Int. J. Anal. Appl. (2024), 22:20

International Journal of Analysis and Applications

Fixed Point Set and Equivariant Map of a $\mathcal{S}$-Topological Transformation Group

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Abstract. The fixed point set and equivariant map of a $\mathcal{S}$-topological transformation group is explored in this work. For any subset K of G, it is established that the fixed point set X^K is clopen in X and for a free $\mathcal{S}$-topological transformation group, it is proved that the fixed point set of K is equal to the fixed point set of closure and interior of the subgroup of G generated by K. Subsequently, it is proved that the map between $\mathrm{STC}_G(X)$ and $\mathrm{STC}_{G'}(X')$ is a homomorphism under a Φ'- equivariant map. Also, it is proved that there is an isomorphism between the quotient topological groups and some basic properties of fixed point set of a $\mathcal{S}$-topological transformation group are studied.

1. Introduction and Preliminaries

Montogomery and Zippin's work on Hilbert's fifth problem created the foundation for exploring the topological transformation group. A topological transformation group is a transformation group that preserves the topological structure of a space. The fixed point set of a transformation group is a subset that retains the given point by the element of G. In the exploration of manifolds and their classification, the fixed point set of a transformation group is significant and they facilitate to determine the distinction of various topological spaces through analyzing the features of their fixed point sets under particular group actions. In 2005, P. Chaoha and A. Phon-on [2] established that the fixed point set of a quasi-nonexpansive selfmap of a nonempty convex subset of a geodesic metric space remains closed, convex, and contractible. Each closed subset in a compact free G-space M is the fixed point set of an autohomeomorphism if the weight of M is not more than the weight of G for a compact connected group G was provided by Karl heinrich Hofmann [6] in 2000. A canonical method for enhancing a G-space from homotopy theoretic information about its fixed point sets was shown by A. D. Elmendorf [3] in 1983. In 1985, Helga Schirmer [8] determined

Received: Oct. 12, 2023 .

2020 *Mathematics Subject Classification.* 54H15.

Key words and phrases. $\mathcal{S}$-topological transformation group; free $\mathcal{S}$-topological transformation group; fixed point set; Φ'-equivariant map.

https://doi.org/10.28924/2291-8639-22-2024-20
ISSN: 2291-8639

the fixed point sets of deformations of pairs of spaces. In 1968, James E West [10] examined the fixed point sets of transformation groups on separable infinite-dimensional frechet spaces. K. P. Hart and J. Vermeer [4] explored fixed-point sets of autohomeomorphisms of compact F-spaces in 1995. In 1975, Karl Heinrich Hofmann and Michael Mislove [5] proved that the fixed point sets of the group of inner automorphisms of a compact connected monoid with zero is connected. In a topological transformation group instead of continuity, we have used semi totally continuous and defined a novel structure called $\mathcal{S}$-topological transformation group. $\mathcal{S}$-topological transformation group is a structure moulded by interconnecting topological group and topological space with a semi totally continuous action, under this structure the map Ψ_h and its inverse are semi totally continuous and the collection of all semi totally continuous functions of X onto itself, denoted by $\mathrm{STC}_G(X)$ forms a discrete space and a topological group. Also we have defined the isotropy group for a $\mathcal{S}$-topological transformation group and the free $\mathcal{S}$-topological transformation group.

In this paper, the main section is determined to the study of the fixed point set and equivariant map of a $\mathcal{S}$-topological transformation group. It is proved that for a $\mathcal{S}$-topological transformation group and a finite frechet space X, the fixed point set of K is both open and closed in X. Later, the fixed point set K is equal to the fixed point set of closure and interior of the subgroup of G generated by K for a free $\mathcal{S}$-topological transformation group. Eventually, it is shown that the topological groups $\mathrm{STC}_G(X)$ and $\mathrm{STC}_{G'}(X')$ are homomorphic under a Φ'-equivariant map. Also, it is proved that for a clopen conjugate subgroups $\mathrm{STC}_H(X)$ and $\mathrm{STC}_K(X)$ of $\mathrm{STC}_G(X)$, $N\Big(\mathrm{STC}_H(X)\Big)$ is conjugate to $N\Big(\mathrm{STC}_K(X)\Big)$. Finally, it is ascertained that the quotient groups $N\Big(\mathrm{STC}_H(X)\Big)\Big/\mathrm{STC}_H(X)$ and $N\Big(\mathrm{STC}_K(X)\Big)\Big/\mathrm{STC}_K(X)$ forms an isomorphism of topological groups.

