

**Mathematical Analysis of Nonlinear Reaction Diffusion Process at Carbon Dioxide Absorption in Concentrated Mixtures of 2-Amino-2-Methyl-1-Proponal and 1,8-Diamino-p-Methane**

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**ABSTRACT.** A mathematical model of carbon dioxide (CO<sub>2</sub>) absorption in an aqueous solution consisting of two reactants, 2-amino-2-methyl-1-proponal and 1,8-diamino-p-methane is considered. Akbari Ganji Method and Differential Transform Method are implemented to resolve the system of nonlinear equations, yielding an analytical formulation for the concentration of carbon dioxide, 2-amino-2-methyl-1-proponal, 1,8-diamino-p-methane and the molar flux in-terms of reaction rate constants. The obtained analytical findings are used to evaluate the different diffusion parameters and compared with numerical results. By assessing Matlab results with analytical findings, a successful outcome is discovered. Moreover, the influence of parameters on molar flux is examined. Also, graphical representations are presented and discussed here. The new analytical results contribute to optimizing the consistency of this model. The ensuring outcomes have been verified utilizing the existing numerical data with prior findings, and we are then presented with an adequate level of agreement.

## 1. Introduction

According to the fact that carbon dioxide in exhaust gases produced by burning fossil fuels is the primary source of air pollution, acid rain, global warming and other environmental issues, the chemical immobilization of CO<sub>2</sub> has recently acquired close consideration as a research area [1]. Chemical reaction engineering practice often deals with multiphase reaction systems.

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These systems are commonly encountered in industrially significant processes such as oxidation, hydroformylation, gas purification, hydrogenation and oxidation. A lot of attention was paid to one of them, the removal of carbon dioxide employing amines in aqueous solutions. The chemical absorption technique is a standard approach to accomplish the CO<sub>2</sub> removal and recovery on an industrial scale. Alkanolamines are chemical compounds that are significant to industry [2]. Refineries, natural gas and synthetic gas sectors commonly employ aqueous alkanolamine solutions to remove CO<sub>2</sub> and other acidic gases from gas streams.

Numerous investigations involving basic mass balance analysis have been conducted on the principles and kinetics of carbon dioxide's interaction with different alkanolamines, leading to the discovery of zwitterion mechanism [3-4]. A class of sterically hindered amines has recently been discovered that exhibits a high amine capacity [5-8] and a reasonably substantial uptake rate, even under enhanced carbon dioxide loading. A prime illustration is 2-amino-2-methyl-1-propanol. Blended amines have been proposed as potentially helpful in facilitating the uptake of acid gases through leveraging the positive aspects of amine [9]

K.J. Oh et al. presented a theoretical and experimental analysis of the absorption of carbon dioxide into aqueous solutions [10]. Paul et al. established the CO<sub>2</sub> extraction in the solution of 2-(1-piperziny) ethylamine [11]. Subramaniam et al. presented the study of mass transfer along CO<sub>2</sub> absorption through phenyl Glycidyl ether solution by Adomian decomposition method [12]. Anitha et al. examines the carbon dioxide concentration in the solutions that used Homotopy analysis method [13].

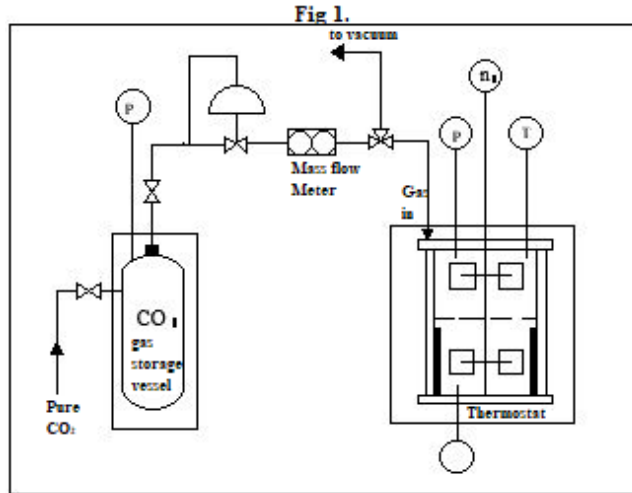
The primary objective of this study is to obtain the analytical approximations regarding the absorption of carbon dioxide into the aqueous solutions along with 2-amino-2-methyl-1-propanol (AMP) and 1,8-diamino-p-methane (DAM), making use of Akbari Ganji Method (AGM) and Differential Transform Method (DTM). Quantitative and graphical statistics are presented to demonstrate these approaches.

## 2. Mathematical formulation

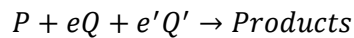
The mathematical model of CO<sub>2</sub> absorption in the stirred semi batch tank with a planar gas liquid was built on the Zwitterion reaction mechanism presented out by K.J. Oh [10]. Figure 1 established the schematic representation of the carbon dioxide absorption into an aqueous solution containing two reactants of 2-amino-2-methyl-1-propanol and 1,8-diamino-p-methane in a stirred semi batch tank with a liquid of planar gas. The range of concentrations of

AMP and DAM was  $0 - 3.0 \text{ k mol/m}^3$  and  $0.02 \text{ k mol/m}^3$  respectively and  $298.2-323.2\text{K}$  at 15% of Carbon dioxide. The mathematical augmentation proportion of  $\text{CO}_2$  was determined by employing the diffusion coefficient, mass transfer coefficient in the solution composed of AMP and DAM. This succeeded by approximating the mass balance solution for the instantaneous and rapid regime in the  $\text{CO}_2 - \text{AMP-DAM}$  system.

