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Identifying the Severity of Criminal Activity in Society Using Picture Fuzzy Baire Space

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Abstract. In this paper, the idea of picture fuzzy Baire space is explored and its properties are examined. The features of picture fuzzy semi-closed and semi-open sets, picture fuzzy nowhere dense sets, picture fuzzy first and second category sets, picture fuzzy residual sets, picture fuzzy submaximal spaces, picture fuzzy strongly irresolvable spaces, picture fuzzy G_δ set, picture fuzzy F_σ set, and picture fuzzy regular closed sets are analyzed. To understand the concepts, some examples are provided. An algorithm using picture fuzzy Baire space is developed to address real-world scenarios. This method is more effective in assessing criminal activity as it identifies an individual who has committed a more serious offense. This algorithmic approach proves its effectiveness in navigating the complexities of practical examples, showcasing its potential for real-world applications.

1. Introduction

L. A. Zadeh introduced a novel category of fuzzy sets by utilizing the principles of vagueness and uncertainty. His contributions to understanding ambiguity significantly aid in solving problems associated with imprecision [17]. The applications of fuzzy sets have been widely expanded and integrated into diverse domains such as information and control [16], as well as robotics [10]. Notably, Chang's significant contribution [6] has demonstrated the applicability of fuzzy sets to topological structures. The evolution of fuzzy set theory took a noteworthy turn with Atanassov's work [2] and [3], where the concept of intuitionistic fuzzy sets is generalized, giving rise to intuitionistic fuzzy set theory. Later, Coker [7] extended the notion of an intuitionistic fuzzy set. While conventional fuzzy set theory focuses on quantifying membership degrees, the intuitionistic fuzzy set introduces the novel idea of aggregating non-membership degrees. Expanding on these

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principles, B. Cong and V. Kerinovich [8] introduced the concept of PFS, derived from both fuzzy and intuitionistic fuzzy sets. This innovative approach expands the theoretical framework and opens up new avenues for addressing complex problems involving ambiguity and uncertainty. Several mathematicians have attempted to apply all the essential principles of general topology to the fuzzy environment.

Abdul Razaaq et al. [1] described the rank of PFTS and properties related to continuous functions. K. Tamilselvan, V. Visalakshi, and Prasanalakshmi Balaji [13] introduced the notions of picture fuzzy filter, grill, picture fuzzy ultrafilter, and discussed their properties, interrelations, etc. René Louis Baire [12] pioneered the concepts of first and second category sets in his doctoral dissertation in 1899. In 1913, Denjoy introduced residual sets as complements of first-category sets. The concept of the Baire space, which pays tribute to René Louis Baire through its name, was initially presented in Bourbaki's work, "Topologies Generale" [9], within classical topology. The notions of Baire spaces have undergone thorough investigation within the field of classical topology [5]. G. Thangaraj [15] and S. Anjalmose [14] introduced and explored Baire spaces in fuzzy environments. K. K. Azad [4] made significant contributions to the field by introducing fuzzy semi-open and semi-closed sets within the context of fuzzy topological spaces. This study focuses on exploring various characterizations of fuzzy Baire space through the lens of fuzzy semi-closed and fuzzy semi-open sets. It aims to establish the conditions under which fuzzy first-category sets can be classified as fuzzy semi-closed sets within a fuzzy topological space. Nguy'n Xuan [11] presented the fundamental concepts of PFTSs and the rough PFS on the crisp approximation space.

The primary aim of this research is to investigate the different classifications of PFBS using picture fuzzy semi-closed and semi-open sets. Section 2 discusses the basics of PFS's and their topological structures. Section 3 delves into the characteristics of systems such as picture fuzzy dense, picture fuzzy nowhere dense, semi-closed and semi-open sets, picture fuzzy first and second category sets, PFRS's, picture fuzzy submaximal spaces, and strongly irresolvable spaces. Section 4 picture fuzzy G_δ set, picture fuzzy F_σ set, picture fuzzy regular closed set established their characteristics using picture fuzzy Baire space. Section 5 demonstrates the practical application of the proposed algorithm for using PFBS's of PFSs. The identification of the severity of criminal activity in society by the accused as having committed a higher crime is central to this analysis. To identify the accused, three primary types of offenses violent crimes (k), property crimes (l), and drug offenses (m)are taken into account, denoted by $X = \{k, l, m\}$. This set of offenses is assumed to be the attributes under consideration. The analysis indicates that the defendant belongs to distinct categories of criminal activity levels, known as picture fuzzy sets (PFS's). These PFS's are determined based on the degree of severity of the criminal activity of the accused person and are categorized as highly dangerous to society, false accusation and less dangerous to society, each playing a distinct role within the context of PFS's. An algorithm designed for PFS's is subsequently applied using PFBS. This application aims to identify the criminal activity of the accused who committed a higher crime, asserting its superiority among other methods for determining the severity of criminal activities in society.

Table 1. The paper employs the following abbreviations and acronyms:

Acronyms	Definitions
PFS	Picture Fuzzy Set
PFS(X)	Collection of all Picture Fuzzy Sets on X
PFTS	Picture Fuzzy Topological Space
PFOS	Picture Fuzzy Open Set
PFCS	Picture Fuzzy Closed Set
int(D)	Interior of D
cl(D)	Closure of D
PF*fcs*	Picture Fuzzy First Category Set
PFRS	Picture Fuzzy Residual Set
PFBS	Picture Fuzzy Baire Space
PFNDS	Picture Fuzzy Nowhere Dense Set
PFDS	Picture Fuzzy Dense Set
PFSOS	Picture Fuzzy Semi-Open Set
PFSCS	Picture Fuzzy Semi-Closed Set
PF*scs*	Picture Fuzzy Second Category Set
PFG_δS	Picture Fuzzy G_δ Set
PFF_σS	Picture Fuzzy F_σ Set
PFRCS	Picture Fuzzy Regular Closed Set

2. Preliminaries

In this section, basic concepts are discussed on the picture fuzzy sets.

Definition 2.1. [11] A PFS A on a universe of discourse X is of the form
$A = \{(x, \varpi_A(x), \omega_A(x), \psi_A(x)) : x \in X\}$, where
$\varpi_A(x) \in [0,1]$ is called the "degree of positive membership of x in A",
$\omega_A(x) \in [0,1]$ is called the "degree of neutral membership of x in A",
$\psi_A(x) \in [0,1]$ is called the "degree of negative membership of x in A",
$\varpi_A(x), \omega_A(x), \psi_A(x)$ satisfy the following condition:
$0 \le \varpi_A(x) + \omega_A(x) + \psi_A(x) \le 1 \forall x \in X$.
Then $\forall x \in X, 1 - (\varpi_A(x) + \omega_A(x) + \psi_A(x))$ is called the degree of refusal members of x in A.

Definition 2.2. [4] Let $\mathcal{D}$ and $\mathcal{E}$ any two PFS's, then

(1) $\mathcal{D} \subseteq \mathcal{E}$ iff $(\forall y \in X, \varpi_{\mathcal{D}}(y) \le \varpi_{\mathcal{E}}(y)$ and $\omega_{\mathcal{D}}(y) \le \omega_{\mathcal{E}}(y)$ and $\psi_{\mathcal{D}}(y) \ge \psi_{\mathcal{E}}(y))$
(2) $\mathcal{D} = \mathcal{E}$ iff $(\mathcal{D} \subseteq \mathcal{E}$ and $\mathcal{E} \subseteq \mathcal{D})$
(3) $\mathcal{D} \cup \mathcal{E}$=$\{(x, \vee(\varpi_{\mathcal{D}}(x), \varpi_{\mathcal{E}}(x)), \wedge(\omega_{\mathcal{D}}(x), \omega_{\mathcal{E}}(x)), \wedge(\psi_{\mathcal{D}}(x), \psi_{\mathcal{E}}(x)))|\forall x \in X\}$
(4) $\mathcal{D} \cap \mathcal{E}$=$\{(x, \wedge(\varpi_{\mathcal{D}}(x), \varpi_{\mathcal{E}}(x)), \wedge(\omega_{\mathcal{D}}(x), \omega_{\mathcal{E}}(x)), \vee(\psi_{\mathcal{D}}(x), \psi_{\mathcal{E}}(x)))|\forall x \in X\}$
(5) $CO(\mathcal{D}) = \bar{D} = \{(\omega_{\mathcal{D}}(x), \varpi_{\mathcal{D}}(x), \psi_{\mathcal{D}}(x))|x \in X\}$

Definition 2.3. [11] Some Special PFSs are as follows:

(1) A constant PFS is the PFS $(\varpi, \omega, \psi) = \{(x, \varpi, \omega, \psi)|x \in X\}$.

