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Single-Valued Neutrosophic Roughness via Ideals

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Abstract. In this paper, we connect the idea of single-valued neutrosophic ideal to the concept of single-valued neutrosophic approximation space to define the concept of single-valued neutrosophic ideal approximation spaces. We present the single-valued neutrosophic ideal approximation interior operator $\mathrm{int}^{\psi}_{\Phi}$ and the single-valued neutrosophic ideal approximation closure operator $\mathrm{cl}^{\psi}_{\Phi}$, and we present the single-valued neutrosophic ideal approximation pre-interior operator $\mathrm{pint}^{\psi}_{\Phi}$ and the single-valued neutrosophic ideal approximation pre-closure operator $\mathrm{pcl}^{\psi}_{\Phi}$ about this concerning single-valued neutrosophic ideal defined on the single-valued neutrosophic approximation space $(\tilde{\chi}, \varphi)$ related with some single-valued neutrosophic set $\psi \in \xi^{\tilde{\chi}}$. Also, we present single-valued neutrosophic separation axioms, single-valued neutrosophic connectedness, and single-valued neutrosophic compactness in single-valued neutrosophic approximation spaces and single-valued neutrosophic ideal approximation spaces as well, and prove the associations in between.

1. Introduction

Sometimes, it is not convenient to apply practical problems to real-life applications. Data in medical sciences, economics, weather, climate changes, etc. always involve various types of uncertainties. To exceed the difficulties in using the traditional classical methods the word neutrosophy is initiated to be a tool for handling problems involving incomplete, indeterminate, and inconsistent information. Smarandache [1] presented the idea of a neutrosophic set as an intuitionistic fuzzy set generalization. Salama et al [2] defined the neutrosophic set theory and neutrosophic crisp set. Correspondingly, Salama and Alblowi [3], introduced neutrosophic topology as they claimed a number of its characteristics. Others as Wang et al [4] defined the single-valued neutrosophic set concept. In (2020, 2021) Saber et al [5–11] introduced and studied the concepts

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of single-valued neutrosophic ideal, single-valued neutrosophic ideal open local function, connectedness in single-valued neutrosophic topological spaces $(£, \tilde{\mathsf{T}}^{\sigma}, \tilde{\mathsf{T}}^{\varsigma}, \tilde{\mathsf{T}}^{\delta})$ and compactness in single-valued neutrosophic ideal topological spaces.

Quite recently, considerable attention has been paid to the approximate operations on sets and approximate enclosure of sets introduced by Pawlak [12]. An approximation space $(\tilde{\mathcal{F}}, \delta)$ is fashioned from a universe set of objects and an equivalence relation on these objects. The boundary among the lower approximation set $(\alpha_n)_\delta$ and the upper approximation set $(\alpha_n)^\delta$ of a set δ in $(\tilde{\mathcal{F}}, \delta)$ termed these rough sets. If the lower and the upper approximation sets are identical, then α_n is then a thorough subset of $\tilde{\mathcal{F}}$, and there is no roughness. Many of their applications appear in the studies [13–19]. Irfan in [20] studied the connections between fuzzy set, rough set, and Soft set notions. Many papers studied the relationship between fuzzy rough set notions and fuzzy topologies [21, 22]. Recently, many researchers have used topological approaches in the study of rough sets and their applications. In [23], it was used the notion of ideal in soft rough ordinary topological space, and in [24], the authors introduced fuzzy soft connectedness in the sense of Chang [25].

In this article, we combined the idea of single-valued neutrosophic ideal $\hbar$ with single-valued neutrosophic approximation space (SVNA-space) $(\tilde{\chi}, \varphi)$ related with single-valued neutrosophic set ψ, and presented the concept of single-valued neutrosophic interior and single-valued neutrosophic closure operators concerning that single-valued neutrosophic ideal The local function $\Phi_\psi(\rho)$ of some $\rho \in \xi^{\tilde{\chi}}$ concerning that single-valued neutrosophic ideal was a base in defining the associated interior and closure operators. Separation axioms in SVNA-space $(\tilde{\chi}, \varphi, \hbar)$ and in SVNA-space $(\tilde{\chi}, \varphi, \hbar)$ have been obtained and we obtain some of their properties. Connectedness in SVNA-space and in SVNIA-space were defined and compared with examples to show the suggestions in between. Compactness in SVNA-space $(\tilde{\chi}, \varphi, \hbar)$ were defined as well.

Definition 1.1. *[1] Let $\tilde{\chi}$ be a non-empty set. A neutrosophic set (briefly, NS) in $\tilde{\chi}$ is an object having the form*

$$\psi = \{\langle \kappa, \tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi \rangle : \omega \in \tilde{\chi}\}$$

where

$$\tilde{\eta} : \tilde{\chi} \to \rfloor^{-}0, 1^{+}\lfloor,\ \tilde{\gamma} : \tilde{\chi} \to \rfloor^{-}0, 1^{+}\lfloor,\ \tilde{\delta} : \tilde{\chi} \to \rfloor^{-}0, 1^{+}\lfloor,$$

and

$$^{-}0 \leq \tilde{\eta}_\psi(\kappa) + \tilde{\gamma}_\psi(\kappa) + \tilde{\delta}_\psi(\kappa) \leq 3^{+}$$

represent the degree of membership ($\tilde{\eta}_\psi$), the degree of indeterminacy ($\tilde{\gamma}_\psi$), and the degree of non-membership ($\tilde{\delta}_\psi$) respectively of any $\kappa \in \tilde{\chi}$ to the set ψ.

Definition 1.2. *[4] Let $\tilde{\chi}$ be a space of points (objects), with a generic element in $\tilde{\chi}$ denoted by κ. Then ψ is called a single-valued neutrosophic set (briefly, SVNS) in $\tilde{\chi}$, if ψ has the form*

$$\psi = \langle \kappa, \tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi \rangle : \kappa \in \tilde{\chi},$$

where, $\tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi : \tilde{\chi} \to [0,1]$. In this case, $\tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi$ are called the truth of membership function, indeterminacy membership function, and falsity membership function, respectively.

For conformist motives and as there is no ambiguity, we denote an SVNS merely as a neutrosophic set throughout this article; we too paraphrase the definition, in order to view it clearly as a function from a non-empty set $\tilde{\chi}$ to $\xi = [0,1]^3$, in the next method

Let X be a nonempty set and $\xi = [0,1]$. A NS on $\tilde{\chi}$ is a mapping defined as $\psi = \langle \tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi \rangle : \tilde{\chi} \to \xi$, where $\xi = I^3$ and $\tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi : \tilde{\chi} \to \xi$ such that $0 \leq \tilde{\eta}_\psi + \tilde{\gamma}_\psi + \tilde{\delta}_\psi \leq 3$.

We denote the set of all neutrosophic sets of $\tilde{\chi}$ by $\xi^{\tilde{\chi}}$ and the neutrosophic sets $\langle 0,1,1 \rangle$ and $\langle 1,0,0 \rangle$ by $\tilde{0}$ and $\tilde{1}$ respectively.

Definition 1.3. *[4,26,27] Let $\tilde{\chi}$ be a non-empty set and let $\psi, \rho \in \xi^{\tilde{\chi}}$ be given by $\psi = \langle \tilde{\eta}_\psi, \tilde{\gamma}_\psi, \tilde{\delta}_\psi \rangle$ and $\rho = \langle \tilde{\eta}_\rho, \tilde{\gamma}_\rho, \tilde{\delta}_\rho \rangle$. Then*

(1) *The complement of ψ (briefly, ψ^c) is given by*

$$\tilde{\eta}_{\psi^c}(\kappa) = \tilde{\delta}_\psi(\kappa),\ \tilde{\gamma}_{\psi^c}(\kappa) = [\tilde{\gamma}_\psi]^c(\kappa),\ \tilde{\delta}_{\psi^c}(\kappa) = \tilde{\eta}_\psi(\kappa).$$

(2) *We say that $\psi \subseteq \rho$ for every $\kappa \in \tilde{\chi}$ if*

$$\tilde{\eta}_\psi(\kappa) \leq \tilde{\eta}_\rho(\kappa),\ \tilde{\gamma}_\psi(\kappa) \geq \tilde{\gamma}_\rho(\kappa),\ \tilde{\delta}_\psi(\kappa) \geq \tilde{\delta}_\rho(\kappa).$$

(3) *The union of ψ and ρ (briefly, $\psi \vee \rho$) is an SVNS in $\tilde{\chi}$ is given by,*

$$\psi \vee \rho = \langle (\tilde{\eta}_\psi \vee \tilde{\eta}_\rho)(\kappa),\ (\tilde{\gamma}_\psi \wedge \tilde{\gamma}_\rho)(\kappa),\ (\tilde{\delta}_\psi \wedge \tilde{\delta}_\rho)(\kappa) \rangle.$$

(4) *The intersection of ψ and ρ (briefly, $\psi \wedge \rho$) is an SVNS in $\tilde{\chi}$ is given by,*

$$\psi \wedge \rho = \langle (\tilde{\eta}_\psi \wedge \tilde{\eta}_\rho)(\kappa),\ (\tilde{\gamma}_\psi \vee \tilde{\gamma}_\rho)(\kappa),\ (\tilde{\delta}_\psi \vee \tilde{\delta}_\rho)(\kappa) \rangle.$$

For any arbitrary collection $\{\psi\}_{i \in j} \in \xi^{\tilde{\chi}}$ of SVNS *the union and intersection are given by*

(5) $\bigwedge_{i \in j} \psi_i = \left\langle \bigwedge_{i \in j} \tilde{\eta}_{\psi_i}(\kappa),\ \bigvee_{i \in j} \tilde{\gamma}_{\psi_i}(\kappa),\ \bigvee_{i \in j} \tilde{\delta}_{\psi_i}(\kappa) \right\rangle$.

(6) $\bigvee_{i \in j} \psi_i = \left\langle \bigvee_{i \in j} \tilde{\eta}_{\psi_i}(\kappa),\ \bigwedge_{i \in j} \tilde{\gamma}_{\psi_i}(\kappa),\ \bigwedge_{i \in j} \tilde{\delta}_{\psi_i}(\kappa) \right\rangle$.

Suppose that single valued neutrosophic relation (for short, SVNR) φ is defined as:

(1) $\tilde{\eta}_\varphi(\kappa,\kappa) = 1, \tilde{\gamma}_\varphi(\kappa,\kappa) = 0, \tilde{\delta}_\delta(\kappa,\kappa) = 0\ \forall\ \kappa \in \tilde{\chi}$,
(2) $\tilde{\eta}_\varphi(\kappa,\upsilon) = \tilde{\eta}_\varphi(\upsilon,\kappa), \tilde{\gamma}_\varphi(\kappa,\upsilon) = \tilde{\gamma}_\varphi(\upsilon,\kappa), \tilde{\delta}_\varphi(\kappa,\upsilon) = \tilde{\delta}_\varphi(\upsilon,\kappa)\ \forall\ \kappa,\upsilon \in \tilde{\chi}$,
(3) $\tilde{\eta}_\varphi(\kappa,\upsilon) \geq \Big(\tilde{\eta}_\varphi(\kappa,\omega) \wedge\ \tilde{\eta}_\varphi(\omega,\upsilon)\Big), \tilde{\gamma}_\varphi(\kappa,\upsilon) \leq \Big(\tilde{\eta}_\varphi(\kappa,\omega) \vee \tilde{\gamma}_\varphi(\omega,\upsilon)\Big)$ and $\tilde{\delta}_\varphi(\kappa,\upsilon) \leq \Big(\tilde{\gamma}_\varphi(\kappa,\omega) \vee \tilde{\gamma}_\varphi(\omega,\upsilon)\Big)\ \forall\ \kappa,\upsilon,\omega \in \tilde{\chi}$.

The pair $(\tilde{\chi}, \varphi)$ is said to be single valued neutrosophic approximation space ((for short, SVNA-space) created on single valued neutrosophic equivalence relation (briefly, SVNER) φ on $\tilde{\chi}$.

Definition 1.4. *For each $\kappa \in \tilde{\chi}$, , define a single valued neutrosophic coset (briefly, SVN-coset) by:*

$$\tilde{\eta}_{[\kappa]}(\upsilon) = \tilde{\eta}_{\varphi}(\kappa, \upsilon), \quad \tilde{\gamma}_{[\kappa]}(\upsilon) = \tilde{\gamma}_{\varphi}(\kappa, \upsilon), \quad \tilde{\delta}_{[\kappa]}(\upsilon) = \tilde{\delta}_{\varphi}(\kappa, \upsilon) \quad \forall \upsilon \in \tilde{\chi}, \qquad (*)$$

All elements $\upsilon \in \tilde{\chi}$ with SVNR value $\tilde{\eta}_{\varphi}(\kappa, \upsilon) > 0, \tilde{\gamma}_{\varphi}(\kappa, \upsilon) \leq 1, \tilde{\delta}_{\varphi}(\kappa, \upsilon) \leq 1$ are points having a membership value in the SVN-coset $[\kappa]$, and any point $\upsilon \in \tilde{\chi}$ with $\tilde{\eta}_{\varphi}(\kappa, \upsilon) = 0$, $\tilde{\gamma}_{\varphi}(\kappa, \upsilon) = 1$ and $\tilde{\delta}_{\varphi}(\kappa, \upsilon) = 1$ is not included in the SVN-coset $[\kappa]$. Any SVN-coset $[\kappa]$ confidently contained within the point $\kappa \in \tilde{\chi}$, and So, $\tilde{\eta}_{\bigvee_{\omega \in \tilde{\chi}}[\kappa]}(\omega) = 1$, $\tilde{\gamma}_{\bigwedge_{\omega \in \tilde{\chi}}[\kappa]}(\omega)) = 0$, $\tilde{\delta}_{\bigwedge_{\omega \in \tilde{\chi}}[\kappa]}(\omega) = 0$, $\forall \kappa \in \tilde{\chi}$. Also, $\tilde{\eta}_{\bigvee_{\omega \in \tilde{\chi}}[\omega]}(\upsilon) = 1$, $\tilde{\gamma}_{\bigwedge_{\omega \in \tilde{\chi}}([\omega]}(\upsilon) = 0$, $\tilde{\delta}_{\bigwedge_{\omega \in \tilde{\chi}}[\omega]}(\upsilon) = 0$, $\forall \upsilon \in \tilde{\chi}$ [i.e. $\bigvee_{\omega \in \tilde{\chi}}([\omega]) = \langle 0, 1, 1 \rangle$]. Clearly, if $\tilde{\eta}_{\varphi}(\kappa, \upsilon) > 0, \tilde{\gamma}_{\varphi}(\kappa, \upsilon) \leq 1, \tilde{\delta}_{\varphi}(\kappa, \upsilon) \leq 1$, then the SVN-cosets $[\kappa], [\upsilon]$ (as SVNSs) are having the same points of $\tilde{\chi}$ with some non zero membership values, and also, if $\tilde{\eta}_{[\upsilon]}(\omega) = 0$, $\tilde{\gamma}_{[\upsilon]}(\omega) = 1$ and $\tilde{\delta}_{[\upsilon]}(\omega) = 1$, then $\tilde{\eta}_{[\kappa]}(\omega) = 0$, $\tilde{\gamma}_{[\kappa]}(\omega) = 1$ and $\tilde{\delta}_{[\kappa]}(\omega) = 1$ whenever $\tilde{\eta}_{\varphi}(\kappa, \upsilon) > 0, \tilde{\gamma}_{\varphi}(\kappa, \upsilon) \leq 1, \tilde{\delta}_{\varphi}(\kappa, \upsilon) \leq 1$. That is very two SVN-cosets are either two SVN-coset containing the same points of $\tilde{\chi}$ with some non-zero membership values or encompassing totally different points of $\tilde{\chi}$ with some non-zero membership values.

