \cap_{i=1}^N F(T_i) \neq \emptyset$ is closed. Let $L = \max\{L_i : i = 1, 2, \dots, N\}$, $\sum_{n=1}^{\infty} \|u_n\| < \infty$, $\sum_{n=1}^{\infty} \mu_n < \infty$, and suppose that there exist $K_i > 0$ such that $\phi_i(\lambda_i) \leq K_i\lambda_i$, $i = 1, 2, \dots, N$. Given $x_1 \in C$, let $\{x_n\}_{n=1}^{\infty}$ be the sequence generated by an implicit iteration scheme (1.4). If $\{\alpha_n\}$ is chosen such that $\alpha_n \in (0, 1)$ with $0 < \tau < \alpha_n < 1$, where τ is some constant. Assume that $\lim_{n\to\infty} \|x_n - T_ix_n\| = 0$, for all $i \in I = \{1, 2, \dots, N\}$. Suppose $\{T_i : i = 1, 2, \dots, N\}$ satisfies condition (B), then the sequence $\{x_n\}$ converges strongly to a common fixed point of the mappings $\{T_i : i = 1, 2, \dots, N\}$.*

Proof. By assumption $\lim_{n\to\infty} \|x_n - T_ix_n\| = 0$, for all $i \in I = \{1, 2, \dots, N\}$. Since $\{T_i : i = 1, 2, \dots, N\}$ satisfies condition (B), so condition (B) guarantees that $\lim_{n\to\infty} f(d(x_n, \mathcal{F})) = 0$. Since f is a non-decreasing function and $f(0) = 0$, it follows that $\lim_{n\to\infty} d(x_n, \mathcal{F}) = 0$. Therefore, Theorem 2.1 implies that $\{x_n\}$ converges strongly to a point in $\mathcal{F}$. This completes the proof. □

Theorem 2.4. *Let E be a real Banach space satisfying Opial's condition and C be a weakly compact subset of E with $C + C \subset C$. Let $\{T_i\}_{i=1}^N$ be a finite family of uniformly L_i-Lipschitzian total asymptotically pseudo contractive self mappings of C into itself such that $\mathcal{F} = \cap_{i=1}^N F(T_i) \neq \emptyset$ is closed. Let $L = \max\{L_i : i = 1, 2, \dots, N\}$, $\sum_{n=1}^{\infty} \|u_n\| < \infty$, $\sum_{n=1}^{\infty} \mu_n < \infty$, and suppose that there exist $K_i > 0$ such that $\phi_i(\lambda_i) \leq K_i\lambda_i$, $i = 1, 2, \dots, N$. Given $x_1 \in C$, let $\{x_n\}_{n=1}^{\infty}$ be the sequence generated by an implicit iteration scheme (1.4) and the sequence $\{\alpha_n\}$ is chosen such that $\alpha_n \in (0, 1)$ with $0 < \tau < \alpha_n < 1$, where τ is some constant. Suppose that $\{T_i : i = 1, 2, \dots, N\}$ has a common fixed point, $I - T_i$ for all $i \in I = \{1, 2, \dots, N\}$ is demiclosed at zero and $\{x_n\}$ is an approximating common fixed point sequence for T_i, that is, $\lim_{n\to\infty} \|x_n - T_ix_n\| = 0$, for all $i \in I = \{1, 2, \dots, N\}$. Then the sequence $\{x_n\}$ defined by (1.4) converges weakly to a common fixed point of the mappings $\{T_i : i = 1, 2, \dots, N\}$.*

Proof. First, we show that $\omega_w(x_n) \subset \mathcal{F}$. Let $x_{n_k} \to x$ weakly. By assumption, we have $\lim_{n\to\infty} \|x_n - T_ix_n\| = 0$. Since $I - T_i$, for all $i \in I = \{1, 2, \dots, N\}$ is demiclosed at zero, $x \in \mathcal{F}$. By Opial's condition, $\{x_n\}$ possesses only one weak limit point, that is, $\{x_n\}$ converges weakly to a common fixed point of $\{T_i\}_{i=1}^N$. This completes the proof. □

Remark 2.5. Theorem 2.1 extends the corresponding result of Chang [2] to the case of more general class of asymptotically nonexpansive mappings and implicit iteration scheme with errors considered in this paper.

Remark 2.6. Theorem 2.1 also extends the corresponding result of Miao et al. [5] to the case of implicit iteration scheme with errors considered in this paper.

Remark 2.7. Theorem 2.1 also extends the corresponding result of Sun [10] to the case of more general class of asymptotically quasi nonexpansive mapping and implicit iteration scheme with errors considered in this paper.

Remark 2.8. Theorem 2.1 also extends the corresponding result of Osilike and Akuchu [8] to the case of more general class of asymptotically pseudo contractive mapping and implicit iteration scheme with errors considered in this paper.

3. **Conclusion**

The class of total asymptotically pseudo contractive mapping is more general than the class of pseudo contractive mapping and it also unify various definitions of classes of mappings associated with the class of asymptotically pseudo contractive mappings. Thus the results obtained in this paper are good improvement and generalization of several known results in the existing literature.

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