(2) Picture fuzzy universe set is 1_X defined as $1_X = (1,0,0) = \{(x,1,0,0)|x \in X\}$.
(3) Picture fuzzy empty set is ϕ defined as $\phi = 0_X = (0,0,1) = \{(x,0,0,1)|x \in X\}$.

Definition 2.4. [11] An picture fuzzy topology (PFT) on a non empty set U is a family $\mathfrak{I}$ of PFS's in U satisfing the following axioms:

(C_1) $\overbrace{\varpi,\omega,\psi} \in \mathfrak{I}$, for all $\overbrace{\varpi,\omega,\psi} \in$ PFS(U)
$(C_2) Z_1 \cap Z_2 \in \mathfrak{I}$ for any $Z_1, Z_2 \in \mathfrak{I}$,
$(C_3) \cup Z_i \in \mathfrak{I}$ for any aribitary family $\{Z_i : i \in I\} \subseteq \mathfrak{I}$.
In this case the pair $(U, \mathfrak{I})$ is called a PFTS and each PFS A in $\mathfrak{I}$ is a PFOS in $(U, \mathfrak{I})$. The complement of a PFS in PFTS $(U, \mathfrak{I})$ is called a PFCS in $(U, \mathfrak{I})$.

Definition 2.5. [11] Let $(U, \mathfrak{I})$ be a PFTS and $W \in$ PFS(U).
Then the picture fuzzy interior and picture fuzzy closure of W are int(W), cl(W) : PFS(U) $\to$ PFS(U) respectively, where $int(W) = \cup\{L : L \text{ is a PFOS and } L \subseteq W\}$, $cl(W) = \cap\{K : K \text{ is a PFCS and } W \subseteq K\}$.

Definition 2.6. [11]

(1) W is a PFOS in $(U, \mathfrak{I})$ iff int(W) = W.
(2) int($\overbrace{\varpi,\omega,\psi}$) $= \overbrace{\varpi,\omega,\psi}$, for all $\overbrace{\varpi,\omega,\psi} \in$ PFS(U),
(3) int(W ∩ Y) = int(W) ∩ int(Y), for all W, Y ∈ PFS(U),
(4) int(int(W)) = int(W), for all $W \in$ PFS(U),
(5) int(W) ⊆ W, for all W ∈ PFS (U).

Definition 2.7. [11]

(1) W is a PFCS in $(U, \mathfrak{I})$ iff cl(W) = W.
(2) cl($\overbrace{\varpi,\omega,\psi}$) $= \overbrace{\varpi,\omega,\psi}$, for all $\overbrace{\varpi,\omega,\psi} \in$ PFS(U),
(3) cl(W ∪ Y) = cl(W) ∪ cl(Y), for all W, Y ∈ PFS(U),
(4) cl(cl(W)) = cl(W), for all $W \in$ PFS(U),
(5) W ⊆ cl(W), for all W ∈ PFS (U).

3. Picture Fuzzy Baire Spaces

This section discusses the properties of PFBS based on the PFS's and few concrete examples are given.

Definition 3.1. A PFS $\mathcal{D}$ in a PFTS is called

(1) picture fuzzy semi-closed iff $int[cl(\mathcal{D})] \subseteq \mathcal{D}$
(2) picture fuzzy semi-open iff $\mathcal{D} \subseteq cl[int(\mathcal{D})]$

Example 3.1. Let $\mathcal{X}$=$\{k\}$ and $\mathfrak{I}$=$\{1_{\mathcal{X}}, 0_{\mathcal{X}}, G, H, I = G \cup H, J = G \cap H\}$, the membership values of $\{G, H, I, J\}$ are provided in the Table 2. Then the PFS $\mathcal{D} = (k, 0.1, 0.1, 0.6)$ is PFSOS $cl[int(\mathcal{D})] = (k, 0.3, 0.2, 0.3)$ but it is not PFOS. $(k, 0.1, 0.1, 0.6) \subseteq (k, 0.3, 0.2, 0.3)$.

Table 2. Membership values of $\mathfrak{I}$

		G	H	I	J	$1_{\mathcal{X}}$	$0_{\mathcal{X}}$
	γ	0.3	0.3	0.3	0.3	1	0
k	η	0.2	0.2	0.2	0.2	0	0
	ν	0.3	0.4	0.3	0.3	0	1

Example 3.2. Let $\mathcal{X}$={k} and $\mathfrak{I}$={$1_{\mathcal{X}}, 0_{\mathcal{X}}, G, H, I = G \cup H, J = G \cap H$}, the membership values of {G, H, I, J} are provided in the Table 3. Then the PFS $\mathcal{D} = (m, 0.4, 0.0, 0.5)$ is PFSCS $int[cl(\mathcal{D})] = (m, 0.4, 0.0, 0.5)$ but it is not PFCS. $(m, 0.4, 0.0, 0.5) \subseteq (m, 0.4, 0.0, 0.5)$.

Table 3. Membership values of $\mathfrak{I}$

		G	H	I	J	$1_{\mathcal{X}}$	$0_{\mathcal{X}}$
	γ	0.3	0.6	0.4	0.5	1	0
m	η	0.2	0.0	0.0	0.0	0	0
	ν	0.4	0.3	0.5	0.4	0	1

Theorem 3.1. For a family $\mathcal{A} = \mathcal{D}_\alpha$ of PFS's of PFTS $\mathcal{X}$.

(1) $\cup_\alpha(cl(\mathcal{D}_\alpha)) \subseteq cl(\cup_\alpha(\mathcal{D}_\alpha))$ in case α is finite set $\cup_\alpha cl(\mathcal{D}_\alpha) = cl(\cup_\alpha(\mathcal{D}_\alpha))$ also
(2) $\cup_\alpha(int(\mathcal{D}_\alpha)) \subseteq int(\cup_\alpha(\mathcal{D}_\alpha))$

Proof. Let $\mathcal{D}_1$ and $\mathcal{D}_2$ are two PFS's. To prove that $\overline{\mathcal{D}_1} \cup \overline{\mathcal{D}_2} \subset \overline{\mathcal{D}_1 \cup \mathcal{D}_2}$, $\mathcal{D}_1 \subset \overline{\mathcal{D}_1}$ and $\mathcal{D}_2 \subset \overline{\mathcal{D}_2}$, $\overline{\mathcal{D}_1} \subset \overline{\mathcal{D}_1 \cup \mathcal{D}_2}$ and $\overline{\mathcal{D}_2} \subset \overline{\mathcal{D}_1 \cup \mathcal{D}_2}$. Therefore, $\overline{\mathcal{D}_1} \cup \overline{\mathcal{D}_2} \subset \overline{\mathcal{D}_1 \cup \mathcal{D}_2}$. To prove $\overline{\mathcal{D}_1 \cup \mathcal{D}_2} \subset \overline{\mathcal{D}_1} \cup \overline{\mathcal{D}_2}$. Given $x \in \overline{\mathcal{D}_1 \cup \mathcal{D}_2}$, we claim $x \in \overline{\mathcal{D}_1} \cup \overline{\mathcal{D}_2}$. If $x \in \overline{\mathcal{D}}$ then there is nothing to prove so assume that $x \notin \overline{\mathcal{D}}$. There exists an open set V such that $x \in V$ and $V \cap \mathcal{D} = \phi$. Now let U be any open set such that $x \in U$ put $W = U \cap V$ then W is open and $x \in W$. $\mathcal{D}_1 \cup \mathcal{D}_2 \cap W \neq \phi$. But $\mathcal{D} \cap W \subset \mathcal{D} \cap V = \phi$. Therefore, if follow that $\phi \neq \mathcal{D}_1 \cap W \subset \mathcal{D}_2 \cap U$. Since this is true for all open sets U such that $x \in U$ $x \in \overline{\mathcal{D}_1}$, $x \in \overline{\mathcal{D}_2}$. Hence $\overline{\mathcal{D}_1 \cup \mathcal{D}_2} \subset \overline{\mathcal{D}_1} \cup \overline{\mathcal{D}_2}$. In general, in case α is finite set $\overline{\mathcal{D}_n \cup \mathcal{D}_m} \subseteq \overline{\mathcal{D}_n} \cup \overline{\mathcal{D}_m}$ therefore $n \neq m$. Similarly, $\cup_\alpha(int(\mathcal{D}_\alpha)) \subseteq int(\cup_\alpha(\mathcal{D}_\alpha))$ □

Definition 3.2. A PFS $\mathcal{D}$ in PFTS $(\mathcal{X}, \mathfrak{I})$ is called PFDS if there exists no PFCS E in $(\mathcal{X}, \mathfrak{I})$ such that $\mathcal{D} \subset E$ that is $cl(\mathcal{D}) = 1_{\mathcal{X}}$.