Let us define the difference between two SVNSs as follows:

$$\tilde{\eta}_{\psi} \overline{\wedge} \tilde{\eta}_{\rho}(\kappa) = \begin{cases} 0, & \text{if} \quad \tilde{\eta}_{\psi}(\kappa) \leq \tilde{\eta}_{\rho}(\kappa), \\ (\tilde{\eta}_{\psi} \wedge \tilde{\eta}_{\rho^c})(\kappa), & \text{otherwise.} \end{cases}$$

$$(\tilde{\gamma}_{\psi} \overline{\vee} \tilde{\gamma}_{\rho}(\kappa) = \begin{cases} 1, & \text{if} \quad \tilde{\gamma}_{\psi}(\kappa) \geq \tilde{\gamma}_{\rho}(\kappa), \\ (\tilde{\gamma}_{\psi} \vee \tilde{\gamma}_{\rho^c})(\kappa), & \text{otherwise.} \end{cases} \tag{1.1}$$

$$\tilde{\delta}_{\psi} \overline{\vee} \tilde{\delta}_{\rho}(\kappa) = \begin{cases} 1, & \text{if} \quad \tilde{\delta}_{\psi}(\kappa) \geq \tilde{\delta}_{\rho}(\kappa), \\ (\tilde{\delta}_{\psi} \vee \tilde{\delta}_{\rho^c})(\kappa), & \text{otherwise.} \end{cases}$$

Definition 1.5. *Let $\psi \in \xi^{\tilde{\chi}}$ and φ a SVNER on $\tilde{\chi}$ and the SVN-cosets [are explained as in (*)]. Therefore, the single-valued neutrosophic lower (briefly, SVNL) ψ_{φ} set, the single-valued neutrosophic upper (briefly, SVNU) ψ^{φ} set and the single-valued neutrosophic boundary region (briefly, SVNBR) ψ^{B} set can be defined as follows:*

$$\begin{aligned} \tilde{\eta}_{\psi_{\varphi}}(\kappa) &= \tilde{\eta}_{\psi}(\kappa) \vee \bigwedge_{\psi^c(\omega) > 0, \omega \neq \kappa} \tilde{\delta}_{[\kappa]}(\omega), \\ \tilde{\gamma}_{\psi_{\varphi}}(\kappa) &= \tilde{\gamma}_{\psi}(\kappa) \wedge \bigvee_{\psi^c(\omega) > 0, \omega \neq \kappa} (1 - \tilde{\gamma}_{[\kappa]})(\omega), \\ \tilde{\delta}_{\psi_{\varphi}}(\kappa) &= \tilde{\delta}_{\psi}(\kappa) \wedge \bigvee_{\psi^c(\omega) > 0, \omega \neq \kappa} \tilde{\eta}_{[\kappa]}(\omega), \\ \tilde{\eta}_{\psi^{\delta}}(\kappa) &= \tilde{\eta}_{\psi}(\kappa) \wedge \bigvee_{\psi(\omega) > 0, \omega \neq \kappa} \tilde{\eta}_{[\kappa]}(\omega), \end{aligned} \tag{1.2}$$

$$\tilde{\gamma}_{\psi^{\varphi}}(\kappa) = \tilde{\gamma}_{\psi}(\kappa) \vee \bigwedge_{\psi(\omega)>0, \omega\neq\kappa} \tilde{\gamma}_{[\kappa]}(\omega), \tag{1.3}$$

$$\tilde{\delta}_{\psi^{\psi}}(\kappa) = \tilde{\delta}_{\psi}(\kappa) \vee \bigwedge_{\psi(\omega)>0, \omega\neq\kappa} \tilde{\delta}_{[\kappa]}(\omega),$$

$$\tilde{\eta}_{(\psi)^{B}}(\kappa) = (\tilde{\eta}_{\psi^{\varphi}} \overline{\wedge} \tilde{\eta}_{\psi_{\varphi}})(\kappa) = \begin{cases} 0, & \text{if} \quad \tilde{\eta}_{\psi^{\varphi}}(\kappa) \leq \tilde{\eta}_{\psi_{\varphi}}(\kappa), \\ (\tilde{\eta}_{\psi^{\delta}} \wedge \tilde{\eta}_{(\psi_{\varphi})^{c}})(\kappa), & \text{otherwise.} \end{cases}$$

$$\tilde{\gamma}_{\psi^{B}}(\kappa) = (\tilde{\gamma}_{\psi^{\delta}} \overline{\vee} \tilde{\gamma}_{\psi_{\varphi}})(\kappa) = \begin{cases} 1, & \text{if} \quad \tilde{\gamma}_{\psi^{\varphi}}(\kappa) \geq \tilde{\gamma}_{\psi_{\varphi}}(\kappa), \\ (\tilde{\gamma}_{\psi_{\varphi}} \vee \tilde{\gamma}_{(\psi_{\varphi})^{c}})(\kappa), & \text{otherwise.} \end{cases} \tag{1.4}$$

$$\tilde{\delta}_{\psi^{B}}(\kappa) = (\tilde{\delta}_{\psi^{\varphi}} \overline{\vee} \tilde{\delta}_{\psi_{\varphi}})(\kappa) = \begin{cases} 1, & \text{if} \quad \tilde{\delta}_{\psi^{\varphi}}(\kappa) \geq \tilde{\delta}_{(\psi_{\varphi}}(\kappa), \\ (\tilde{\delta}_{\psi_{\delta}} \vee \tilde{\delta}_{(\psi_{\varphi})^{c}})(\kappa), & \text{otherwise.} \end{cases}$$

Lemma 1.1. *For every* `SVNS` $\psi \in \xi^{\tilde{\chi}}$ *we obtain simply that:*

(1) $(\psi \vee \rho)_{\varphi} \geq \psi_{\varphi} \vee \rho_{\varphi}$,
(2) $(\varphi \wedge \rho)^{\varphi} \leq \psi^{\varphi} \wedge \rho^{\varphi}$,
(3) *if* $\psi \leq \rho$ *then* $\psi^{\varphi} \leq \rho^{\varphi}$ *and* $\psi_{\varphi} \leq \rho_{\varphi}$,
(4) $(\psi \vee \rho)^{\varphi} = \psi^{\varphi} \vee \rho^{\varphi}$,
(5) $(\psi \wedge \rho)_{\varphi} = \psi_{\varphi} \wedge \rho_{\varphi}$,
(6) $(\psi^{\varphi})^{c} = (\psi^{c})_{\varphi}$ *and* $(\psi_{\varphi})^{c} = (\psi^{c})^{\varphi}$,
(7) $(\psi_{\varphi})_{\varphi} = \psi_{\varphi}$,
(8) $(\psi^{\varphi})^{\varphi} = \psi^{\varphi}$.

Associated with a `SVNS` $\psi \in \xi^{\tilde{\chi}}$ in a `SVNA-space` $(\tilde{\chi}, \varphi)$, we can define a single-valued neutrosophic interior operator $\text{int}^{\psi}_{\varphi} : \xi^{\tilde{\chi}} \to \xi^{\tilde{\chi}}$ as follows:

$$\text{int}^{\psi}_{\varphi}(\rho) = \psi_{\varphi} \wedge \rho_{\varphi}, \forall\ \rho \neq \langle 1,0,0\rangle \text{ and } \text{int}^{\psi}_{\varphi}(\langle 1,0,0\rangle) = \langle 1,0,0\rangle. \tag{1.5}$$

Also, it was defined a $\text{cl}^{\psi}_{\varphi} : \xi^{\tilde{\chi}} \to \xi^{\tilde{\chi}}$ as next:

$$\text{cl}^{\psi}_{\varphi}(\rho) = (\psi_{\varphi})^{c} \vee \rho^{\varphi}, \forall\ \rho \neq \langle 0,1,1\rangle \text{ and } \text{cl}^{\psi}_{\varphi}(\langle 0,1,1\rangle) = \langle 0,1,1\rangle. \tag{1.6}$$

Remembrance that:

$$\text{cl}^{\psi}_{\varphi}(\rho^{\varphi}) = \text{cl}^{\psi}_{\varphi}(\rho), \forall \rho \in \xi^{\tilde{\chi}}, \text{int}^{\psi}_{\varphi}(\rho_{\varphi}) = \text{int}^{\psi}_{\varphi}(\rho), \forall \rho \in \xi^{\tilde{\chi}}, \tag{1.7}$$

$$\text{int}^{\psi}_{\varphi}(\rho^{c}) = [\text{cl}^{\psi}_{\varphi}(\rho)]^{c} \text{ and } \text{cl}^{\psi}_{\varphi}(\rho^{c}) = [\text{int}^{\psi}_{\varphi}(\psi)]^{c},\ \forall \rho \in \xi^{\tilde{\chi}}. \tag{1.8}$$

Definition 1.6. *Let* $(\tilde{\chi}, \varphi)$ *be a* `SVNA-space` *Subsequently, for every* $\psi \in \xi^{\tilde{\chi}}$. *Therefore,*

(1) ρ *is single-valued neutrosophic preopen* (`SVN-preopen`) *[resp. preclosed* (`SVN-preclosed`)*] set iff* $\rho \leq \text{int}^{\psi}_{\varphi}(\text{cl}^{\psi}_{\varphi}(\rho))$ *[resp.* $\rho \geq \text{cl}^{\psi}_{\varphi}(\text{int}^{\psi}_{\varphi}(\rho))$*].*
(2) *The single-valued neutrosophic preinterior of* ρ *(abbreviated,* $\text{pint}^{\psi}_{\varphi}$*) can be defined as follows:*

$$\text{pint}^{\psi}_{\varphi}(\rho) = \bigvee \{\pi \in \xi^{\tilde{\chi}} : \rho \geq \pi, \pi \text{ is } \texttt{SVN-preopen}\}.$$

(3) *The single-valued neutrosophic preclosure of ρ (abbreviated, $\mathrm{pcl}^{\psi}_{\varphi}$) can be defined as follows:*

$$\mathrm{pcl}^{\psi}_{\varphi}(\rho) = \bigwedge\{\pi \in \xi^{\tilde{\chi}} : \rho \leq \pi, \pi \text{ is } \texttt{SVN-preclosed}\}.$$

2. Single-valued neutrosophic ideal approximation spaces

In this section, we first introduce and analyze the single-valued neutrosophic ideal approximation space (abbreviated, `SVNIA-space`) and single-valued neutrosophic operator $\mathrm{cl}^{\psi}_{\Phi}$, $\mathrm{pcl}^{\psi}_{\Phi}$, $\mathrm{int}^{\psi}_{\Phi}$. Subsequently, we analyze the local single valued neutrosophic closed set ($\Phi_{\psi}(\rho)(\varphi,\hbar)$, for brevity) and local single valued neutrosophic preclosed set ($\Phi^{p}_{\psi}(\rho)(\varphi,\hbar)$, for brevity).

Definition 2.1. *A subset $\hbar \subseteq \xi^{\tilde{\chi}}$ is known as the single-valued neutrosophic ideal (`SVNI`) on $\tilde{\chi}$ if it meets the next criteria.*

(1) $\langle 0,1,1\rangle \in \hbar$,

(2) *If $\tilde{\eta}_{\psi}(\kappa) \leq \tilde{\eta}_{\rho}(\kappa), \tilde{\gamma}_{\psi}(\kappa) \geq \tilde{\gamma}_{\rho}(\kappa), \tilde{\delta}_{\psi}(\kappa) \geq \tilde{\delta}_{\rho}(\kappa)$ and $\psi \in \hbar$, then, $\rho \in \hbar, \forall \kappa \in \tilde{\chi}$ and $\psi,\rho \in \xi^{\tilde{\chi}}$,*

(3) *If $\psi \in \hbar$ and $\rho \in \hbar$, then $\langle (\tilde{\eta}_{\psi} \vee \tilde{\eta}_{\rho})(\kappa), (\tilde{\gamma}_{\psi} \wedge \tilde{\gamma}_{\rho})(\kappa), (\tilde{\delta}_{\psi} \wedge \tilde{\delta}_{\rho})(\kappa)\rangle \in \hbar, \forall \kappa \in \tilde{\chi}$ and $\psi,\rho \in \xi^{\tilde{\chi}}$.*

If $\hbar_1$ and $\hbar_2$ are `SVNI`s on $\tilde{\chi}$, we have $\hbar_1$ is finer than $\hbar_1$ [$\hbar_2$ is coarser than $\hbar_1$] if $\hbar_1 \supseteq \hbar_2$. The triple $(\tilde{\chi},\varphi,\hbar)$ is known as an single valued neutrosophic ideal approximation space (abbreviated, `SVNIA-space`). Occasionally, $\hbar = \langle 0,1,1\rangle$ is written as $\hbar^{\circ}$ herein to avoid ambiguity.

