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Applications of Bipolar Fuzzy Almost Ideals in Semigroups

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Abstract. In this paper, we give the concept of bipolar fuzzy almost ideal, minimal almost bipolar fuzzy ideals, and prime (semiprime, strongly prime) bipolar fuzzy almost ideals in semigroups. We investigate the basic properties of bipolar fuzzy almost ideals in semigroups. Finally, we study the relationship between almost ideals and bipolar fuzzy almost ideals in semigroups.

1. Introduction

Fuzzy sets are a kind of proper mathematical structure to represent a collection of objects whose boundary is vague, which was introduced by Zaden in 1965 [19]. In 1994, Zhang [20] extended the concept of the fuzzy set to bipolar fuzzy sets, which is an extension of fuzzy sets whose membership degree range is $[-1,0] \cup [0,1]$. A bipolar fuzzy set is the membership degree of an element means that the element is irrelevant to the related property, the membership degree of an element indicates that the element somewhat satisfies the property, and the membership degree of an element indicates that the element somewhat satisfies the implicit counter-property. In 2000, Lee [10] used the term bipolar valued fuzzy sets and applied it to algebraic structures. In 2013 Yaqoob and Ansari [18] studied bipolar (λ,δ)-fuzzy ideals in ternary semigroups. Leter in 2015 Ansari and Masmail [1] gave the concept ternary semigroups in terms of bipolar (λ,δ)-fuzzy ideals. The ideals, introduced by her are still central concepts in ring theory, and the notion of a one-sided ideal of any algebraic structure is a generalization of the notion of an ideal. The almost ideal theory in semigroups was studied by Grosek and Satko in 1980 [4–6]. In 1981, Bogdanvic, [2]

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established definitions of almost bi-ideals in semigroups and studied properties of almost bi-ideals in semigroups. Later, Chinram gives definition definitions of the types of almost ideals in semigroups such that almost quasi-ideal [16], almost ideal, (m,n)-almost ideal. In 2020, Chinram et. al [7] introduced almost interior and weakly almost interior ideals in semigroups and studied the properties of its. In 2022, R. Chinrm et. al [3] studied the almost bi-quasi-interior ideal and fuzzy almost bi-quasi-interior ideal in semigroups. Moreover, related research to various types of almost ideals has been studied in other algebraic structures (see, e.g., [9, 11, 12, 14–17]). Recently N. Sarasit et al. [13] studied almost ternary subsemirings.

In this paper, we define bipolar fuzzy almost ideals, minimal almost bipolar fuzzy ideals and prime (semiprime, strongly prime) almost bipolar fuzzy ideals in semigroups. We investigate the properties of bipolar fuzzy A-ideals and the relationship between almost ideals and bipolar fuzzy A-ideals in semigroups.

2. Preliminaries

In this section we give some concepts and results, which will be helpful in later sections. A *subsemigroup* of a semigroup E is a non-empty set K of E such that $KK \subseteq K$. A *left (right) ideal* of a semigroup E is a non-empty set K of E such that $EK \subseteq K$ $(KE \subseteq K)$. By an *ideal* of a semigroup E, we mean a non-empty set of E which is both a left and a right ideal of E. A subsemigroup K of a semigroup E is called a *bi-ideal* of S if $KEK \subseteq K$. An *almost ideal* K of a semigroup E if $tK \cap K \neq \emptyset$ and $Kr \cap K \neq \emptyset$ for all $t, r \in E$.

Theorem 2.1. *[16] Every ideal K of a semigroup E is an almost ideal of E.*

For any $h_i \in [0,1]$, $i \in \mathcal{F}$, define

$$\underset{i\in\mathcal{F}}{\vee} h_i := \sup_{i\in\mathcal{F}}\{h_i\} \quad \text{and} \quad \underset{i\in\mathcal{F}}{\wedge} h_i := \inf_{i\in\mathcal{F}}\{h_i\}.$$

We see that for any $h, r \in [0,1]$, we have

$$h \vee r = \max\{h, r\} \quad \text{and} \quad h \wedge r = \min\{h, r\}.$$

A **fuzzy set (fuzzy subset)** of a non-empty set E is a function
$\vartheta : E \to [0,1]$.