Example 3.3. Let $\mathcal{X} = \{l, m\}$ and PFS in $\mathfrak{I} = \{1_{\mathcal{X}}, 0_{\mathcal{X}}, G, H, I, J\}$ where $I = G \cap H$ and $J = G \cup H$, the membership value of $G, H, I, J, 1_{\mathcal{X}}, 0_{\mathcal{X}}$ are provided in the Table 4 and 5. Thus $(\mathcal{X}, \mathfrak{I})$ is a PFTS. Here, H is a PFDS.

Definition 3.3. A PFS $\mathcal{D}$ in PFTS $(\mathcal{X}, \mathfrak{I})$ is called PFNDS if there exists no non-zero PFOS E in $(\mathcal{X}, \mathfrak{I})$ such that $E \subset cl(\mathcal{D})$ that is $int[cl(\mathcal{D})] = 0_{\mathcal{X}}$.

Example 3.4. Let $\mathcal{X} = \{l, m\}$ and $\mathfrak{I} = \{1_{\mathcal{X}}, 0_{\mathcal{X}}, G, H, I, J\}$ where $I = G \cap H$ and $J = G \cup H$, the membership value of $G, H, I, J, 1_{\mathcal{X}}, 0_{\mathcal{X}}$ are provided in the Table 6 and 7. Thus $(\mathcal{X}, \mathfrak{I})$ is a PFTS. H is a PFNDS.

TABLE 4. Membership values of $\mathfrak{I}$

		G	H	I	J	1_X	0_X
	γ	0.9	0.8	0.8	0.9	1	0
l	η	0.1	0.0	0.0	0.0	0	0
	ν	0.0	0.2	0.2	0.0	0	1
	γ	0.8	0.6	0.6	0.8	1	0
m	η	0.2	0.0	0.0	0.0	0	0
	ν	0.0	0.4	0.4	0.0	0	1

TABLE 5. Membership values of $\mathfrak{I}^c$

		$\overline{G}$	$\overline{H}$	$\overline{I}$	$\overline{J}$	$\overline{1}_X$	$\overline{0}_X$
	γ	0.0	0.2	0.2	0.0	0	1
l	η	0.1	0.0	0.0	0.0	0	0
	ν	0.9	0.8	0.8	0.9	1	0
	γ	0.0	0.4	0.4	0.0	0	1
m	η	0.2	0.0	0.0	0.0	0	0
	ν	0.8	0.6	0.6	0.8	1	0

TABLE 6. Membership values of $\mathfrak{I}$

		G	H	I	J	1_X	0_X
	γ	0.2	0.7	0.2	0.7	1	0
l	η	0.1	0.0	0.0	0.0	0	0
	ν	0.3	0.2	0.3	0.2	0	1
	γ	0.3	0.6	0.3	0.6	1	0
m	η	0.2	0.0	0.0	0.0	0	0
	ν	0.4	0.3	0.4	0.3	0	1

TABLE 7. Membership values of $\mathfrak{I}^c$

		$\overline{G}$	$\overline{H}$	$\overline{I}$	$\overline{J}$	$\overline{1}_X$	$\overline{0}_X$
	γ	0.3	0.2	0.3	0.2	0	1
l	η	0.1	0.0	0.0	0.0	0	0
	ν	0.2	0.7	0.2	0.7	1	0
	γ	0.4	0.3	0.4	0.3	0	1
m	η	0.2	0.0	0.0	0.0	0	0
	ν	0.3	0.6	0.3	0.6	1	0

Definition 3.4. A PFS $\mathcal{D}$ in PFTS $(\mathcal{X}, \mathfrak{I})$ is called PF*fcs* if $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$ where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Any other PFS in $(\mathcal{X}, \mathfrak{I})$ is said to be PF*scs*.

Theorem 3.2. If a non-zero PFS $\mathcal{D}$ in a PFTS $(\mathcal{X}, \mathfrak{I})$ is PFNDS. Then $\mathcal{D}$ is PFSCS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ be a PFNDS in PFTS $(\mathcal{X}, \mathfrak{I})$, then $int[cl(\mathcal{D})] = 0_{\mathcal{X}}$ and $int[cl(\mathcal{D})] \subseteq \mathcal{D}$. Hence $\mathcal{D}$ is PFSCS in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.3. If $\mathcal{D}$ is a PF*fcs* in a PFTS $(\mathcal{X}, \mathfrak{I})$. Then $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$ where $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ is a PF*fcs* in a PFTS $(\mathcal{X}, \mathfrak{I})$. Then $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$, where $\mathcal{D}_i$ are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. By Theorem 3.2, the PFNDS $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$ and hence $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$, where $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.4. Let $\mathcal{D}$ be a PFSCS in PFTS $(\mathcal{X}, \mathfrak{I})$. If $\mathcal{D}$ is not a PFNDS in $(\mathcal{X}, \mathfrak{I})$, then $int(\mathcal{D}) \neq 0_{\mathcal{X}}$.

Proof. Since $\mathcal{D}$ be a PFSCS in $(\mathcal{X}, \mathfrak{I})$, we have $int[cl(\mathcal{D})] \subseteq \mathcal{D}$. If $\mathcal{D}$ is not a PFNDS in $(\mathcal{X}, \mathfrak{I})$, then $int[cl(\mathcal{D})] \neq 0_{\mathcal{X}}$. Let $int[cl(\mathcal{D})] = E$ and hence E is a PFOS in $(\mathcal{X}, \mathfrak{I})$. E $\subseteq cl(\mathcal{D})$, $int(E) \subseteq int[cl(\mathcal{D})]$, E $\subseteq int[cl(\mathcal{D})] \subseteq \mathcal{D}$, E $\subseteq \mathcal{D}$, $int(\mathcal{D}) \neq 0_{\mathcal{X}}$. □

Definition 3.5. Let $\mathcal{D}$ is a PF*fcs* in a PFTS $(\mathcal{X}, \mathfrak{I})$. Then $\mathcal{D}^c$ is called a PFRS in $(\mathcal{X}, \mathfrak{I})$. The complement of PF*fcs* is called a PFRS in $(\mathcal{X}, \mathfrak{I})$.

Theorem 3.5. If $\mathcal{D}$ is a PFRS in a PFTS $(\mathcal{X}, \mathfrak{I})$, then $\mathcal{D} = \bigcap_{i=1}^{\infty} Ei$ where (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - \mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ and hence $1_{\mathcal{X}} - \mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}i$ where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Then, $\mathcal{D} = 1_{\mathcal{X}} - \bigcup_{i=1}^{\infty} \mathcal{D}_i = \bigcap_{i=1}^{\infty} (1_{\mathcal{X}} - \mathcal{D}_i)$. By Theorem 3.2, the PFNDS's $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$ and hence $1_{\mathcal{X}} - \mathcal{D}_i$ are PFSOS's in $(\mathcal{X}, \mathfrak{I})$. Let $E_i = 1_{\mathcal{X}} - \mathcal{D}_i$ then $\mathcal{D} = \bigcap_{i=1}^{\infty} E_i$. (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$. □

Remark 3.1. Let $\mathcal{D}$ be a PFS in a PFTS $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - cl(\mathcal{D}) = int(1_{\mathcal{X}} - \mathcal{D})$ and $1_{\mathcal{X}} - int(\mathcal{D}) = cl(1_{\mathcal{X}} - \mathcal{D})$