Definition 2.2. *Let $(\tilde{\chi},\varphi,\hbar)$ be an `SVNIA-space` related with $\psi \in \xi^{\tilde{\chi}}$. Therefore*

(1) *The local single valued neutrosophic closed set $\Phi_{\psi}(\rho)(\varphi,\hbar)$ of a set $\rho \in \xi^{\tilde{\chi}}$ is defined by:*

$$\Phi_{\psi}(\rho)(\varphi,\hbar) = \bigwedge\{\pi \in \xi^{\tilde{\chi}} : \rho\overline{\wedge}\pi = \langle \tilde{\eta}_{\rho}\overline{\wedge}\tilde{\eta}_{\pi}(\kappa), \tilde{\gamma}_{\rho}\overline{\vee}\tilde{\gamma}_{\pi}(\kappa), \tilde{\delta}_{\rho}\overline{\vee}\tilde{\delta}_{\pi}(\kappa)\rangle \in \hbar, \mathrm{cl}^{\psi}_{\varphi}(\pi) = \pi\}. \tag{2.1}$$

Occasionally, $\Phi_{\psi}(\rho)(\varphi,\hbar)$ is written as $\Phi_{\psi}(\rho)$ or $\Phi_{\psi}(\rho)(\hbar)$ herein to avoid ambiguity.

(2) *The local single valued neutrosophic pre-closed set $\Phi^{p}_{\psi}(\rho)(\varphi,\hbar)$ of a set $\rho \in \xi^{\tilde{\chi}}$ is defined by:*

$$\Phi^{p}_{\psi}(\rho)(\varphi,\hbar) = \bigwedge\{\pi \in \xi^{\tilde{\chi}} : \rho\overline{\wedge}\pi = \langle \tilde{\eta}_{\rho}\overline{\wedge}\tilde{\eta}_{\pi}(\kappa), \tilde{\gamma}_{\rho}\overline{\vee}\tilde{\gamma}_{\pi}(\kappa), \tilde{\delta}_{\rho}\overline{\vee}\tilde{\delta}_{\pi}(\kappa)\rangle \in \hbar, \mathrm{pcl}^{\psi}_{\varphi}(\pi) = \pi\}. \tag{2.2}$$

We will write $\Phi^{p}_{\psi}(\rho)$ or $\Phi^{p}_{\psi}(\rho)(\hbar)$ instead of $\Phi^{p}_{\psi}(\rho)(\varphi,\hbar)$.

Lemma 2.1. *Let $(\tilde{\chi},\varphi,\hbar^{\diamond})$ be a `SVNIA-space`, $\psi \in \xi^{\tilde{\chi}}$. Then for any $\rho \in \xi^{\tilde{\chi}}$ we have $\Phi_{\psi}(\rho) = \mathrm{cl}^{\psi}_{\varphi}(\rho)$ and $\Phi^{p}_{\psi}(\rho) = \mathrm{pcl}^{\psi}_{\varphi}(\rho)$.*

Proposition 2.1. *Let $(\tilde{\chi},\varphi,\hbar)$ be a `SVNIA-space` related with $\psi \in \xi^{\tilde{\chi}}$. Then,*

(1) *$\rho \leq \pi$ implies $\Phi_{\psi}(\rho) \leq \Phi_{\psi}(\pi)$ and $\Phi^{p}_{\psi}(\rho) \leq \Phi^{p}_{\psi}(\pi)$.*

(2) *If $\hbar_1$ and $\hbar_2$ are `SVNIs` on $\tilde{\chi}$ and $\hbar_1 \subseteq \hbar_2$, then $\Phi_{\psi}(\rho)(\hbar_1) \geq \Phi_{\psi}(\rho)(\hbar_2)$ and $\Phi^{p}_{\psi}(\rho)(\hbar_1) \geq \Phi^{p}_{\psi}(\rho)(\hbar_2)$.*

(3) *$\Phi^{p}_{\psi}(\rho) \leq \Phi_{\psi}(\rho) \leq \mathrm{cl}^{\psi}_{\varphi}(\Phi_{\psi}(\rho)) \leq \mathrm{cl}^{\psi}_{\varphi}(\rho)$ and $\Phi^{p}_{\psi}(\rho) \leq \mathrm{pcl}^{\psi}_{\varphi}(\Phi^{p}_{\psi}(\rho)) \leq \mathrm{pcl}^{\psi}_{\varphi}(\rho) \leq \mathrm{cl}^{\psi}_{\varphi}(\rho)$.*

(4) *$\Phi_{\psi}(\Phi_{\psi}(\rho)) \leq \mathrm{cl}^{\psi}_{\varphi}(\Phi_{\psi}(\rho)) = \Phi_{\psi}(\rho)$.*

(5) *$\Phi^{p}_{\psi}(\Phi^{p}_{\psi}(\rho)) \leq \mathrm{pcl}^{\psi}_{\varphi}(\Phi^{p}_{\psi}(\rho)) = \Phi^{p}_{\psi}(\rho)$.*

(6) *$\Phi_{\psi}(\rho) \vee \Phi_{\psi}(\pi) \leq \Phi_{\psi}(\rho \vee \pi)$ and $\Phi_{\psi}(\rho) \wedge \Phi_{\psi}(\pi) \geq \Phi_{\psi}(\rho \wedge \pi)$.*

Proof. Easy, so omitted. □

Definition 2.3. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be a* `SVNIA-space` *associated with* $\psi \in \xi^{\tilde{\chi}}$. *Then for each* $\rho \in \xi^{\tilde{\chi}}$ *define the single valued neutrosophic operator* $\mathrm{cl}^{\psi}_{\Phi}$, $\mathrm{pcl}^{\psi}_{\Phi}$, $\mathrm{int}^{\psi}_{\Phi}$, $\mathrm{pint}^{\psi}_{\Phi} : \xi^{\tilde{\chi}} \to \xi^{\tilde{\chi}}$ *as next:*

$$\mathrm{cl}^{\psi}_{\Phi}(\rho) = \rho \vee \Phi_{\psi}(\rho) \qquad \mathrm{pcl}^{\psi}_{\Phi}(\rho) = \rho \vee \Phi^{p}_{\psi} \quad \forall \rho \in \xi^{\tilde{\chi}}. \tag{2.3}$$

$$\mathrm{int}^{\psi}_{\Phi}(\rho) = \rho \wedge (\Phi_{\psi}(\rho^{c}))^{c} \qquad \mathrm{pint}^{\psi}_{\Phi}(\rho) = \rho \wedge (\Phi^{p}_{\psi}(\rho^{c}))^{c} \quad \forall \rho \in \xi^{\tilde{\chi}}. \tag{2.4}$$

Proposition 2.2. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be a* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$. *Then, for any* $\rho, \pi \in \xi^{\tilde{\chi}}$, *we have:*

(1) $\mathrm{int}^{\psi}_{\varphi}(\rho) \leq \mathrm{pint}^{\psi}_{\Phi}(\rho) \leq \mathrm{int}^{\psi}_{\Phi}(\rho) \leq \rho \leq \mathrm{pcl}^{\psi}_{\Phi}(\rho) \leq \mathrm{cl}^{\psi}_{\Phi}(\rho) \leq \mathrm{cl}^{\psi}_{\varphi}(\rho)$.
(2) $\mathrm{cl}^{\psi}_{\Phi}(\rho^{c}) = (\mathrm{int}^{\psi}_{\Phi}(\rho))^{c}$ *and* $\mathrm{int}^{\psi}_{\Phi}(\rho^{c}) = (\mathrm{cl}^{\psi}_{\Phi}(\rho))^{c}$.
(3) $\mathrm{cl}^{\psi}_{\Phi}(\rho \vee \pi) \geq \mathrm{cl}^{\psi}_{\Phi}(\rho) \wedge \mathrm{cl}^{\psi}_{\Phi}(\pi)$ *and* $\mathrm{cl}^{\psi}_{\Phi}(\rho \wedge \pi) \leq \mathrm{cl}^{\psi}_{\Phi}(\rho) \wedge \mathrm{cl}^{\psi}_{\Phi}(\pi)$.
(4) $\mathrm{int}^{\psi}_{\Phi}(\rho \vee \pi) \geq \mathrm{int}^{\psi}_{\Phi}(\rho) \wedge \mathrm{int}^{\psi}_{\Phi}(\pi)$ *and* $\mathrm{int}^{\psi}_{\Phi}(\rho \wedge \pi) \leq \mathrm{int}^{\psi}_{\Phi}(\rho) \wedge \mathrm{int}^{\psi}_{\Phi}(\pi)$.
(5) $\mathrm{cl}^{\psi}_{\Phi}(\mathrm{cl}^{\psi}_{\Phi}(\rho)) \geq \mathrm{cl}^{\psi}_{\Phi}(\rho)$ *and* $\mathrm{int}^{\psi}_{\Phi}(\mathrm{int}^{\psi}_{\Phi}(\rho)) \leq \mathrm{int}^{\psi}_{\Phi}(\rho)$.
(6) *If* $\rho \leq \pi$, *then* $\mathrm{cl}^{\psi}_{\Phi}(\rho) \leq \mathrm{cl}^{\psi}_{\Phi}(\pi)$ *and* $\mathrm{int}^{\psi}_{\Phi}(\rho) \leq \mathrm{int}^{\psi}_{\Phi}(\pi)$.
(7) $\mathrm{pcl}^{\psi}_{\Phi}(\rho) \leq \mathrm{pcl}^{\psi}_{\varphi}(\rho)$.

Proof. From (7): Suppose that $\mathrm{pcl}^{\psi}_{\Phi}(\rho) \nleq \mathrm{pcl}^{\psi}_{\varphi}(\rho)$, and if $\mathrm{pcl}^{\psi}_{\varphi}(\rho) = \pi$, then $\rho \leq \pi$ and π is `SVN-preclosed` set with $\mathrm{pcl}^{\psi}_{\Phi}(\rho) \nleq \pi$. But $\rho \leq \pi$ implies $\rho \overline{\wedge} \pi = \langle \tilde{\eta}_{\rho} \overline{\wedge} \tilde{\eta}_{\pi}(\kappa), \tilde{\gamma}_{\rho} \overline{\vee} \tilde{\gamma}_{\pi}(\kappa), \tilde{\delta}_{\rho} \overline{\vee} \tilde{\delta}_{\pi}(\kappa) \rangle \in \hbar$, and thus $\Phi^{P}_{\psi}(\rho) \leq \pi$ which means that $\mathrm{pcl}^{\psi}_{\varphi}(\rho) = \rho \vee \Phi^{P}_{\psi}(\rho) \leq \rho \wedge \pi \leq \pi$, which is a contradiction. Hence,$\mathrm{pcl}^{\psi}_{\Phi}(\rho) \nleq \mathrm{pcl}^{\psi}_{\varphi}(\rho)$.
(1)-(6): Clear. □

Definition 2.4. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be a* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$. *Then,*

(1) $\rho \in \xi^{\tilde{\chi}}$ *is termed a single valued neutrosophic ideal pre-open (*`SVNI-preopen`*, abbreviated) if* $\rho \leq \mathrm{int}^{\psi}_{\varphi}(\mathrm{cl}^{\psi}_{\Phi}(\rho))$.
(2) $\rho \in \xi^{\tilde{\chi}}$ *is termed a single valued neutrosophic* Φ*-open (*`SVNΦ-open`*, abbreviated) if* $\rho \leq \mathrm{int}^{\psi}_{\varphi}(\Phi_{\psi}(\rho))$.
The complement of `SVNΦ-closed` *(resp,*`SVNI-preclosed`*) is termed a* `SVNΦ-open` *(resp,* `SVNI-preopen`*).*
(3) $\rho \in \xi^{\tilde{\chi}}$ *is termed a single valued neutrosophic dense in itself (*`SVN-dense in itself`*, abbreviated) if* $\rho \leq \Phi_{\psi}(\rho)$.

Theorem 2.1. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be a* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$. *Then,*

(1) *If* $\rho \in \xi^{\tilde{\chi}}$ *is* `SVNΦ-closed`*, then is* $\rho \geq \Phi_{\psi}(\mathrm{int}^{\psi}_{\varphi}(\rho))$.
(2) *If* $\rho \in \xi^{\tilde{\chi}}$ *is* `SVNI-preclosed`*, then is* $\rho \geq \mathrm{cl}^{\psi}_{\varphi}(\mathrm{int}^{\psi}_{\Phi}(\rho))$.

Proof. For (1): Let$\rho \in \xi^{\tilde{\chi}}$ is `SVNΦ-closed`. Then,

$$\begin{aligned} \rho^{c} \leq \mathrm{int}^{\psi}_{\varphi}(\Phi_{\psi}(\rho^{c})) &\leq \mathrm{int}^{\psi}_{\varphi}(\mathrm{cl}^{\psi}_{\varphi}(\rho^{c})) = \mathrm{int}^{\psi}_{\varphi}((\mathrm{int}^{\psi}_{\varphi}(\rho))^{c}) \\ &= (\mathrm{cl}^{\psi}_{\varphi}(\mathrm{int}^{\psi}_{\varphi}(\rho)))^{c} \leq (\Phi_{\varphi}(\mathrm{int}^{\psi}_{\varphi}(\rho)))^{c}. \end{aligned}$$

Hence, $\Phi_{\varphi}(\mathrm{int}^{\psi}_{\varphi}(\rho)) \le \rho$.