For any two fuzzy sets ϑ and ξ of a non-empty set E, define the symbol as follows:

(1) $\vartheta \geq \xi \Leftrightarrow \vartheta(h) \geq \xi(h)$ for all $h \in E$,
(2) $\vartheta = \xi \Leftrightarrow \vartheta \geq \xi$ and $\xi \geq \vartheta$,
(3) $(\vartheta \wedge \xi)(h) = \min\{\vartheta(h), \xi(h)\} = \vartheta(h) \wedge \xi(h)$ for all $h \in E$,
(4) $(\vartheta \vee \xi)(h) = \max\{\vartheta(h), \xi(h)\} = \vartheta(h) \vee \xi(h)$ for all $h \in E$.
(5) $\vartheta \subseteq \xi$ if $\vartheta(h) \leq \xi(h)$,
(6) $(\vartheta \cup \xi)(h) = \max\{\vartheta(h), \xi(h)\}$ and $(\vartheta \cap \xi)(h) = \min\{\vartheta(h), \xi(h)\}$ for all $h \in E$.
(7) the *support* of ϑ instead of $\mathrm{supp}(\vartheta) = \{h \in E \mid \vartheta(h) \neq 0\}$ For the symbol $\vartheta \leq \xi$, we mean $\xi \geq \vartheta$.

Definition 2.1. *[10] A **bipolar fuzzy set** (BF set) ϑ on a non-empty set E is an object having the form*

$$\vartheta := \{(h, \vartheta^p(h), \vartheta^n(h)) \mid h \in E\},$$

where $\vartheta^p : E \to [0,1]$ and $\vartheta^n : E \to [-1,0]$.

Remark 2.1. *For the sake of simplicity we shall use the symbol $\vartheta = (E; \vartheta^p, \vartheta^n)$ for the BF set $\vartheta = \{(h, \vartheta^p(h), \vartheta^n(h)) \mid h \in E\}$.*

The following example of a BF set.

Example 2.1. *Let $E = \{41, 42, 43...\}$. Define $\vartheta^p : S \to [0,1]$ is a function*

$$\vartheta^p(u) = \begin{cases} 0 & \text{if } h \text{ is old number} \\ 1 & \text{if } h \text{ is even number} \end{cases}$$

and $\vartheta^n : S \to [-1,0]$ is a function

$$\vartheta^n(u) = \begin{cases} -1 & \text{if } h \text{ is old number} \\ 0 & \text{if } h \text{ is even number.} \end{cases}$$

Then $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF set.

For $h \in E$, define $F_h = \{(h_1, h_2) \in E \times E \mid h = h_1 h_2\}$.

Define products $\vartheta^p \circ \xi^p$ and $\vartheta^n \circ \xi^n$ as follows: For $h \in E$

$$(\vartheta^p \circ \xi^p)(h) = \begin{cases} \bigvee\limits_{(h_1,h_2)\in F_h} \{\vartheta^p(h_1) \wedge \xi^p(h_2)\} & \text{if } h = h_1 h_2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$(\vartheta^n \circ \xi^n)(h) = \begin{cases} \bigwedge\limits_{(h_1,h_2)\in F_h} \{\vartheta^n(h_1) \vee \xi^n(h_2)\} & \text{if } h = h_1 h_2 \\ 0 & \text{if otherwise.} \end{cases}$$

Definition 2.2. *[10] Let K be a non-empty set of a semigroup E. A positive characteristic function and a negative characteristic function are respectively defined by*

$$\lambda_K^p : E \to [0,1], h \mapsto \lambda_K^p(h) := \begin{cases} 1 & h \in K, \\ 0 & h \notin K, \end{cases}$$

and

$$\lambda_K^n : E \to [-1,0], h \mapsto \lambda_K^n(h) := \begin{cases} -1 & h \in K, \\ 0 & h \notin K. \end{cases}$$

Lemma 2.1. *[8] Let K and L be non-empty subsets of a semigroup E. Then the following holds.*

(1) $\lambda_K^p \wedge \lambda_L^p = \lambda_{K\cap L}^p$.
(2) $\lambda_K^n \vee \lambda_L^n = \lambda_{K\cup L}^p$.
(3) $\lambda_K^p \circ \lambda_L^p = \lambda_{KL}^n$.

(4) $\lambda^n_K \wedge \lambda^n_L = \lambda^n_{KL}$.

Remark 2.2. *For the sake of simplicity we shall use the symbol* $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ *for the BF set* $\lambda_K := \{(h, \lambda^p_K(h), \lambda^n_K(h)) \mid h \in K\}$.