**Figure 1. Diagrammatic depiction of the carbon dioxide absorber**



The irreversible reactions involving reactant and  $\text{CO}_2$  can be described as,



where  $P, Q, Q'$  represents  $\text{CO}_2$ , 2-amino-2-methyl-1-proponal, 1-8-diamino-p-methane respectively and  $e, e'$  are stoichiometric coefficients of species.

The following pre-assumptions form the basis of the mathematical equations that govern for the assimilation of  $\text{CO}_2$  over the aqueous solution. (i) The circumstance essentially isothermal (ii) Henry's law constitutes applicable (iii) AMP and DAM species are non-removal (iv) First order reactions exist concerning  $P$  and  $Q$  as well as between  $P$  and  $Q'$  with regard to  $Q$  and  $Q'$  respectively.

The following set of nonlinear differential equations describes the fundamental mass balance computations of  $\text{CO}_2$  absorption in the solution of AMP with DAM.

$$\mathcal{D}_{\text{CO}_2} \frac{d^2 R_{\text{CO}_2}(\zeta)}{d\zeta^2} = a R_{\text{CO}_2}(\zeta) R_{\text{AMP}}(\zeta) + a' R_{\text{AMP}}(\zeta) R_{\text{DAM}}(\zeta) \quad (2.1)$$

$$\mathcal{D}_{\text{AMP}} \frac{d^2 R_{\text{AMP}}(\zeta)}{d\zeta^2} = e a R_{\text{CO}_2}(\zeta) R_{\text{AMP}}(\zeta) \quad (2.2)$$

$$\mathcal{D}_{\text{DAM}} \frac{d^2 R_{\text{DAM}}(\zeta)}{d\zeta^2} = e' a' R_{\text{AMP}}(\zeta) R_{\text{DAM}}(\zeta) \quad (2.3)$$

The associated boundary conditions are demonstrated as,

$$R_{CO_2}(0) = R_{CO_{2i}}, R_{AMP}(0) = R_{AMP_i}, R_{DAM}(0) = R_{DAM_i} \quad (2.4)$$

$$R_{CO_2}(l) = 0, R_{AMP}(l) = R_{AMP_0}, R_{DAM}(l) = R_{DAM_0} \quad (2.5)$$

Defining the molar flux as,

$$F_{CO_2} = -\mathcal{D}_{CO_2} \frac{dR_{CO_2}(0)}{dz} \quad (2.6)$$

### Non-dimensional version of the problem

To generate the aforementioned nonlinear differential equations in non-dimensional manner based on establishing their subsequent parameters in order to compare the analytical findings with the simulation results.

$$\begin{aligned} \tilde{R}_{CO_2} &= \frac{R_{CO_2}}{R_{CO_{2i}}}; \tilde{R}_{AMP} = \frac{R_{AMP}}{R_{AMP_i}}; \tilde{R}_{DAM} = \frac{R_{DAM}}{R_{DAM_i}}; \xi = \frac{z}{l}; \\ \delta_1 &= \frac{l^2 a R_{AMP_i}}{\mathcal{D}_{CO_2}}; \delta_2 = \frac{l^2 a' R_{DAM_i}}{\mathcal{D}_{CO_2}}; \delta_3 = \frac{l^2 a R_{CO_{2i}}}{\mathcal{D}_{AMP}}; \delta_4 = \frac{l^2 a' R_{CO_{2i}}}{\mathcal{D}_{DAM}}; \end{aligned} \quad (2.7)$$

The following are the normalized version of the equations (2.1) -(2.3)

$$\frac{d^2 \tilde{R}_{CO_2}(\xi)}{d\xi^2} = \delta_1 \tilde{R}_{CO_2}(\xi) \tilde{R}_{AMP}(\xi) + \delta_2 \tilde{R}_{AMP}(\xi) \tilde{R}_{DAM}(\xi) \quad (2.8)$$

$$\frac{d^2 \tilde{R}_{AMP}(\xi)}{d\xi^2} = e \delta_3 \tilde{R}_{CO_2}(\xi) \tilde{R}_{AMP}(\xi) \quad (2.9)$$

$$\frac{d^2 \tilde{R}_{DAM}(\xi)}{d\xi^2} = e' \delta_4 \tilde{R}_{CO_2}(\xi) \tilde{R}_{DAM}(\xi) \quad (2.10)$$

The relevant dimensionless boundary conditions are often expressed as

$$\tilde{R}_{CO_2}(0) = 1; \tilde{R}_{AMP}(0) = 1; \tilde{R}_{DAM}(0) = 1 \quad (2.11)$$

$$\tilde{R}_{CO_2}(1) = 1; \tilde{R}_{AMP}(1) = \frac{R_{AMP_0}}{R_{AMP_i}} = \alpha; \tilde{R}_{DAM}(1) = \frac{R_{DAM_0}}{R_{DAM_i}} = \beta \quad (2.12)$$

The normalized flux becomes,

$$\Omega_{CO_2} = -\frac{\mathcal{D}_{CO_2} R_{CO_{2i}}}{l} \frac{d\tilde{R}_{CO_2}(0)}{d\xi} \quad (2.13)$$

### 3. Derivation of analytical formulation for the concentrations

In recent years, there have been numerous efforts to develop analytical techniques to deal with nonlinear problems. A certain number of significant analytical techniques used for solved nonlinear equations consists of Akbari Ganji Method [14-21], Homotopy Perturbation Method [22-25], Taylor Series Method [26-28] and Differential Transform Method [29-35].