(2), it is easy. □

It is clear that:

$$\begin{array}{ccc} \texttt{SVN}\Phi\texttt{-open}(\texttt{SVN}\Phi\texttt{-open}) & \longrightarrow & \texttt{SVNI-preopen}(\texttt{SVNI-preclosed}) \\ \downarrow & & \downarrow \\ \texttt{SVNI-preopen}(\texttt{SVNI-preclosed}) & & \texttt{SVNI-preopen}(\texttt{SVNI-preclosed}) \end{array}$$

Example 2.1. *Let χ be SVNR on a set $\tilde{\chi} = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ defined as next.*

φ	κ_1	κ_2	κ_3	κ_4
ω_1	(1, 0, 0)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
ω_2	(1, 0, 0)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
ω_3	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(0.6, 0.4, 0.6)
ω_4	(0, 1, 1)	(0, 1, 1)	(0.6, 0.4, 0.6)	(1, 0, 0)

Table 1. Caption

Assume that $\psi = \langle(1,0,0),(1,0,0),(0.5,0.5,0.5),(0.5,0.5,0.5)\rangle$. Then,

$$\tilde{\eta}_{(\psi)^{\varphi}}(\kappa_1) = \tilde{\eta}_{\psi}(\kappa_1) \wedge \bigvee_{\psi(\omega)>0,\omega\neq\kappa_1} \tilde{\eta}_{[\kappa]}(\omega) = 1,$$

$$\tilde{\gamma}_{(\psi)^{\varphi}}(\kappa_1) = \tilde{\gamma}_{\psi}(\kappa_1) \vee \bigwedge_{\psi(\omega)>0,\omega\neq\kappa_1} (\tilde{\gamma}_{[\kappa]})(\omega) = 0,$$

$$\tilde{\delta}_{(\psi)^{\varphi}}(\kappa_1) = \tilde{\delta}_{\psi}(\kappa_1) \vee \bigwedge_{\psi(\omega)>0,\omega\neq\kappa_1} \tilde{\delta}_{[\kappa]}(\omega) = 0,$$

Hence, $(\psi)^{\varphi}(\kappa_1) = (0,1,1)$ and similarly, we can obtain $(\psi)^{\varphi}(\kappa_2) = (0,1,1)$ and

$$\tilde{\eta}_{(\psi)^{\varphi}}(\kappa_3) = \tilde{\eta}_{\psi}(\kappa_3) \wedge \bigvee_{\psi(\omega)>0,\omega\neq\kappa_3} \tilde{\eta}_{[\kappa]}(\omega) = 0.5,$$

$$\tilde{\gamma}_{(\psi)^{\varphi}}(\kappa_3) = \tilde{\gamma}_{\psi}(\kappa_3) \vee \bigwedge_{\psi(\omega)>0,\omega\neq\kappa_3} (\tilde{\gamma}_{[\kappa]})(\omega) = 0.5,$$

$$\tilde{\delta}_{(\psi)^{\varphi}}(\kappa_3) = \tilde{\delta}_{\psi}(\kappa_3) \vee \bigwedge_{\psi(\omega)>0,\omega\neq\kappa_3} \tilde{\delta}_{[\kappa]}(\omega) = 0.6.$$

Hence, $\psi^{\varphi}(\kappa_3) = (0.5,0.5,0.6)$ and similarly, we can obtain $\psi^{\varphi}(\kappa_4) = (0.5,0.5,0.6)$. Thus and by equation (1.3), we have

$$\psi^{\psi} = \langle(1,0,0),(1,0,0),(0.5,0.5,0.6),(0.5,0.5,0.6)\rangle,$$

$$\psi_{\delta} = \langle(1,0,0),(1,0,0),(0.6,0.5,0.5),(0.6,0.5,0.5)\rangle$$

$$(\psi_\varphi)^c = \langle(0,1,1),(0,1,1),(0.5,0.5,0.6),(0.5,0.5,0.6)\rangle,$$

Let, $\rho = \langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0,1,1),(0,1,1)\rangle$, *then by equations (3) and (4), we obtain* $\rho^\varphi = \langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0,1,1),(0,1,1)\rangle$ *and* $\rho_\varphi = \langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.6,0.6,0.6),(0.6,0.6,0.6)\rangle$. *Hence,*

$$\mathrm{cl}^\psi_\varphi(\rho) = (\psi_\varphi)^c \vee \rho^\varphi = \langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.5,0.5,0.6),(0.5,0.5,0.6)\rangle$$

implies that

$$\rho \leq \mathrm{int}^\psi_\varphi(\mathrm{cl}^\psi_\varphi(\rho)) = \langle(0.3,0.3,0.3),(0.3,0.3,0.3),(0.5,0.6,0.6),(0.5,0.6,0.6)\rangle.$$

Thus, ρ *is* `SVN-preopen`.

Assume that a `SVNI` *is defined on* $\tilde{\chi}$ *as next*

$$\hbar = \left\{\pi_n \in \xi^{\tilde{\chi}} : \pi_n \leq \langle(0.5,0.3,0.3),(0.5,0.3,0.3),(1,0,0),(1,0,0)\rangle\right\}.$$

By equations (1.1), (1.2) and (1.3) we get that, ρ *it is neither* `SVNI-preopen` *nor* `SVNΦ-open`.

Theorem 2.2. *Let* $(\tilde{\chi},\varphi,\hbar)$ *be a* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$*.Then, the next are equivalent.*

(1) $\rho \in \xi^{\tilde{\chi}}$ *is* `SVNΦ-open`.

(2) $\rho \in \xi^{\tilde{\chi}}$ *is* `SVNI-preopen` *and* `SVN-dense in itself`

Proof. (1) $\Rightarrow$ (2): It is very easy to see that all `SVNΦ-open` set is `SVNI-preopen`. On the other hand $\rho\mathrm{int}^\psi_\varphi(\Phi_\varphi(\rho))$, which means ρ is `SVN-dense in itself`.

(2) $\Rightarrow$ (1): By assumption, $\rho \leq \mathrm{int}^\psi_\varphi(\mathrm{cl}^\psi_\Phi(\rho)) = \mathrm{int}^\psi_\varphi(\rho \vee \Phi_\varphi(\rho)) = \mathrm{int}^\psi_\varphi(\Phi_\varphi(\rho))$, and hence ρ is `SVNΦ-open`. □

3. Separation axioms in single-valued neutrosophic ideal approximation spaces

The goal of this unit is to familiarize the concepts of $\mathtt{SVN}T_0^{(t,s)}$, $\mathtt{SVN}T_1^{(t,s)}$, $\mathtt{SVN}T_2^{(t,s)}$ and $\mathtt{SVNI}T_0^{(t,s)}$, $\mathtt{SVNI}T_1^{(t,s)}$, $\mathtt{SVNI}T_2^{(t,s)}$.

Definition 3.1. *Let* $(\tilde{\chi},\varphi,\hbar)$ *be a* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$*. Then,*

(1) *A* `SVNIA-space` $(\tilde{\chi},\varphi,\hbar)$ *(resp. a* `SVNA-space` $(\tilde{\chi},\varphi)$*) is termed a* $\mathtt{SVNI}T_0^{(t,s)}$ *(resp.* $\mathtt{SVN}T_0^{(t,s)}$*) if for every* $x \neq y \in \tilde{\chi}$*, there exists* $\rho \in \xi^{\tilde{\chi}}$*,* $t \in \xi_0$ *with* $\tilde{\eta}_{\mathrm{int}^\psi_\Phi(\rho)}(x) \geq t$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\Phi(\rho)}(x) \leq t$ *and* $\tilde{\delta}_{\mathrm{int}^\psi_\Phi(\rho)}(x) \leq t$ *[resp.* $\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \geq t$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \leq t$*,* $\tilde{\delta}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \leq t$*] such that* $\tilde{\eta}_\rho(y) < t$*,* $\tilde{\gamma}_\rho(y) > t$ $\tilde{\delta}_\rho(y) > t$ *or there exists* $\pi \in \xi^{\tilde{\chi}}$*,* $s \in \xi_0$ *with* $\tilde{\eta}_{\mathrm{int}^\psi_\Phi(\pi)}(y) \geq s$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\Phi(\pi)}(y) \leq s$ *and* $\tilde{\delta}_{\mathrm{int}^\psi_\Phi(\pi)}(y) \leq s$ *[resp.* $\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \geq s$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \leq s$ *and* $\tilde{\delta}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \leq s$*] such that* $\tilde{\eta}_\pi(x) < s$*,* $\tilde{\gamma}_\pi(x) > s$*,* $\tilde{\delta}_\pi(x) > s$*.*

(2) *A* `SVNIA-space` $(\tilde{\chi},\varphi,\hbar)$ *(resp. a* `SVNA-space` $(\tilde{\chi},\varphi)$*) is termed a* $\mathtt{SVNI}T_1^{(t,s)}$ *(resp.* $\mathtt{SVN}T_1^{(t,s)}$*) if for every* $x \neq y \in \tilde{\chi}$*, there exists* $\rho,\pi \in \xi^{\tilde{\chi}}$*,* $t,s \in \xi_0$ *with* $\tilde{\eta}_{\mathrm{int}^\psi_\Phi(\rho)}(x) \geq t$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\Phi(\rho)}(x) \leq t$*,* $\tilde{\delta}_{\mathrm{int}^\psi_\Phi(\rho)}(x) \leq t$ *and* $\tilde{\eta}_{\mathrm{int}^\psi_\Phi(\pi)}(y) \geq s$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\Phi(\pi)}(y) \leq s$*,* $\tilde{\delta}_{\mathrm{int}^\psi_\Phi(\pi)}(y) \leq s$ *[resp.* $\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \geq t$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \leq t$*,* $\tilde{\delta}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \leq t$ *and* $\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \geq s$*,* $\tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \leq s$*,* $\tilde{\delta}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \leq s$*] such that* $\tilde{\eta}_\rho(y) < t$*,* $\tilde{\gamma}_\rho(y) > t$ $\tilde{\delta}_\rho(y) > t$ *and* $\tilde{\eta}_\pi(x) < s$*,* $\tilde{\gamma}_\pi(x) > s$*,* $\tilde{\delta}_\pi(x) > s$*.*

(3) *A* `SVNIA-space` $(\tilde{\chi}, \varphi, \hbar)$ *(resp. a* `SVNA-space` $(\tilde{\chi}, \varphi)$*) is termed a* $\texttt{SVNIT}_2^{(t,s)}$ *(resp.* $\texttt{SVNT}_2^{(t,s)}$*) if for every* $x \neq y \in \tilde{\chi}$*, there exists* $\rho, \pi \in \xi^{\tilde{\chi}}$*,* $t, s \in \xi_0$ *with* $\tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(\rho)}(x) \geq t$*,* $\tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(\rho)}(x) \leq t$*,* $\tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(\rho)}(x) \leq t$ *and* $\tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(\pi)}(y) \geq s$*,* $\tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(\pi)}(y) \leq s$*,* $\tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(\pi)}(y) \leq s$ *[resp.* $\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi}(\rho)}(x) \geq t$*,* $\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi}(\rho)}(x) \leq t$*,* $\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi}(\rho)}(x) \leq t$ *and* $\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi}(\pi)}(y) \geq s$*,* $\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi}(\pi)}(y) \leq s$*,* $\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi}(\pi)}(y) \leq s$*] such that* $\tilde{\eta}_{\sup(\rho\cap\pi)} < (t \cap s), \tilde{\gamma}_{\sup(\rho\cap\pi)} > (t \cap s), \tilde{\delta}_{\sup(\rho\cap\pi)} > (t \cap s)$*.*

Remark 3.1. ***It is clear that:***

$$\begin{array}{ccccc}
\texttt{SVNT}_2^{(t,s)} & \longrightarrow & \texttt{SVNT}_1^{(t,s)} & \longrightarrow & \texttt{SVNT}_0^{(t,s)} \\
\downarrow & & \downarrow & & \downarrow \\
\texttt{SVNIT}_2^{(t,s)} & \longrightarrow & \texttt{SVNIT}_1^{(t,s)} & \longrightarrow & \texttt{SVNIT}_0^{(t,s)}
\end{array}$$

Example 3.1. *Let* φ *be a* `SVNR` *on a set* $\tilde{\chi} = \{x, y, z\}$ *as shown down.*

φ	x	y	z
x	(1, 0, 0)	(0.3, 0.7, 0.3)	(0, 1, 1)
y	(0.3, 0.7, 0.3)	(1, 0, 0)	(0.1, 0.9, 0.1)
z	(0, 1, 1)	(0.1, 0.9, 0.1)	(1, 0, 0)

Assume that $\psi = \langle(1,0,0), (0.8,0.8,0.2), (0,1,1)\rangle$ *and* $t = s = 0.5$*. Then,*

$$\tilde{\eta}_{\psi_\varphi}(x) = \tilde{\eta}_{\psi}(x) \vee \bigwedge_{\psi^c(\omega)>0, \omega\neq x} \tilde{\delta}_{[x]}(\omega) = 1,$$

$$\tilde{\gamma}_{\psi_\varphi}(x) = \tilde{\gamma}_{\psi}(x) \wedge \bigvee_{\psi^c(\omega)>0, \omega\neq x} (1 - \tilde{\gamma}_{[x]})(\omega) = 0,$$

$$\tilde{\delta}_{\psi_\varphi}(x) = \tilde{\delta}_{\psi}(x) \wedge \bigvee_{\psi^c(\omega)>0, \omega\neq x} \tilde{\eta}_{[x]}(\omega) = 0,$$

$$\tilde{\eta}_{\psi_\varphi}(y) = \tilde{\eta}_{\psi}(y) \vee \bigwedge_{\psi^c(\omega)>0, \omega\neq y} \tilde{\delta}_{[y]}(\omega) = 0.8,$$

$$\tilde{\gamma}_{\psi_\varphi}(y) = \tilde{\gamma}_{\psi}(y) \wedge \bigvee_{\psi^c(\omega)>0, \omega\neq y} (1 - \tilde{\gamma}_{[y]})(\omega) = 0.3,$$

$$\tilde{\delta}_{\psi_\varphi}(y) = \tilde{\delta}_{\psi}(y) \wedge \bigvee_{\psi^c(\omega)>0, \omega\neq y} \tilde{\eta}_{[y]}(\omega) = 0.2,$$

and

$$\tilde{\eta}_{\psi_\varphi}(z) = \tilde{\eta}_{\psi}(z) \vee \bigwedge_{\psi^c(\omega)>0, \omega\neq z} \tilde{\delta}_{[z]}(\omega) = 0.1,$$

$$\tilde{\gamma}_{\psi_\varphi}(z) = \tilde{\gamma}_\psi(z) \wedge \bigvee_{\psi^c(\omega)>0, \omega \neq z} (1 - \tilde{\gamma}_{[z]})(\omega) = 0.1,$$

$$\tilde{\delta}_{\psi_\varphi}(z) = \tilde{\delta}_\psi(z) \wedge \bigvee_{\psi^c(\omega)>0, \omega \neq z} \tilde{\eta}_{[z]}(\omega) = 0.1.$$

