

Neutrosophic Regression Modeling with Dummy Variables: Applications and Simulations

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ABSTRACT. In this paper, we introduce a regression model using dummy variables within the framework of neutrosophic statistics. This model is designed for regression analysis under conditions of uncertainty, extending the classical regression model with dummy variables. We also present regression and analysis of variance under neutrosophic statistics. The application of our model is demonstrated through simulation and comparative studies, showing that the results differ from those obtained using classical regression. Our findings indicate that the degree of uncertainty significantly impacts the predicted and residual values.

1. Introduction

In general, regression analysis does not account for categorical variables in modeling and prediction. To address this, regression with dummy variables is employed. In this method, specific categories are assigned to variables, and regression models are created for these categories to facilitate prediction. The primary advantage of using dummy variables is that they enable the inclusion of categorical data in the analysis. The application of regression with dummy variables for analyzing structural change is demonstrated by [1]. [2] provides a detailed overview of dummy variables in regression analysis. [3] applied this technique for rainfall forecasting. [4] used it for insurance data analysis. [5] explored its use in a probabilistic environment with applications in quality control. [6] evaluated and applied the regression with

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dummy variables. [7] presented its application in analyzing students' learning data. Further applications can be found in [8], [9], [10] and [11].

Neutrosophic statistics, a branch of mathematical science, is essential for managing, analyzing, presenting, and interpreting uncertain data. This field extends classical statistics by integrating degrees of uncertainty often neglected in traditional methods. Introduced by [12], neutrosophic statistics has shown increased flexibility for dealing with imprecise data, as evidenced by numerous subsequent studies. Recent research highlights the effectiveness of neutrosophic statistical analysis, especially with complex or e-commerce data. Significant contributions include studies by [13], [14], and [15], and [16]. [17] explored neutrosophic multiple regression analysis, while [18] examined split-plot design for neutrosophic data analysis. Additionally, [19] looked into the analysis of covariance for imprecise data, and [20] studied neutrosophic statistical analysis in the context of temperature variations across cities. [21] introduced neutrosophic kernel regression for mean estimation, further broadening the applications of neutrosophic statistical methods.

Upon reviewing the literature, we discovered a substantial body of work on dummy regression within classical statistics. However, existing dummy regression methods are applicable only when all data observations are precise. In practice, statistical data often contains imprecision or intervals, rendering classical dummy regression methods unsuitable under these conditions of uncertainty. To the best of the author's knowledge, there has been no research on dummy regression using neutrosophic statistics. This paper aims to address this gap by proposing a dummy regression model under neutrosophic statistics. We will introduce a dummy regression framework that accommodates uncertainty and present an analysis of variance within this context. Additionally, we will conduct extensive simulation studies and apply the proposed regression model to real-world data. Our findings will demonstrate the significant impact of varying degrees of uncertainty on the predicted and residual values from the model.

2. Neutrosophic Random Variables

Consider two neutrosophic random variables, $X_N = X_L + X_L I_N$ and $Y_N = Y_L + Y_L I_N$, each made up of two components. The terms X_L and Y_L represent the determinate parts, akin to those in classical statistics. The terms $X_L I_N$ and $Y_L I_N$ represent the indeterminate parts, with $(I_N \in [I_L, I_U])$ denoting the degree of indeterminacy or uncertainty. When $I_N \in [I_L, I_U]$, these neutrosophic

random variables reduce to classical statistical variables. Assume X_L and Y_L follow normal distributions with means μ_x and μ_y , and variances σ_x^2 and σ_y^2 , respectively, as stated by [22]. Neutrosophic logic extends fuzzy logic, with $I_N^2 = I_N$ and $I_N^n = I_N$ for $n \in N$. Given this context, we outline some properties of these proposed neutrosophic random variables.

$$E(X_N) = E(X_L + X_L I_N) = (1 + I_N)\mu_x \quad \text{and} \quad E(Y_N) = E(Y_L + Y_L I_N) = (1 + I_N)\mu_y$$

$$\text{Var}(X_N) = \text{Var}(X_L + X_L I_N) = (1 + I_N)^2 \sigma_x^2 \quad \text{and} \quad \text{Var}(Y_N) = \text{Var}(Y_L + Y_L I_N) = (1 + I_N)^2 \sigma_y^2$$

$$\text{Var}(X_N + Y_N) = (1 + I_N)^2 \sigma_x^2 + (1 + I_N)^2 \sigma_y^2$$

3. Methodology

As previously discussed, regression with dummy variables is used for categorical variables. Under classical statistics, this type of regression can only be applied when data are precise. It is not suitable for use in situations involving uncertainty or imprecise data. In this section, we will modify the traditional regression with dummy variables by incorporating neutrosophic statistics. Our goal is to develop a regression model with dummy variables that remains effective even when observations are imprecise or uncertain. Assume the dependent variable $Y_N = Y_L + Y_U I_N$ is a neutrosophic random variable, composed of a determinate part Y_L and an indeterminate part $Y_U I_N$, with I_N representing the degree of indeterminacy. The neutrosophic regression with two dummy variables is then defined as follows:

$$Y_L + Y_U I_N = \beta_0 + \beta_1 X + \beta_2 D_i + \epsilon_i; I_N \in [I_L, I_U], i = 1, 2 \quad (1)$$