#### Akbari Ganji Method

AGM signifies a striking progress in the field of nonlinear sciences. Analytically simulating nonlinear differential problems can be considerably more difficult than addressing

linear differential equations. This method is an extremely innovative approach to overcoming such issues in this sense. Within the AGM, a solution function containing a newly discovered constant coefficient satisfies the initial and boundary limitations. Employing this approach, nonlinear equations (2.8) - (2.10) may be solved to get the straightforward analytical expressions for the concentration of species.

The AGM proceeds by presumed that the hyperbolic function governs the solution to the equations (2.8) - (2.10).

$$\tilde{R}_{CO_2}(\xi) = \mathcal{M}_{CO_2} \cosh(\varphi_1 \xi) + \mathcal{N}_{CO_2} \sinh(\varphi_1 \xi) \quad (3.1)$$

$$\tilde{R}_{AMP}(\xi) = \mathcal{M}_{AMP} \cosh(\varphi_2 \xi) + \mathcal{N}_{AMP} \sinh(\varphi_2 \xi) \quad (3.2)$$

$$\tilde{R}_{DAM}(\xi) = \mathcal{M}_{DAM} \cosh(\varphi_3 \xi) + \mathcal{N}_{DAM} \sinh(\varphi_3 \xi) \quad (3.3)$$

Where  $\mathcal{M}_{CO_2}, \mathcal{N}_{CO_2}, \mathcal{M}_{AMP}, \mathcal{N}_{AMP}, \mathcal{M}_{DAM}, \mathcal{N}_{DAM}$  and  $\varphi_i$  are acted as constants.

When boundary conditions (2.11) -(2.12) have been substituted in (3.1) -(3.3), the value of the constants are

$$\mathcal{M}_{CO_2} = 1; \quad \mathcal{N}_{CO_2} = -\coth(\varphi_1); \quad (3.4)$$

$$\mathcal{M}_{AMP} = 1; \quad \mathcal{N}_{AMP} = \frac{\alpha - \cosh(\varphi_2)}{\sinh(\varphi_2)}; \quad (3.5)$$

$$\mathcal{M}_{DAM} = 1; \quad \mathcal{N}_{DAM} = \frac{\beta - \cosh(\varphi_3)}{\sinh(\varphi_3)}. \quad (3.6)$$

Equation (3.1) -(3.3) now reduces to

$$\tilde{R}_{CO_2}(\xi) = \frac{\sinh(\varphi_1(1-\xi))}{\sinh(\varphi_1)} \quad (3.7)$$

$$\tilde{R}_{AMP}(\xi) = \frac{\alpha \sinh(\varphi_2 \xi) + \sinh(\varphi_2(1-\xi))}{\sinh(\varphi_2)} \quad (3.8)$$

$$\tilde{R}_{DAM}(\xi) = \frac{\beta \sinh(\varphi_3 \xi) + \sinh(\varphi_3(1-\xi))}{\sinh(\varphi_3)} \quad (3.9)$$

Substituting (3.7) -(3.9) in (2.8) -(2.10) and at  $\xi = 0$  yields the constants

$$\varphi_1 = \sqrt{\delta_1 + \delta_2} \quad (3.10)$$

$$\varphi_2 = \sqrt{e\delta_3} \quad (3.11)$$

$$\varphi_3 = \sqrt{e'\delta_4} \quad (3.12)$$

The analytical expression for the concentration of carbon dioxide, 2-amino-2-methyl-1-proponal and 1,8-diamino-p-methane for all dimensionless parameters is derived by substituting equations (3.10) -(3.12) into (3.7) -(3.9)

$$\tilde{R}_{CO_2}(\xi) = \frac{\sinh(\sqrt{\delta_1 + \delta_2}(1-\xi))}{\sinh(\sqrt{\delta_1 + \delta_2})} \quad (3.13)$$

$$\tilde{R}_{AMP}(\xi) = \frac{\alpha \sinh(\sqrt{e\delta_3}\xi) + \sinh(\sqrt{e\delta_3}(1-\xi))}{\sinh(\sqrt{e\delta_3})} \quad (3.14)$$

$$\tilde{R}_{DAM}(\xi) = \frac{\beta \sinh(\sqrt{e\delta_4}\xi) + \sinh(\sqrt{e\delta_4}(1-\xi))}{\sinh(\sqrt{e\delta_4})} \quad (3.15)$$

The nondimensional current is determine as

$$\Omega_{CO_2} = \frac{D_{CO_2} R_{CO_2i}}{l} (\sqrt{\delta_1 + \delta_2} \coth(\sqrt{\delta_1 + \delta_2})) \quad (3.16)$$

### Differential Transform Method

The differential transform method had been first proposed by Zhou in 1986 and since then, it has been explained in several literatures to solve different types of integral and differential problems. With the help of differential transform approach, one may find the coefficient of the Taylor series expansion explored for determining differential equations.