Theorem 3.6. Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS. Then a PFS $\mathcal{D}$ is a PFNDS in $(\mathcal{X}, \mathfrak{I})$, iff $1_{\mathcal{X}} - cl(\mathcal{D})$ is a PFOS and PFDS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ be a PFNDS in $(\mathcal{X}, \mathfrak{I})$. Then $int[cl(\mathcal{D})]=0_{\mathcal{X}}$. Now $cl(\mathcal{D})$ is a PFCS in $(\mathcal{X}, \mathfrak{I})$, $1_{\mathcal{X}} - cl(\mathcal{D})$ is a PFOS in $(\mathcal{X}, \mathfrak{I})$. Also $cl[1_{\mathcal{X}} - cl(\mathcal{D})] = 1_{\mathcal{X}} - int[cl(\mathcal{D})] = 1_{\mathcal{X}} - 0_{\mathcal{X}} = 1_{\mathcal{X}}$, implies that $1_{\mathcal{X}} - cl(\mathcal{D})$ is a PFDS in $(\mathcal{X}, \mathfrak{I})$. Thus, $1_{\mathcal{X}} - cl(\mathcal{D})$ is a PFOS and PFDS in $(\mathcal{X}, \mathfrak{I})$. Conversely, let $1_{\mathcal{X}} - cl(\mathcal{D})$ be PFOS and PFDS in $(\mathcal{X}, \mathfrak{I})$. Then $cl[1_{\mathcal{X}} - cl(\mathcal{D})] = 1_{\mathcal{X}} - int[cl(\mathcal{D})] = 1_{\mathcal{X}}$ and hence $int[cl(\mathcal{D})] = 0_{\mathcal{X}}$. Therefore $\mathcal{D}$ is a PFNDS in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.7. Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS. If $[1_{\mathcal{X}} - cl(\mathcal{D})]$ is a PFOS and PFDS for a PFS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$, then $\mathcal{D}$ is a PFSCS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $[1_{\mathcal{X}} - cl(\mathcal{D})]$ is a PFOS and PFDS, for a PFS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$. Then by Theorem 3.6, $\mathcal{D}$ is PFNDS $(\mathcal{X}, \mathfrak{I})$ and hence by Theorem 3.2, $\mathcal{D}$ is a PFSCS in $(\mathcal{X}, \mathfrak{I})$. □

Definition 3.6. Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS. Then $(\mathcal{X}, \mathfrak{I})$ is called a PFBS if $int(\bigcup_{i=1}^{\infty}(\mathcal{D}_i)) = 0_{\mathcal{X}}$ where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$.

Theorem 3.8. Let$(\mathcal{X}, \mathfrak{I})$ be a PFTS, then the following are equivalent:

(1) $(\mathcal{X}, \mathfrak{I})$ is a PFBS.
(2) $int(\mathcal{D}) = 0_{\mathcal{X}}$ for a every PF*fcs* in $(\mathcal{X}, \mathfrak{I})$.
(3) $cl(E) = 1_{\mathcal{X}}$ for ever PFRS in $(\mathcal{X}, \mathfrak{I})$.

Proof. (1) ⇒ (2)
Let $\mathcal{D}$ be a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$ where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Now $int(\mathcal{D}) = int(\bigcup_{i=1}^{\infty}(\mathcal{D}_i)) = 0_{\mathcal{X}}$, since $(\mathcal{X}, \mathfrak{I})$ is a PFBS. Therefore $int(\mathcal{D}) = 0_{\mathcal{X}}$.
(2) ⇒ (3)
Let E be a PFRS in $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - E$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. By the hypothesis, $int(1_{\mathcal{X}} - E) = 0_{\mathcal{X}}$, implies that $1_{\mathcal{X}} - cl(E) = 0_{\mathcal{X}}$, $cl(E) = 1_{\mathcal{X}}$. Hence $(\mathcal{X}, \mathfrak{I})$ is a PFRS.
(3) ⇒ (1)
Let $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$ where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Now $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. $1_{\mathcal{X}} - D$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. By the hypothesis $cl(1_{\mathcal{X}} - \mathcal{D}) = 1_{\mathcal{X}}$, $1_{\mathcal{X}} - int(\mathcal{D}) = 1_{\mathcal{X}}$, $int(\mathcal{D}) = 0_{\mathcal{X}}$ that is $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$ where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Hence $(\mathcal{X}, \mathfrak{I})$ is a PFBS. □

Theorem 3.9. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS then $int(\bigcup_{i=1}^{\infty} \mathcal{D}_i) = 0_{\mathcal{X}}$, where $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS. By Theorem 3.8, $int(\mathcal{D}) = 0_{\mathcal{X}}$ for a PF*fcs* $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$ and by Theorem 3.3, $D = \bigcup_{i=1}^{\infty} \mathcal{D}_i$, where $\mathcal{D}_i$ are PFSCS's in $(\mathcal{X}, \mathfrak{I})$. Therefore $int(\bigcup_{i=1}^{\infty} \mathcal{D}_i) = 0_{\mathcal{X}}$, where $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.10. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS then $int(\bigcap_{i=1}^{\infty} E_i) = 1_{\mathcal{X}}$, where (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS. By Theorem 3.8, $cl(\mathcal{D}) = 1_{\mathcal{X}}$ for a PFRS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$ and by Theorem 3.5, $\mathcal{D} = \bigcap_{i=1}^{\infty} E_i$, where (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$. Therefore $cl(\bigcap_{i=1}^{\infty} E_i) = 1_{\mathcal{X}}$, where (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.11. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS then $cl(\bigcap_{i=1}^{\infty} E_i) = 1_{\mathcal{X}}$, where (E_i)'s are PFDS's in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS. By Theorem 3.10, $cl(\bigcap_{i=1}^{\infty}(E_i)) = 1_{\mathcal{X}}$, where (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$. Now $cl(\bigcap_{i=1}^{\infty} E_i) \subseteq \bigcap_{i=1}^{\infty}(cl(E_i))$, implies that $1_{\mathcal{X}} \subseteq \bigcap_{i=1}^{\infty}(cl(E_i))$. That is, $\bigcap_{i=1}^{\infty}(cl(E_i)) = 1_{\mathcal{X}}$ and hence $cl(E_i) = 1_{\mathcal{X}}$. Therefore, $cl(\bigcap_{i=1}^{\infty} E_i) = 1_{\mathcal{X}}$, where (E_i)'s are PFDS's in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.12. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS then $cl(\bigcap_{i=1}^{\infty} E_i) = 1_{\mathcal{X}}$, where the PFS's (E_i)'s are such that $cl(int(E_i)) = 1_{\mathcal{X}}$ in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS. By Theorem 3.10, $cl(\bigcap_{i=1}^{\infty} E_i) = 1_{\mathcal{X}}$ in $(\mathcal{X}, \mathfrak{I})$, where (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$. Now $cl(\bigcap_{i=1}^{\infty} E_i) \subseteq \bigcap_{i=1}^{\infty}(cl(E_i))$ implies that, $1_{\mathcal{X}} \subseteq \bigcap_{i=1}^{\infty}(cl(E_i))$. That is $\bigcap_{i=1}^{\infty} cl(E_i) = 1_{\mathcal{X}}$ and $cl(E_i) = 1_{\mathcal{X}}$. Since (E_i)'s are PFSOS's in $(\mathcal{X}, \mathfrak{I})$, $E_i \subseteq (cl[int(E_i)])$. Then $cl(E_i) \subseteq cl(cl[int(E_i)])$ and hence $1_{\mathcal{X}} \subseteq (cl[int(E_i)])$. That is $cl[int(E_i)] = 1_{\mathcal{X}}$. Therefore $cl(\bigcap_{i=1}^{\infty}(E_i)) = 1_{\mathcal{X}}$ where the PFS's (E_i)'s are such that $cl[int(E_i)] = 1_{\mathcal{X}}$ in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.13. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PF*fcs* then $(\bigcup_{i=1}^{\infty} \mathcal{D}_i) = 1_{\mathcal{X}}$, where $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PF*fcs*. Then $(\bigcup_{i=1}^{\infty} \mathcal{D}_i) = 1_{\mathcal{X}}$, where $(\mathcal{D}_i)$'s are PFNDS's set in $(\mathcal{X}, \mathfrak{I})$. By Theorem 3.2, the PFNDS's $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$. Hence $(\bigcup_{i=1}^{\infty} \mathcal{D}_i) = 1_{\mathcal{X}}$, where $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X}, \mathfrak{I})$. □

Remark 3.2. If $\mathcal{D} \subseteq E$ and E is PFNDS in a PFTS $(\mathcal{X}, \mathfrak{I})$ then $\mathcal{D}$ is also a PFNDS in $(\mathcal{X}, \mathfrak{I})$.