Note that D_i represents the category, X is the independent variable, and β_0 , β_1 , and β_2 , are the intercept and coefficients of the model, respectively. ϵ_i is the random error. Let $D = 1$ indicate defective items and $D = 0$ indicate non-defective items, with $I_L = 0$. The proposed regression model for defective items can be written as follows:

$$Y_L = \beta_0 + \beta_1 X + \beta_2 + \epsilon_i \quad (2)$$

The proposed regression model for non-defective items can be expressed as follows:

$$Y_L = \beta_0 + \beta_1 X + \epsilon_i \quad (3)$$

Let $I_N = I_U$. The proposed regression model for defective items can be expressed as follows:

$$Y_L + Y_U I_U = \beta_0 + \beta_1 X + \beta_2 + \epsilon_i \quad (4)$$

The proposed regression model for non-defective items can be formulated as follows:

$$Y_L + Y_U I_U = \beta_0 + \beta_1 X + \epsilon_i \quad (5)$$

Note that the proposed regression with dummy variables simplifies to the classical regression with dummy variables when $I_L=0$.

3.1 Neutrosophic Analysis of Variance (NANOVA)

In this section, we will extend the existing analysis of variance (ANOVA) under classical statistics by incorporating neutrosophic statistics. The neutrosophic analysis of variance (NANOVA) for regression with a dummy variable is given in Table 1.

Table 1: NANOVA table

	df	Sum of square	Mean square error	F_N test
Regression	k	RSS_N	$\frac{RSS_N}{k}$	$\frac{RSS_N}{k} / \frac{ESS_N}{n-k-1}$
Residual	$n-k-1$	ESS_N	$\frac{ESS_N}{n-k-1}$	
Total	$N-1$	RSS_N		

Note here that error/residual sum of square, say ESS_N is given by

$$ESS_N = \sum (y_N - \hat{Y}_N)^2 \quad (6)$$

where \hat{Y}_N denotes the predicted values.

The regression of sum square, say RSS_N is given by

$$RSS_N = \sum (\hat{Y}_N - \bar{y}_N)^2 \quad (7)$$

The neutrosophic F-test is given by

$$F_N = \frac{RSS_L}{k} / \frac{ESS_L}{n-k-1} + \frac{RSS_L}{k} / \frac{ESS_L}{n-k-1} I_N; I_N \in [I_L, I_U] \quad (8)$$

The proposed F-test simplifies to the classical F-test when $I_L=0$.

4. Application

In this section, we demonstrate the application of the proposed regression with dummy variables using neutrosophic data on expected defective item counts from three different machines, alongside the hours of operation for each machine. Notably, the anticipated defective counts from these machines yield intervals rather than exact figures. These data are detailed in Table 2. Notably, the imprecision in the defective item counts renders classical statistical regression with dummy variables infeasible. Hence, employing the proposed regression method within neutrosophic statistics becomes imperative. Table 3 presents a summary of our applied regression with dummy variables, revealing multiple R values ranging from 0.7110 to 0.7229, and standard error between 4.3262 to 4.2286. Table 4 depicts the analysis of variance

(ANOVA), showing F-test values ranging from 2.38 to 2.55, all below the significance level of 0.05, indicating insignificance in the results. The neutrosophic predicted and residual values are delineated in Table 5, illustrating a notable disparity between regression methodologies. Figures 2-3 further elucidate the discrepancies in predicted and residual values between the proposed and existing methods. This study underscores the differential performance of regression with dummy variables under conditions of indeterminacy, advocating for the adoption of the proposed method in data scenarios fraught with uncertainty. The neutrosophic F-test is given by

$$F_N = 2.38 + 2.55I_N; I_N \in [0, 0.06667]$$

Table 2: The data of number of defectives

Number of defectives	Number of hour	Machines
[6,7]	3	A
[8,12]	4	A
[5,8]	3	A
[9,11]	6	A
[7,10]	9	B
[3,6]	2	A
[12,15]	8	B
[2,3]	1	C
[14,16]	7	C
[19,20]	8	B
[13,16]	3	B