The paper is sectioned as follows. The first section pertains to the introduction and preliminaries. The section two is examined with the basic properties of fixed point set and equivariant map of a $\mathcal{S}$-topological transformation group and followed by the conclusion.

Definition 1.1. [7] A topological group is a nonempty set G that satisfies the following conditions:
(1) G forms a group.
(2) G is a topological space.
(3) The maps $\varphi : G \times G \to G$ and $\alpha : G \to G$ defined by $\varphi(h_1, h_2) = h_1 h_2$ and $\alpha(h) = h^{-1}$ are both continuous.

Definition 1.2. [1] A topological transformation group is a triplet (G, X, ζ), in which G is a topological group, X is a topological space, the map $\zeta : G \times X \to X$ is continuous, and satisfies the following conditions,
(1) $\zeta(e, \mathtt{x}) = \mathtt{x}$, for every $\mathtt{x} \in X$, where e is the identity element of G.
(2) $\zeta(h_2, \Psi(h_1, \mathtt{x})) = \zeta(h_2 h_1, \mathtt{x})$, for all $h_1, h_2 \in G$ and $\mathtt{x} \in X$.

Definition 1.3. [9] Let X and Y be topological spaces. A semi-totally continuous function $f : X \to Y$ in which the inverse image of all semi-open subsets of Y is clopen in X.

2. Fixed Point Set and Equivariant Map of a $\mathcal{S}$-Topological Transformation Group

Definition 2.1. Let $\left(G, X, \Psi\right)$ be a $\mathcal{S}$-topological transformation group. For a subset K of G, $X^K = \{x \in X | K \subset G_{\mathbf{x}}\}$ is called the fixed point set of K.

Example 2.1. Let $\left(GL_2(\mathbb{Z}_2), \mathbb{Z}_2^2, \Psi\right)$ be a $\mathcal{S}$-topological transformation group with a discrete topology under matrix multiplication, where $GL_2(\mathbb{Z}_2) = \left\{p = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, q = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, r = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, s = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, u = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}\right\}$ and $\mathbb{Z}_2^2 = \left\{a = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, d = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right\}$. Then $G_a = \{p, q\}, G_b = \{p, t\}, G_c = \{p, s\}$ and $G_d = \{p, q, r, s, t, u\}$ are the isotropy groups of $\left(GL_2(\mathbb{Z}_2), \mathbb{Z}_2^2, \Psi\right)$. Now, let $K = \left\{q, r, s, t, u\right\}$ be the subset of $GL_2(\mathbb{Z}_2)$. Then the fixed point set of K is $X^K = \left\{d\right\}$.

Example 2.2. Consider a group, $G = \left\{e, i, j, ij \middle| i^2 = e, j^2 = e, ij = ji\right\}$, $K = \left\{e, i\right\}$ be a subgroup of G and $G/K = h_1K = \left\{eK, jK\right\}$. The map $\Psi : G \times G/K \to G/K$ given by $\Psi(h', h_1K) = h'h_1K$ is a semi-totally continuous action and $\left(G, G\middle/K, \Psi\right)$ forms a $\mathcal{S}$-topological transformation group. Then $G_{eK} = \{e, i\}$ and $G_{jK} = \{e, i\}$ are the isotropy groups of $\left(G, G\middle/K, \Psi\right)$. Now, for any subset K of G, the fixed point set of K is either $\emptyset$ or $\left\{eK, jK\right\}$.

Proposition 2.1. Let $\left(G, X, \Psi\right)$ be a $\mathcal{S}$-topological transformation group and X be a finite frechet space. Then for any $K \subset G$, X^K is clopen in X.

Proof. For $h \in G$, define a map $k_h : X \to X \times X$ given by $k_h(x) = (x, \Psi_h(x))$. By the definition of product topology, k_h is continuous. Let the subset Δ of the direct product space $X \times X$ is defined as $\Delta = \{(x, x) | x \in X \subset X \times X\}$ and since X is a finite frechet space, Δ is clopen in $X \times X$. Since k_h is continuous, $k_h^{-1}(\Delta) = X^k$ is clopen in X and hence $X^K = \bigcap\limits_{h \in K} X^h$ is clopen in X. □

Proposition 2.2. For a free $\mathcal{S}$-topological transformation group and for any subset K of G, we have $X^K = X^{\overline{K}}$ and $X^K = X^{K^\circ}$.

Proof. Since $\left(G, X, \Psi\right)$ be a free $\mathcal{S}$-topological transformation group, we have $G_x = \{e\}$ and the proof follows. □

Proposition 2.3. $X^K = X^{<K>}$.