Theorem 3.14. If $\mathcal{D} \subseteq E$ and E is PF*fcs* in a PFTS $(\mathcal{X}, \mathfrak{I})$ then $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let E is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then $E = (\bigcup_{i=1}^{\infty} \mathcal{D}_i)$, where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Now $\mathcal{D} \subseteq E$, implies that $\mathcal{D} = \mathcal{D} \cap E = \mathcal{D} \cap [(\bigcup_{i=1}^{\infty} \mathcal{D}_i)] = \bigcup_{i=1}^{\infty}[\mathcal{D} \cap \mathcal{D}_i]$. Since $\mathcal{D} \cap \mathcal{D}_i \subseteq \mathcal{D}_i$ and by Remark 3.2, $[\mathcal{D} \cap \mathcal{D}_i]$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Hence $\mathcal{D} = \bigcup_{i=1}^{\infty}[\mathcal{D} \cap \mathcal{D}_i]$, where $[\mathcal{D} \cap \mathcal{D}_i]$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Hence $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.15. If $\mathcal{D} \subseteq E$ and $\mathcal{D}$ is PFRS in a PFTS $(\mathcal{X}, \mathfrak{I})$ then E is a PFRS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ is PFRS in a $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - \mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Now $\mathcal{D} \subseteq E$ implies that, $1_{\mathcal{X}} - \mathcal{D} \supseteq 1_{\mathcal{X}} - E$. By Theorem 3.14, $1_{\mathcal{X}} - E$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ and hence E is PFRS in a $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.16. Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS. Then the following are equivalent:

(1) $(\mathcal{X}, \mathfrak{I})$ is a PFBS.
(2) Each non-zero PFOS is a PF*scs* in $(\mathcal{X}, \mathfrak{I})$.

Proof. $(1) \Rightarrow (2)$.

Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS. Suppose that $\mathcal{D}$ is a non-zero PFOS in $(\mathcal{X}, \mathfrak{I})$ such that $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$, where $\mathcal{D}_i$ are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Then $int(\mathcal{D}) = int[\bigcup_{i=1}^{\infty}(\mathcal{D}_i)]$. Since $\mathcal{D}$ is a non-zero PFOS in $(\mathcal{X}, \mathfrak{I})$, $\text{int}(\mathcal{D}) = \mathcal{D} \neq 0_{\mathcal{X}}$ and hence $int[\bigcup_{i=1}^{\infty}(\mathcal{D}_i)] \neq 0_{\mathcal{X}}$, a contradiction to $(\mathcal{X}, \mathfrak{I})$ being a PFBS. Hence, for the PFOS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$, $\mathcal{D} \neq \bigcup_{i=1}^{\infty}(\mathcal{D}_i)$, where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. That is, no non-zero PFOS is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ and hence each non-zero PFOS is a PF*scs* in $(\mathcal{X}, \mathfrak{I})$.

$(2) \Rightarrow (1)$.

Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS in which each non-zero PFOS is a PF*scs* in $(\mathcal{X}, \mathfrak{I})$. We claim that $(\mathcal{X}, \mathfrak{I})$ is a PFBS. Suppose not. Then, by Theorem 3.8, $int(\mathcal{D}) \neq 0_{\mathcal{X}}$, for a PF*fcs* $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$ and hence there exists a non-zero PFOS E in $(\mathcal{X}, \mathfrak{I})$ such that $E \subseteq \mathcal{D}$. Since $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ and $E \subseteq \mathcal{D}$, by Theorem 3.14, E is also a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$, a contradiction to the hypothesis. Hence $int(\mathcal{D}) = 0_{\mathcal{X}}$, for each PF*fcs* $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$ and therefore by Theorem 3.8, $(\mathcal{X}, \mathfrak{I})$ is a PFBS. □

Definition 3.7. A PFTS $(\mathcal{X}, \mathfrak{I})$ is called a picture fuzzy submaximal space if $cl(\mathcal{D}) = 1_{\mathcal{X}}$, for any non-zero PFS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$, then $(\mathcal{D} \in \mathfrak{I})$.

Definition 3.8. A PFTS $(\mathcal{X}, \mathfrak{I})$ is said to be a picture fuzzy strongly irresolvable space if $cl[int(\mathcal{D})] = 1_{\mathcal{X}}$, for each PFDS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$.

Theorem 3.17. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS and picture fuzzy submaximal space and if $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ then $\mathcal{D}$ is a PFSCS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ be a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - D$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. By Theorem 3.8, $cl(1_{\mathcal{X}} - \mathcal{D}) = 1_{\mathcal{X}}$ in $(\mathcal{X}, \mathfrak{I})$. Again since $(\mathcal{X}, \mathfrak{I})$ is a picture fuzzy submaximal space, the PFDS $1_{\mathcal{X}} - \mathcal{D}$ is a PFOS in $(\mathcal{X}, \mathfrak{I})$. Then $\mathcal{D}$ is a PFCS in $(\mathcal{X}, \mathfrak{I})$ and hence $cl(\mathcal{D}) = \mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$. By Theorem 3.8, $int(\mathcal{D}) = 0_{\mathcal{X}}$. Now $int[cl(\mathcal{D})] = int(\mathcal{D}) = 0_{\mathcal{X}}$ and hence $\mathcal{D}$ is a PFNDS in $(\mathcal{X}, \mathfrak{I})$. Then by Theorem 3.2, the PFNDS $\mathcal{D}$ is a PFSCS in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 3.18. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS and picture fuzzy strongly irresolvable space and if E is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ then $\mathcal{D}$ is a PFSCS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Let $\mathcal{D}$ be a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - \mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. By Theorem 3.8, $cl(1_{\mathcal{X}} - \mathcal{D}) = 1_{\mathcal{X}}$ in $(\mathcal{X}, \mathfrak{I})$. Since $(\mathcal{X}, \mathfrak{I})$ is a picture fuzzy strongly irresolvable space, for the PFDS $1_{\mathcal{X}} - \mathcal{D}$, $cl(int(1_{\mathcal{X}} - \mathcal{D})) = 1_{\mathcal{X}}$ in $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - int[cl(\mathcal{D})] = 1_{\mathcal{X}}$, implies that $int(cl(\mathcal{D})) = 0_{\mathcal{X}}$ and hence $\mathcal{D}$ is a PFNDS in $(\mathcal{X}, \mathfrak{I})$. Then by Theorem 3.2, the PFNDS $\mathcal{D}$ is a PFSCS in $(\mathcal{X}, \mathfrak{I})$. □

4. Applications of Picture Fuzzy F_{σ} and G_{δ}-Sets in Picture Fuzzy Baire Space

Definition 4.1. Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS and $\mathcal{D}$ be a PFS in $\mathcal{X}$. Then $\mathcal{D}$ is called a PFG_{δ}S if $\mathcal{D} = \bigcap_{i=1}^{\infty}(\mathcal{D}_i)$, for each $\mathcal{D}_i \in \mathfrak{I}$.

Definition 4.2. Let $(\mathcal{X}, \mathfrak{I})$ be a PFTS and $\mathcal{D}$ be a PFS in $\mathcal{X}$. Then $\mathcal{D}$ is called a PFF_σS if $\mathcal{D} = \bigcup_{i=1}^{\infty}(\mathcal{D}_i)$, for each $\mathcal{D}_i \in \mathfrak{I}^C$.

Definition 4.3. A PFS $\mathcal{D}$ of a PFTS $\mathcal{X}$ is called a PFRCS of $\mathcal{X}$ if $cl(int(\mathcal{D})) = \mathcal{D}$.

Theorem 4.1. If $\mathcal{D}$ is a PFD and PFG_δS in a PFTS $(\mathcal{X}, \mathfrak{I})$, then $1_\mathcal{X} - \mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$.