Equations (2.8) - (2.10) is rewritten as the transform equation for the DTM solution process as,

$$(n+1)(n+2)\tilde{R}_{CO_2}(n+2) - \delta_1 \sum_{t=0}^n \tilde{R}_{CO_2}(t)\tilde{R}_{AMP}(n-t) + \delta_2 \sum_{t=0}^n \tilde{R}_{AMP}(t)\tilde{R}_{DAM}(n-t) = 0 \quad (3.17)$$

$$(n+1)(n+2)\tilde{R}_{AMP}(n+2) - e\delta_3 \sum_{t=0}^n \tilde{R}_{CO_2}(t)\tilde{R}_{AMP}(n-t) = 0 \quad (3.18)$$

$$(n+1)(n+2)\tilde{R}_{DAM}(n+2) - e\delta_4 \sum_{t=0}^n \tilde{R}_{CO_2}(t)\tilde{R}_{DAM}(n-t) = 0 \quad (3.19)$$

From (3.17) -(3.19), we obtain the following recurrence relations,

$$\tilde{R}_{CO_2}(n+2) = \frac{1}{(n+1)(n+2)} [\delta_1 \sum_{t=0}^n \tilde{R}_{CO_2}(t)\tilde{R}_{AMP}(n-t) + \delta_2 \sum_{t=0}^n \tilde{R}_{AMP}(t)\tilde{R}_{DAM}(n-t)] \quad (3.20)$$

$$\tilde{R}_{AMP}(n+2) = \frac{1}{(n+1)(n+2)} [e\delta_3 \sum_{t=0}^n \tilde{R}_{CO_2}(t)\tilde{R}_{AMP}(n-t)] \quad (3.21)$$

$$\tilde{R}_{DAM}(n+2) = \frac{1}{(n+1)(n+2)} [e\delta_4 \sum_{t=0}^n \tilde{R}_{CO_2}(t)\tilde{R}_{DAM}(n-t)] \quad (3.22)$$

Let as assume that  $\tilde{R}_{CO_2}(1) = v_1$ ,  $\tilde{R}_{AMP}(1) = v_2$ ,  $\tilde{R}_{DAM}(1) = v_3$

Using recurrence relations (3.20)-(3.22) along with boundary conditions (2.11)-(2.12) at  $n = 0$ , we

$$\text{have } \tilde{R}_{CO_2}(2) = \frac{\delta_1 + \delta_2}{2}; \tilde{R}_{AMP}(2) = \frac{e\delta_3}{2}; \tilde{R}_{DAM}(2) = \frac{e\delta_4}{2} \quad (3.23)$$

In DTM Approach, the exact solution set is considered as,

$$\tilde{R}_{CO_2}(\xi) = \sum_{n=0}^2 \tilde{R}_{CO_2}(n) \xi^n = 1 + v_1 \xi + \left(\frac{\delta_1 + \delta_2}{2}\right) \xi^2 \quad (3.24)$$

$$\tilde{R}_{AMP}(\xi) = \sum_{n=0}^2 \tilde{R}_{AMP}(n) \xi^n = 1 + v_2 \xi + \left(\frac{e\delta_3}{2}\right) \xi^2 \quad (3.25)$$

$$\tilde{R}_{DAM}(\xi) = \sum_{n=0}^2 \tilde{R}_{DAM}(n) \xi^n = 1 + v_3 \xi + \left(\frac{e\delta_4}{2}\right) \xi^2 \quad (3.26)$$

At  $\xi = 1$ , we obtain the values,

$$v_1 = -1 - \left(\frac{\delta_1 + \delta_2}{2}\right); v_2 = \alpha - 1 - \frac{e\delta_3}{2}; v_3 = \beta - 1 - \frac{e'\delta_4}{2}$$

After substituting the constants in (3.24) -(3.26), we are able to compute the analytical formulation that follows.

$$\tilde{R}_{CO_2}(\xi) = 1 - \xi - \left(\frac{\delta_1 + \delta_2}{2}\right)\xi + \left(\frac{\delta_1 + \delta_2}{2}\right)\xi^2 \tag{3.27}$$

$$\tilde{R}_{AMP}(\xi) = 1 + (\alpha - 1 - \frac{e\delta_3}{2})\xi + \frac{e\delta_3}{2}\xi^2 \tag{3.28}$$

$$\tilde{R}_{DAM}(\xi) = 1 + (\beta - 1 - \frac{e'\delta_4}{2})\xi + \frac{e'\delta_4}{2}\xi^2 \tag{3.29}$$

We determine the analytical equation for the current expressed as,

$$\Omega_{CO_2} = \frac{\mathcal{D}_{CO_2} R_{CO_2i}}{l} \left(1 + \frac{\delta_1 + \delta_2}{2}\right) \tag{3.30}$$

#### 4. Evaluation of analytical findings with numerical simulation

The numerical technique provides an approximation to solve a mathematical issue. Additionally, the validation of analytical findings is also advantageous. Figure (2-6) summaries the findings of concentration predictions for the nonlinear differential equation that was numerically solved using MATLAB software to examine the appropriateness of these analytical approaches. Table (1-3) compares the simulation results with AGM and DTM results. The maximum average error of CO<sub>2</sub>, AMP and DAM is reported as 0.01%, 0.01%, 0.02% by applying AGM and 0.01%, 0.02% and 0.2% using DTM respectively. It provides a good agreement for all parameter values upon comparison.

**Table 1.** Deviation between Numerical result (2.8) and Analytical results (3.13) and (3.27) of the concentration of Carbon dioxide for different parameters.