Proof. Clearly $X^K \supset X^{<K>}$. For any h of $< K >$, we have $h = p_1^{n_1} p_2^{n_2} ... p_r^{n_r}, p_i \in K, n_i \in Z$. Then $p_i^{-1} x = x$, for any $x \in X^K$. Hence by induction $hx = x$, $x \in X^h$. Since $h \in< K >$, $x \in X^{<K>}$. Thus $X^K \subset X^{<K>}$. □

Theorem 2.1. Let $\left(G, X, \Psi\right)$ be a $\mathcal{S}$-topological transformation group and X be a finite frechet space. Then for any $K \subset G, X^K = X^{\overline{<K>}}$ and $X^K = X^{<K>^\circ}$.

Proof. By the Proposition 2.2 and 2.3, the proof follows. □

Theorem 2.2. Let $\Phi' : \mathrm{STC}_G(X) \to \mathrm{STC}_{G'}(X)$ be a homomorphism of topological groups and $\gamma : X \to X'$ is a Φ'-equivariant map such that $\gamma(hx) = \Phi'(\Psi_h)\gamma(x)$, for all $h \in G$ and $x \in X$. Then the map $\rho : \mathrm{STC}_G(X) \to \mathrm{STC}_{G'}(X')$ defined by $\rho(\Psi_h(x)) = \Psi'_{\gamma\left(\Psi_h(x)\right)}$ is also a homomorphism of topological groups.

Proof. Let $\Psi_{h_1}, \Psi_{h_2} \in \mathrm{STC}_G(X)$. Then

$$\begin{aligned}
\rho(\Psi_{h_1} \circ \Psi_{h_2})(x) &= \Psi'_{\gamma(\Psi_{h_1}\Psi_{h_2})(x)} \\
&= \Psi'_{\Psi_{h'_1}\Psi_{h'_2}(x')} \\
&= \Psi' \circ i_{\Psi_{h'_1}\Psi_{h'_2}}(x') \\
&= \Psi'(\Psi_{h'_1}\Psi_{h'_2}, x')) \\
&= \Psi'(\Psi_{h'_1}, \Psi'(\Psi_{h'_2}, x')) \\
&= \Psi'(\Psi_{h'_1}, \Psi'_{\Psi_{h'_2}})(x') \\
&= \Psi'(\Psi_{h'_1}, \Psi' \circ i_{h'_2})(x') \\
&= \Psi'_{\Psi_{h'_1 x'}} \circ \Psi'_{\Psi_{h'_2 x'}} \\
&= \rho(\Psi_{h_1}) \circ \rho(\Psi_{h_2})(x)
\end{aligned}$$

□

Proposition 2.4. Let $\mathrm{STC}_H(X)$ and $\mathrm{STC}_K(X)$ are clopen conjugate subgroups of $\mathrm{STC}_G(X)$ and for any $\Psi_g \in \mathrm{STC}_G(X)$, we have $\mathrm{STC}_H(X) = \Psi_g \mathrm{STC}_K(X)\Psi_{g^{-1}}$. Then $N\big(\mathrm{STC}_H(X)\big) = N\big(\mathrm{STC}_K(X)\big)$.

Proof. Let $N\big(\mathrm{STC}_H(X)\big) = \{\Psi_p \mathrm{STC}_H(X) = \mathrm{STC}_H(X)\Psi_p, \Psi_p \in \mathrm{STC}_G(X)\}$. Then

$$\begin{aligned}
\Psi_g N(\mathrm{STC}_H(X))\Psi_{g^{-1}} &= \Psi_g\{\Psi_p \mathrm{STC}_H(X) = \mathrm{STC}_H(X)\Psi_p, \Psi_p \in \mathrm{STC}_G(X)\}\Psi_{g^{-1}} \\
&= \{\Psi_g\Psi_p\Psi_{g^{-1}} \in \mathrm{STC}_G(X) \big| \Psi_p \mathrm{STC}_H(X) = \mathrm{STC}_H(X)\Psi_h\} \\
&= \{\Psi_h = \Psi_g\Psi_p\Psi_{g^{-1}} \in \mathrm{STC}_G(X) \big| \Psi_p \mathrm{STC}_H(X)\Psi_{p^{-1}} = \mathrm{STC}_H(X)\} \\
&= \{\Psi_h \in \mathrm{STC}_G(X) \big| \Psi_{g^{-1}}\Psi_h\Psi_g \mathrm{STC}_H(X)\Psi_{g^{-1}}\Psi_{h^{-1}}\Psi_g = \mathrm{STC}_H(X)\} \\
&= \{\Psi_h \in \mathrm{STC}_G(X) \big| \Psi_{g^{-1}}\Psi_h \mathrm{STC}_K(X)\Psi_{h^{-1}}\Psi_g = \mathrm{STC}_H(X)\} \\
&= \{\Psi_h \in \mathrm{STC}_G(X) \big| \Psi_h STC_K(X)\Psi_{h^{-1}} = \Psi_g \mathrm{STC}_H(X)\Psi_{g^{-1}} = \mathrm{STC}_K(X)\} \\
&= \{\Psi_h \in \mathrm{STC}_G(X) \big| \Psi_h \mathrm{STC}_K(X) = \mathrm{STC}_K(X)\Psi_h\} \\
&= N\big(\mathrm{STC}_K(X)\big)
\end{aligned}$$