For $h \in E$ and $(t,s) \in [0,1] \times [-1,0]$, a BF point $h_{(t,s)} = (E; x^p_t, x^n_s)$ of a set E is a bipolar set of E defined by

$$x^p_t(h) = \begin{cases} t & \text{if } h = x \\ 0 & \text{if } h \neq x \end{cases}$$

and

$$x^n_s(h) = \begin{cases} s & \text{if } h = x \\ 0 & \text{if } h \neq x. \end{cases}$$

Definition 2.3. *[8] A BF set* $\vartheta = (E; \vartheta^p, \vartheta^n)$ *on a semigroup E is called a BF subsemigroup on E if it satisfies the following conditions:* $\vartheta^p(hr) \geq \vartheta^p(h) \wedge \vartheta^p(r)$ *and* $\vartheta^n(hr) \leq \vartheta^n(h) \vee \vartheta^n(r)$ *for all* $h, r \in E$.

The following example of a BF subsemigroup.

Example 2.2. *Let E be a semigroup defined by the following table:*

$\cdot$	a	b	c	d	e
a	a	a	a	a	a
a	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	c	c	e

Define a BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ *on E as follows :*

S	a	b	c	d	e
ϑ^p	0.9	0.8	0.5	0.3	0.3
ϑ^n	−0.8	−0.8	−0.6	−0.5	−0.3

Then $\vartheta = (E; \vartheta^p, \vartheta^n)$ *is a BF subsemigroup.*

Definition 2.4. *[8] A BF set* $\vartheta = (E; \vartheta_p, \vartheta_n)$ *on a semigroup E is called a BF left (right) ideal on E if it satisfies the following conditions:* $\vartheta^p(hr) \geq \vartheta^p(r)$ $(\vartheta^p(hr) \geq \vartheta^p(h))$ *and* $\vartheta^n(hr) \leq \vartheta^n(r)$ $(\vartheta^n(hr) \leq \vartheta^n(h))$ *for all* $h, r \in E$.

3. Main Results

In this section, we define the bipolar fuzzy almost ideal and discuss properties of bipolar fuzzy almost ideal in semigroups.

Definition 3.1. *A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a* BF almost left ideal *of E if $(x_t^p \circ \vartheta^p) \wedge \vartheta^p \neq 0$ and $(x_s^n \circ \vartheta^n) \vee \vartheta^n \neq 0$ for any BF point $x_t^p, x_s^n \in E$.*

Definition 3.2. *A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a* BF almost right ideal *of E if $(\vartheta^p \circ x_t^p) \wedge \vartheta^p \neq 0$ and $(\vartheta^n \circ x_s^n) \vee \vartheta^n \neq 0$ for any BF point $x_t^p, x_s^n \in E$.*

Example 3.1. *Every BF left (right) ideal of of a semigroup E is a BF left (right) alomst ideal of E.*

Theorem 3.1. *If $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost left (right) ideal of a semigroup E and $\xi = (E; \xi^p, \xi^n)$ is a BF subset of E such that $\vartheta \subseteq \xi$, then ξ is a BF almost left (right) ideal of E.*

Proof. Suppose that $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost left ideal of a semigroup E and $\xi = (E; \xi^p, \xi^n)$ is a BF subset of E such that $\vartheta \subseteq \xi$. Then for any BF points $x_t^p, x_s^n \in E$, we obtain that $(x_t^p \circ \vartheta^p) \wedge \vartheta^p \neq 0$ and $(x_s^n \circ \vartheta^n) \vee \vartheta^n \neq 0$. Thus,

$$(x_t^p \circ \vartheta^p) \wedge \vartheta^p \subseteq (x_t^p \circ \xi^p) \wedge \xi^p \neq 0.$$

and

$$(x_s^n \circ \vartheta^n) \vee \vartheta^n \subseteq (x_s^n \circ \xi^n \circ) \vee \xi^n \neq 0.$$

Hence $(x_t^p \circ \xi^p \circ) \wedge \xi^p \neq 0$ and $(x_s^n \circ \xi^n \circ) \vee \xi^n \neq 0$. Therefore, ξ is a BF almost left ideal of E. □

Theorem 3.2. *Let $\vartheta = (E; \vartheta^p, \vartheta^n)$ and $\xi = (E; \xi^p, \xi^n)$ be BF almost left (right) ideals of a semigroup E. Then $\vartheta \vee \xi$ is also a BF almost left (right) ideal of E.*