$\delta_1 = 0.01, \delta_2 = 0.1$						$\delta_1 = 0.1, \delta_2 = 0.1$				
$\xi$	Num. Value	AGM of (3.13)	Error % of AGM	DTM of (3.27)	Error of DTM	Num. Value	AGM of (3.13)	Error % of AGM	DTM of (3.27)	Error of DTM
0.1	0.8977	0.8999	0.0022	0.8951	0.0026	0.8952	0.8997	0.0045	0.891	0.0042
0.2	0.7961	0.7999	0.0038	0.7912	0.0049	0.7918	0.7945	0.0027	0.7840	0.0078
0.3	0.6948	0.6999	0.0051	0.6885	0.0063	0.6845	0.6994	0.0149	0.6790	0.0055
0.4	0.5937	0.5999	0.0062	0.5868	0.0069	0.5880	0.5993	0.0113	0.5760	0.012
0.5	0.4927	0.4999	0.0072	0.4863	0.0064	0.4872	0.4994	0.0122	0.4750	0.0122
0.6	0.3919	0.3999	0.008	0.3868	0.0051	0.3870	0.3995	0.0125	0.3760	0.011
0.7	0.2912	0.2999	0.0087	0.2885	0.0027	0.2873	0.2996	0.0123	0.2790	0.0083
0.8	0.1907	0.1999	0.0092	0.1912	0.0005	0.1880	0.1997	0.0117	0.1840	0.004
0.9	0.0903	0.0999	0.0096	0.0950	0.0047	0.0889	0.0999	0.011	0.0910	0.0021
<b>Average Error %</b>			<b>0.0067</b>		<b>0.0045</b>			<b>0.0103</b>		<b>0.0075</b>

**Table 2.** Comparison of numerical findings (2.9) and new analytical findings (3.14) and (3.28) of the concentration of 2-amino-2-methyl-1-proponal for various values of parameters.

$\delta_3 = 1, e = 0.1, \alpha = 1$						$\delta_3 = 0.1, e = 0.1, \alpha = 1$				
$\xi$	Num. Value	AGM of (3.14)	Error % of AGM	DTM of (3.28)	Error of DTM	Num. Value	AGM of (3.14)	Error % of AGM	DTM of (3.28)	Error of DTM
0.1	0.9972	0.9955	0.0017	1.0055	0.0083	0.9997	0.9995	0.0002	1.0005	0.0008
0.2	0.9952	0.9920	0.0032	1.0120	0.0168	0.9995	0.9992	0.0003	1.0012	0.0017
0.3	0.9941	0.9896	0.0045	1.0195	0.0254	0.9994	0.9989	0.0005	1.0019	0.0025
0.4	0.9937	0.9881	0.0056	1.0280	0.0343	0.9994	0.9988	0.0006	1.0028	0.0034
0.5	0.9939	0.9876	0.0063	1.0375	0.0436	0.9994	0.9987	0.0007	1.0038	0.0044
0.6	0.9946	0.9881	0.0065	1.0480	0.0534	0.9995	0.9988	0.0007	1.0048	0.0053
0.7	0.9956	0.9896	0.006	1.0595	0.0639	0.9996	0.9989	0.0007	1.0059	0.0063
0.8	0.997	0.9920	0.005	1.0720	0.075	0.9997	0.9992	0.0005	1.0072	0.0075
0.9	0.9985	0.9955	0.003	1.0855	0.087	0.9999	0.9995	0.0004	1.0086	0.0087
<b>Average Error %</b>			<b>0.0046</b>		<b>0.0453</b>			<b>0.0005</b>		<b>0.0045</b>

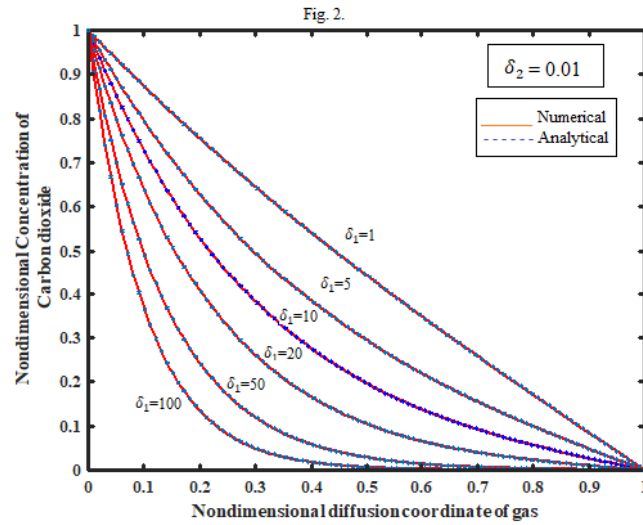
**Table 3.** Comparative analysis of numerical solution (2.10) and analytical solutions (3.15) and (3.29) of the concentration of 1,8-diamino-p-methane for different parameter values.

$\delta_3 = 1, e = 0.5, \alpha = 1$						$\delta_3 = 0.1, e = 0.5, \alpha = 1$				
$\xi$	Num. Value	AGM of (3.15)	Error % of AGM	DTM of (3.29)	Error of DTM	Num. Value	AGM of (3.15)	Error % of AGM	DTM of (3.29)	Error of DTM
0.1	0.9862	0.9784	0.0078	1.0275	0.0289	0.9986	0.9977	0.0009	1.0028	0.0042
0.2	0.9768	0.9618	0.015	1.0600	0.0624	0.9976	0.9960	0.0016	1.0060	0.0084
0.3	0.9714	0.9500	0.0214	1.0975	0.1004	0.9971	0.9947	0.0024	1.0098	0.0127
0.4	0.9694	0.9429	0.0265	1.1400	0.1431	0.9969	0.9940	0.0029	1.0140	0.0171
0.5	0.9703	0.9405	0.0298	1.1875	0.1905	0.9970	0.9937	0.0033	1.0188	0.0218
0.6	0.9735	0.9429	0.0306	1.2400	0.2427	0.9973	0.9940	0.0033	1.0240	0.0267
0.7	0.9787	0.9500	0.0287	1.2975	0.2997	0.9978	0.9947	0.0031	1.0298	0.032
0.8	0.9853	0.9618	0.0235	1.3600	0.3615	0.9985	0.9960	0.0025	1.0360	0.0375
0.9	0.9928	0.9784	0.0144	1.4275	0.4282	0.9993	0.9977	0.0016	1.0428	0.0435
<b>Average Error %</b>			<b>0.0219</b>		<b>0.2064</b>			<b>0.0024</b>		<b>0.0227</b>