Hence, $\psi_\varphi = \langle (1,0,0), (0.8,0.3,0.2), (0.1,0.1,0.1) \rangle$ *and by equation (3), we have*

$$\psi^\varphi = \langle (0.3,0.7,0.3), (0.3,0.8,0.2), (0,1,1) \rangle,$$

and hence, we get

$$(\psi_\varphi)^c = \langle (0,1,1), (0.2,0.7,0.8), (0.1,0.9,0.1) \rangle.$$

Now, for the case $x \neq y$*, there exists* $\rho = \langle (0.8,0.2,0.8), (0,1,1), (0.6,0.4,0.6) \rangle$*, and then*

$$\rho_\varphi = \langle (0.8,0.2,0.3), (0.1,0.3,0.3), (0.6,0.1,0.1) \rangle,$$

which means $\mathrm{int}^\psi_\varphi(\rho) = \psi_\varphi \wedge \rho_\varphi = \langle (0.8,0.2,0.3), (0.1,0.3,0.3), (0.1,0.1,0.1) \rangle$*, and thus*

$$\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \geq 0.5, \quad \tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \leq 0, \quad \tilde{\delta}_{\mathrm{int}^\psi_\varphi(\rho)}(x) \leq 0.5,$$

with $\tilde{\eta}_\rho(y) < 0.5$, $\tilde{\gamma}_\rho(y) > 0.5$ $\tilde{\delta}_\rho(y) > 0.5$.

Also, we can find $\pi = \langle (0,1,1), (0.6,0.4,0.4), (0.1,0.1,0.1) \rangle$*, and then*

$$\pi_\varphi = \langle (0,1,1), (0.6,0.3,0.3), (0.1,0.1,0.1) \rangle$$

which means $\mathrm{int}^\psi_\varphi(\pi) = \psi_\varphi \wedge \pi\varphi = \langle (0,1,1), (0.6,0.3,0.3), (0.1,0.1,0.1) \rangle$*, and thus*

$$\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \geq 0.5, \quad \tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \leq 0.5, \quad \tilde{\delta}_{\mathrm{int}^\psi_\varphi(\pi)}(y) \leq 0.5,$$

with $\tilde{\eta}_\pi(x) < 0.5$, $\tilde{\gamma}_\pi(x) > 0.5$ $\tilde{\delta}_\pi(x) > 0.5$.

For the cases $x \neq z$ *and* $y \neq z$ *we can find* $\sigma \in \xi^{\tilde{\chi}}$ *with*

$$\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\sigma)}(x) \geq 0.5, \quad \tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\sigma)}(x) \leq 0.5, \quad \tilde{\delta}_{\mathrm{int}^\psi_\varphi(\sigma)}(x) \leq 0.5,$$

or

$$\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\sigma)}(y) \geq 0.5, \quad \tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\sigma)}(y) \leq 0.5, \quad \tilde{\delta}_{\mathrm{int}^\psi_\varphi(\sigma)}(y) \leq 0.5,$$

such that $\tilde{\eta}_\sigma(z) < 0.5$, $\tilde{\gamma}_\sigma(z) > 0.5$ $\tilde{\delta}_\sigma(z) > 0.5$*, while we can not find* $\sigma \in \xi^{\tilde{\chi}}$ *with*

$$\tilde{\eta}_{\mathrm{int}^\psi_\varphi(\sigma)}(z) \geq 0.5, \quad \tilde{\gamma}_{\mathrm{int}^\psi_\varphi(\sigma)}(z) \leq 0.5, \quad \tilde{\delta}_{\mathrm{int}^\psi_\varphi(\sigma)}(z) \leq 0.5.$$

Hence, $(\tilde{\chi}, \varphi)$ *is a single-valued neutrosophic approximation* $T_0^{(0.5,0.5)}$ *− space related with* ψ. $(\tilde{\chi}, \varphi)$ *but its not be a single-valued neutrosophic approximation* $T_1^{(0.5,0.5)}$ *− space or* $T_2^{(0.5,0.5)}$ *− space.*

In this example given to show that $(\tilde{\chi}, \varphi, \hbar)$ *is a single-valued neutrosophic ideal approximation* $T_i^{(0.5,0.5)}$ *− space,* $i = 1,2,3$ *while* $(\tilde{\chi}, \varphi)$ *is even not single-valued neutrosophic approximation* $T_1^{(0.5,0.5)}$ *− space.*

If $(\tilde{\chi},\varphi)$ and $(\tilde{\mathcal{F}},\varphi^*)$ are SVNA-spaces related with $\psi\in\xi^{\tilde{\chi}}$ and $\rho\in\xi^{\tilde{\mathcal{F}}}$, respectively, then a mapping $f:(\tilde{\chi},\varphi)\longrightarrow(\tilde{\mathcal{F}},\varphi^*)$ is said to be single-valued neutrosophic approximation continuous (SVNAC-map), if

$$\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\sigma))\geq f^{-1}(\mathrm{int}^{\psi}_{\varphi^*}(\sigma)),\forall,\sigma\in\xi^{\tilde{\mathcal{F}}}.$$

Obviously, it corresponds to

$$\mathrm{cl}^{\psi}_{\varphi}(f^{-1}(\sigma))\leq f^{-1}(\mathrm{cl}^{\psi}_{\varphi^*}(\sigma)),\forall,\sigma\in\xi^{\tilde{\mathcal{F}}}$$

Now, with respect to $\psi\in\xi^{\tilde{\chi}}$ and $\rho\in\xi^{\tilde{\mathcal{F}}}$, provided that $\hbar$, $\hbar^*$ are SVNIs on $\tilde{\chi},\tilde{\mathcal{F}}$, respectively, then a map $f:(\tilde{\chi},\varphi,\hbar)\longrightarrow(\tilde{\chi},\varphi^*)$ is termed single-valued neutrosophic ideal approximation continuous (birfy, SVNIAC-map), provided that $\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\sigma))\geq f^{-1}(\mathrm{int}^{\psi}_{\varphi^*}(\sigma))$ for every $\sigma\in\xi^{\tilde{\mathcal{F}}}$. Obviously, it corresponds to $\mathrm{cl}^{\psi}_{\Phi}(f^{-1}(\sigma))\leq f^{-1}(\mathrm{cl}^{\psi}_{\varphi^*}(\sigma))$ $\forall$, $\sigma\in\xi^{\tilde{\mathcal{F}}}$.

Also, let us call $f:(\tilde{\chi},\varphi)\longrightarrow(\tilde{\mathcal{F}},\varphi^*)$ is termed single-valued neutrosophic approximation open (birfy, SVNAO-map), provided that $\mathrm{int}^{\psi}_{\varphi^*}(f(\pi))\geq f(\mathrm{int}^{\psi}_{\varphi}(\pi))$ for all $\pi\in\xi^{\tilde{\chi}}$, $f:(\tilde{\chi},\varphi)\longrightarrow(\tilde{\mathcal{F}},\varphi^*,\hbar^*)$ is termed single-valued neutrosophic ideal approximation open (birfy, SVNIAO-map), provided that $\mathrm{int}^{\psi}_{\Phi}(f(\pi))\geq f(\mathrm{int}^{\psi}_{\varphi}(\pi))$ for all $\pi\in\xi^{\tilde{\chi}}$.

From (1) in Proposition 2, we can prove that (SVNAC-map) (resp. (SVNAO-map)) will be (SVNIAC-map) (resp. (SVNIAO-map)).

Theorem 3.1. *Let $(\tilde{\chi},\varphi)$, $(\tilde{\mathcal{F}},\varphi^*)$ be SVNA-spaces related with $\psi\in\xi^{\tilde{\chi}}$, $\rho\in\xi^{\tilde{\mathcal{F}}}$, respectively. Subsequently, let $\hbar$ be a SVNI on $\tilde{\chi}$, $f:(\tilde{\chi},\varphi)\longrightarrow(\tilde{\mathcal{F}},\varphi^*)$ is an injective (SVNAC-map) with $f(\psi)=\rho$. Therefore, $(\tilde{\chi},\varphi,\hbar)$ is a single-valued neutrosophic ideal approximation $T_i^{(t,s)}-space$ if $(\tilde{\mathcal{F}},\varphi^*)$ is a single-valued neutrosophic approximation $T_i^{(t,s)}-space$, $i=1,2,3$.*

Proof. Because $\ell\neq\jmath$ in $\tilde{\chi}$, we get than $f(\ell)\neq f(\jmath)$ in $\tilde{\mathcal{F}}$, wing to $\tilde{\mathcal{F}}$ is a single-valued neutrosophic approximation $T_2^{(t,s)}-space$, then there are $\sigma,\varrho\in\xi^{\tilde{\mathcal{F}}}$ with $t\leq\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi^*}(\sigma)}(f(\ell))$, $t\geq\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi^*}(\sigma)}(f(\ell))$, $t\geq\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi^*}(\sigma)}(f(\ell))$ and $s\leq\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi^*}(\varrho)}(f(\jmath))$, $s\geq\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi^*}(\varrho)}(f(\jmath))$, $s\geq\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi^*}(\varrho)}(f(\jmath))$ such that $\tilde{\eta}_{\sup(\sigma\cap\varrho)}<(t\cap s)$, $\tilde{\gamma}_{\sup(\sigma\cap\varrho)}>(t\cap s)$, $\tilde{\delta}_{\sup(\sigma\cap\varrho)}>(t\cap s)$, this implies, $[t\leq f^{-1}(\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi^*}(\sigma)})(\ell)$, $t\geq f^{-1}(\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi^*}(\sigma)})(\ell)$, $t\geq f^{-1}(\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi^*}(\sigma)})(\ell)$ and $s\leq f^{-1}(\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi^*}(\varrho)})(\jmath)$, $s\geq f^{-1}(\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi^*}(\varrho)})(\jmath)$, $s\geq f^{-1}(\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi^*}(\varrho)})(\jmath)]$ and hence

$$t\leq f^{-1}(\tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(\sigma)})(\ell),\quad t\geq f^{-1}(\tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(\sigma)})(\ell),\quad t\geq f^{-1}(\tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(\sigma)})(\ell),$$

$$s\leq f^{-1}(\tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(\varrho)})(\jmath),\quad s\geq f^{-1}(\tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(\varrho)})(\jmath),\quad s\geq f^{-1}(\tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(\varrho)})(\jmath).$$

Since f is SVNAC-map, we have $t\leq\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\sigma))}(\ell)$, $t\geq\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\sigma))}(\ell)$, $t\geq\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\sigma))}(\ell)$ and $s\leq\tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\varrho))}(\jmath)$, $s\geq\tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\varrho))}(\jmath)$, $s\geq\tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\varrho))}(\jmath)$ and hence

$$t\leq\tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\sigma))}(\ell),\quad t\geq\tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\sigma))}(\ell),\quad t\geq\tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\sigma))}(\ell),$$

$$s\leq\tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\varrho))}(\jmath),\quad s\geq\tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\varrho))}(\jmath),\quad s\geq\tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(f^{-1}(\varrho))}(\jmath).$$

That is, there is $£ = f^{-1}(\sigma)$, $\varpi = f^{-1}(\varrho)$ with $t \leq \tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(£)}(\ell)$, $t \geq \tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(£)}(\ell)$, $t \geq \tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(£)}(\ell)$ and $s \leq \tilde{\eta}_{\mathrm{int}^{\psi}_{\Phi}(\varrho)}(\jmath)$, $s \geq \tilde{\gamma}_{\mathrm{int}^{\psi}_{\Phi}(\varrho)}(\jmath)$, $s \geq \tilde{\delta}_{\mathrm{int}^{\psi}_{\Phi}(\varrho)}(\jmath)$. Hence, $(\tilde{\chi}, \varphi, \hbar)$ is a single-valued neutrosophic ideal approximation $T_2^{(t,s)}$ *− space*. Likewise, we can establish that other cases have a similar line of reasoning. □

Theorem 3.2. *Let $(\tilde{\chi}, \varphi)$, $(\tilde{\mathcal{F}}, \varphi^*)$ be a `SVNA-spaces` related with $\psi \in \xi^{\tilde{\chi}}$, $\rho \in \xi^{\tilde{\mathcal{F}}}$, respectively. Subsequently, let $\hbar$ be a `SVNI` on $\tilde{\mathcal{F}}$ and $f : (\tilde{\chi}, \varphi) \longrightarrow (\tilde{\mathcal{F}}, \varphi^*)$ is an surjective (`SVNAO`-map) with $f^{-1}(\rho) = \psi$. Therefore, $(\tilde{\chi}, \varphi^*, \hbar^*)$ is a single-valued neutrosophic ideal approximation $T_i^{(t,s)}$ − space if $(\tilde{\mathcal{F}}, \varphi)$ is a single-valued neutrosophic approximation $T_i^{(t,s)}$ − space, $i = 1, 2, 3$.*