Table 3: SUMMARY OUTPUT

<i>Regression Statistics</i>	
Multiple R	[0.7110,0.7229]
R Square	[0.5055,0.5226]
Adjusted R Square	[0.2935,0.3180]
Standard Error	[4.3262,4.2286]
Observations	11

Table 4: ANOVA table

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	3	[134,137]	[44.63,45.67]	[2.38,2.55]	[0.1549,0.1385]
Residual	7	[131,125]	[18.72,17.88]		
Total	10	[265,262]			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	[3.0976,5.6829]	[2.9450,2.8786]	[1.05,1.97]	[0.3278,0.0889]
Hours	[0.8618,0.8659]	[0.6168,0.6028]	[1.40,1.44]	[0.2050,0.1941]
Machine B	[3.6199,3.5061]	[3.5804,3.4996]	[1.01,1.00]	[0.3457,0.3498]
Machine C	[1.4553,0.3537]	[3.6279,3.5461]	[0.40,0.10]	[0.7003,0.9234]

Table 5: RESIDUAL OUTPUT

<i>Observation</i>	<i>Predicted Defectives</i>	<i>Residuals</i>
1	[5.6829,8.2805]	[0.3171,-1.2805]
2	[6.5447,9.1463]	[1.4553,2.8537]
3	[5.6829,8.2805]	[-0.6829,-0.2805]
4	[8.2683,10.8780]	[0.7317,0.1220]
5	[14.4736,16.9817]	[-7.4736,-6.9817]
6	[4.8211,7.4146]	[-1.8211,-1.4146]
7	[13.6118,16.1159]	[-1.6118,-1.1159]
8	[5.4146,6.9024]	[-3.4146,-3.9024]
9	[10.5854,12.0976]	[3.4146,3.9024]
10	[13.6118,16.1159]	[5.3882,3.8841]
11	[9.3028,11.7866]	[3.6972,4.2134]