## 5. Results and discussion

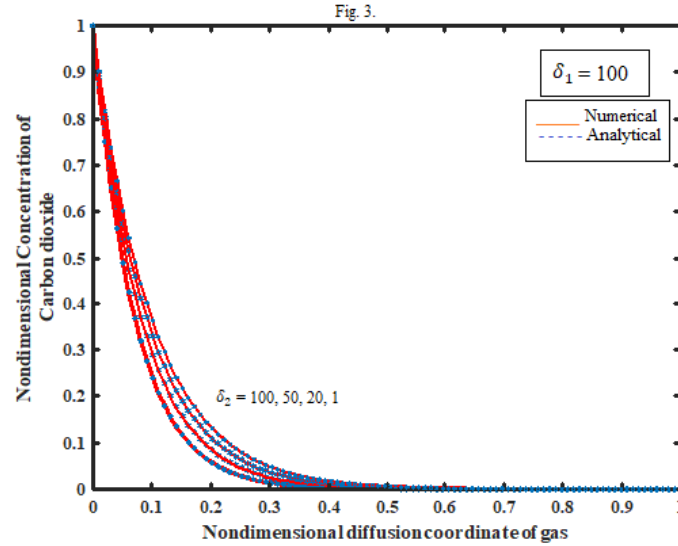
Equations (3.13)-(3.15) and (3.27)-(3.29) are the newly developed approximative analytical expressions for the concentration of species in Akbari Ganji Method and Differential Transform Method respectively. The dimensionless reaction rate constant  $\delta_i$  and stoichiometric coefficients  $e$  and  $e'$  affects the species concentration. The normalized current of  $\text{CO}_2$  is derived by using the concentration of carbon dioxide which expressed in equations (3.16) and (3.30).



**Figure 2:** Graph of Analytical and Numerical solutions for different values of parameter  $\delta_1$  and the fixed value of  $\delta_2$ .

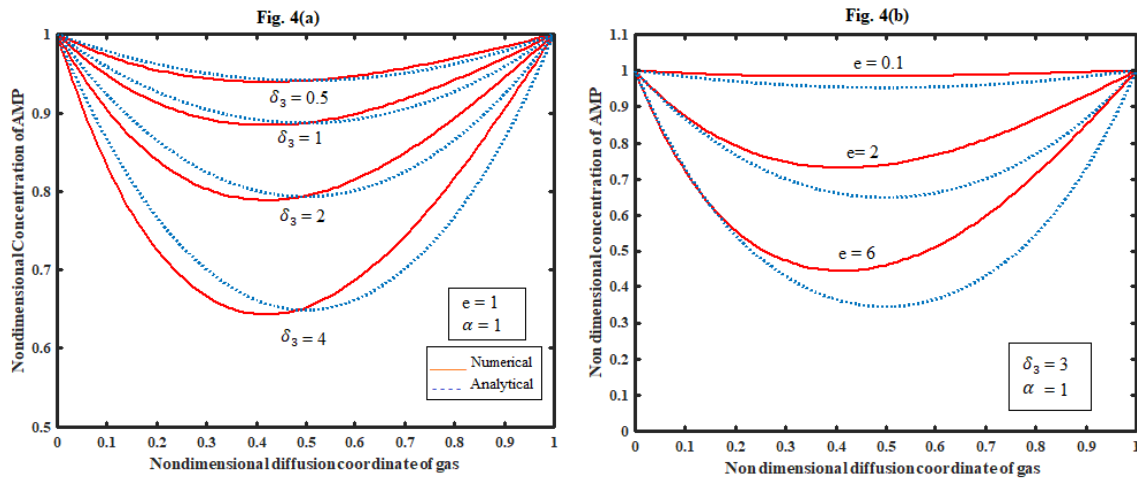
Figure 2 represents that the nondimensional concentration of carbon dioxide  $\tilde{R}_{\text{CO}_2}$  versus non dimensional diffusion coordinate of gas  $\xi$  for the various values of the dimensionless parameter  $\delta_1$  using (3.13). Figure 2 demonstrates that with a sense of decreasing amount of  $\delta_1$ , the concentration of carbon dioxide reduces into an aqueous solution. That means, the reaction rate constant  $\delta_1$  is inversely proportional to the concentration profile of  $\text{CO}_2$  for all minimal amount of  $\delta_2$ .

The concentration  $\tilde{R}_{\text{CO}_2}$  approaches the steady state regarding the range of  $\delta_1 \geq 100$  whereas the gas's diffusion coordinate exceeds in the bound  $0.4 \leq \xi \leq 1$ . At the initial state of the diffusion coordinate of gas,  $\tilde{R}_{\text{CO}_2}$  attains its optimum value. A linear tandem occurs among the concentration and reaction rate constant  $\delta_2$  meanwhile  $\tilde{R}_{\text{CO}_2}$  is minimal and closes in steady state.