Therefore $N\big(\mathrm{STC}_K(X)\big) = \Psi_g N\big(\mathrm{STC}_H(X)\big)\Psi_{g^{-1}}$.

□

Theorem 2.3. Let $\mathrm{STC}_H(X)$ and $\mathrm{STC}_K(X)$ be the clopen subgroups of $\mathrm{STC}_G(X)$ with $\mathrm{STC}_K(X) = \Psi_h \mathrm{STC}_H(X)\Psi_{h^{-1}}$, for some $\Psi_h \in \mathrm{STC}_G(X)$. Then the map $\Gamma : N\big(\mathrm{STC}_H(X)\big)\big/\mathrm{STC}_H(X) \to$

$N\Big(\mathrm{STC}_K(X)\Big)\Big/\mathrm{STC}_K(X)$ defined by $\Gamma(\Psi_p\mathrm{STC}_H(X)) = (\Psi_h\Psi_p\Psi_{h^{-1}})\mathrm{STC}_K(X)$, for $\Psi_p \in N\Big(\mathrm{STC}_H(X)\Big)$ gives an isomorphism of topological groups.

Proof. Let $\Psi_p, \Psi_{p'} \in N\Big(\mathrm{STC}_H(X)\Big)$ be such that $\Psi_p\mathrm{STC}_H(X) = \Psi_{p'}\mathrm{STC}_H(X)$. Then there exists $\Psi_g \in \mathrm{STC}_H(X)$ with $\Psi_{p'} = \Psi_p\Psi_g$ and we have,

$$\begin{aligned}
\Gamma(\Psi_{p'}\mathrm{STC}_H(X)) &= (\Psi_h\Psi_{p'}\Psi_{h^{-1}})\mathrm{STC}_H(X) \\
&= (\Psi_h\Psi_p\Psi_g\Psi_{h^{-1}})(\Psi_h\mathrm{STC}_H(X)\Psi_{h^{-1}}) \\
&= \Psi_h\Psi_p\Psi_g\mathrm{STC}_H(X)\Psi_{h^{-1}} \\
&= \Psi_h\Psi_p\mathrm{STC}_H(X)\Psi_{h^{-1}} \\
&= (\Psi_h\Psi_p\Psi_{h^{-1}})(\Psi_h\mathrm{STC}_H(X)\Psi_{h^{-1}}) \\
&= (\Psi_h\Psi_p\Psi_{h^{-1}})\mathrm{STC}_K(X) \\
&= \Gamma(\Psi_p\mathrm{STC}_H(X))
\end{aligned}$$

Therefore Γ is well-defined. Now, the continuity of Γ follows from the commutative diagram,

$$\begin{array}{ccc}
N\Big(\mathrm{STC}_H(X)\Big) & \xrightarrow{\;I_h\big|N\big(\mathrm{STC}_H(X)\big)\;} & N\Big(\mathrm{STC}_K(X)\Big) \\
\Big\downarrow \pi & & \Big\downarrow \pi \\
N\Big(\mathrm{STC}_H(X)\Big)\Big/\mathrm{STC}_H(X) & \xrightarrow[\Gamma]{} & N\Big(\mathrm{STC}_K(X)\Big)\Big/\mathrm{STC}_K(X)
\end{array}$$