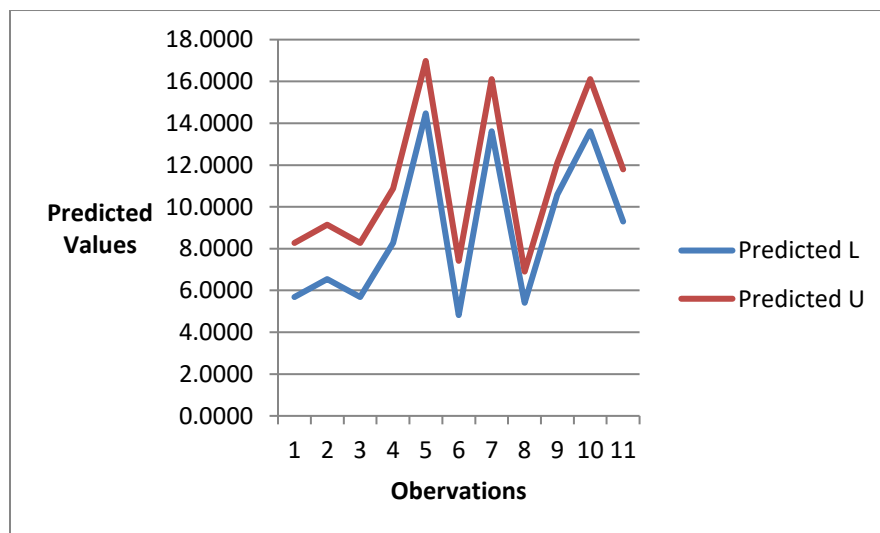


Figure 1: The neutrosophic predicted values for the data

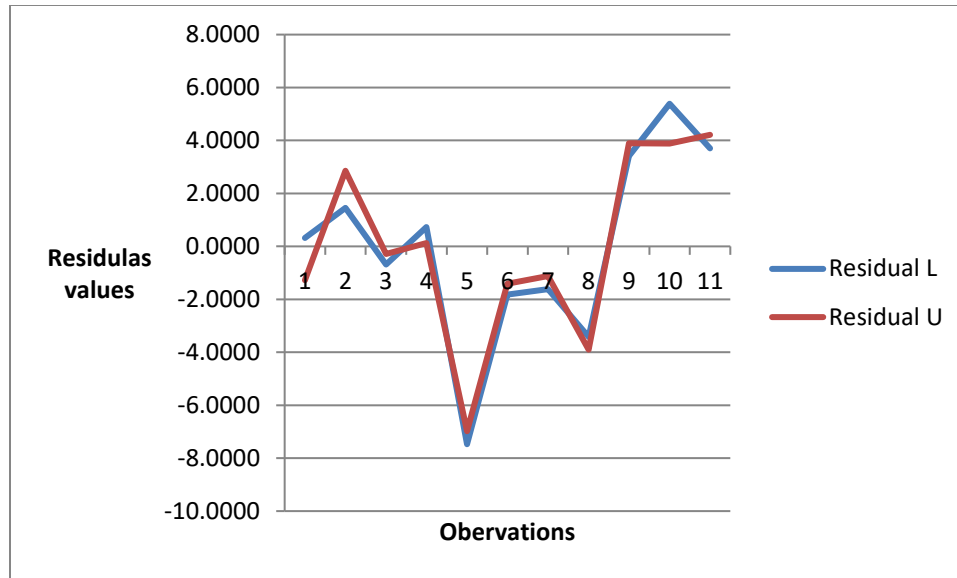


Figure 2: The neutrosophic residuals values for the data

5. Simulation

This section presents a simulation study examining how the degree of indeterminacy (I_N) affects key statistics, such as predicted values, residual values, percentiles, and the number of defectives. Utilizing data from Table 1 on the number of defectives, we explore various values of I_N to observe their impact on these statistics. Predicted values derived from the model are reported in Table 6, while Table 7 displays residual values. Percentile values are detailed in Table 8, and the number of defectives is outlined in Table 9. An analysis of Table 6 reveals that as the degree of indeterminacy increases, predicted values exhibit an upward trend. For instance, with $I_N=0.1$, the predicted value is 9.1085, whereas with $I_N=1$, it rises to 16.5610. This behavior is further illustrated in Figure 3, where the predicted value curves for $I_N=0.1$ is notably lower than for other I_N values. Similarly, Table 7 demonstrates that increasing I_N correlates with rising residual values. For instance, with $I_N=0.1$, the residual value is 4.2134, while with $I_N=1$, it increases to 8.4268. Figure 4 visually represents this trend, with the residual value curve for $I_N=0.1$ notably lower compared to other I_N values. Examining Table 8, we observe minimal fluctuation in percentile values with varying degrees of indeterminacy. For instance, when $I_N=0.1$, the percentile value is 4.5455, consistent with the value when $I_N=1$. Figure 5 further confirms this stability, with the percentile value curve for $I_N=0.1$ mirroring that of other I_N values. In contrast, Table 9 shows a clear increase in the number of defectives as the degree of indeterminacy rises. For instance, with $I_N=0.1$, there are 3 defectives, whereas with $I_N=1$, this

figure grows to 6. Figure 6 illustrates this trend graphically, with the defective value curve for $I_N=0.1$ notably lower than for other I_N values. In conclusion, this study highlights the impact of uncertainty on predicted and residual values within the model. Therefore, decision-makers should exercise caution when employing regression with dummy variables in uncertain conditions.

Table 6: Effect on Predicted values when $n=11$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
8.2805	9.1085	9.9366	10.7646	11.5927	12.4207	13.2488	14.0768	14.9049	15.7329	16.5610
9.1463	10.0610	10.9756	11.8902	12.8049	13.7195	14.6341	15.5488	16.4634	17.3780	18.2927
8.2805	9.1085	9.9366	10.7646	11.5927	12.4207	13.2488	14.0768	14.9049	15.7329	16.5610
10.8780	11.9659	13.0537	14.1415	15.2293	16.3171	17.4049	18.4927	19.5805	20.6683	21.7561
16.9817	18.6799	20.3780	22.0762	23.7744	25.4726	27.1707	28.8689	30.5671	32.2652	33.9634
7.4146	8.1561	8.8976	9.6390	10.3805	11.1220	11.8634	12.6049	13.3463	14.0878	14.8293
16.1159	17.7274	19.3390	20.9506	22.5622	24.1738	25.7854	27.3970	29.0085	30.6201	32.2317
6.9024	7.5927	8.2829	8.9732	9.6634	10.3537	11.0439	11.7341	12.4244	13.1146	13.8049
12.0976	13.3073	14.5171	15.7268	16.9366	18.1463	19.3561	20.5659	21.7756	22.9854	24.1951
16.1159	17.7274	19.3390	20.9506	22.5622	24.1738	25.7854	27.3970	29.0085	30.6201	32.2317
11.7866	12.9652	14.1439	15.3226	16.5012	17.6799	18.8585	20.0372	21.2159	22.3945	23.5732