Proof. Since $\mathcal{D}$ is a PFG_δS in $(\mathcal{X}, \mathfrak{I})$, $\mathcal{D} = \bigcap_{i=1}^{\infty} \mathcal{D}_i$ where $\mathcal{D}_i \in \mathfrak{I}$. And since $\mathcal{D}$ is a PFDS in $(\mathcal{X}, \mathfrak{I})$, $cl(\mathcal{D}) = 1_\mathcal{X}$. Then $cl\left(\bigcap_{i=1}^{\infty} \mathcal{D}_i\right) = 1_\mathcal{X}$. But $cl\left(\bigcap_{i=1}^{\infty} \mathcal{D}_i\right) \subseteq \bigcap_{i=1}^{\infty} cl(\mathcal{D}_i)$. Hence $1_\mathcal{X} \subseteq \bigcap_{i=1}^{\infty} cl(\mathcal{D}_i)$. That is, $\bigcap_{i=1}^{\infty} cl(\mathcal{D}_i) = 1_\mathcal{X}$. Then $cl(\mathcal{D}) = 1_\mathcal{X}$ for each $\mathcal{D}_i \in \mathfrak{I}$ and hence $cl(int(\mathcal{D}_i)) = 1_\mathcal{X}$ which implies that $1_\mathcal{X} - (\mathcal{D}_i)$ is PFNDS's for each $\mathcal{D}_i \in \mathfrak{I}$. Consider $1_\mathcal{X} - \mathcal{D} = 1_\mathcal{X} - \bigcap_{i=1}^{\infty} \mathcal{D}_i = \bigcup_{i=1}^{\infty}(1_\mathcal{X} - \mathcal{D}_i)$. Therefore $1_\mathcal{X} - \mathcal{D} = \bigcup_{i=1}^{\infty}(1_\mathcal{X} - \mathcal{D}_i)$ where $(1_\mathcal{X} - \mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Hence $1_\mathcal{X} - \mathcal{D}_i$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 4.2. If $\mathcal{D}$ is a PFD and PFG_δS in a PFTS $(\mathcal{X}, \mathfrak{I})$, then $\mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$.

Proof. Since $\mathcal{D}$ is a PFD and PFG_δS in $(\mathcal{X}, \mathfrak{I})$, by Theorem 4.1, $1_\mathcal{X} - \mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Hence $\mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 4.3. If a PFTS $(\mathcal{X}, \mathfrak{I})$ has a PFD and PFG_δS then $(\mathcal{X}, \mathfrak{I})$ is a PFBS.

Proof. Let $\mathcal{D}$ be a PFD and PFG_δS in $(\mathcal{X}, \mathfrak{I})$. Then by Theorem 4.1, $1_\mathcal{X} - \mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ and $(1_\mathcal{X} - \mathcal{D}) = (1_\mathcal{X} - \bigcap_{i=1}^{\infty} \mathcal{D}_i)$, $(1_\mathcal{X} - \mathcal{D}) = \bigcup_{i=1}^{\infty}(1_\mathcal{X} - \mathcal{D}_i)$, where $(1_\mathcal{X} - \mathcal{D}_i)$'s are PFNDSs in $(\mathcal{X}, \mathfrak{I})$. But $int(1_\mathcal{X} - \mathcal{D}) = 1_\mathcal{X} - cl(\mathcal{D}) = 1_\mathcal{X} - 1_\mathcal{X} = 0_\mathcal{X}$ (since $\mathcal{D}$ is PFD, $cl(\mathcal{D}) = 1_\mathcal{X}$). Then $int\left(\bigcup_{i=1}^{\infty}(1_\mathcal{X} - \mathcal{D}_i)\right) = int(1_\mathcal{X} - \mathcal{D}) = 0_\mathcal{X}$. Hence $(\mathcal{X}, \mathfrak{I})$ is a PFBS. □

Theorem 4.4. If $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then there is a PF$F_\sigma$S, G in $(\mathcal{X}, T)$ such that $\mathcal{D} \subseteq G$.

Proof. Let $\mathcal{D}$ be a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then, $\mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$, where $\mathcal{D}_i$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Now $1_\mathcal{X} - cl(\mathcal{D}_i)$ is a PFOS in $(\mathcal{X}, \mathfrak{I})$. Then $\bigcap_{i=1}^{\infty}[1_\mathcal{X} - cl(\mathcal{D}_i)]$ is a PFG_δS in $(\mathcal{X}, \mathfrak{I})$. Let $E = \bigcap_{i=1}^{\infty}[1_\mathcal{X} - cl(\mathcal{D}_i)]$. Now $\bigcap_{i=1}^{\infty}[1_\mathcal{X} - cl(\mathcal{D}_i)] = 1_\mathcal{X} - \bigcup_{i=1}^{\infty}(cl\mathcal{D}_i) \subseteq 1_\mathcal{X} - \bigcup_{i=1}^{\infty}(\mathcal{D}_i) = 1_\mathcal{X} - \mathcal{D}$. Hence $E \subseteq 1_\mathcal{X} - \mathcal{D}$. Then $\mathcal{D} \subseteq 1_\mathcal{X} - E$. Let $\eta = 1_\mathcal{X} - E$. Since E is a PFG_δS in $(\mathcal{X}, \mathfrak{I})$, G is a PFF_σS in $(\mathcal{X}, \mathfrak{I})$. Therefore, if $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$, then there is a PF$F_\sigma$S G in $(\mathcal{X}, \mathfrak{I})$ such that $\mathcal{D} \subseteq G$.

□

Theorem 4.5. If $(\mathcal{X}, \mathfrak{I})$ is a PFBS and $\bigcup_{i=1}^{\infty}(\mathcal{D}_i) = 1_\mathcal{X}$, where $\mathcal{D}_i$'s are PFRCS's in $(\mathcal{X}, \mathfrak{I})$ then $cl\left(\bigcup_{i=1}^{\infty}(\mathcal{D}_i)\right) = 1_\mathcal{X}$.

Proof. Suppose that int$(\mathcal{D}_i) = 0_{\mathcal{X}}$ for each $i \in I$. Now $\mathcal{D}_i$ is a PFRCS in $(\mathcal{X}, \mathfrak{I})$ implies that $\mathcal{D}_i$ is PFCS's in $(\mathcal{X}, \mathfrak{I})$. Also $int(\mathcal{D}_i) = 0_{\mathcal{X}}$ implies that $int(cl(\mathcal{D}_i)) = 0_{\mathcal{X}}$ and hence $\mathcal{D}_i$ is a PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Since $\bigcup\limits_{i=1}^{\infty}(\mathcal{D}_i) = 1_{\mathcal{X}}$, $int\left(\bigcup\limits_{i=1}^{\infty}(\mathcal{D}_i)\right) = int(1) = 1_{\mathcal{X}}$. Since $(\mathcal{X}, \mathfrak{I})$ is a PFBS, $int\left(\bigcup\limits_{i=1}^{\infty}(\mathcal{D}_i)\right) = 0_{\mathcal{X}}$, where $\mathcal{D}_i$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Therefore $0_{\mathcal{X}} = 1_{\mathcal{X}}$, which is a contradiction. Hence $int(\mathcal{D}_i) \neq 0$, for atleast one i. Hence $\left(\bigcup\limits_{i=1}^{\infty} int(\mathcal{D}_i)\right) \neq 0_{\mathcal{X}}$. Consider $\left(\bigcup\limits_{i=1}^{\infty} cl(\mathcal{D}_i)\right) \subseteq cl\left(\bigcup(\mathcal{D}_i)\right)$ implies that $\left(\bigcup\limits_{i=1}^{\infty} cl(int(\mathcal{D}_i))\right) \subseteq cl\left(\bigcup\limits_{i=1}^{\infty} int(\mathcal{D}_i)\right)$. That is, $cl\left(\bigcup\limits_{i=1}^{\infty} int(\mathcal{D}_i)\right) \supseteq \left(\bigcup\limits_{i=1}^{\infty} cl(int(\mathcal{D}_i))\right) = \left(\bigcup\limits_{i=1}^{\infty}(\mathcal{D}_i)\right) = 1_{\mathcal{X}}$. Then $cl\left(\bigcup\limits_{i=1}^{\infty} int(\mathcal{D}_i)\right) \supseteq 1_{\mathcal{X}}$. Hence $cl\left(\bigcup\limits_{i=1}^{\infty} int(\mathcal{D}_i)\right) = 1_{\mathcal{X}}$. □