Proof. Since $\vartheta \subseteq \vartheta \vee \xi$, by Theorem 3.1, $\vartheta \vee \xi$ is a BF almost left (right) ideal of E. □

Theorem 3.3. *Let K be a nonempty subset of a semigroup E. Then K is an almost left (right) ideal of E if and only if $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost left (right) ideal of E.*

Proof. Suppose that K is an almost left ideal of a semigroup E. Then $xK \cap K \neq \emptyset$ for all $x \in E$. Thus there exists $c \in E$ such that $c \in xK$ and $c \in K$. Let $x \in E$ and $t \in (0, 1]$ and $s \in [-1, 0)$. Then $(x_t^p \circ \lambda_K^p)(c) \neq 0$, $(x_s^n \circ \lambda_K^n)(c) \neq 0$ and $\lambda_K^p(c) = 1$ and $\lambda_K^n(c) = -1$. Thus, $((x_t^p \circ \lambda_K^p) \wedge \lambda_K^p)(c) \neq 0$ and $((x_s^n \circ \lambda_K^n) \vee \lambda_K^n)(c) \neq 0$ so $(x_t^p \circ \lambda_K^p) \wedge \lambda_K^p \neq 0$ and $(x_s^n \circ \lambda_K^n) \vee \lambda_K^n \neq 0$. Hence, $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost left ideal of E.

Conversely, suppose that $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost left ideal of E and let $x \in E$ and $t \in (0, 1]$ and $s \in [-1, 0)$. Then $(x_t^p \circ \lambda_K^p) \wedge \lambda_K^p \neq 0$ and $(x_s^n \circ \lambda_K^n) \vee \lambda_K^n \neq 0$. Thus there exists $c \in E$ such that $((x_t^p \circ \lambda_K^p) \wedge \lambda_K^p)(c) \neq 0$ and $((x_s^n \circ \lambda_K^n) \vee \lambda_K^n)(c) \neq 0$. Hence $c \in xK \cap K$. So $xK \cap K \neq \emptyset$. We conclude that K is an almost left ideal of E. □

Theorem 3.4. *Let $\vartheta = (E; \vartheta^p, \vartheta^n)$ be a BF subset of a semigroup E. Then $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost left (right) ideal of E if and only if* $\mathrm{supp}(\vartheta)$ *is an almost left (right)ideal of E.*

Proof. Assume that $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost left ideal of a semigroup E and let $x \in E$ and $t \in (0,1]$ and $s \in [-1,0)$. Then $(x_t^p \circ \vartheta^p) \neq 0$, $(x_s^n \circ \vartheta^n) \neq 0$. Thus there exists $z \in E$ such that $(x_t^p \circ \vartheta^p)(z) \neq 0$, $(x_s^n \circ \vartheta^n)(z) \neq 0$. So $\vartheta^p(z) \neq 0$, $\vartheta^n(z) \neq 0$ and $\vartheta^n(z) \neq 0$, $\vartheta^n(z) \neq 0$ there exists $w \in E$ such that $z = xw$ and $\vartheta^p(w) \neq 0$, and $\vartheta^n(w) \neq 0$. Thus $((x_t^p \circ \lambda^p_{\text{supp}(\vartheta)}) \wedge \lambda^p_{\text{supp}(\vartheta)})(z) \neq 0$ and $((x_s^n \circ \lambda^n_{\text{supp}(\vartheta)}) \vee \lambda^n_{\text{supp}(\vartheta)})(z) \neq 0$. Hence, $((x_t^p \circ \lambda^p_{\text{supp}(\vartheta)}) \wedge \lambda^p_{\text{supp}(\vartheta)}) \neq 0$ and $((x_s^n \circ \lambda^n_{\text{supp}(\vartheta)}) \vee \lambda^n_{\text{supp}(\vartheta)}) \neq 0$. Therefore, $\lambda^p_{\text{supp}(\vartheta)}$ is a BF almost left ideal of E. By Theorem 3.3, $\text{supp}(\vartheta)$ is an almost left ideal of E.

