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A Vague Rough Set Approach for Failure Mode and Effect Analysis in Medical Accident

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Abstract. Identifying the risk factor is a key to success for the Failure model. This paper presents a new FMEA (Failure Mode and Effect Analysis) in medical accident for the hospital management by using vague rough matrix information captured through vague rough set. Using β-product and β-complement product into fuzzy relation matrix and vague set to find the new lower and upper approximation of vague rough set. This method will provide an easy methodology to find the weight of risk factor. Using an example we validate our proposed approach.

1. Introduction

US aerospace industry introduced the concept of FMEA during 1960. The implementation of this technique can be classified as a risk management strategy and used for mission successes. The primary aim of this methodology is to identify and prioritizes eventual failures modes alongside the targeted system and risk mitigation strategies. The FMEA process is conducted using the risk priority number

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(RPN). The outcome of the calculation is derived through the mathematical operation of multiplying the respective magnitudes of occurrence (O), severity (S) and detection (D).

The application of a systematic and structured approach is evident in various industries, such as manufacturing, engineering, health care, and other fields. This methodology is employed to ascertain and allocate precedence to potential failure modes within a product, process, or system. The application of FMEA allows teams to proactively detect and mitigate potential issues before they occur, there by enhancing the reliability, safety, and overall performance of a product or process. The following is a comprehensive exposition on the functioning of FMEA and its fundamental constituents.

The initial stage of the FMEA involves the identification of components, processes, or systems that necessitate analysis. The entity in question may encompass a product, a manufacturing process, a service, or any other intricate system. The implementation of FMEA commonly involves the formation of a cross-functional team consisting of persons possessing diverse expertise and viewpoints. This practise ensures that multiple facets of the product or the consideration of various processes is undertaken. The methodology of identifying failure modes is conducted by the team, encompassing all potential ways in which the component, process, or system may experience failure. This entails the consideration of both prospective functional failures, i.e., potential malfunctions, and the potential consequences of such failures, i.e., how they might affect the end-user or subsequent operations. The severity ratings are assigned to each identified failure mode using a predetermined scale. The rating provided quantifies the potential consequences of a failure on the end-user, the organization, or the environment. In general, higher severity ratings are indicative of more significant repercussions. The occurrence ratings are assigned by the team with the aim to evaluate and analyse probability or frequency of each failure mode occurring. Occurrence ratings evaluate the frequency at which a failure is anticipated to occur, often measured on a numerical scale ranging from 1 (indicating a low probability) to 10 (indicating a high probability). Detection ratings are utilised to assess the probability of identifying a failure mode prior to it reaching the end-user or resulting in any kind of harm. The presence of low detection ratings implies that the failure in question possesses a level of intricacy that renders it challenging to identify, whilst high ratings show that the failure is more readily discernible. The RPN is determined for each failure mode through the multiplication of severity, occurrence, and detection ratings, resulting in a numerical value. RPN offers a quantitative measure that aids in ascertaining the hierarchy of precedence. for addressing various failure modes. Elevated RPN readings are indicative of increased risk levels and necessitate a heightened focus on mitigation efforts. The team utilizes the RPN values to establish a hierarchy of actions aimed at minimising or preventing failures. Greater RPN levels are indicative of a heightened requirement for prompt attention. Teams have the option to engage in the redesigning of processes, enhancing safety measures, or implementing other steps in order to mitigate the risks associated with failure modes that are of high priority. The team proceeds to adopt preventative and corrective measures in response to the identified activities, with the aim of mitigating the risk associated with failure modes. This may entail the reconfiguration of components, modification of processes, augmentation of training protocols, or revision of quality control measures. The periodic review of FMEA necessitates an ongoing and iterative approach, rather than a singular event. The document ought to undergo regular review and revision in light of emerging data or modifications to the product or process. Periodic evaluations guarantee that potential hazards are consistently assessed and resolved.

The FMEA is an indispensable technique utilized for the enhancement of quality, effective risk management, and the assurance of reliability in products or processes. The implementation of proactive measures enables organisations to effectively identify and proactively resolve possible issues, resulting in improved levels of safety, quality, and customer satisfaction.

Pawlak introduced rough sets [11], and since then, numerous researchers have produced various extensions of this concept. The notions of rough matrix theory were introduced by Vijayabalaji and Balaji [16, 17, 18], who subsequently utilised the aforementioned ideas across numerous domains. Vague Rough Sets can be seen as an expanded version of classical Rough Sets theory, specifically developed to address the challenges posed by ambiguity and uncertainty in a more efficient manner. Fuzzy logic is employed in the field of data processing, specifically in scenarios involving imprecise or missing data. Vague Rough Sets aim to comprehensively collect and represent imprecise or uncertain data in a more nuanced manner as compared to conventional Rough Sets.

The following are few fundamental components of Vague Rough Sets.