Proof. Because f is surjective, we obtain $a \neq b$ in $\tilde{\mathcal{F}}$ which implies $f^{-1}(a) \neq f^{-1}(b)$ in $\tilde{\chi}$. Owing to $(\tilde{\chi}, \varphi)$ is single-valued neutrosophic approximation $T_2^{(t,s)}$ *− space*, `SVNSs`, $\alpha, \beta \in \xi^{\tilde{\chi}}$ with $t \leq \tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi}(\alpha)}(f^{-1}(a))$, $t \geq \tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi}(\alpha)}(f^{-1}(a))$, $t \geq \tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi}(\alpha)}(f^{-1}(a))$ and $s \leq \tilde{\eta}_{\mathrm{int}^{\psi}_{\varphi}(\beta)}(f^{-1}(b))$, $s \geq \tilde{\gamma}_{\mathrm{int}^{\psi}_{\varphi}(\beta)}(f^{-1}(b))$, $s \geq \tilde{\delta}_{\mathrm{int}^{\psi}_{\varphi}(\beta)}(f^{-1}(b))$ exists such that $\tilde{\eta}_{\sup(\alpha \cap \beta)} < (t \cap s)$, $\tilde{\gamma}_{\sup(\alpha \cap \beta)} > (t \cap s)$, $\tilde{\delta}_{\sup(\alpha \cap \beta)} > (t \cap s)$, and since f is surjective, we obtain $t \leq \tilde{\eta}_{f(\mathrm{int}^{\psi}_{\varphi}(\alpha))}(a)$, $t \geq \tilde{\gamma}_{f(\mathrm{int}^{\psi}_{\varphi}(\alpha))}(a)$, $t \geq \tilde{\delta}_{f(\mathrm{int}^{\psi}_{\varphi}(\alpha))}(a)$ and $s \leq \tilde{\eta}_{f(\mathrm{int}^{\psi}_{\varphi}(\beta))}(b)$, $s \geq \tilde{\gamma}_{f(\mathrm{int}^{\psi}_{\varphi}(\beta))}(b)$, $s \geq \tilde{\delta}_{f(\mathrm{int}^{\psi}_{\varphi}(\beta))}(b)$. Subsequently, based on f is `SVNAO`-map,

$$t \leq \tilde{\eta}_{\mathrm{int}^{\rho}_{\varphi^*}(f(\alpha))}(a), \quad t \geq \tilde{\gamma}_{\mathrm{int}^{\rho}_{\varphi^*}(f(\alpha))}(a), \quad t \geq \tilde{\delta}_{f(\mathrm{int}^{\rho}_{\varphi^*}(f(\alpha))}(a),$$

$$s \leq \tilde{\eta}_{\mathrm{int}^{\rho}_{\varphi^*}(f(\beta))}(b), \quad s \geq \tilde{\gamma}_{\mathrm{int}^{\rho}_{\varphi^*}(f(\beta))}(b), \quad s \geq \tilde{\delta}_{\mathrm{int}^{\rho}_{\varphi^*}(f(\beta))}(b).$$

Therefore,

$$t \leq \tilde{\eta}_{\mathrm{int}^{\rho}_{\Phi}(f(\alpha))}(a), \quad t \geq \tilde{\gamma}_{f(\mathrm{int}^{\rho}_{\Phi}(f(\alpha))}(a), \quad t \geq \tilde{\delta}_{\mathrm{int}^{\rho}_{\Phi}(f(\alpha))}(a),$$

$$s \leq \tilde{\eta}_{\mathrm{int}^{\rho}_{\Phi}(f(\beta))}(b), \quad s \geq \tilde{\gamma}_{\mathrm{int}^{\rho}_{\Phi}(f(\beta))}(b), \quad s \geq \tilde{\delta}_{\mathrm{int}^{\rho}_{\Phi}(f(\beta))}(b).$$

That is, there exist $\sigma = f(\alpha)$ and $\varrho = f(\beta)$ with

$$t \leq \tilde{\eta}_{\mathrm{int}^{\rho}_{\Phi}(\sigma)}(a), \quad t \geq \tilde{\gamma}_{f(\mathrm{int}^{\rho}_{\Phi}(\sigma)}(a), \quad t \geq \tilde{\delta}_{\mathrm{int}^{\rho}_{\Phi}(\sigma)}(a),$$

$$s \leq \tilde{\eta}_{\mathrm{int}^{\rho}_{\Phi}(\varrho)}(b), \quad s \geq \tilde{\gamma}_{\mathrm{int}^{\rho}_{\Phi}(\varrho)}(b), \quad s \geq \tilde{\delta}_{\mathrm{int}^{\rho}_{\Phi}(\varrho)}(b),$$

and

$$\tilde{\eta}_{\sup(\sigma \cap \varrho)} < (t \cap s), \quad \tilde{\gamma}_{\sup(\sigma \cap \varrho)} > (t \cap s), \quad \tilde{\delta}_{\sup(\sigma \cap \varrho)} > (t \cap s).$$

Thus, $(\tilde{\chi}, \varphi^*, \hbar^*)$ is a single-valued neutrosophic ideal approximation $T_i^{(t,s)}$ *− space*. Likewise, we can establish that other cases have a similar line of reasoning. □

4. Connected single-valued neutrosophic ideal approximation spaces

Definition 4.1. *Let $(\tilde{\chi}, \varphi)$ be an `SVNA-spaces` related with $\psi \in \xi^{\tilde{\chi}}$. Therefore,*

(1) *two non-null `SVNSs` $\rho, \pi \in \xi^{\tilde{\chi}}$ are single-valued neutrosophic approximation preseparated (abbreviated, `SVNA-preseparated`) [resp. separated, (abbreviated, `SVNA-separated`)] sets if*

$$\mathrm{pcl}^{\psi}_{\varphi}(\rho) \wedge \pi = \rho \wedge \mathrm{pcl}^{\psi}_{\varphi}(\pi) = \langle 0, 1, 1 \rangle, \ \Big[resp.\ \mathrm{cl}^{\psi}_{\varphi}(\rho) \wedge \pi = \rho \wedge \mathrm{cl}^{\psi}_{\varphi}(\pi) = \langle 0, 1, 1 \rangle \Big].$$

(2) *a non-null* `SVNS` $\rho \in \xi^{\tilde{\chi}}$ *is termed single-valued neutrosophic approximation predisconnected (abbreviated,* `SVNA-predisconnected`*) [resp. disconnected,(abbreviated,* `SVNA-disconnected`*)] set if there exist* `SVNA-preseparated` *[resp.* `SVNA-separated`*] sets* $\rho, \pi \in \xi^{\tilde{\chi}}$ *such that*

$$\rho \wedge \pi = \sigma.$$

A `SVNS` $\sigma \in \xi^{\tilde{\chi}}$ *is termed single-valued neutrosophic approximation preconnected (abbreviated,* `SVNA-preconnected`*) [resp. connected, (abbreviated,* `SVNA-preconnected`*)] if it is not* `SVNA-predisconnected` *[resp.* `SVNA-disconnected`*].*

(3) $(\tilde{\chi}, \varphi)$ *is termed* `SVNA-predisconnected` *[resp.* `SVNA-disconnected`*] space if there exist* `SVNA-preseparated` *[resp.* `SVNA-separated`*] sets* $\rho, \pi \in \xi^{\tilde{\chi}}$*, such that*

$$\rho \wedge \pi = \langle 1, 0, \rangle.$$

A `SVNA-space` $(\tilde{\chi}, \varphi)$ *is termed* `SVNA-preconnected` *[resp.* `SVNA-connected`*] space if it is not* `SVNA-predisconnected` *[resp.* `SVNA-disconnected`*].*

Definition 4.2. *Let* $(\tilde{\chi}, \varphi)$ *be an* `SVNA-spaces` *related with* $\psi \in \xi^{\tilde{\chi}}$*. Therefore,*

(1) *two non-null* `SVNSs` $\rho, \pi \in \xi^{\tilde{\chi}}$ *are single-valued neutrosophic ideal approximation preseparated (abbreviated,* `SVNIA-preseparated`*) [resp. separated, (abbreviated,* `SVNIA-separated`*)] sets if*

$$\mathrm{pcl}^{\psi}_{\Phi}(\rho) \wedge \pi = \rho \wedge \mathrm{pcl}^{\psi}_{\Phi}(\pi) = \langle 0, 1, 1 \rangle,$$

$$\Big[resp.\ \mathrm{cl}^{\psi}_{\varphi}(\rho) \wedge \pi = \rho \wedge \mathrm{cl}^{\psi}_{\Phi}(\pi) = \langle 0, 1, 1 \rangle \Big],$$

(2) *a non-null* `SVNS` $\rho \in \xi^{\tilde{\chi}}$ *is termed single-valued neutrosophic ideal approximation predisconnected (abbreviated,* `SVNIA-predisconnected`*) [resp.disconnected, (abbreviated,* `SVNIA-disconnected`*)] set if there exist* `SVNIA-preseparated` *[resp.*`SVNIA-separated`*] sets* $\rho, \pi \in \xi^{\tilde{\chi}}$ *such that*

$$\rho \wedge \pi = \sigma.$$

A `SVNS` $\sigma \in \xi^{\tilde{\chi}}$ *is termed single-valued neutrosophic ideal approximation preconnected (abbreviated,* `SVNIA-preconnected`*) [resp. connected, (abbreviated,* `SVNIA-preconnected`*) if it is not* `SVNIA-predisconnected` *[resp.* `SVNIA-disconnected`*.*

(3) $(\tilde{\chi}, \varphi)$ *is termed* `SVNIA-predisconnected` *[resp.* `SVNIA-disconnected`*] space if there exist* `SVNIA-preseparated` *[resp.* `SVNIA-separated` *sets* $\rho, \pi \in \xi^{\tilde{\chi}}$*, such that*

$$\rho \wedge \pi = \langle 1, 0, 0 \rangle.$$

A `SVNIA-space` $(\tilde{\chi}, \varphi)$ *is termed* `SVNIA-preconnected` *[resp.* `SVNIA-connected`*] space if it is not* `SVNIA-predisconnected` *[resp.* `SVNIA-disconnected`*].*

Remark 4.1. *We have the following implications.*

$$\begin{array}{ccc} \texttt{SVNA-preseparated} & \longrightarrow & \texttt{SVNIA-separated} \\ \downarrow & & \downarrow \\ \texttt{SVNA-preseparated} & \longrightarrow & \texttt{SVNIA-preseparated} \end{array}$$

and hence

$$\begin{array}{ccc} \texttt{SVNIA-preconnected} & \longrightarrow & \texttt{SVNA-preconnected} \\ \downarrow & & \downarrow \\ \texttt{SVNIA-connected} & \longrightarrow & \texttt{SVNA-connected} \end{array}$$

Example 4.1. *Let φ be a* SVNR *on $\tilde{\chi}$, $\tilde{\chi} = \{a, b, c, d, e\}$ be a set defined by*

φ	a	b	c	d	e
a	(1, 0, 0)	(1, 0, 0)	(0.4, 0.2, 0.2)	(0, 1, 1)	(0, 1, 1)
b	(1, 0, 0)	(1, 0, 0)	(0.4, 0.2, 0.2)	(0, 1, 1)	(0, 1, 1)
c	(0.4, 0.2, 0.2)	(0.4, 0.2, 0.2)	(1, 0, 0)	(0, 1, 1)	(0, 1, 1)
d	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(1, 0, 0)
e	(0, 1, 1)	(0, 1, 1)	(0, 1, 1)	(1, 0, 0)	(1, 0, 0)

spouse that $\psi = \langle(0,1,1),(0,1,1),(0.4,0.4,0.4),(0.8,0.8,0.8),(0,1,1)\rangle$. *Then,*

$$\psi_\varphi = \langle(0,1,1),(0,1,1),(0.4,0.4,0.4),(0.8,0.8,0.8),(0,1,1)\rangle,$$

$$(\psi_\varphi)^c = \langle(1,0,0),(1,0,0),(0.4,0.6,0.4),(0.8,0.2,0.8),(1,0,0)\rangle.$$

Now, for

$$\rho = \langle(0.6,0.6,0.6),(0,1,1),(0,1,1),(0,1,1),(0,1,1)\rangle,$$

$$\pi = \langle(0,1,1),(0.6,0.6,0.6),(0,1,1),(0,1,1),(0,1,1)\rangle$$

Then,

$$\rho^\varphi = \langle(0.6,0.6,0.6),(0,1,1),(0,1,1),(0,1,1),(0,1,1)\rangle,$$

$$\pi^\varphi = \langle(0,1,1),(0.6,0.6,0.6),(0,1,1),(0,1,1),(0,1,1)\rangle.$$

Consequently,

$$\mathrm{cl}_\varphi^\psi(\rho) = (\psi_\varphi)^c \vee \rho^\varphi = \langle(1,0,0),(1,0,0),(0.4,0.6,0.4),(0.8,0.2,0.8),(1,0,0)\rangle,$$

and

$$\mathrm{cl}_\varphi^\psi(\pi) = (\psi_\varphi)^c \vee \pi^\varphi = \langle(1,0,0),(1,0,0),(0.4,0.6,0.4),(0.8,0.2,0.8),(1,0,0)\rangle.$$

Furthermore,

$$\rho_\varphi = \langle(0.6, 0.6, 0.6), (0, 1, 1), (0, 1, 1), (0.2, 0.8, 0.4), (0, 1, 1)\rangle,$$

$$\pi_\varphi = \langle(0, 1, 1), (0.6, 0.6, 0.6), (0, 1, 1), (0.2, 0.8, 0.4), (0, 1, 1)\rangle.$$

Consequently, $\mathrm{int}^{\psi}_{\varphi}(\rho) = \psi_\varphi \wedge \rho_\varphi = \langle(0, 1, 1), (0, 1, 1), (0, 1, 1), (0.2, 0.8, 0.8), (0, 1, 1)\rangle$ *and* $\mathrm{int}^{\psi}_{\varphi}(\pi) = \psi_\varphi \wedge \pi_\varphi = \langle(0, 1, 1), (0, 1, 1), (0, 1, 1), (0.2, 0.8, 0.8), (0, 1, 1)\rangle$. *Therefore,*