**Figure 3:** Plot of the solutions both Analytically and Numerically various amounts of values of  $\delta_2$  for all  $\delta_1$ .

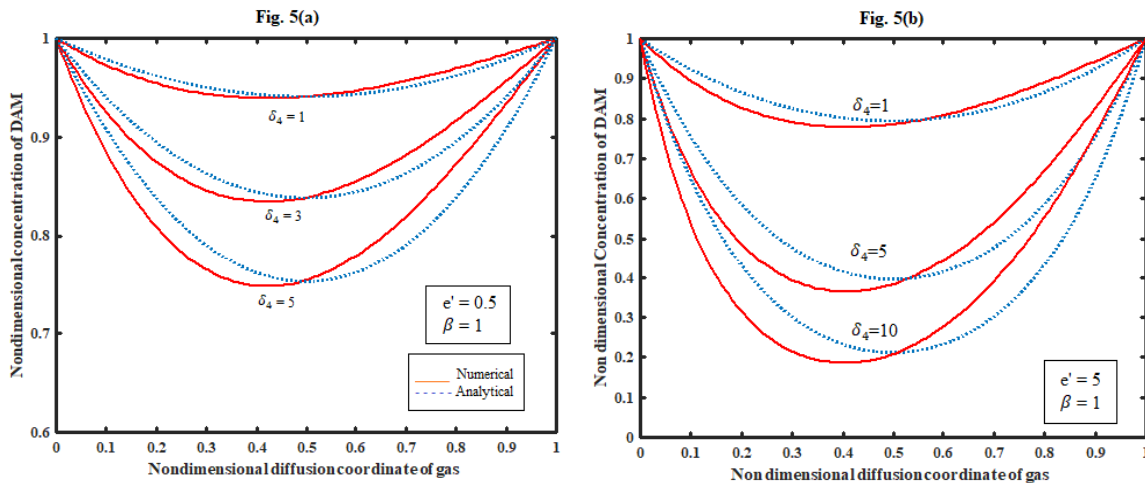
Figure 3 shows that the normalized concentration of carbon dioxide for the different values of  $\delta_2$  and for fixed value of  $\delta_1$ . Figure 3 leads to the conclusion that  $\bar{R}_{CO_2}$  decreases when  $\delta_2$  falls depends on the diffusion coordinate of gas  $\xi$ . Even with increasing diffusion film thickness or the reaction rate constant of  $CO_2$  and the maximum value of  $\delta_2 \geq 100$ , the concentration of  $CO_2$  remains constant. Additionally considering the diffusion gas approaches  $\xi \leq 0.1$ , the inclined curve of the concentration reflects that the elevated level of concentration of  $CO_2$ . For any significant amount of the reaction rate constant, the curve succeeds the value of steady state within  $0.5 \leq \xi \leq 1$ .



**Figure 4(a)-(b).** Plot of dimensionless concentration of AMP versus dimensionless diffusion coordinate of gas. (a)for fixed  $e$  and various  $\delta_3$  (b)fixed value of  $\delta_3$  and numerous value of  $e$ .

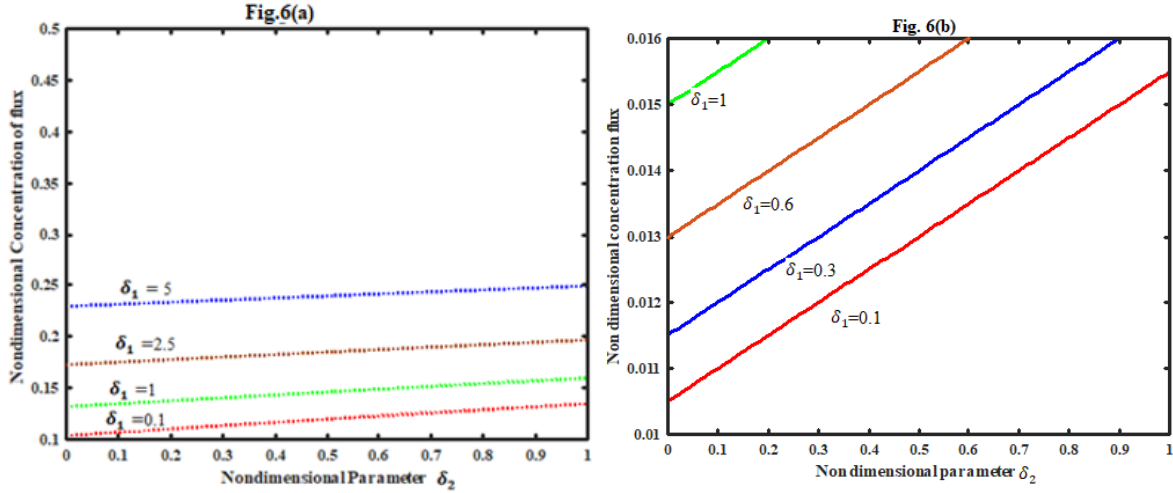
Figure 4(a)-(b) depicts that the compact influence on the concentration of 2-amino-2-methyl-1-proponal due to the nondimensional parameter  $\delta_3$  and the stoichiometric coefficient  $e$ . From Figure 4(a), it evident that the concentration of AMP decreases for maximal value of reaction rate constant  $\delta_3$  and the stoichiometric coefficient  $e = 1$ . The analytical and numerical outcomes are coincide at the equilibrium value of the diffusion coordinate of gas  $\xi = 0.5$ . The Concentration profile  $\tilde{R}_{AMP}$  attains its peak value whenever the range of  $\xi$  is both of  $\xi \leq 0.01$  and  $\xi \geq 1$ .