The notion of Vague Sets pertains to a mathematical framework that facilitates the depiction and management of imprecise or uncertain data. Concealed within the context of traditional rough set theory, information is classified into crisp sets, where objects are assigned to a particular set based on whether they are considered to belong or not belong to it. In contrast, Vague Sets enable the depiction of progressive or hazy membership. This approach proves to be particularly advantageous when confronted with data that possesses intrinsic uncertainty or imprecision. The concept of granularity in Vague Rough Sets encompasses the consideration of several levels of information granularity. In the larger context of traditional rough sets, the classification of information is often divided into lower and upper approximations. However, Vague Rough Sets introduce the concept of allowing for multiple degrees of granularity that exist between these approximations. The ability to adapt and be versatile is crucial when confronted with data that lacks clarity or precision. Degrees of membership are assigned to objects in Vague Rough Sets with the objective to offer a more comprehensive depiction of the relationship between objects and sets. Instead of employing a binary framework of membership, when objects are either deemed to belong or not belong to sets, it is possible to consider a scenario where objects exhibit partial belonging to sets. Vague Rough Sets offer enhanced procedures for effectively managing incomplete or missing information. The significance of this aspect becomes particularly obvious in practical scenarios when data may exhibit incompleteness or uncertainty. The integration of fuzzy logic is commonly observed in Vague Rough Sets, where concepts like fuzzy membership functions are employed to effectively represent and measure the inherent uncertainty or vagueness present in data. Vague Rough Sets have been applied in diverse domains, including in the field of medical diagnosis, it is common for patient symptoms and test findings to exhibit a degree of vagueness and uncertainty. Vague rough sets have the potential to contribute to the modelling and analysis of such data, hence facilitating accurate diagnosis. In the field of natural language processing, which deals with the inherent vagueness and ambiguity of language, the utilisation of Vague Rough Sets might be beneficial in managing linguistic uncertainty present in textual data. Environmental monitoring is a field that frequently encounters ambiguity and imprecision in the data collected. Vague rough sets have the capability to analyse data and facilitate decision-making processes. In the realm of business, the accuracy and comprehensiveness of client data can sometimes be ambiguous or inaccurate. The application of Vague Rough Sets in the context of customer relationship management enables the generation of informed judgements.

In the field of image and pattern recognition, Vague Rough Sets can be employed for feature analysis when dealing with objects that may only partially conform to a given pattern or demonstrate a certain level of resemblance. Vague Rough Sets broaden the scope of Rough Sets theory by accommodating situations characterised by intrinsic vagueness, uncertainty, or ambiguity in the data. This extension offers a more sophisticated framework for analysing data and making decisions in complex scenarios.

The FMEA methodology has been extensively utilised across many sectors, including aerospace, nuclear, automotive, and health industries. Numerous writers have made significant contributions to the advancement of FMEA. In their study, Geum et al. [6] presented a structured approach for identifying and evaluating probable failures. This approach involved the use of service-specific FMEA along with the grey relational analysis (GRA) tool. Through the use of the intuitionistic fuzzy hybrid weighted Euclidean distance method, Liu et al. [8] originated a risk evaluation and prioritisation method to analyse modes of failure in FMEA. Additionally, Liu et al.[9] incorporated the Interval 2-Tuple Hybrid Weighted Distance and Interval 2-Tuple Linguistic Variables with GRA [10] in their analysis. Chang et al. [2] employed a fuzzy technique and grey theory in order to identify the RPNs. Safari et al. [13] proposed an expansion of the VIKOR method known as the Fuzzy VIKOR-based FMEA model. Emovon et al. [4] effectively examined the failure modes related to marine equipment systems in their research. The methodology commonly referred to as the fuzzy technique for order preference by similarity to ideal solution (TOPSIS) for failure mode and effects analysis (FMEA) was initially introduced by Braglia et al. [1]. The risk assessment problem in Failure Mode and Effects Analysis (FMEA) was addressed by Chang et al.[3] through the development by TOPSIS approach. In their study, Wang et al. [19, 20] developed an enhanced Failure Mode, Effects, and Criticality Analysis (FMECA) feed system for a Computer Numerical Control (CNC) machine. In their study, Liu et al.[7] introduced a unique FMEA model. This methodology incorporates the use of fuzzy digraph and matrix techniques to assess and prioritise the risks associated with different failure modes. Wang et al. [21] pioneered the soft set-based FMEA technique. Song et al. [15] proposed an FMEA model based on rough set theory.

In this research, we put forward a novel hybrid FMEA model known as the vague rough set approach for analysing failure factors in hospital settings, as indicated by the aforementioned analysis. Based on the vague set and fuzzy equivalence matrix, first we construct the vague rough lower approximation and upper approximation values for each failure factors of each expert. Nextwe calculate the roughness measure for each failure factor. Finally we rank the failure factor and give the conclusion.

The present paper is organised in the following manner. In section 2, a fundamental definition pertaining to our methodology is provided. In the third section, we present the construction of our proposed FMEA model and algorithms. An illustrative instance is furnished within section 4, serving as a manifestation of our proposed methodology. The culmination of the research findings is expounded upon in Section 5.

2. Preliminaries

Definition 2.1. [22]: A fuzzy set A is defined by $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0, 1]\}$. In the pair $(x, \mu_A(x))$, the first element x belong to the classical set A, the second element $\mu_A(x)$, belong to the interval $[0, 1]$, called membership function.

Definition 2.2. [5]: Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse, x_i denotes a generic element of U. A vague set A in the universe of discourse U is characterized by a truth-membership function t_A and false-membership function f_A given by $t_A : U \to [0, 1]$ and $f_A : U \to [0, 1]$, where $t_A(x_i)$ is a lower bound on the grade of membership of x_i derived from "the evidence for x_i", $f_A(x_i)$ is a lower bound on the negation of x_i derived from "the evidence against x_i", and $t_A(x_i) + f_A(x_i) \leq 1$. Thus the grade of membership of x_i in the vague set A is bounded to a subinterval $[t_A(x_i), 1 - f_A(x_i)]$ of $[0, 1]$. This indicates that if the actual grade of membership is $\mu_A(x_i)$, then $t_A(x_i) \leq \mu_A(x_i) \leq 1 - f_A(x_i)$. In general, the vague set A is written as $A = \{< x, t_A(x), f_A(x) >: x \in U\}$, where the interval $[t_A(x_i), 1 - f_A(x_i)]$ is called the vague value of x in A.