Conversely, suppose that $\text{supp}(\vartheta)$ is an almost left ideal of E. By Theorem 3.3, $\lambda^p_{\text{supp}(\vartheta)}$ is a BF almost ideal of E. Then for any BF points $x_t^p, x_s^n \in E$, we have $(x_t^p \circ \lambda^p_{\text{supp}(\vartheta)}) \wedge \lambda^p_{\text{supp}(\vartheta)} \neq 0$ and $(x_s^n \circ \lambda^n_{\text{supp}(\vartheta)}) \wedge \lambda^n_{\text{supp}(\vartheta)} \neq 0$. Thus there exists $c \in E$ such that $[(x_t^p \circ \lambda^p_{\text{supp}(\vartheta)}) \wedge \lambda^p_{\text{supp}(\vartheta)}](c) \neq 0$ and $[(x_s^p \circ \lambda^n_{\text{supp}(\vartheta)}) \vee \lambda^n_{supp(\vartheta)}](c) \neq 0$. Hence, $(x_t^p \circ \lambda^p_{\text{supp}(\vartheta)})(c) = 0$, $\lambda^p_{supp(\vartheta)}(c) \neq 0$ and $(x_s^n \circ \lambda^n_{\text{supp}(\vartheta)})(c) = 0$, $\lambda^n_{\text{supp}(\vartheta)}(c) \neq 0$. Then there exists $b \in \text{supp}(\vartheta)$ such that $c = xby$. Thus $\vartheta^p(c) \neq 0, \vartheta^p(b) \neq 0$ and $\vartheta^n(c) \neq 0, \vartheta^n(b) \neq 0$. So $(x_t^p \circ \vartheta^p) \neq 0$, $(x_s^n \circ \vartheta^n) \neq 0$. Therefore, $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost left ideal of E. □

Next, we investigate minimal BF almost left (right) ideals in semigroups and study relationships between minimal almost left (right) ideals and minimal BF almost left (right) ideals of semigroups.

Definition 3.3. *An almost left (right) ideal K of a semigroup E is called* minimal *if for any almost left (right) ideal M of E if whenever $M \subseteq K$, then $M = K$.*

Definition 3.4. *A BF almost left (right) ideal $\vartheta = (E; \vartheta^p, \vartheta^n)$ of a semigroup E is called* minimal *if for any BF almost left (right) ideal $\xi = (E; \xi^p, \xi^n)$ of E if whenever $\xi \subseteq \vartheta$, then* $\text{supp}(\xi) = \text{supp}(\vartheta)$.

Theorem 3.5. *Let K be a nonempty subset of a semigroup E. Then K is a minimal almost left (right) ideal of E if and only if $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost left (right) ideal of E.*

Proof. Assume that K is a minimal almost left ideal of E. Then K is an almost left ideal of E. Thus by Theorem 3.3, $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost ideal of E. Let $\xi = (E; \xi^p, \xi^n)$ be a BF almost left ideal of E such that $\xi \subseteq \lambda_K$ Then $\text{supp}(\xi) \subseteq \text{supp}(\lambda_K) = K$. By Theorem 3.4, $\text{supp}(\xi)$ is an almost left ideal of E. Since K is minimal we have $\text{supp}(\xi) = K = \text{supp}(\lambda_K)$. Therefore, $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is minimal BF almost left ideal of E.

Conversely, suppose that $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost left ideal of E. Then $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost left ideal of E. Thus by Theorem 3.3, K is an almost left ideal of E. Let M be an almost left ideal of E such that $M \subseteq K$. Then λ_M is a BF almost left ideal of E such that $\lambda_M \subseteq \lambda_K$. Thus $\text{supp}(\lambda_M) \subseteq \text{supp}(\lambda_K)$. Since $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a minimal BF almost left ideal of E we have $\text{supp}(\lambda_M) = \text{supp}(\lambda_K)$. Thus, $M = \text{supp}(\lambda_M) = \text{supp}(\lambda_K) = K$. Hence K is minimal almost left ideal of E. □

Corollary 3.1. *Let E be a semigroup. Then $\mathcal{E}$ has no proper almost ideal if and only if* $\text{supp}(\vartheta) = E$ *for every BF almost ideal $\vartheta = (E; \vartheta^p, \vartheta^n)$ of E.*

Proof. Suppose that E has no proper almost ideal and let $\vartheta = (E; \vartheta^p, \vartheta^n)$ be a BF almost ideal of E. Then by Theorem 3.4, $\mathrm{supp}(\vartheta)$ is an almost ideal of E. By assumption, $\mathrm{supp}(\vartheta) = E$.