(1) ρ, π *are* `SVNA-preseparated` *sets but not* `SVNA-separated` *sets.*

(2) *We define an* `SVNI` $\hbar$ *on* $\tilde{\chi}$ *as follows:* $\sigma \in \hbar$ *for all* $\sigma = \langle(0.6, 0.6, 0.6), (0.6, 0.6, 0.6), (0.6, 0.6, 0.6), (0.6, 0.6, 0.6), (0.6, 0.6, 0.6)\rangle$. *Then* $\rho \in \hbar$ *and* $\pi \in \hbar$ *which means that* $\Phi_\psi(\rho) = \langle 0, 1, 1\rangle$ *and* $\Phi_\psi(\pi) = \langle 0, 1, 1\rangle$ *and then* $\mathrm{cl}^{\psi}_{\Phi}(\rho) = \rho$ *and* $\mathrm{cl}^{\psi}_{\Phi}(\pi) = \pi$. *Therefore,* $\mathrm{cl}^{\psi}_{\varphi}(\rho) \wedge \pi = \langle 0, 1, 1\rangle$ *and* $\mathrm{cl}^{\psi}_{\varphi}(\pi) \wedge \rho = \langle 0, 1, 1\rangle$. *Hence,* ρ, π *are* `SVNIA-separated` *sets but not* `SVNA-separated` *sets*

Theorem 4.1. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be an* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$. *Therefore, the following statements are equivalent.*

(1) $(\tilde{\chi}, \varphi, \hbar)$ *is* `SVNIA-preconnected` *space.*

(2) $\rho \wedge \pi = \langle 0, 1, 1\rangle$, $\mathrm{pint}^{\psi}_{\Phi}(\rho) = \rho$, $\mathrm{pint}^{\psi}_{\Phi}(\pi) = \pi$ *and* $\rho \vee \pi = \langle 1, 0, 0\rangle$ *imply* $\rho = \langle 0, 1, 1\rangle$ *or* $\pi = \langle 0, 1, 1\rangle$.

(3) $\rho \wedge \pi = \langle 0, 1, 1\rangle$, $\mathrm{pcl}^{\psi}_{\Phi}(\rho) = \rho$, $\mathrm{pcl}^{\psi}_{\Phi}(\pi) = \pi$ *and* $\rho \vee \pi = \langle 1, 0, 0\rangle$ *imply* $\rho = \langle 0, 1, 1\rangle$ *or* $\pi = \langle 0, 1, 1\rangle$.

Proof. (1) $\Rightarrow$ (2): Let $\rho, \pi \in \xi^{\tilde{\chi}}$ with $\mathrm{pint}^{\psi}_{\Phi}(\rho) = \rho$, $\mathrm{pint}^{\psi}_{\Phi}(\pi) = \pi$ such that $\rho \wedge \pi = \langle 0, 1, 1\rangle$ and $\rho \vee \pi = \langle 1, 0, 0\rangle$. Therefor, $\mathrm{pcl}^{\psi}_{\Phi}(\rho) = \mathrm{pcl}^{\psi}_{\Phi}(\pi^c) = (\mathrm{pcl}^{\psi}_{\Phi}(\pi))^c = \pi^c = \rho$, $\mathrm{pcl}^{\psi}_{\Phi}(\pi) = \mathrm{pcl}^{\psi}_{\Phi}(\rho^c) = (\mathrm{pcl}^{\psi}_{\Phi}(\rho))^c = \rho^c = \pi$. Thus, $\mathrm{pcl}^{\psi}_{\Phi}(\rho) \wedge \pi = \rho \wedge \mathrm{pcl}^{\psi}_{\Phi}(\pi) = \rho \wedge \pi = \langle 0, 1, 1\rangle$. Consequently, ρ, π are `SVNIA-separated` sets so that $\rho \vee \pi = \langle 1, 0, 0\rangle$. Because $(\tilde{\chi}, \varphi, \hbar)$ is `SVNIA-preconnected` space we obtain $\rho = \langle 0, 1, 1\rangle$ or $\pi = \langle 0, 1, 1\rangle$.

(2) $\Rightarrow$ (3):, (3) $\Rightarrow$ (1): Clear. □

Theorem 4.2. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be an* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$ *for each* $\rho \in \xi^{\tilde{\chi}}$. *Therefore, the following properties are equivalent:*

(1) ρ *is* `SVNIA-preconnected` *set.*

(2) *If* π, σ *are* `SVNIA-preseparated` *sets with* $\rho \leq \pi \vee \sigma$, *then* $\rho \wedge \pi = \langle 0, 1, 1\rangle$ *or* $\rho \wedge \sigma = \langle 0, 1, 1\rangle$.

(3) *If* π, σ *are* `SVNIA-preseparated` *sets with* $\rho \leq \pi \vee \sigma$, *then* $\rho \leq \pi$ *or* $\rho \leq \sigma$.

Proof. (1) $\Rightarrow$ (2): Let π, σ are `SVNIA-preseparated` sets with $\rho \leq \pi \vee \sigma$. Therefore, $\mathrm{pcl}^{\psi}_{\Phi}(\pi) \wedge \sigma = \pi \wedge \mathrm{pcl}^{\psi}_{\Phi}(\sigma) = \langle 0, 1, 1\rangle$ so that $\rho \leq \pi \vee \sigma$. Because,

$$\begin{aligned}\mathrm{pcl}^{\psi}_{\Phi}(\rho \wedge \pi) \wedge (\rho \wedge \sigma) &= \mathrm{pcl}^{\psi}_{\Phi}(\rho) \wedge \mathrm{pcl}^{\psi}_{\Phi}(\pi) \wedge (\rho \wedge \sigma) = \mathrm{pcl}^{\psi}_{\Phi}(\rho) \wedge \rho \wedge \mathrm{pcl}^{\psi}_{\Phi}(\pi) \wedge \sigma \\ &= \rho \wedge \langle 0, 1, 1\rangle = \langle 0, 1, 1\rangle.\end{aligned}$$

$$\begin{aligned}\mathrm{pcl}^{\psi}_{\Phi}(\rho \wedge \sigma) \wedge (\rho \wedge \pi) &= \mathrm{pcl}^{\psi}_{\Phi}(\rho) \wedge \mathrm{pcl}^{\psi}_{\Phi}(\sigma) \wedge (\rho \wedge \pi) = \mathrm{pcl}^{\psi}_{\Phi}(\rho) \wedge \rho \wedge \mathrm{pcl}^{\psi}_{\Phi}(\sigma) \wedge \pi \\ &= \rho \wedge \langle 0, 1, 1\rangle = \langle 0, 1, 1\rangle.\end{aligned}$$

Hence, $(\rho \wedge \sigma)$ and $(\rho \wedge \pi)$ are `SVNIA-preseparated` sets with $\rho = (\rho \wedge \sigma) \vee (\rho \wedge \pi)$. But ρ is `SVNIA-preconnected` set means that $\rho \wedge \sigma = \langle 0,1,1\rangle$ or $\rho \wedge \pi = \langle 0,1,1\rangle$.

(2) $\Rightarrow$ (3): If $\rho \wedge \pi = \langle 0,1,1\rangle$ and $\rho \leq \pi \vee \sigma$ means that $\rho = \rho \wedge (\pi \vee \sigma) = (\rho \wedge \pi) \vee (\rho \wedge \sigma) = \rho \wedge \sigma$ and hence $\rho \leq \sigma$. Also, if $\rho \wedge \sigma = \langle 0,1,1\rangle$, then $\rho \leq \pi$.

(3) $\Rightarrow$ (1): Clear. □

Lemma 4.1. *Let $(\tilde{\chi}, \varphi)$ be an `SVNA-space` related with $\psi \in \xi^{\tilde{\chi}}$ for each $\rho \in \xi^{\tilde{\chi}}$. Therefore, the following properties are equivalent:*

(1) *ρ is `SVNA-preconnected` set.*
(2) *If π, σ are `SVNIA-preseparated` sets with $\rho \leq \pi \vee \sigma$, then $\rho \wedge \pi = \langle 0,1,1\rangle$ or $\rho \wedge \sigma = \langle 0,1,1\rangle$.*
(3) *If π, σ are `SVNA-preseparated` sets with $\rho \leq \pi \vee \sigma$, then $\rho \leq \pi$ or $\rho \leq \sigma$.*

Theorem 4.3. *Let $(\tilde{\chi}, \varphi)$ and $(\tilde{\mathcal{F}}, \varphi^*)$ are `SVNA-spaces` related with $\psi \in \xi^{\tilde{\chi}}$ and $\rho \in \xi^{\tilde{\mathcal{F}}}$, respectively, $\hbar$ a `SVNI` on $\tilde{\chi}$, and $f : (\tilde{\chi}, \varphi, \hbar) \longrightarrow (\tilde{\mathcal{F}}, \varphi^*)$ is single-valued neutrosophic mapping such that $\mathrm{pcl}^{\psi}_{\Phi}(f^{-1}(\pi)) \leq f^{-1}(\mathrm{pcl}^{\rho}_{\varphi^*}(\pi))$ for all $\pi \in \xi^{\tilde{\mathcal{F}}}$. Then, $f(\varrho) \in \xi^{\tilde{\mathcal{F}}}$ is a `SVNA-preconnected` set if ϱ is a `SVNIA-preconnected` set in $\tilde{\chi}$.*

Proof. Let $\pi, \sigma \in \xi^{\tilde{\mathcal{F}}}$ be `SVNA-preseparated` sets with $f(\varrho) = \pi \vee \sigma$. Therefore, $\mathrm{pcl}^{\rho}_{\varphi^*}(\pi) \wedge \sigma = \pi \wedge \mathrm{pcl}^{\rho}_{\varphi*}(\sigma) = \langle 0,1,1\rangle$. Then, $\varrho \leq (f^{1}(\pi) \vee f^{-1}(\sigma))$, and by hypothesis of f, we get that

$$\begin{aligned}\mathrm{pcl}^{\psi}_{\Phi}(f^{-1}(\pi)) \wedge f^{-1}(\sigma) \quad &\leq f^{-1}(\mathrm{pcl}^{\rho}_{\varphi^*}(\pi)) \wedge f^{-1}(\sigma) = f^{-1}(\mathrm{pcl}^{\rho}_{\varphi^*}(\pi) \wedge \sigma) \\ &= f^{-1}(\langle 0,1,1\rangle) = \langle 0,1,1\rangle,\end{aligned}$$

and in similar way,

$$\begin{aligned}\mathrm{pcl}^{\psi}_{\Phi}(f^{-1}(\sigma)) \wedge f^{-1}(\pi) \quad &\leq f^{-1}(\mathrm{pcl}^{\rho}_{\varphi^*}(\sigma)) \wedge f^{-1}(\pi) = f^{-1}(\mathrm{pcl}^{\rho}_{\varphi^*}(\sigma) \wedge \pi) \\ &= f^{-1}(\langle 0,1,1\rangle) = \langle 0,1,1\rangle.\end{aligned}$$

Therefore, $f^{-1}(\pi)$ and $f^{-1}(\sigma)$ are `SVNIA-preseparated` sets in $\tilde{\chi}$. Consequently, $\varrho \leq (f^{1}(\pi) \vee f^{-1}(\sigma))$. Because ϱ is is a `SVNIA-preconnected` set in $\tilde{\chi}$, then by Theorem 6 (3), we get that $\varrho \leq f^{1}(\pi)$ or $\varrho \leq f^{1}(\sigma)$ implies that $f(\varrho) \leq \pi$ or $f(\varrho) \leq \sigma$. Hence, from Corollary 2, $f(\varrho) \in \xi^{\tilde{\mathcal{F}}}$ is a `SVNA-preconnected` set in $\tilde{\mathcal{F}}$. □

5. Compactness in single-valued neutrosophic ideal approximation spaces

Definition 5.1. *Let $(\tilde{\chi}, \varphi, \hbar)$ be an `SVNIA-space` related with $\psi \in \xi^{\tilde{\chi}}$. Therefore, $\tilde{\chi}$ is termed single-valued neutrosophic regular (abbreviated, `SVN-regular`) [resp. ideal regular (abbreviated, `SVNI-regular`)] space if for every $\varrho \in \xi^{\tilde{\chi}}$ with $\mathrm{int}^{\psi}_{\varphi}(\varrho) = \varrho$,*

$$\varrho = \bigvee_{i \in \Gamma} \left\{\varrho_i : \mathrm{int}^{\psi}_{\varphi}(\varrho_i) = \varrho_i,\ \mathrm{cl}^{\psi}_{\varphi}(\varrho_i) \leq \varrho\right\},$$

$$\left[\textit{resp. } \varrho = \bigvee_{i \in \Gamma} \{\varrho_i : \mathrm{int}^{\psi}_{\varphi}(\varrho_i) = \varrho_i,\ \mathrm{cl}^{\psi}_{\Phi}(\varrho_i) \leq \varrho\}\right].$$

For each `SVN-regular` *space is a* `SVNI-regular` *space the validity of this relation is clear based on Definition 4.1. If* $\hbar = \langle 0,1,1\rangle$ *then the notions of* `SVN-regular` *and* `SVNI-regular` *are equivalent.*

Definition 5.2. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be an* `SVNIA-space` *related with* $\psi \in \xi^{\tilde{\chi}}$. *Therefore,*

(1) ρ *is an single-valued neutrosophic approximation compact (abbreviated,* `SVNA-compact`*) [resp. ideal approximation compact (abbreviated,* `SVNIA-compact`*)] if for every family* $\rho_i \in \xi^{\tilde{\chi}} : \operatorname{int}^{\psi}_{\varphi}(\rho_i) = \rho_i,\ i \in \Gamma\}$ *with* $\rho \leq \bigvee_{i\in\Gamma} \rho_i$ *there exists a finite* $\Gamma_0 \subseteq \Gamma$ *such that* $\rho \leq \bigvee_{i\in\Gamma_0} \rho_i$ $\Big[$*resp.* $\rho \bar{\wedge} (\bigvee_{i\in\Gamma_0} \rho_i) \in \hbar\Big]$.