Table 7: Effect on Residual values when $n=11$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
-1.2805	-1.4085	-1.5366	-1.6646	-1.7927	-1.9207	-2.0488	-2.1768	-2.3049	-2.4329	-2.5610
2.8537	3.1390	3.4244	3.7098	3.9951	4.2805	4.5659	4.8512	5.1366	5.4220	5.7073
-0.2805	-0.3085	-0.3366	-0.3646	-0.3927	-0.4207	-0.4488	-0.4768	-0.5049	-0.5329	-0.5610
0.1220	0.1341	0.1463	0.1585	0.1707	0.1829	0.1951	0.2073	0.2195	0.2317	0.2439
-6.9817	-7.6799	-8.3780	-9.0762	-9.7744	-10.4726	-11.1707	-11.8689	-12.5671	-13.2652	-13.9634
-1.4146	-1.5561	-1.6976	-1.8390	-1.9805	-2.1220	-2.2634	-2.4049	-2.5463	-2.6878	-2.8293
-1.1159	-1.2274	-1.3390	-1.4506	-1.5622	-1.6738	-1.7854	-1.8970	-2.0085	-2.1201	-2.2317
-3.9024	-4.2927	-4.6829	-5.0732	-5.4634	-5.8537	-6.2439	-6.6341	-7.0244	-7.4146	-7.8049
3.9024	4.2927	4.6829	5.0732	5.4634	5.8537	6.2439	6.6341	7.0244	7.4146	7.8049
3.8841	4.2726	4.6610	5.0494	5.4378	5.8262	6.2146	6.6030	6.9915	7.3799	7.7683
4.2134	4.6348	5.0561	5.4774	5.8988	6.3201	6.7415	7.1628	7.5841	8.0055	8.4268

Table 8: Effect on Percentiles values when $n=11$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
4.545455	4.5455	4.545455	4.5455	4.5455	4.5455	4.545455	4.5455	4.5455	4.5455	4.5455
13.63636	13.6364	13.63636	13.6364	13.6364	13.6364	13.63636	13.6364	13.6364	13.6364	13.6364
22.72727	22.7273	22.72727	22.7273	22.7273	22.7273	22.72727	22.7273	22.7273	22.7273	22.7273
31.81818	31.8182	31.81818	31.8182	31.8182	31.8182	31.81818	31.8182	31.8182	31.8182	31.8182
40.90909	40.9091	40.90909	40.9091	40.9091	40.9091	40.90909	40.9091	40.9091	40.9091	40.9091
50	50	50	50	50	50	50	50	50	50	50
59.09091	59.0909	59.09091	59.0909	59.0909	59.0909	59.09091	59.0909	59.0909	59.0909	59.0909
68.18182	68.1818	68.18182	68.1818	68.1818	68.1818	68.18182	68.1818	68.1818	68.1818	68.1818
77.27273	77.2727	77.27273	77.2727	77.2727	77.2727	77.27273	77.2727	77.2727	77.2727	77.2727
86.36364	86.3636	86.36364	86.3636	86.3636	86.3636	86.36364	86.3636	86.3636	86.3636	86.3636
95.45455	95.4545	95.45455	95.4545	95.4545	95.4545	95.45455	95.4545	95.4545	95.4545	95.4545

Table 9: Effect on Number of defectives when $n=11$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
3	3	4	4	4	5	5	5	5	6	6
6	7	7	8	8	9	10	10	11	11	12
7	8	8	9	10	11	11	12	13	13	14
8	9	10	10	11	12	13	14	14	15	16
10	11	12	13	14	15	16	17	18	19	20
11	12	13	14	15	17	18	19	20	21	22
12	13	14	16	17	18	19	20	22	23	24
15	17	18	20	21	23	24	26	27	29	30
16	18	19	21	22	24	26	27	29	30	32
16	18	19	21	22	24	26	27	29	30	32
20	22	24	26	28	30	32	34	36	38	40