Figure 4(b) illustrates that the dimensionless concentration of 2-amino-2-methyl-1-proponal according to the various amount of stoichiometric coefficient  $e$  with the fixed value of  $\delta_3 = 3$ . It is observable that  $\tilde{R}_{AMP}$  decreases for large values of  $\delta_3$  and the stoichiometric coefficient rises. For the very minimal amount of  $e \leq 0.01$ , it clear that the AMP concentration is uniform. The effect of the stoichiometric coefficient which is inversely proportional to the amount of 2-amino-2-methyl-1-proponal.



**Figure 5(a)-(b).** Graph of nondimensional DAM concentration with the dimensionless distance for numerous amount of constant  $\delta_4$ .

Figure 5(a)-(b) exhibits a graph of the concentration of 1,8-diamino-p-methane contrasted with the gas's diffusion coordinate considering various reaction rate constant  $\delta_4$  and fixed values of the stoichiometric coefficient  $e'$ . It is clear that DAM concentration drops at the highest  $\delta_4$  values. A comparison of figure 5(a) and 5(b) shows that for all maximal  $e'$ , the concentration profile  $\tilde{R}_{DAM} \leq 0.5$ . According to figure 5(a)-(b), the stoichiometric coefficient  $e'$  which is inverted with respect to  $\tilde{R}_{DAM}$  but immediately correlated to the reaction rate constant  $\delta_4$ .



**Figure 6(a)-(b)** Graph of nondimensional concentration of flux for  $CO_2$  determined by the various nondimensional reaction rate constants  $\delta_1$  and (a) for fixed  $\mathcal{D}_{CO_2} = 0.1, R_{CO_{2i}} = 1, l = 1$ . (b) for fixed  $\mathcal{D}_{CO_2} = 0.01, R_{CO_{2i}} = 1, l = 1$ .

Figure 6(a)-(b) suggests the normalized current response fluctuations under many different kinds of parameter values  $\delta_1$  and  $\delta_2$ . It represents how the non-dimensional parameters  $\delta_1$  and  $\delta_2$  impact current circumstances profiles. The following information indicates that as the reaction rate constant  $\delta_1$  is increased, the molar flux climbs at a modest diffusion coefficient  $\mathcal{D}_{CO_2}$ . The variation of normalized current  $\Omega_{CO_2} \leq 1$  for numerous values of reaction rate constant  $\delta_2$  and  $\mathcal{D}_{CO_2} \geq 0.1$ . For any maximum value of the diffusion coefficient  $\mathcal{D}_{CO_2} \leq 0.01$ , the normalized current becomes lower in the range of  $\Omega_{CO_2} \leq 0.1$ .

From figure 6(a)-(b), it depicts

that  $\Omega_{CO_2}$  is exactly correlated to the diffusion coefficient  $\mathcal{D}_{CO_2}$  and the reaction rate constant  $\delta_2$ .

### Nomenclature

$\mathcal{D}_{CO_2}$	Diffusion coefficient of $CO_2$ $\mu m^2/s$	$l$	Diffusion film thickness $m$
$\mathcal{D}_{AMP}$	Diffusion coefficient of AMP $\mu m^2/s$	$F_{CO_2}$	Molar flux
$\mathcal{D}_{DAM}$	Diffusion coefficient of DAM $\mu m^2/s$	$\delta_i$	Dimensionless parameters
$R_{CO_2}$	Concentration of $CO_2$ $\mu m$	$\tilde{R}_{CO_2}$	Dimensionless Concentration of $CO_2$
$R_{AMP}$	Concentration of AMP $\mu m$	$\tilde{R}_{AMP}$	Dimensionless concentration of AMP
$R_{DAM}$	Concentration of DAM $\mu m$	$\tilde{R}_{DAM}$	Dimensionless concentration of DAM
$\zeta$	Diffusion coordinate of gas $m$	$\xi$	Dimensionless diffusion coordinate of gas
$a, a'$	Reaction rate constants ( $m^3/kmol.s$ )	$\Omega_{CO_2}$	Dimensionless normalized current
$e, e'$	Stoichiometric coefficients of species	$\phi_i$	Relevant flux factor

## 6. Conclusion

Mathematical formulation of Carbon dioxide absorption into an aqueous solution were discussed. A steady state behavior of system of non-linear equations were solved analytically by using Akbari Ganji Method and Differential Transform Method in order to obtain the analytical solution for the concentrations of CO<sub>2</sub>, AMP and DAM for all parameter values. The normalized current was also analytically expressed in simple closed form. A remarkable outcome might be observed graphically while comparing the new analytical results of AGM and DTM along with numerical simulation results for differing parameter reliability. The absorption rate of CO<sub>2</sub> in an aqueous solution was examined by the new approximate result of the diffusion model. These excellent outcomes are used to estimate a removal of gases released from power plant flues for the purpose of using Carbon dioxide. There was an extensive comprehension of the system while these outcomes were in good accordance our two techniques are simple to use in addition to the possibility that solve other nonlinear equations.

**Conflicts of Interest:** The authors declare that there are no conflicts of interest regarding the publication of this paper.

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