Definition 2.3. [12]: Using the indiscernibility relation the two operations are defined as $B_*(X) = \{x \in U : B(x) \subseteq X\}$ and $B^*(X) = \{x \in U : B(x) \cap X \neq \phi\}$ assigning to every subset $X \subseteq U$ two sets $B_*(X)$ and $B^*(X)$, called the B-lower and B-upper approximation of X respectively. More over the sets, $Pos_B(X) = B_*(X)$, $Neg_B(X) = U - B^*(X)$, $Bnd_B(X) = B^*(X) - B_*(X)$ are referred as the B-positive, B-negative and B-boundary region of X respectively. If the boundary region of X is empty then X is crisp with respect to B , otherwise the set is rough.

Definition 2.4. [14]: For the finite universes $U = \{x_1, x_2, \ldots, x_n\}$ and $V = \{y_1, y_2, \ldots, y_m\}$ the fuzzy relation $R \in (U, V)$ may be expressed by the fuzzy matrix, i.e., if the element $r_{ij} = R(u_i, v_j)$, then the matrix $R = (r_{ij})_{n\times m}$ represents a fuzzy relation on $U \times V$.

Definition 2.5. [14]: Suppose $\beta : [0,1]\times[0,1] \to [0,1]$, for all $a, b \in [0,1]$, we have

$$a\beta b = \begin{cases} \frac{b}{a} & \text{if} \quad a > b \\ 1 & \text{if} \quad a \leq b \end{cases}$$

the mapping β is called the β-operator and

$$a\beta^* b = \begin{cases} 1 - \frac{b}{a} & \text{if} \quad a > b \\ 0 & \text{if} \quad a \leq b \end{cases}$$

The mapping $\beta^* : [0,1]\times[0,1] \to [0,1]$ is called β-complement operator.

Definition 2.6. [14]: Let $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{n\times s}$ be two fuzzy matrices, the β-product of the matrices A and B is defined as $A\beta B = C = (c_{ij})_{m\times s}$, where $c_{ij} = \wedge_{k=1}^{n}((a_{ik} \bigvee b_{kj})\beta b_{kj})$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, s$.

Definition 2.7. [14]: Let $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{n\times s}$ be two fuzzy matrices, the β-complement product of the matrices A and B is defined as $A\beta^* B = D = (d_{ij})_{m\times s}$, where $d_{ij} = \bigvee_{k=1}^{n}((a_{ik}\beta^*(1 - b_{kj}))$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, s$.

Example 2.8. [14]: Let $A = \begin{bmatrix} 0.3 & 0.5 \\ 0.2 & 0.7 \\ 0.5 & 0.4 \end{bmatrix}$ and $B = \begin{bmatrix} 0.5 & 0.6 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}$ then

$$A\beta B = \begin{bmatrix} \frac{1}{5} & 1 & \frac{2}{5} \\ \frac{1}{7} & 1 & \frac{2}{7} \\ \frac{1}{4} & 1 & \frac{1}{2} \end{bmatrix} \text{ and } A\beta^* B = \begin{bmatrix} 0 & \frac{2}{5} & 0 \\ 0 & \frac{4}{7} & 0 \\ 0 & \frac{1}{4} & 0 \end{bmatrix}.$$

Definition 2.9. [14]: Let $A = \{(x, t_A(x), f_A(x)) : x \in U\}$ be a vague set of the universe set U and R be a fuzzy equivalence relation on U. The lower approximation $\underline{A}$ and the upper approximation $\overline{A}$ of A in the fuzzy approximation space (U, R) are defined as

$\underline{A} = \{(x, (x, t_{\underline{A}}(x), f_{\underline{A}}(x)) : x \in U)\}$ and $\overline{A} = \{(x, (x, t_{\overline{A}}(x), f_{\overline{A}}(x)) : x \in U)\}$, where $\forall\ x \in U$.

$t_{\underline{A}}(x) = \bigwedge_{x\in U}((R(x, y)\bigvee t_A(y))\beta t_A(y))$ and $f_{\underline{A}}(x) = \bigvee_{x\in U}(R(x, y)\beta^*(1 - f_A(y)))$,

$t_{\overline{A}}(x) = \bigvee_{x\in U}(R(x, y)\beta^*(1 - t_A(y)))$ and $f_{\overline{A}}(x) = \bigwedge_{x\in U}((R(x, y)\bigvee f_A(y))\beta f_A(y))$.

Generally, the pair $(\underline{A}, \overline{A})$ can be called vague rough sets in the fuzzy approximation space.

Definition 2.10. [14]: A roughness measure $\rho_A^{\alpha\beta}$ of the vague set A of U with respect to the parameters α, β in the fuzzy approximation space (U, R), is defined as
$\rho_A^{\alpha\beta} = 1 - \frac{|\underline{A_{\alpha\beta}}|}{|\overline{A_{\alpha\beta}}|}$. Especially, $\rho_A^{\alpha\beta} = 0$ when $|\overline{A_{\alpha\beta}}| = 0$.