Conversely, suppose that $\mathrm{supp}(\vartheta) = E$ and M is a proper almost ideal of E. Then by Theorem 3.3, $\lambda_M = (E; \lambda^p_M, \lambda^n_M)$ is a BF almost ideal of E. Thus $\mathrm{supp}(\lambda_M) = M \neq E$. It is a contradiction. Hence E has no proper alomst ideal. □

We give definition of prime (resp., semiprime, strongly prime) almost ideal and prime (resp., semiprime strongly prime) BF almost ideal. We study the relationships between prime (resp., semiprime strongly prime) almost ideals and their bioplar fuzzification of semigroups

Definition 3.5. *Let K be an almost ideal of semigroup E. Then we called*

*(1) K is a **prime** if for any almost ideals M and L of E such that $ML \subseteq K$ implies that $M \subseteq K$ or $L \subseteq K$.*
*(2) K is a **semiprime** if for any almost ideal M of E such that $M^2 \subseteq K$ implies that $M \subseteq K$.*
*(3) K is a **strongly prime** if for any almost ideals M and L of E such that $ML \cap LM \subseteq K$ implies that $M \subseteq K$ or $L \subseteq K$.*

Definition 3.6. *A BF almost ideal $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E. Then we called*

*(1) ϑ is a **prime** if for any two BF almost ideals $\xi = (E; \xi^p, \xi^{n)}$ and $\nu = (E; \nu^p, \nu^n)$ of E such that $\xi^p \circ \nu^p \leq \vartheta^p$ and $\xi^n \circ \nu^n \geq \vartheta^n$ implies that $\xi^p \leq \vartheta^p$ and $\xi^n \geq \vartheta^n$ or $\nu^p \leq \vartheta^p$ and $\nu^n \geq \vartheta^n$.*
*(2) ϑ is a **semiprime** if for any BF almost ideal $\xi = (E; \xi^p, \xi^n)$ of E such that $\xi^p \circ \xi^p \leq \vartheta^p$ and $\xi^n \circ \xi^n \leq \vartheta^n$ implies that $\xi^p \leq \vartheta^p$ or $\xi^n \geq \vartheta^n$.*
*(3) ϑ is a **strongly prime** if for any two BF almost ideals $\xi = (E; \xi^p, \xi^n)$ and $\nu = (E; \nu^p, \nu^n)$ of E such that $(\xi^p \circ \nu^p) \wedge (\nu^p \circ \xi^p) \leq \vartheta^p$ and $(\xi^n \circ \nu^n) \vee (\nu^n \circ \xi^n) \leq \vartheta^n$ implies that $\xi^p \subseteq \vartheta^p$ and $\xi^n \geq \vartheta^n$ or $\nu^p \leq \vartheta^p$ and $\nu^n \geq \vartheta^n$.*

It is clearly, every BF strongly prime almost ideal of a semigroup is a BF prime almost ideal, and every BF prime almost ideal of a semigroup is a BF semiprime almost ideal.

Theorem 3.6. *Let K be a nonempty subset of a semigroup E. Then K is a prime (resp., semiprime) almost ideal of E if and only if $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a prime (resp., semiprime) BF almost ideal of E.*

Proof. Suppose that K is a prime almost ideal of a semigroup E. Then K is an almost ideal of E. Thus by Theorem 3.3, $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF almost ideal of E. Let $\vartheta = (E; \vartheta^p_K, \vartheta^n_K)$ and $\xi = (E; \xi^p_K, \xi^n_K)$ be BF almost ideals such that $\vartheta^p \circ \xi^p \leq \lambda^p_K$ and $\vartheta^n \circ \xi^n \geq \lambda^n_K$. Assume that $\vartheta^p \nleq \lambda^p_K$ and $\vartheta^n \ngeq \lambda^n_K$ or $\xi^p \nleq \lambda^p_K$ and $\xi^n \ngeq \lambda^n_K$. Then there exist $h, r \in E$ such that $\vartheta^p(h) \neq 0$, $\vartheta^n(h) \neq 0$ and $\xi^p(r) \neq 0$, $\xi^n(r) \neq 0$. While $\lambda^p_K(r) = 0$, $\lambda^n_K(r) = 0$ and $\lambda^p_K(r) = 0$, $\lambda^n_K(r) = 0$. Thus $h \in \mathrm{supp}(\vartheta)$ and $r \in \mathrm{supp}(\xi)$, but $h, r \notin K$. So $\mathrm{supp}(\vartheta) \nsubseteq K$ and $\mathrm{supp}(\xi) \nsubseteq K$. Since $\mathrm{supp}(\vartheta)$ and $\mathrm{supp}(\xi)$ are almost ideals of E we have $\mathrm{supp}(\vartheta)\,\mathrm{supp}(\xi) \nsubseteq K$. Thus there exists $m = de$ for some $d \in \mathrm{supp}(\vartheta)$ and $e \in \mathrm{supp}(\xi)$ such that $m \in K$. Hence $\lambda^p_K(m) = 0$ and $\lambda^n_K(m) = 0$ implies that $(\vartheta^p \circ \xi^p)(m) = 0$ and $(\vartheta^n \circ \xi^n)(m) = 0$. Since $\vartheta^p \circ \xi^p \leq \lambda^p_K$ and $\vartheta^n \circ \xi^n \leq \lambda^n_K$. we have $d \in \mathrm{supp}(\vartheta)$ and $e \in \mathrm{supp}(\xi)$.