(2) ρ *is an single-valued neutrosophic almost approximation compact (abbreviated,* `SVNAA-compact`*) [resp. almost ideal approximation compact (abbreviated,* `SVNAIA-compact`*)] if for every family* $\rho_i \in \xi^{\tilde{\chi}} : \operatorname{int}^{\psi}_{\varphi}(\rho_i) = \rho_i,\ i \in \Gamma\}$ *with* $\rho \leq \bigvee_{i\in\Gamma} \rho_i$ *there exists a finite* $\Gamma_0 \subseteq \Gamma$ *such that* $\rho \leq \bigvee_{i\in\Gamma_0} \operatorname{cl}^{\psi}_{\varphi}(\rho_i)$ $\Big[$*resp.* $\rho \bar{\wedge} (\bigvee_{i\in\Gamma_0} \operatorname{cl}^{\psi}_{\Phi}(\rho_i)) \in \hbar\Big]$.

(3) ρ *is an single-valued neutrosophic nearly approximation compact (abbreviated,* `SVNNA-compact`*) [resp. nearly ideal approximation compact (abbreviated,* `SVNNIA-compact`*)] if for every family* $\rho_i \in \xi^{\tilde{\chi}} : \operatorname{int}^{\psi}_{\varphi}(\rho_i) = \rho_i,\ i \in \Gamma\}$ *with* $\rho \leq \bigvee_{i\in\Gamma} \rho_i$ *there exists a finite* $\Gamma_0 \subseteq \Gamma$ *such that* $\rho \leq \bigvee_{i\in\Gamma_0} \operatorname{int}^{\psi}_{\varphi}(\operatorname{cl}^{\psi}_{\varphi}((\rho_i))$ $\Big[$*resp.* $\rho \bar{\wedge} (\bigvee_{i\in\Gamma_0} \operatorname{int}^{\psi}_{\varphi}(\operatorname{cl}^{\psi}_{\Phi}(\rho_i))) \in \hbar\Big]$.

The `SVNA-space` $(\tilde{\chi}, \varphi)$ *[resp.* `SVNIA-space` $(\tilde{\chi}, \varphi, \hbar)$*] will be termed* `SVNA-compact`*,* `SVNAA-compact`*,* `SVNNA-compact` *[resp.* `SVNIA-compact`*,* `SVNAIA-compact`*,* `SVNNIA-compact`*] if we replaced* ρ *with* $\langle 1,0,0\rangle$.

It is clear that:

$$\begin{array}{ccccc}
\texttt{SVNA-compact} & \longrightarrow & \texttt{SVNAA-compact} & \longrightarrow & \texttt{SVNNA-compact} \\
\downarrow & & \downarrow & & \downarrow \\
\texttt{SVNIA-compact} & \longrightarrow & \texttt{SVNAIA-compact} & \longrightarrow & \texttt{SVNNIA-compact}
\end{array}$$

Theorem 5.1. *Let* $(\tilde{\chi}, \varphi, \hbar)$ *be* `SVNAIA-compact` *and* `SVNI-regular`. *Therefore,* $\tilde{\chi}$ *is a* `SVNIA-compact` *space.*

Proof. Assume a family $\{\rho_i \in \xi^{\tilde{\chi}} : \operatorname{int}^{\psi}_{\varphi}(\rho_i) = \rho_i,\ i \in \Gamma\}$ with $\bigvee_{i\in\Gamma} \rho_i = \langle 1,0,0\rangle$. By `SVNI-regularity` of $\tilde{\chi}$ then for every $\operatorname{int}^{\psi}_{\varphi}(\rho_i) = \rho_i$, we have

$$\rho_i = \bigvee_{i_k \in \Gamma_K} \{\rho_{i_k} : \operatorname{int}^{\psi}_{\varphi}(\rho_{i_k}),\ \operatorname{cl}^{\psi}_{\Phi}(\rho_{i_k}) \leq \rho_i\}.$$

Thus, $\bigvee_{i\in\Gamma}(\bigvee_{i_k\in\Gamma_K} \rho_{i_k}) = \langle 1,0,0\rangle$, because, $\tilde{\chi}$ is `SVNAIA-compact`,therefore there exists a finite index subset $\Gamma_0 \times \Gamma_K$ of $\Gamma \times \Gamma$ such that

$$\langle 1,0,0\rangle \bar{\wedge} (\bigvee_{i\in\Gamma_0} (\bigvee_{i_k\in\Gamma_K} (\operatorname{cl}^{\psi}_{\Phi}(\rho_{i_k}))) \in \hbar.$$

Since for any $i \in \gamma_0$ we have $\bigvee_{i_k\in\Gamma_K}(\operatorname{cl}^{\psi}_{\Phi}(\rho_{i_k})) \leq \rho_i$ and hence we get that

$$\langle 1,0,0\rangle \bar{\wedge} (\bigvee_{i\in\Gamma_0} (\bigvee_{i_k\in\Gamma_K} (\operatorname{cl}^{\psi}_{\Phi}(\rho_{i_k}))) \geq \langle 1,0,0\rangle \bar{\wedge} (\bigvee_{i\in\Gamma_0} \rho_i).$$

Thus, $\langle 1,0,0\rangle \bar{\wedge}(\bigvee_{i\in\Gamma_0}\rho_i) \in \hbar$, and hence $\tilde{\chi}$ is a `SVNIA-compact` space. □

Theorem 5.2. *Let $(\tilde{\chi},\varphi,\hbar)$ be `SVNNIA-compact` and `SVNI-regular`. Therefore, $\tilde{\chi}$ is a `SVNNIA-compact` space.*

Proof. Similar to Theorem 8. □

Theorem 5.3. *Let $f:(\tilde{\chi},\varphi,\hbar_1)\longrightarrow(\tilde{\mathcal{F}},\varphi^*,\hbar_2)$ be injective (`SVNAC`-map) between two `SVNA-spaces` associated with $\psi\in\xi^{\tilde{\chi}}$ and $\rho\in\xi^{\tilde{\mathcal{F}}}$, respectively, and $\pi\in\hbar_1\Rightarrow f(\pi)\in\hbar_2$, for all $\sigma\in\xi^{\tilde{\chi}}$ is a `SVNIA-compact`. Then, $f(\sigma)$ is `SVNIA-compact` as well.*

Proof. Let $\{\varrho_i\in\xi^{\tilde{\mathrm{F}}}: \ \mathrm{int}^{\rho}_{\varphi^*}(\varrho_i)=\varrho_i,\ i\in\Gamma\}$ be a family with $f(\sigma)\leq\bigvee_{i\in\Gamma}\varrho_i$. Because of f is a `SVNAC`-map, then $\mathrm{int}^{\psi}_{\varphi}(f^{-1}(\varrho_i))=f^{-1}(\varrho_i)$ with $\sigma\leq\bigvee_{i\in\Gamma}f^{-1}(\varrho_i)$. By `SVNIA-compactness` of σ there exists a finite $\Gamma_0\subseteq\Gamma$ such that $\sigma\bar{\wedge}(\bigvee_{i\in\Gamma_0}(f^{-1}(\varrho_i)))\in\hbar_1$. Since $\pi\in\hbar_1\Rightarrow f(\pi)\in\hbar_2$, for all $\sigma\in\xi^{\tilde{\chi}}$ then $f(\sigma\bar{\wedge}(\bigvee_{i\in\Gamma_0}(f^{-1}(\varrho_i))))\in\hbar_2$. Because of f is a injective, therefore, $f(\sigma\bar{\wedge}(\bigvee_{i\in\Gamma_0}(f^{-1}(\varrho_i))))=f(\sigma)\bar{\wedge}(\bigvee_{i\in\Gamma_0}(\varrho_i))$. Hence, $f(\sigma)\bar{\wedge}(\bigvee_{i\in\Gamma_0}(\varrho_i))\in\hbar_2$. Thus, $f(\sigma)$ is `SVNIA-compact`. □

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References

[1] F. Smarandache, A Unifying Field in Logics, Neutrosophy: Neutrosophic Probability, Set and Logic, American Research Press, Rehoboth, (1999).

[2] A.A. Salama, F. Smarandache, Neutrosophic Crisp Set Theory, The Educational Publisher, Columbus, 2015.

[3] A.A. Salama, S.A. Alblowi, Neutrosophic Set and Neutrosophic Topological Spaces, IOSR J. Math. 3 (2012), 31–35.

[4] H. Wang, F.Y. Smarandache, Q. Zhang, Single valued neutrosophic sets, Multispace Multistruct. 4 (2010), 410–413.

[5] Y. Saber, F. Alsharari, F. Smarandache, On Single-Valued Neutrosophic Ideals in Šostak Sense, Symmetry. 12 (2020), 193. https://doi.org/10.3390/sym12020193.

[6] Y. Saber, F. Alsharari, F. Smarandache, M. Abdel-Sattar, Connectedness and Stratification of Single-Valued Neutrosophic Topological Spaces, Symmetry. 12 (2020), 1464. https://doi.org/10.3390/sym12091464.

[7] F. Alsharari, Y.M. Saber, F. Smarandache, Compactness on Single-Valued Neutrosophic Ideal Topological Spaces, Neutrosophic Sets Syst. 41 (2021), 127–145.

[8] Y. Saber, F. Alsharari, F. Smarandache, M. Abdel-Sattar, On Single Valued Neutrosophic Regularity Spaces, Computer Model. Eng. Sci. 130 (2022), 1625–1648. https://doi.org/10.32604/cmes.2022.017782.

[9] F. Alsharari, Y.M. Saber, $\mathcal{G}\Theta^{\star\tau_j}_{\tau_i}$-Fuzzy Closure Operator, New Math. Nat. Comput. 16 (2020), 123–141. https://doi.org/10.1142/s1793005720500088.

[10] F. Alsharari, Y.M. Saber, Separation axioms on fuzzy ideal topological spaces in Šostak sense, Int. J. Adv. Appl. Sci. 7 (2020), 78–84. https://doi.org/10.21833/ijaas.2020.02.011.

[11] Y.M. Saber, F. Alsharari, Generalized Fuzzy Ideal Closed Sets on Fuzzy Topological Spaces in Šostak Sense, Int. J. Fuzzy Logic Intell. Syst. 18 (2018), 161–166. https://doi.org/10.5391/ijfis.2018.18.3.161.

[12] Z. Pawlak, Rough sets, Int. J. Comput. Inf. Sci. 11 (1982), 341–356. https://doi.org/10.1007/bf01001956.

[13] X. Chen, Q. Li, Construction of Rough Approximations in Fuzzy Setting, Fuzzy Sets Syst. 158 (2007), 2641–2653. https://doi.org/10.1016/j.fss.2007.05.016.

[14] D. Dubois, H. Prade, Rough Fuzzy Sets and Fuzzy Rough Sets, Int. J. Gen. Syst. 17 (1990), 191–209. https://doi.org/10.1080/03081079008935107.

[15] Z. Li, T. Xie, Roughness of Fuzzy Soft Sets and Related Results, Int. J. Comput. Intell. Syst. 8 (2015), 278–296. https://doi.org/10.1080/18756891.2015.1001951.

[16] G. Liu, Generalized Rough Sets Over Fuzzy Lattices, Inf. Sci. 178 (2008), 1651–1662. https://doi.org/10.1016/j.ins.2007.11.010.

[17] W. Wu, Generalized Fuzzy Rough Sets, Inf. Sci. 151 (2003), 263–282. https://doi.org/10.1016/s0020-0255(02)00379-1.

[18] Z. Pei, D. Pei, L. Zheng, Topology vs Generalized Rough Sets, Int. J. Approx. Reason. 52 (2011), 231–239. https://doi.org/10.1016/j.ijar.2010.07.010.

[19] I. Ibedou, S.E. Abbas, Fuzzy Rough Sets With a Fuzzy Ideal, J. Egypt. Math. Soc. 28 (2020), 36. https://doi.org/10.1186/s42787-020-00096-2.

[20] M. Irfan Ali, A Note on Soft Sets, Rough Soft Sets and Fuzzy Soft Sets, Appl. Soft Comput. 11 (2011), 3329–3332. https://doi.org/10.1016/j.asoc.2011.01.003.

[21] D. Boixader, J. Jacas, J. Recasens, Upper and Lower Approximations of Fuzzy Sets, Int. J. Gen. Syst. 29 (2000), 555–568. https://doi.org/10.1080/03081070008960961.

[22] W.Z. Wu, A Study on Relationship Between Fuzzy Rough Approximation Operators and Fuzzy Topological Spaces, in: L. Wang, Y. Jin (Eds.), Fuzzy Systems and Knowledge Discovery, Springer, Berlin, Heidelberg, 2005: pp. 167–174. https://doi.org/10.1007/11539506_21.

[23] A.M. Abd El-Latif, Generalized Soft Rough Sets and Generated Soft Ideal Rough Topological Spaces, J. Intell. Fuzzy Syst. 34 (2018), 517–524. https://doi.org/10.3233/jifs-17610.

[24] J. Mahanta, P.K. Das, Fuzzy Soft Topological Spaces, J. Intell. Fuzzy Syst. 32 (2017), 443–450. https://doi.org/10.3233/jifs-152165.

[25] C.L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968), 182–190. https://doi.org/10.1016/0022-247x(68)90057-7.

[26] J. Ye, A Multicriteria Decision-Making Method Using Aggregation Operators for Simplified Neutrosophic Sets, J. Intell. Fuzzy Syst. 26 (2014), 2459–2466. https://doi.org/10.3233/ifs-130916.

[27] H.L. Yang, Z.L. Guo, Y. She, X. Liao, On Single Valued Neutrosophic Relations, J. Intell. Fuzzy Syst. 30 (2016), 1045–1056. https://doi.org/10.3233/ifs-151827.