3. The Suggestive FMEA Paradigm

This section presents a novel risk prioritisation model for FMEA that makes use of vague set theory and rough set called fuzzy rough approximation model. Figure.1 depicts the flowchart of the proposed FMEA algorithm. Consider a multidisciplinary FMEA group, including l team members TM_k $(k = 1, 2, \ldots, l)$ has to asses m failure modes FM_i $(i = 1, 2, \ldots, m)$ based on n risk factors RF_j $(j = 1, 2, \ldots, n)$. The specific stages of the proposed FMEA methodology are outlined.

Step 1. The primary aims of risk assessment encompass the identification and evaluation of prospective risks, as well as the determination of their likelihood and potential impact. Furthermore, the development of strategies to effectively reduce or manage these risks is a crucial component of the risk assessment process.

Step 2. To initiate the establishment of a FMEA team, it is important to compile a comprehensive list of likely failure modes.

Step 3. To ascertain the fuzzy relation matrix for each failure scenario, it is necessary to perform an analysis.

Step 4. Construct expert opinions in the form of vague set for each failure modes.

Step 5. Construct fuzzy rough set (lower and upper approximations in the form of vague values) on each failure modes.

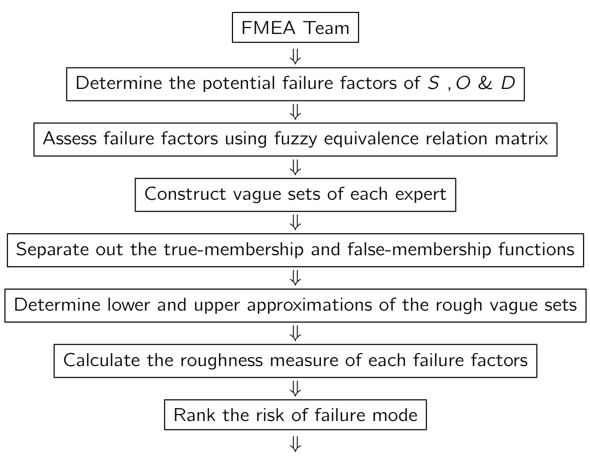

Figure 1. Conceptualized FMEA framework flowchart

Step 6. Tabulate experts' opinions values of O, S & D in the form of vague rough set.

Step 7. Find the roughness measure on each failure modes.

Step 8. Rank the roughness measures.

Step 9. Conclusion.

4. An Example

The application of FMEA might prove to be a valuable asset within the healthcare sector, specifically when considering patient safety and the mitigation of medical mishaps. The FMEA methodology is employed in a systematic manner to identify and assign priority to potential failure modes present in healthcare processes and systems. This approach aims to comprehensively understand the causes and implications of various failure patterns, afterwards adopt preventative measures to mitigate the risk of medical errors and accidents. This paper discusses the application of FMEA as a preventive measure for medical incidents.

The integration of FMEA into the organizational culture of healthcare institutions is vital in order to uphold their dedication to ensuring patient safety. Through a methodical process of identifying and mitigating potential failure modes, healthcare providers have the ability to decrease the likelihood of medical mishaps, enhance patient outcomes, and elevate the overall quality of care.

Medical risk management uses the recommended FMEA paradigm.

Step 1. The first step involves mitigating medical errors and non-iatrogenic disorders in order to prevent medical accidents. The risk study was conducted by medical professionals, including doctors, anesthetists a, and nurses. A team consisting of two decision makers, referred to as DM1 and DM2, has been established inside the hospital to conduct examination of discovered failure modes. The FMEA team has collected relevant information through interviews, discussions, and the review of public materials.

Step 2. The research team has successfully identified the six primary potential failure modes for the ongoing examination. The failure modes that have been chosen for analysis include arterial gas embolism (FM1), esophageal intubation (FM2), respiratory depression (FM3), inadequate surgical planning (FM4), incorrect blood transfusion (FM5), and visceral injury (FM6).

Step 3. The third stage of the proposed framework involves the identification of risk factors, represented as O, S, and D, using fuzzy relation matrices.

For the fuzzy relation matrix of O is

$$O = \begin{bmatrix} 1 & 0.8 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.8 & 1 & 0.85 & 0.2 & 0.85 & 0.2 \\ 0.8 & 0.85 & 1 & 0.2 & 0.9 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.2 \\ 0.8 & 0.85 & 0.9 & 0.2 & 1 & 0.9 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.9 & 1 \end{bmatrix}$$

For the fuzzy relation matrix of S is

$$S = \begin{bmatrix} 1 & 0.6 & 0.75 & 0.9 & 0.8 & 0.4 \\ 0.6 & 1 & 0.6 & 0.8 & 0.5 & 0.3 \\ 0.75 & 0.6 & 1 & 0.75 & 0.4 & 0.5 \\ 0.9 & 0.8 & 0.75 & 1 & 0.6 & 0.25 \\ 0.8 & 0.5 & 0.4 & 0.6 & 1 & 0.2 \\ 0 & 0.3 & 0.5 & 0.25 & 0.2 & 1 \end{bmatrix}$$

For the fuzzy relation matrix of D is

$$D = \begin{bmatrix} 1 & 0.8 & 0.7 & 0.9 & 0.6 & 0.85 \\ 0.8 & 1 & 0.95 & 0.75 & 0.8 & 0.65 \\ 0.7 & 0.95 & 1 & 0.6 & 0.75 & 0.9 \\ 0.9 & 0.75 & 0.6 & 1 & 0.4 & 0.7 \\ 0.6 & 0.8 & 0.75 & 0.4 & 1 & 0.6 \\ 0.85 & 0.65 & 0.9 & 0.7 & 0.6 & 1 \end{bmatrix}$$