Thus $\vartheta^p(d) \neq 0$, $\vartheta^n(d) \neq 0$ and $\xi^p(e) \neq 0$, $\xi^n(e) \neq 0$. It implies that

$$(\vartheta^p \circ \xi^p)(m) = \bigvee_{(de)\in F_m} \{\vartheta^p(d) \wedge \xi^p(e)\} \neq 0$$

and

$$(\vartheta^n \circ \xi^n)(m) = \bigwedge_{(de)\in F_m} \{\vartheta^n(d) \vee \xi^n(e)\} \neq 0.$$

It is a contradiction so $\vartheta^p \leq \lambda_K^p$ and $\vartheta^n \geq \lambda_K^n$ or $\xi^p \leq \lambda_K^p$ and $\xi^n \geq \lambda_K^n$. Therefore $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a prime BF almost ideal of E.

Conversely, suppose that $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a prime BF almost ideal of E. Then $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost ideal of E. Thus by Theorem 3.3, K is an almost ideal of E. Let M and L be almost ideals of E such that $ML \subseteq K$. Then $\lambda_M = (E; \lambda_M^p, \lambda_M^n)$ and $\lambda_L = (E; \lambda_L^p, \lambda_L^n)$ are BF almost ideals of E. By Lemma 2.1 $\lambda_M^p \circ \lambda_L^p = \lambda_{ML}^p \leq \lambda_K^p$ and $\lambda_M^n \circ \lambda_L^n = \lambda_{ML}^n \geq \lambda_K^n$. By assumption, $\lambda_M^p \leq \lambda_K^p$ and $\lambda_M^n \geq \lambda_K^n$ or $\lambda_L^p \leq \lambda_K^p$ and $\lambda_L^n \geq \lambda_K^n$. Thus $M \subseteq K$ or $L \subseteq K$. We conclude that K is a prime almost ideal of E. □

Theorem 3.7. *Let K be a nonempty subset of a semigroup E. Then K is a strongly prime almost ideal of E if and only if $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF strongly prime almost ideal of E.*

Proof. Suppose that K is a strongly prime almost ideal of a semigroup E. Then K is an almost ideal of E. Thus by Theorem 3.3, $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ is a BF almost ideal of E. Let $\vartheta = (E; \vartheta_K^p, \vartheta_K^n)$ and $\xi = (E; \xi_K^p, \xi_K^n)$ be BF almost ideals of E such that $(\vartheta^p \circ \xi^p) \wedge (\xi^p \circ \vartheta^p) \leq \lambda_K^p$ and $(\vartheta^n \circ \xi^n) \vee (\xi^n \circ \vartheta^n) \geq \lambda_K^n$. Assume that $\vartheta^p \nleq \lambda_K^p$ and $\vartheta^n \ngeq \lambda_K^n$ or $\xi^p \nleq \lambda_K^p$ and $\xi^n \ngeq \lambda_K^n$. Then there exist $h, r \in E$ such that $\vartheta^p(h) \neq 0$, $\vartheta^n(h) \neq 0$ and $\xi^p(r) \neq 0$, $\vartheta^n(h) \neq 0$. While $\lambda_K^p(h) = 0$, $\lambda_K^n(h) = 0$ and $\lambda_K^p(r) = 0$, $\lambda_K^n(r) = 0$. Thus $h \in$supp(ϑ) and $r \in$ supp(ξ), but $h, r \notin K$. So supp$(\vartheta) \nsubseteq K$ and supp$(\xi) \nsubseteq K$. Hence, there exists $m \in [\text{supp}(\vartheta)\,\text{supp}(\xi)] \cap [\text{supp}(\vartheta)\,\text{supp}(\xi)]$ such that $m \notin K$. Thus $\lambda_K^p(m) = 0$ and $\lambda_K^n(m) = 0$.