Since the map $I_h : G \to G$ is continuous, its restriction $I_h\Big|N\Big(\mathrm{STC}_H(X)\Big) \to N\Big(\mathrm{STC}_K(X)\Big)$ is also continuous and since each π is a surjective open continuous map, Γ is continuous. Also, the inverse map of Γ is defined as $\Gamma' : N\Big(\mathrm{STC}_K(X)\Big)\Big/\mathrm{STC}_K(X) \to N\Big(\mathrm{STC}_H(X)\Big)\Big/\mathrm{STC}_H(X)$ by $\Gamma'(\Psi_p STC_K(X)) = (\Psi_{h^{-1}}\Psi_p\Psi_h)\mathrm{STC}_H(X)$, for $\Psi_p \in N\Big(\mathrm{STC}_K(X)\Big)$. Then the inverse continuous of Γ can be verified by the following commutative diagram,

$$\begin{array}{ccc}
N\Big(\mathrm{STC}_K(X)\Big) & \xrightarrow{\;I_{h'}\big|N\big(\mathrm{STC}_K(X)\big)\;} & N\Big(\mathrm{STC}_H(X)\Big) \\
\Big\downarrow \pi' & & \Big\downarrow \pi' \\
N\Big(\mathrm{STC}_K(X)\Big)\Big/\mathrm{STC}_K(X) & \xrightarrow[\Gamma']{} & N\Big(\mathrm{STC}_H(X)\Big)\Big/\mathrm{STC}_H(X)
\end{array}$$

Since the map $I_{h'} : G \to G$ is continuous, its restriction $I_{h'}\Big|N\Big(\mathrm{STC}_K(X)\Big) : N\Big(\mathrm{STC}_K(X)\Big) \to N\Big(\mathrm{STC}_H(X)\Big)$ is also continuous and since each π' is a surjective open continuous map, Γ' is continuous. Now, for any $\Psi_p, \Psi_{p'} \in N\Big(\mathrm{STC}_H(X)\Big)$, we have

$$\begin{aligned}
\Gamma\Big(\Psi_p\mathrm{STC}_H(X)\Psi_{p'}\mathrm{STC}_H(X)\Big) &= \Gamma\Big(\Psi_p\Psi_{p'}\mathrm{STC}_H(X)\Big) \\
&= (\Psi_h\Psi_p\Psi_{p'}\Psi_{h^{-1}})\mathrm{STC}_K(X) \\
&= (\Psi_h\Psi_p\Psi_{p'}\Psi_{h^{-1}})(\Psi_h\mathrm{STC}_H(X)\Psi_{h^{-1}}) \\
&= \Psi_h\Psi_p\Psi_{p'}\mathrm{STC}_H(X)\Psi_{h^{-1}} \\
&= (\Psi_h\Psi_p\mathrm{STC}_H(X)\Psi_{p'}\mathrm{STC}_H(X)\Psi_{h^{-1}}) \\
&= (\Psi_h\Psi_p\Psi_{h^{-1}})(\Psi_h\mathrm{STC}_H(X)\Psi_{h^{-1}})(\Psi_h\Psi_{p'}\Psi_{h^{-1}})(\Psi_h\mathrm{STC}_H(X)\Psi_{h^{-1}}) \\
&= \Gamma(\Psi_p\mathrm{STC}_H(X)) \circ \Gamma(\Psi_{p'}\mathrm{STC}_H(X))
\end{aligned}$$

$$\begin{aligned}
\text{Also, } \Gamma'\Big(\Gamma(\Psi_p\mathrm{STC}_H(X))\Big) &= \Gamma'\Big((\Psi_h\Psi_p\Psi_{h^{-1}})\mathrm{STC}_K(X)\Big) \\
&= (\Psi_{h^{-1}}\Psi_h\Psi_p\Psi_{h^{-1}}\Psi_h)\mathrm{STC}_H(X) \\
&= \Psi_p\mathrm{STC}_H(X).
\end{aligned}$$

Hence Γ is homomorphism and $\Gamma' \circ \Gamma = id, \Gamma \circ \Gamma' = id$. Therefore Γ is an isomorphism of topological groups. □

3. Conclusion

Some fundamental algebraic and topological attributes have been examined and presented pertaining to the fixed point set and equivariant map of a $\mathcal{S}$-topological transformation group and it is determined that for a finite frechet space X, the fixed point set of K is clopen. Also, it is established that the topological groups $\mathrm{STC}_G(X)$ and $\mathrm{STC}_{G'}(X')$ are homomorphic. Finally, it is shown that there is a homeomorphism between the quotient groups $N(\mathrm{STC}_H(X))\Big/\mathrm{STC}_H(X)$ and $N(\mathrm{STC}_K(X))\Big/\mathrm{STC}_K(X)$. In a future work, we explore some other properties of fixed point set and equivariant map of $\mathcal{S}$-topological transformation group. The results described in this work have the capacity to induce subsequent research and enables the way to orbit spaces.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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