Step 4. The experts opinion of vague values for O are

$$O_{E1} = \left\{ \frac{[0.2,0.5]}{FM1}, \frac{[0.7,0.8]}{FM2}, \frac{[0.4,0.6]}{FM3}, \frac{[0.1,0.4]}{FM4}, \frac{[0.6,0.9]}{FM5}, \frac{[0.2,0.4]}{FM6} \right\}$$

$$O_{E2} = \left\{ \frac{[0.4,0.6]}{FM1}, \frac{[0.3,0.7]}{FM2}, \frac{[0.3,0.6]}{FM3}, \frac{[0.2,0.5]}{FM4}, \frac{[0.4,0.8]}{FM5}, \frac{[0.2,0.5]}{FM6} \right\}$$

The experts opinion of vague values for S are

$$S_{E1} = \left\{ \frac{[0.3,0.6]}{FM1}, \frac{[0.2,0.4]}{FM2}, \frac{[0.4,0.6]}{FM3}, \frac{[0.2,0.6]}{FM4}, \frac{[0.3,0.7]}{FM5}, \frac{[0.4,0.8]}{FM6} \right\}$$

$$S_{E2} = \left\{ \frac{[0.2,0.8]}{FM1}, \frac{[0.3,0.9]}{FM2}, \frac{[0.4,0.6]}{FM3}, \frac{[0.1,0.5]}{FM4}, \frac{[0.4,0.8]}{FM5}, \frac{[0.2,0.7]}{FM6} \right\}$$

The experts opinion of vague values for D are

$$D_{E1} = \left\{ \frac{[0.3,0.6]}{FM1}, \frac{[0.2,0.5]}{FM2}, \frac{[0.3,0.7]}{FM3}, \frac{[0.4,0.8]}{FM4}, \frac{[0.2,0.6]}{FM5}, \frac{[0.3,0.6]}{FM6} \right\}$$

$$D_{E2} = \left\{ \frac{[0.4,0.6]}{FM1}, \frac{[0.5,0.8]}{FM2}, \frac{[0.2,0.5]}{FM3}, \frac{[0.3,0.6]}{FM4}, \frac{[0.2,0.6]}{FM5}, \frac{[0.1,0.4]}{FM6} \right\}$$

Step 5. First separate out the true membership values and false membership values. For

$'O' t_{O_{E1}} = \{0.2, 0.7, 0.4, 0.1, 0.6, 0.2\}$, $f_{O_{E1}} = \{0.5, 0.2, 0.4, 0.6, 0.1, 0.6\}$

$t_{O_{E2}} = \{0.4, 0.3, 0.3, 0.2, 0.4, 0.2\}$, $f_{O_{E2}} = \{0.4, 0.3, 0.4, 0.5, 0.2, 0.5\}$

Find the vague rough lower and upper approximations of $'O'$ (using Definition.2.5 and Definition. 2.6.)

$$\underline{t_{O_{E1}}} = O\beta(t_{O_{E1}})^T = \begin{bmatrix} 1 & 0.8 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.8 & 1 & 0.85 & 0.2 & 0.85 & 0.2 \\ 0.8 & 0.85 & 1 & 0.2 & 0.9 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.2 \\ 0.8 & 0.85 & 0.9 & 0.2 & 1 & 0.9 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.9 & 1 \end{bmatrix} \beta \begin{bmatrix} 0.2 \\ 0.7 \\ 0.4 \\ 0.1 \\ 0.6 \\ 0.2 \end{bmatrix}$$

$$\underline{t_{O_{E1}}} = [0.2, 0.25, 0.25, 0.1, 0.22, 0.2]^T$$

$$\overline{t_{O_{E1}}} = O\beta^*(t_{O_{E1}})^T = \begin{bmatrix} 1 & 0.8 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.8 & 1 & 0.85 & 0.2 & 0.85 & 0.2 \\ 0.8 & 0.85 & 1 & 0.2 & 0.9 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.2 \\ 0.8 & 0.85 & 0.9 & 0.2 & 1 & 0.9 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.9 & 1 \end{bmatrix} \beta^* \begin{bmatrix} 0.2 \\ 0.7 \\ 0.4 \\ 0.1 \\ 0.6 \\ 0.2 \end{bmatrix}$$

$$\overline{t_{O_{E1}}} = [0.625, 0.7, 0.647, 0.1, 0.647, 0.55]^T$$

$$\underline{f_{O_{E1}}} = O\beta^*(f_{O_{E1}})^T = \begin{bmatrix} 1 & 0.8 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.8 & 1 & 0.85 & 0.2 & 0.85 & 0.2 \\ 0.8 & 0.85 & 1 & 0.2 & 0.9 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.2 \\ 0.8 & 0.85 & 0.9 & 0.2 & 1 & 0.9 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.9 & 1 \end{bmatrix} \beta^* \begin{bmatrix} 0.5 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.1 \\ 0.6 \end{bmatrix}$$