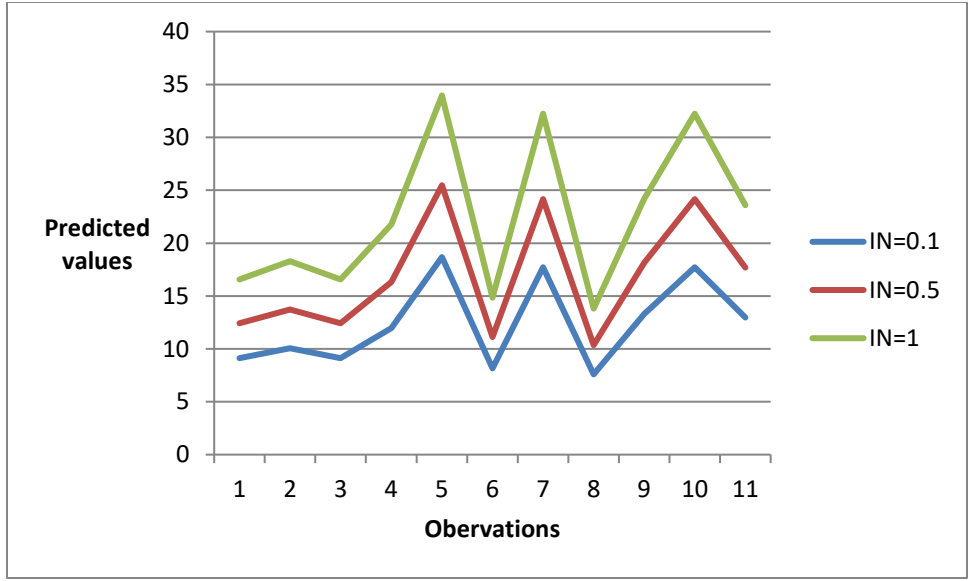


Figure 3: The neutrosophic predicted values for the simulated data

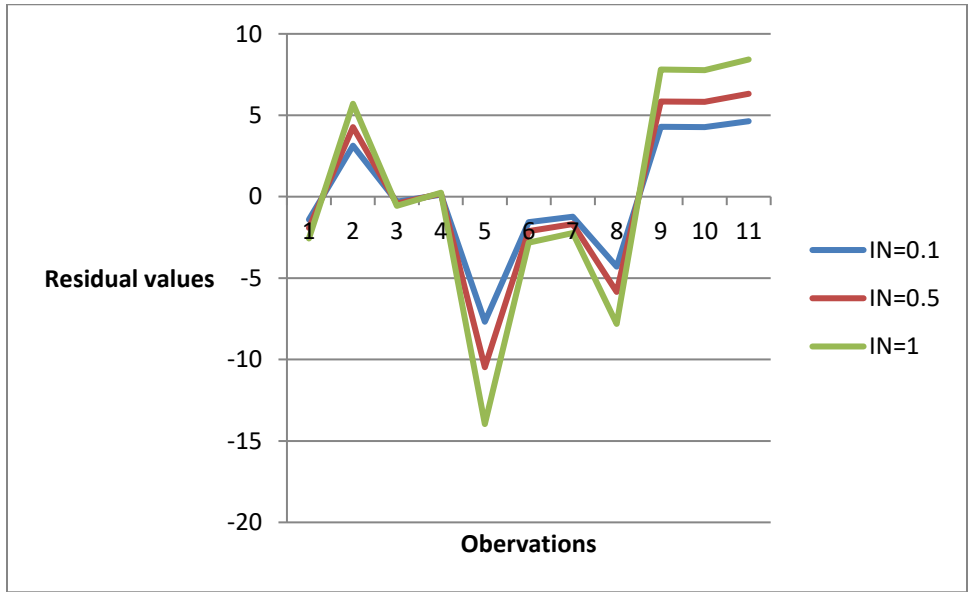


Figure 4: The neutrosophic residual values for the simulated data

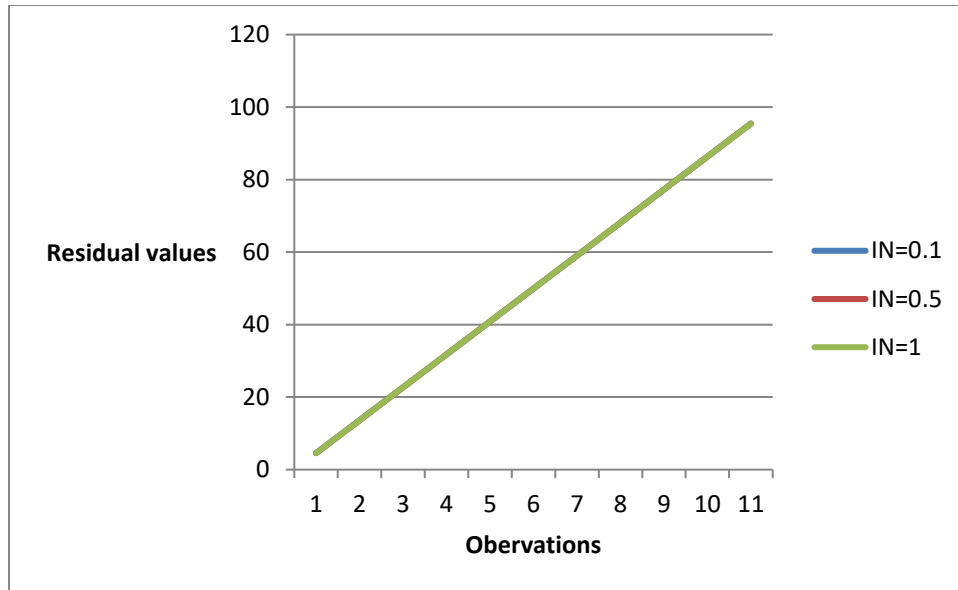


Figure 5: The neutrosophic Percentiles values for the simulated data

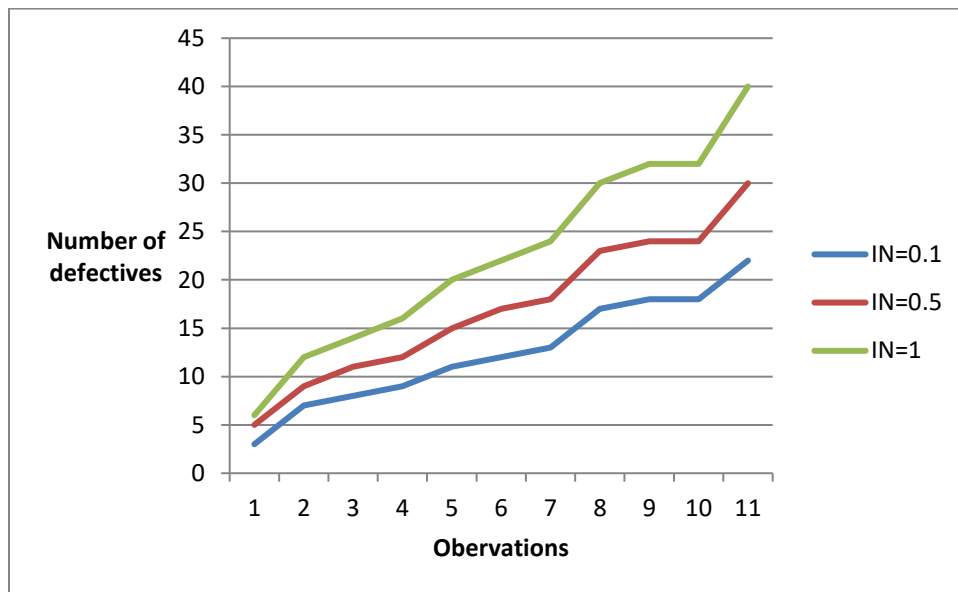


Figure 6: The neutrosophic number of defectives for the simulated data

6. Comparative Study

In this section, we present the results obtained using the proposed regression model with dummy variables and compare them with the regression model with dummy variables under classical statistics. As previously mentioned, the proposed regression model reduces to the classical regression model with dummy variables when there is no uncertainty, i.e., $I_L=0$. The

results for the existing regression with dummy variables are reported in Tables 6-9. These tables show an increasing trend in the predicted values, residual values, and the number of defectives as I_N increases from 0 to other values. For instance, when $I_L=0$, the predicted value from Table 6 is 11.7866, and when $I_N=0.20$, the predicted value is 14.1439. Similarly, the residual value from Table 7 is 4.2134 when $I_L=0$, and 5.0561 when $I_N=0.20$. Additionally, the number of defectives from Table 9 is 20 when $I_L=0$, and 24 when $I_N=0.20$. These trends in predicted values, residuals, and the number of defectives are illustrated in Figures 7-9, which show that the curves for $I_L=0$ are consistently lower than those for $I_N=0.20$.