Theorem 4.6. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS, then a PFS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$ is a PFRS iff there exists a PFD and PFG_δS E in $(\mathcal{X}, \mathfrak{I})$ such that $E \subseteq \mathcal{D}$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS and $\mathcal{D}$ be a PFRS in $(\mathcal{X}, \mathfrak{I})$. Then $1_{\mathcal{X}} - \mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$ and hence $1_{\mathcal{X}} - \mathcal{D} = \bigcup_{i=1}^{\infty} \mathcal{D}_i$, where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Let $E = \bigcap\limits_{i=1}^{\infty}[1_{\mathcal{X}} - cl(\mathcal{D}_i)]$. Then E is a PFG_δS in $(\mathcal{X}, \mathfrak{I})$. Now $1_{\mathcal{X}} - cl(\mathcal{D}_i) \subseteq 1_{\mathcal{X}} - \mathcal{D}_i$ implies that, $\bigcap\limits_{i=1}^{\infty}[1_{\mathcal{X}} - cl(\mathcal{D}_i)] \subseteq \bigcap\limits_{i=1}^{\infty}(1_{\mathcal{X}} - \mathcal{D}_i)$. Hence $\bigcap\limits_{i=1}^{\infty}[1_{\mathcal{X}} - cl(\mathcal{D}_i)] \subseteq 1_{\mathcal{X}} - \bigcup\limits_{i=1}^{\infty}(\mathcal{D}_i)$. Thus $E \subseteq 1_{\mathcal{X}} - (1_{\mathcal{X}} - \mathcal{D})$. That is $E \subseteq \mathcal{D}$. Since $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$, $[cl(\mathcal{D}_i)]$'s are PFNDS's in $(\mathcal{X}, \mathfrak{I})$. Now $E = \bigcup\limits_{i=1}^{\infty}[1_{\mathcal{X}} - cl(\mathcal{D}_i)]$ implies that, $1_{\mathcal{X}} - E = \bigcup_{i=1}^{\infty} cl(\mathcal{D}_i)$ and hence $1_{\mathcal{X}} - E$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Since $(\mathcal{X}, \mathfrak{I})$ is a PFBS, by Theorem 3.8, $int(1_{\mathcal{X}} - E) = 0_{\mathcal{X}}$ and $1_{\mathcal{X}} - cl(E) = int(1_{\mathcal{X}} - E)$, implies that $1_{\mathcal{X}} - cl(E) = 0_{\mathcal{X}}$. Thus, E is a PFD and PFG_δS in $(\mathcal{X}, \mathfrak{I})$ such that $E \subseteq \mathcal{D}$. Conversely, let E be a PFD and PFG_δS in $(\mathcal{X}, \mathfrak{I})$ such that $E \subseteq \mathcal{D}$. By Theorem 4.2, E is a PFRS in $(\mathcal{X}, \mathfrak{I})$. Since $E \subseteq \mathcal{D}$ and E is a PFRS in $(\mathcal{X}, \mathfrak{I})$, by Theorem 3.15, $\mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. □

Theorem 4.7. If a PFTS $(\mathcal{X}, \mathfrak{I})$ is a PFBS then a PFS $\mathcal{D}$ in $(\mathcal{X}, \mathfrak{I})$ is a PF*fcs* iff there exists a PFF_σS, G with int$(G) = 0_{\mathcal{X}}$, in $(\mathcal{X}, \mathfrak{I})$ such that $\mathcal{D} \subseteq G$.

Proof. Let $(\mathcal{X}, \mathfrak{I})$ be a PFBS and $\mathcal{D}$ be a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. Then, $1_{\mathcal{X}} - \mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. By Theorem 4.6, there exists a PFD and PFG_δS E in $(\mathcal{X}, \mathfrak{I})$ such that $E \subseteq 1_{\mathcal{X}} - \mathcal{D}$. Then $\mathcal{D} \subseteq 1_{\mathcal{X}} - E$. Since E is a PFG_δS in $(\mathcal{X}, \mathfrak{I})$, $1_{\mathcal{X}} - E$ is a PFF_σS in $(\mathcal{X}, \mathfrak{I})$. Also $cl(E) = 1_{\mathcal{X}}$ implies that, $int(1_{\mathcal{X}} - E) = 1_{\mathcal{X}} - cl(E) = 1_{\mathcal{X}} - 1_{\mathcal{X}} = 0_{\mathcal{X}}$. Let $G = 1_{\mathcal{X}} - E$, then G is an PFF_σS with $int(G) = 0_{\mathcal{X}}$, in $(\mathcal{X}, \mathfrak{I})$ such that $\mathcal{D} \subseteq G$. Conversely, let G be a PFF_σS with $int(G) = 0_{\mathcal{X}}$, in $(\mathcal{X}, \mathfrak{I})$ such that $\mathcal{D} \subseteq G$. Then $1_{\mathcal{X}} - \gamma$ is a PFD (since $cl(1_{\mathcal{X}} - G) = 1_{\mathcal{X}} - int(G) = 1_{\mathcal{X}} - 0_{\mathcal{X}} = 1_{\mathcal{X}}$) and PF$G_\delta$S in $(\mathcal{X}, \mathfrak{I})$ such that $1_{\mathcal{X}} - G \subseteq 1_{\mathcal{X}} - \mathcal{D}$. Then by Theorem 4.6, $1_{\mathcal{X}} - \mathcal{D}$ is a PFRS in $(\mathcal{X}, \mathfrak{I})$. Hence $\mathcal{D}$ is a PF*fcs* in $(\mathcal{X}, \mathfrak{I})$. □

Remark 4.1. If $\mathcal{D}$ is a PFSOS in a PFTS $(\mathcal{X}, \mathfrak{I})$ then cl$(\mathcal{D})$ is a PFRCS in $(\mathcal{X}, \mathfrak{I})$. For $\mathcal{D}$ is a PFSOS in $(\mathcal{X}, \mathfrak{I})$ implies that, $\mathcal{D} \subseteq cl(int(\mathcal{D}))$. Then $cl(\mathcal{D}) \subseteq cl(cl(int(\mathcal{D}))) = cl(int(\mathcal{D}) \subseteq cl(intcl(\mathcal{D}))$ and $cl(intcl(\mathcal{D})) \subseteq cl(cl(\mathcal{D})) = cl(\mathcal{D})$. This implies that $cl(int(cl(\mathcal{D}))) = cl(\mathcal{D})$ and hence $cl(\mathcal{D})$ is a PFRCS in $(\mathcal{X}, \mathfrak{I})$.

Theorem 4.8. If a PFTS $(\mathcal{X},\mathfrak{I})$ is a PFBS then $\bigcup\limits_{i=1}^{\infty}(cl(E_i)) = 1_{\mathcal{X}}$, where $(cl(E_i))$'s are PFRCS's in $(\mathcal{X},\mathfrak{I})$.

Proof. Let $(\mathcal{X},\mathfrak{I})$ be a PFBS. Then $int\left[\bigcup_{i=1}^{\infty}(\mathcal{D}_i)\right] = 0_{\mathcal{X}}$, where $(\mathcal{D}_i)$'s are PFNDS's in $(\mathcal{X},\mathfrak{I})$. By Theorem 3.2, the PFNDS's $(\mathcal{D}_i)$'s are PFSCS's in $(\mathcal{X},\mathfrak{I})$ and hence $(1_{\mathcal{X}} - \mathcal{D}_i)$'s are PFSOSs in $(\mathcal{X},\mathfrak{I})$ and by Remark 4.1, $cl(1_{\mathcal{X}} - \mathcal{D}_i)$'s are PFRCS's in $(\mathcal{X},\mathfrak{I})$. Now $int\left[\bigcup_{i=1}^{\infty}(\mathcal{D}_i)\right] = 0_{\mathcal{X}}$ implies that, $1_{\mathcal{X}} - \text{int}\left[\bigcup_{i=1}^{\infty}(\mathcal{D}_i)\right] = 1_{\mathcal{X}}$. Then $cl\left[\bigcup\limits_{i=1}^{\infty}(1_{\mathcal{X}} - \mathcal{D}_i)\right] = 1_{\mathcal{X}}$. But $cl\left[\bigcup\limits_{i=1}^{\infty}(1_{\mathcal{X}} - \mathcal{D}_i)\right] \subseteq \bigcup\limits_{i=1}^{\infty}[cl(1_{\mathcal{X}} - \mathcal{D}_i)]$ implies that, $\bigcup\limits_{i=1}^{\infty}[cl(1_{\mathcal{X}} - \mathcal{D}_i)] = 1_{\mathcal{X}}$ and hence $cl(1_{\mathcal{X}} - \mathcal{D}_i) = 1_{\mathcal{X}}$. Let $E_i = 1_{\mathcal{X}} - \mathcal{D}_i$. Then $\bigcup\limits_{i=1}^{\infty}(cl(E_i)) = 1_{\mathcal{X}}$, where $(cl(E_i))$'s are PFRCS's in $(\mathcal{X},\mathfrak{I})$. □

5. Utilizing PFBS to Assess Societal Criminal Activity Levels: An Algorithmic Approach.

In this section, the applications of PFBS are discussed. The methodology demonstrates the numerical identification of criminal activity levels based on PFBS's. The analysis reveals the crime level of the accused in society.