$$\underline{f_{O_{E1}}} = [0.5, 0.375, 0.4, 0.6, 0.556, 0.6]^T$$

$$\overline{f_{O_{E1}}} = O\beta(f_{O_{E1}})^T = \begin{bmatrix} 1 & 0.8 & 0.8 & 0.2 & 0.8 & 0.2 \\ 0.8 & 1 & 0.85 & 0.2 & 0.85 & 0.2 \\ 0.8 & 0.85 & 1 & 0.2 & 0.9 & 0.2 \\ 0.2 & 0.2 & 0.2 & 1 & 0.2 & 0.2 \\ 0.8 & 0.85 & 0.9 & 0.2 & 1 & 0.9 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.9 & 1 \end{bmatrix} \beta \begin{bmatrix} 0.5 \\ 0.2 \\ 0.4 \\ 0.6 \\ 0.1 \\ 0.6 \end{bmatrix}$$

$$\overline{f_{O_{E1}}} = [0.125, 0.118, 0.111, 0.5, 0.235, 0.111]^T$$

We have vague rough lower approximation and upper approximations. There are

$$\underline{O_{E1}} = \left\{ \frac{[0.2,0.5]}{FM1}, \frac{[0.25,0.625]}{FM2}, \frac{[0.25,0.6]}{FM3}, \frac{[0.1,0.4]}{FM4}, \frac{[0.22,0.44]}{FM5}, \frac{[0.2,0.4]}{FM6} \right\}$$

$$\overline{O_{E1}} = \left\{ \frac{[0.625,0.875]}{FM1}, \frac{[0.7,0.882]}{FM2}, \frac{[0.647,0.889]}{FM3}, \frac{[0.1,0.5]}{FM4}, \frac{[0.647,0.9]}{FM5}, \frac{[0.55,0.889]}{FM6} \right\}$$

Similarly we get, $\underline{O_{E2}} = \left\{ \frac{[0.375,0.6]}{FM1} \frac{[0.3,0.765]}{FM2}, \frac{[0.3,0.6]}{FM3}, \frac{[0.2,0.5]}{FM4}, \frac{[0.22,0.56]}{FM5}, \frac{[0.2,0.5]}{FM6} \right\}$

$$\overline{O_{E2}} = \left\{ \frac{[0.4,0.75]}{FM1}, \frac{[0.3,0.765]}{FM2}, \frac{[0.33,0.78]}{FM3}, \frac{[0.2,0.5]}{FM4}, \frac{[0.4,0.8]}{FM5}, \frac{[0.33,0.78]}{FM6} \right\}$$

$$\underline{S_{E1}} = \left\{ \frac{[0.111,0.556]}{FM1}, \frac{[0.125,0.625]}{FM2}, \frac{[0.133,0.6]}{FM3}, \frac{[0.1,0.5]}{FM4}, \frac{[0.167,0.8]}{FM5}, \frac{[0.2,0.7]}{FM6} \right\}$$

$$\overline{S_{E1}} = \left\{ \frac{[0.25,0.833]}{FM1}, \frac{[0.3,0.9]}{FM2}, \frac{[0.4,0.833}{FM3}, \frac{[0.2,0.875]}{FM4}, \frac{[0.4,0.8]}{FM5}, \frac{[0.2,0.7]}{FM6} \right\}$$

$$\underline{S_{E2}} = \left\{ \frac{[0.22,0.6]}{FM1}, \frac{[0.2,0.4]}{FM2}, \frac{[0.267,0.6]}{FM3}, \frac{[0.2,0.5]}{FM4}, \frac{[0.3,0.7]}{FM5}, \frac{[0.4,0.8]}{FM6} \right\}$$

$$\overline{S_{E2}} = \left\{ \frac{[0.3,0.625]}{FM1}, \frac{[0.2,0.5]}{FM2}, \frac{[0.4,0.6]}{FM3}, \frac{[0.22,0.6]}{FM4}, \frac{[0.3,0.7]}{FM5}, \frac{[0.4,0.8]}{FM6} \right\}$$

$$\underline{D_{E1}} = \left\{ \frac{[0.25,0.6]}{FM1}, \frac{[0.2,0.5]}{FM2}, \frac{[0.211,0.526]}{FM3}, \frac{[0.267,0.667]}{FM4}, \frac{[0.2,0.4]}{FM5}, \frac{[0.3,0.4]}{FM6} \right\}$$

$$\overline{D_{E1}} = \left\{ \frac{[0.3,0.778]}{FM1}, \frac{[0.263,0.733]}{FM2}, \frac{[0.3,0.7]}{FM3}, \frac{[0.4,0.8]}{FM4}, \frac{[0.2,0.6]}{FM5}, \frac{[0.3,0.714]}{FM6} \right\}$$

$$\underline{D_{E2}} = \left\{ \frac{[0.118,0.471]}{FM1}, \frac{[0.514,0.526]}{FM2}, \frac{[0.111,0.444]}{FM3}, \frac{[0.143,0.571]}{FM4}, \frac{[0.167,0.6]}{FM5}, \frac{[0.1,0.4]}{FM6} \right\}$$

$$\overline{D_{E2}} = \left\{ \frac{[0.4,0.75]}{FM1}, \frac{[0.5,0.8]}{FM2}, \frac{[0.474,0.789]}{FM3}, \frac{[0.333,0.73]}{FM4}, \frac{[0.375,0.75]}{FM5}, \frac{[0.294,0.692]}{FM6} \right\}$$

Step 6. The expert's opinions of O, S & D are listed in the following tables.

Step 7. Find the roughness measure.