Since $m \in \text{supp}(\vartheta)\,\text{supp}(\xi)$ and $m \in \text{supp}(\xi)\,\text{supp}(\vartheta)$ we have $m = d_1e_1$ and $m = e_2d_2$ for some $d_1, d_2 \in \text{supp}(\vartheta)$ and for some $e_1, e_2 \in \text{supp}(\xi)$. we have

$$(\vartheta^p \circ \xi^p)(m) = \bigvee_{(d_1e_1)\in F_m} \{\vartheta^p(d_1) \wedge \xi^p(e_1)\} \neq 0$$

and

$$(\vartheta^n \circ \xi^n)(m) = \bigwedge_{(d_1e_1)\in F_m} \{\vartheta^n(d_1) \vee \xi^n(e_1)\} \neq 0.$$

Similarly

$$(\xi^p \circ \vartheta^p)(m) = \bigvee_{(e_2d_2)\in F_m} \{\xi^p(e_2) \wedge \vartheta^p(d_2)\} \neq 0$$

and

$$(\vartheta^n \circ \xi^n)(m) = \bigwedge_{(e_2d_2) in F_m} \{\xi^n(e_2) \vee \vartheta^n(d_2)\} \neq 0.$$

So $(\vartheta^p \circ \xi^p)(m) \wedge (\vartheta^p \circ \xi^p)(m) \neq 0$ and $(\vartheta^n \circ \xi^n)(m) \vee (\vartheta^n \circ \xi^n)(m) \neq 0$. It is a contradiction so $(\vartheta^p \circ \xi^p)(m) \wedge (\xi^p \circ \vartheta^p)(m) = 0$ and $(\vartheta^n \circ \xi^n)(m) \vee (\xi^n \circ \vartheta^n)(m) = 0$. Hence, $\vartheta^p \leq \lambda^p_K$ and $\vartheta^n \geq \lambda^n_K$ or $\xi^p \leq \lambda^p_K$ and $\xi^n \geq \lambda^n_K$. Therefore $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF strongly prime almost ideal of E.

Conversely, suppose that $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF strongly prime almost ideal of E. Then $\lambda_K = (E; \lambda^p_K, \lambda^n_K)$ is a BF almost ideal of E. Thus by Theorem 3.3, K is an almost ideal of E. Let M and L be almost ideals of E such that $ML \cap LM \leq K$. Then $\lambda_M = (E; \lambda^p_M, \lambda^n_K)$ and $\lambda_L = (E; \lambda^p_L, \lambda^n_L)$ are BF almost ideals of E. By Lemma 2.1 $\lambda^p_{ML} = \lambda^p_M \circ \lambda^p_L, \lambda^p_{LM} = \lambda^p_L \circ \lambda^p_M$ and $\lambda^n_{ML} = \lambda^n_M \circ \lambda^n_L, \lambda^n_{LM} = \lambda^n_L \circ \lambda^n_M$. Thus $(\lambda^p_M \circ \lambda^p_L) \wedge (\lambda^p_L \circ \lambda^p_M) = \lambda^p_{ML} \wedge \lambda^p_{LM} = \lambda^p_{ML \cap LM} \leq \lambda^p_K$ and $(\lambda^n_M \circ \lambda^n_L) \vee (\lambda^n_L \circ \lambda^n_M) = \lambda^n_{ML} \vee \lambda^n_{LM} = \lambda^n_{ML \cup LM} \geq \lambda^n_K$. By assumption, $\lambda^p_M \leq \lambda^p_K$ and $\lambda^n_M \geq \lambda^n_K$ or $\lambda^p_L \subseteq \lambda^p_K$ and $\lambda^n_L \geq \lambda^n_K$. Thus $M \subseteq K$ or $L \subseteq K$. We conclude that K is a strongly prime almost ideal of E. □

4. Conclusion

In this paper, we give the concept of BF almost ideals in semigroup and we study properties of BF almost ideals in semigroups. Moreover, we prove relationship between BF almost ideals and almost ideals. In the future we extend study other kinds of almost ideals and interval valued fuzzy set or class of kinds fuzzy sets.

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