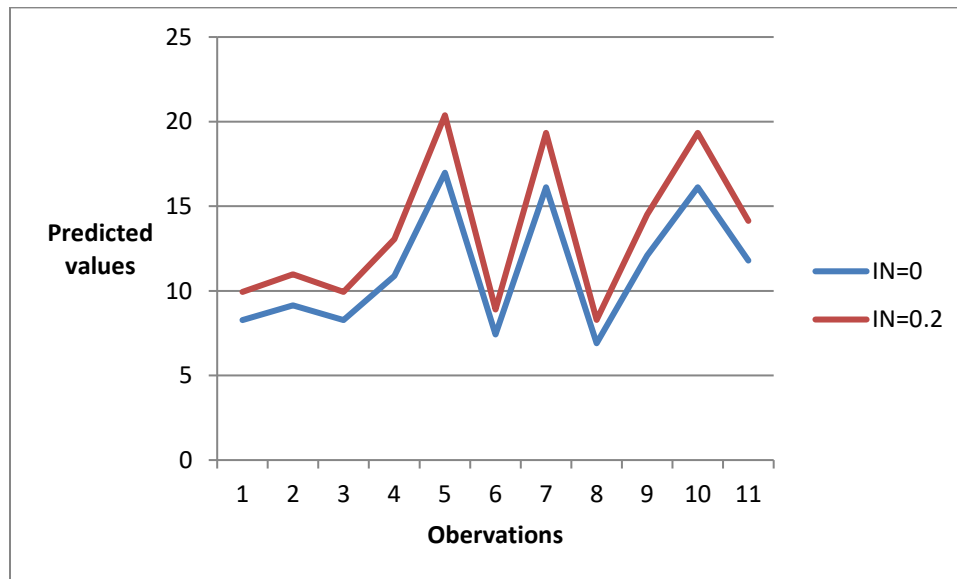


Figure 7: The predicted values from the proposed and predicted values

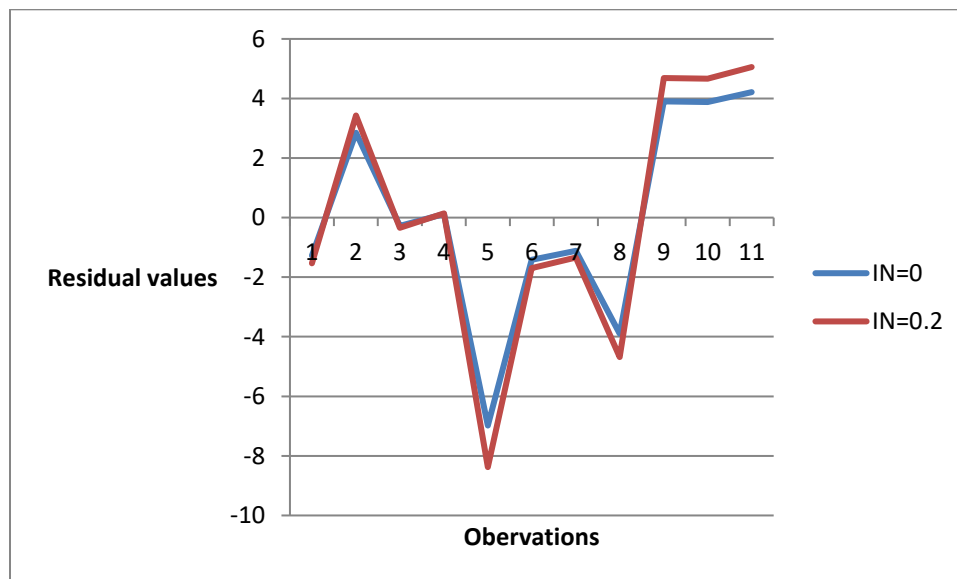


Figure 8: The residual values from the proposed and predicted values

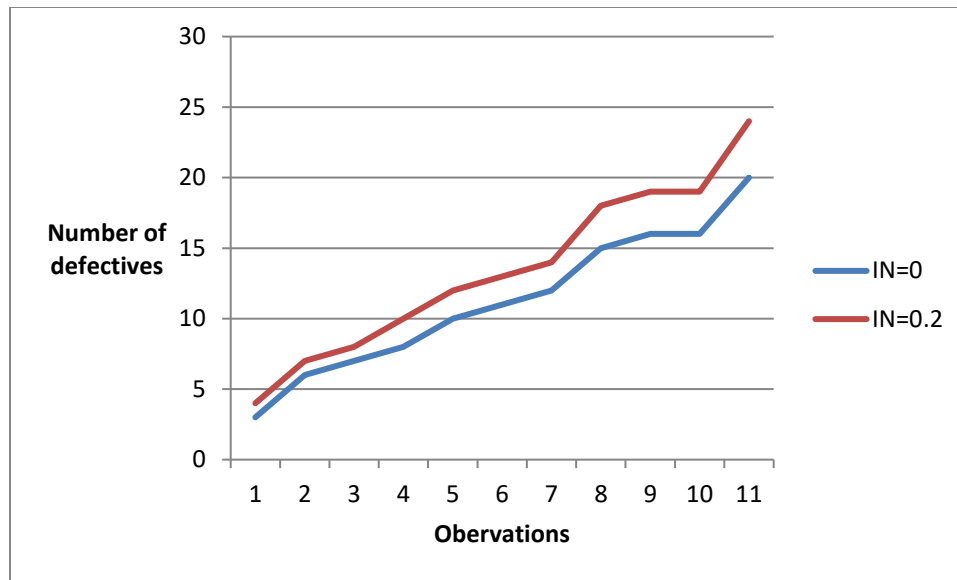


Figure 9: The number of defectives from the proposed and predicted values

7. Concluding Remarks

In this paper, we introduced a regression model using dummy variables within the framework of neutrosophic statistics. This proposed model is designed for regression analysis under conditions of uncertainty, extending the classical regression model with dummy variables. We demonstrated the application of our model through simulation and comparative studies, showing that the results differ from those obtained using classical regression. Our findings indicate that the degree of uncertainty significantly impacts the predicted and residual values. We recommend that decision-makers in fields such as metrology, business, industry, medicine, and education apply this regression model cautiously when dealing with uncertainty. The proposed regression model with dummy variables is suitable for uncertain environments, and future research could explore other regression models using this method.

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