5.1. **Proposed algorithm using PFBS's:**

Step 1. Let $\{C_1, C_2, ..., C_n\}$ be a set of PFS's in $X = \{x_1, x_2, ..., x_m\}$.
Step 2. Next step is to find PFNDS which should satisfy the condition $int[cl(\overline{C_i})] = 0_{\mathcal{X}}$, where $\overline{C_i}$ is the closed set $C_i (i = 1, .., n)$.
Step 3. Verifying the structure satisfy PFBS through the condition $int(\bigcup\limits_{i=1}^{\infty}(C_i)) = 0_{\mathcal{X}}$.
Step 4. Find the $C_{i_{max}}$ that corresponds to the PFNDS's C_i obtained in step-3.
Step 5. Compute the complement of $\bigcup\limits_{i=1}^{\infty}(C_i))$. That PFS will reveal the corresponding open set. From this, we can identify the higher membership of the required alternative.

5.2. **Illustration: Assessing Level of Criminal Activity of the accused person in the society.**

The concept of PFBS revolves around the notion of the empty set, denoted as $0_{\mathcal{X}}$, in this space. This method is commonly used to identify individuals who commit serious crimes while minimizing the risks to society. It is advisable to use the following algorithm to ensure the best decisions in this scenario.

Illustration

The analysis indicates that the accused committed a crime falling within specific categories of criminal activity levels, considered to be PFS's. The PFS's are determined based on the degree of severity of the criminal activity and are categorized as highly dangerous to the society, false accusations, and less dangerous to society. To identify the accused, three types of offenses are considered: violent crimes (k), property crimes (l), and drug offenses (m), denoted by $X = \{k, l, m\}$. Effective management of these categories is essential for establishing a sustainable system that reduces crime frequency. In the process of crime frequency analysis, it is crucial to assess the positive, neutral, and negative aspects of each category. This evaluation is integral to ensuring the development of a system that leads to identifying severity of criminal activities in society. A PFBS of picture fuzzy sets is defined by the variety of crimes committed by the accused person.

Step 1. Let $\sigma = \{1_{\mathcal{X}}, 0_{\mathcal{X}}, C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9\}$ be the PFT where
$C_1 = \{(k, 0.5, 0.1, 0.0), (l, 0.4, 0.1, 0.3), (m, 0.4, 0.2, 0.4)\}$,
$C_2 = \{(k, 0.7, 0.2, 0.1), (l, 0.5, 0.1, 0.4), (m, 0.6, 0.2, 0.2)\}$,
$C_3 = \{(k, 0.4, 0.1, 0.3), (l, 0.6, 0.1, 0.1), (m, 0.6, 0.2, 0.1)\}$,
$C_4 = C_1 \cup C_2, C_5 = C_1 \cup C_3, C_6 = C_2 \cup C_3, C_7 = C_1 \cap C_2, C_8 = C_1 \cap C_3, C_9 = C_2 \cap C_3$ and (X, σ) be the PFTS. Then $\sigma^c = \{\overline{1_{\mathcal{X}}}, \overline{0_{\mathcal{X}}}, \overline{C_1}, \overline{C_2}, \overline{C_3}, \overline{C_4}, \overline{C_5}, \overline{C_6}, \overline{C_7}, \overline{C_8}, \overline{C_9}\}$. C_i degree of severity of the accused persons.

Table 8. Membership value of PFCS's in PFTS $(\mathcal{X}, \mathfrak{I})$

		$\overline{C_1}$	$\overline{C_2}$	$\overline{C_3}$	$\overline{C_4}$	$\overline{C_5}$	$\overline{C_6}$	$\overline{C_7}$	$\overline{C_8}$	$\overline{C_9}$	$\overline{0_X}$	$\overline{1_X}$
	γ	0.0	0.1	0.3	0.0	0.0	0.1	0.1	0.3	0.3	1	0
k	η	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0
	ν	0.5	0.7	0.4	0.7	0.5	0.7	0.5	0.4	0.4	0	1
	γ	0.3	0.4	0.1	0.3	0.1	0.1	0.4	0.3	0.4	1	0
l	η	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0	0
	ν	0.4	0.5	0.6	0.5	0.6	0.6	0.4	0.4	0.5	0	1
	γ	0.4	0.2	0.1	0.2	0.1	0.1	0.4	0.4	0.2	1	0
m	η	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0	0
	ν	0.4	0.6	0.6	0.6	0.6	0.6	0.4	0.4	0.6	0	1

Step 2. $int[cl(\overline{C}_1)] \neq 0_{\mathcal{X}}$, $int[cl(\overline{C}_7)] \neq 0_{\mathcal{X}}$, $int[cl(\overline{C}_8)] \neq 0_{\mathcal{X}}$ is not a nowhere dense sets. $int[cl(\overline{C}_2)] = 0_{\mathcal{X}}$, $int[cl(\overline{C}_3)] = 0_{\mathcal{X}}$, $int[cl(\overline{C}_4)] = 0_{\mathcal{X}}$, $int[cl(\overline{C}_5)] = 0_{\mathcal{X}}$, $int[cl(\overline{C}_6)] = 0_{\mathcal{X}}$, and $int[cl(\overline{C}_9)] = 0_{\mathcal{X}}$. Thus, $\overline{C}_2, \overline{C}_3, \overline{C}_4, \overline{C}_5, \overline{C}_6$ & $\overline{C}_9$ are PFNDS's.

Step 3. $int[\overline{C}_2 \cup \overline{C}_3 \cup \overline{C}_4 \cup \overline{C}_5 \cup \overline{C}_9] = 0_{\mathcal{X}}$. $int(\overline{C}_2 \cup \overline{C}_3 \cup \overline{C}_4 \cup \overline{C}_5 \cup \overline{C}_9) \subseteq \overline{C}_6$.

Step 4. We can identify the $C_{i_{max}}$ set that corresponds to the picture fuzzy set for the given values of $\overline{C}_6 = \{(k, 0.1, 0.1, 0.7), (l, 0.1, 0.1, 0.6), (m, 0.1, 0.2, 0.6)\}$. This satisfies the condition of the PFBS. By following step 4 in the algorithm, we convert the set into an open set which is named as C_6 .

Step 5. From this $C_6 = \{(k, 0.7, 0.1, 0.1), (l, 0.6, 0.1, 0.1), (m, 0.6, 0.2, 0.1)\}$. We can identify the person who has committed a higher crime in society. $C_6 = C_2 \cup C_3$, where C_2&C_3 whose degree of criminal activity is higher in society. The monitoring activity will reduce criminal activity in society. The utilization of the PFBS is essential in identifying whether an individual has perpetrated a more severe offense.

6. Conclusions

This paper introduces a new class of sets called PFDS and nowhere dense set, picture fuzzy semi-open and semi-closed sets, picture fuzzy first and second category sets, PFRS and PFBS.

Moreover, this work includes some characterizations of PFBS. An algorithm has been revised for identifying the PFBS and has been discussed with an illustration to demonstrate the implementation of PFBS. The primary goal is to identify individuals who have committed severe crimes in society, highlighting its superiority over alternative methods for measuring criminal activity levels. In the future, this methodology will be applied to explore other fuzzy environments and properties in topological spaces.

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