If $\alpha = 0.2, \beta = 0.4$ (Using Definition. 2.9.)

$$\underline{F_{M1}} = \{\ [0.25, 0.6], [0.375, 0.6], [0.22, 0.6]\ \}$$

$$\overline{F_{M1}} = \{\ [0.625, 0.875], [0.25, 0.833], [0.3, 0.778], [0.4, 0.75], [0.3, 0.625], [0.4, 0.75]\ \}$$

Table 1. Vague rough lower and upper approximation for Expert-1 (Opinion of O, S & D

-	O	S	D
FAILURE MODE	LA UA	LA UA	LA UA
FM1	[0.2,0.5] [0.625,0.875]	[0.111,0.556] [0.25,0.833]	[0.25,0.6] [0.3,0.778]
FM2	[0.25,0.625] [0.7,0.882]	[0.125,0.625] [0.3,0.9]	[0.2,0.5] [0.263,0.733]
FM3	[0.25,0.6] [0.647,0.889]	[0.133,0.6] [0.4,0.833]	[0.211,0.526] [0.3,0.7]
FM4	[0.1,0.4] [0.1,0.5]	[0.1,0.5] [0.2,0.875]	[0.267,0.667] [0.4,0.8]
FM5	[0.22,0.44] [0.647,0.9]	[0.167,0.8] [0.4,0.8]	[0.2,0.4] [0.2,0.6]
FM6	[0.2,0.4] [0.55,0.889]	[0.2,0.7] [0.2,0.7]	[0.3,0.4] [0.3,0.714]

Table 2. Vague rough lower and upper approximation for Expert-2 (Opinion of O, S & D

-	O	S	D
FAILURE MODE	LA UA	LA UA	LA UA
FM1	[0.375,0.6] [0.4,0.75]	[0.22,0.6] [0.3,0.625]	[0.118,0.471] [0.4,0.75]
FM2	[0.3,0.765] [0.3,0.765]	[0.2,0.4] [0.2,0.5]	[0.154,0.526] [0.5,0.8]
FM3	[0.3,0.6] [0.33,0.78]	[0.267,0.6] [0.4,0.6]	[0.111,0.444] [0.474,0.789]
FM4	[0.2,0.5] [0.2,0.5]	[0.2,0.5] [0.22,0.6]	[0.143,0.571] [0.333,0.73]
FM5	[0.22,0.56] [0.4,0.8]	[0.3,0.7] [0.3,0.7]	[0.167,0.6] [0.375,0.75]
FM6	[0.2,0.5] [0.33,0.78]	[0.4,0.8] [0.4,0.8]	[0.1,0.4] [0.294,0.692]

Roughness measure of the failure mode $FM1 = 1^{\smile}\frac{|\underline{FM1}|}{|\overline{FM1}|} = 1 - \frac{3}{6} = 0.5$

$\underline{FM2} = \{\ [0.25, 0.625], [0.3, 0.765]\ \}$

$\overline{FM2} = \{\ [0.7, 0.882], [0.3, 0.9], [0.263, 0.733], [0.3, 0.765], [0.2, 0.5], [0.5, 0.8]\ \}$

Roughness measure of the failure mode $FM2 = 1^{\smile}\frac{|\underline{FM2}|}{|\overline{FM2}|} = 1 - \frac{2}{6} = 0.667$

$\underline{FM3} = \{\ [0.25, 0.6], [0.211, 0.526], [0.3, 0.6], [0.267, 0.6]\ \}$

$\overline{FM3} = \{\ [0.647, 0.889], [0.4, 0.833], [0.3, 0.7], [0.33, 0.78], [0.4, 0.6], [0.474, 0.789]\ \}$

Roughness measure of the failure mode $FM3 = 1^{\smile}\frac{|\underline{FM3}|}{|\overline{FM3}|} = 1 - \frac{4}{6} = 0.333$

$\underline{FM4} = \{\ [0.267, 0.667]\ \}$

$\overline{FM4} = \{\ [0.2, 0.875], [0.4, 0.8], [0.2, 0.5], [0.22, 0.6], [0.333, 0.73]\ \}$

Roughness measure of the failure mode $FM4 = 1\breve{}\,\frac{|\underline{FM4}|}{|\overline{FM4}|} = 1 - \frac{1}{5} = 0.8$

$\underline{FM5} = \{\ [0.22, 0.44], [0.22, 0.56], [0.3, 0.7]\ \}$

$\overline{FM5} = \{\ [0.647, 0.9], [0.4, 0.8], [0.2, 0.6], [0.4, 0.8], [0.3, 0.7], [0.375, 0.75]\ \}$

Roughness measure of the failure mode $FM5 = 1\breve{}\,\frac{|\underline{FM5}|}{|\overline{FM5}|} = 1 - \frac{3}{6} = 0.5$

$\underline{FM6} = \{\ [0.3, 0.4], [0.4, 0.8]\ \}$

$\overline{FM6} = \{\ [0.55, 0.889], [0.2, 0.7], 0.3, 0.714], [0.33, 0.78], [0.4, 0.8], [0.294, 0.692]\ \}$

Roughness measure of the failure mode $FM6 = 1\breve{}\,\frac{|\underline{FM6}|}{|\overline{FM6}|} = 1 - \frac{2}{6} = 0.667$

Step 8. The hospital has identified the highest priority risk, which will be subsequently addressed.

$$FM4\ >\ FM6\ \geq FM2\ >\ FM5 \geq FM1\ >\ FM3.$$

Step 9. $FM3$ have the minimum value, so give first priority for respiratory depression.

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References

[1] M. Braglia, M. Frosolini, R. Montanari, Fuzzy Topsis Approach for Failure Mode, Effects and Criticality Analysis, Qual. Reliab. Eng. Int. 19 (2003), 425–443. https://doi.org/10.1002/qre.528.

[2] C. Chang, C. Wei, Y. Lee, Failure Mode and Effects Analysis Using Fuzzy Method and Grey Theory, Kybernetes. 28 (1999), 1072–1080. https://doi.org/10.1108/03684929910300295.

[3] K.H. Chang, A Novel General Risk Assessment Method Using the Soft TOPSIS Approach, J. Ind. Product. Eng. 32 (2015), 408–421. https://doi.org/10.1080/21681015.2015.1070375.

[4] I. Emovon, R.A. Norman, A. Murphy J., K. Pazouki, An Integrated Multicriteria Decision Making Methodology Using Compromise Solution Methods for Prioritising Risk of Marine Machinery Systems, Ocean Eng. 105 (2015), 92–103. https://doi.org/10.1016/j.oceaneng.2015.06.005.

[5] W.L. Gau, D.J. Buehrer, Vague Sets, IEEE Trans. Syst., Man, Cybern. 23 (1993), 610–614. https://doi.org/10.1109/21.229476.

[6] Y. Geum, Y. Cho, Y. Park, A Systematic Approach for Diagnosing Service Failure: Service-Specific FMEA and Grey Relational Analysis Approach, Math. Comput. Model. 54 (2011), 3126–3142. https://doi.org/10.1016/j.mcm.2011.07.042.

[7] H.C. Liu, Y.Z. Chen, J.X. You, H. Li, Risk Evaluation in Failure Mode and Effects Analysis Using Fuzzy Digraph and Matrix Approach, J. Intell. Manuf. 27 (2014), 805–816. https://doi.org/10.1007/s10845-014-0915-6.

[8] H.C. Liu, L. Liu, P. Li, Failure Mode and Effects Analysis Using Intuitionistic Fuzzy Hybrid Weighted Euclidean Distance Operator, Int. J. Syst. Sci. 45 (2013), 2012–2030. https://doi.org/10.1080/00207721.2012.760669.

[9] H.C. Liu, J.X. You, X.Y. You, Evaluating the Risk of Healthcare Failure Modes Using Interval 2-Tuple Hybrid Weighted Distance Measure, Comput. Ind. Eng. 78 (2014), 249–258. https://doi.org/10.1016/j.cie.2014.07.018.

[10] H.C. Liu, P. Li, J.X. You, Y.Z. Chen, A Novel Approach for FMEA: Combination of Interval 2-Tuple Linguistic Variables and Gray Relational Analysis, Qual. Reliab. Eng. Int. 31 (2014), 761–772. https://doi.org/10.1002/qre.1633.

[11] Z. Pawlak, Rough Sets, Int. J. Comput. Inf. Sci. 11 (1982), 341–356. https://doi.org/10.1007/bf01001956.

[12] Z. Pawlak, Rough Sets and Fuzzy Sets, Fuzzy Sets Syst. 17 (1985), 99–102. https://doi.org/10.1016/s0165-0114(85)80029-4.

[13] H. Safari, Z. Faraji, S. Majidian, Identifying and Evaluating Enterprise Architecture Risks Using FMEA and Fuzzy VIKOR, J. Intell. Manuf. 27 (2014), 475–486. https://doi.org/10.1007/s10845-014-0880-0.

[14] Y. Shen, F. Wang, Rough Approximations of Vague Sets in Fuzzy Approximation Space, Int. J. Approx. Reason. 52 (2011), 281–296. https://doi.org/10.1016/j.ijar.2010.08.013.

[15] W. Song, X. Ming, Z. Wu, B. Zhu, A Rough TOPSIS Approach for Failure Mode and Effects Analysis in Uncertain Environments, Qual. Reliab. Engng. Int. 30 (2013), 473–486. https://doi.org/10.1002/qre.1500.

[16] S. Vijayabalaji, P. Balaji, Rough Matrix Theory and Its Decision Making, Int. J. Pure Appl. Math. 87 (2013), 845–853. https://doi.org/10.12732/ijpam.v87i6.13.

[17] S. Vijayabalaji, P. Balaji, Construction of QRM using Dense Factor in Quadtree, Asian J. Res. Soc. Sci. Human. 6 (2016), 2573–2581. https://doi.org/10.5958/2249-7315.2016.00764.4.

[18] S. Vijayabalaji, P. Balaji, Best'11 Strategy in Cricket Using MCDM, Rough Matrix and Assignment Model, J. Intell. Fuzzy Syst. 39 (2020), 7431–7447. https://doi.org/10.3233/jifs-200784.

[19] J. Wang, S.Y. Liu, J. Zhang, Roughness of a Vague Set, Int. J. Comput. Cognition. 3 (2005), 83–87.

[20] X. Wang, Y. Zhang, G. Shen, An Improved FMECA for Feed System of CNC Machining Center Based on ICR and DEMATEL Method, Int. J. Adv. Manuf. Technol. 83 (2015), 43–54. https://doi.org/10.1007/s00170-015-7551-y.

[21] Z.L. Wang, J.X. You, H.C. Liu, S.M. Wu, Failure Mode and Effect Analysis Using Soft Set Theory and COPRAS Method, Int. J. Comput. Intell. Syst. 10 (2017), 1002. https://doi.org/10.2991/ijcis.2017.10.1.67.

[22] L.A. Zadeh, Fuzzy Sets, Inf. Control. 8 (1965), 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x.