Int. J. Anal. Appl. (2024), 22:117

International Journal of Analysis and Applications

Prediction of Stochastic Transportation Problem with Fixed Charge in Multi-Objective Rough Interval Environment

P. Indira, M. Jayalakshmi*

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore-14, India

**Corresponding author:* m.jayalakshmi@vit.ac.in

ABSTRACT. Many problems appear to be arising in the present as a result of variations in transportation networks. The stochastic fixed-charge transportation problem (SFCTP) is one such problem. The SFCTP is transformed into a chance-constrained programming (CCP) problem where supply and demand are stochastic and objective functions are in a rough interval. In this model, to analyze the multi-objective rough interval stochastic fixed-charge transportation problem (MORISFCTP), where the objective function coefficients are represented by rough intervals and the supply and destination factors are probabilistic constraints. This model operates an expected value operator to deal with uncertainty, in which the coefficient of the objective functions in the fuzzy is changed to a crisp form, and the probabilistic constraints are converted to a deterministic form by the Weibull distribution. To produce the optimal compromise solutions of the proposed model, three distinct methods are used: the fuzzy programming approach, the method of a linear weighted sum, and the €-constraint method. Lastly, the paper delivers a practical illustration of a MORISFCTP to demonstrate the usefulness and feasibility of the suggested methodology.

1. Introduction

The growth and development of a country's economy become quicker through the advancement of corporations, trade, and factories. The enhancement in them, once again, is dependent on a transitory system. As a result, transportation is the strength of a country's budget. As a result, transport system research is a critical component of a country's economic progress.

Received May 22, 2024

2020 Mathematics Subject Classification. 90B06, 90C15, 90C29.

Key words and phrases. rough intervals; multi-objective transportation problem; stochastic fixed-charge transportation problem; Weibull distribution; fuzzy programming; linear weighted sum; €-constraint method.

https://doi.org/10.28924/2291-8639-22-2024-117

ISSN: 2291-8639

Hitchcock gave the first mathematical solution to a transportation problem (TP) in 1941 (Hitchcock 1941). Many researchers followed, including Koopmans (1949), Charnes and Cooper (1954), Roy et al. (2018), Das et al. (2019), Agrawal and Ganesh (2020), Garg and Rizk-Allah (2021), Biswas et al. (2022), Mardanya and Roy (2023), and others.

The *rough set* model, created by Pawlak (1982), has frequently been shown to be an exceptional mathematical device for the study of ambiguous descriptions of things. The complexity of the social and economic context in some authentic transportation situations necessitates the unambiguous examination of many composed functions rather than a particular objective function. The incommensurate and conflicting nature of this problem extends to its complexity. In *multi-objective transportation problems* (MOTP), the thought of the best solution is replaced by the conception of a Pareto optimal solution or non-inferior solution.

The *fixed-charge transportation problem* (FCTP) is an extension of the TP. When the parameters that represent transportation cost assume a positive value, a fixed cost, additionally referred to as set-up cost, is incurred in numerous real-world circumstances. This particular kind of problem is known as FCTP. The MOFCTP is an extension of the classic FCTP that is beneficial when dealing with FCTP with multiple objective functions. As a consequence, the advancing problem would be more complex than in standard FCTP.

One of the most essential distributions in risk analysis, actuarial science, and engineering is the Weibull distribution. It is the distribution that has gained the most attention in recent decades. Numerous studies have been written depicting Weibull distribution applications across multiple sciences. It is frequently employed in systems engineering to analyze the cumulative loss of performance of a complicated system. A *stochastic* framework can be used to deal with uncertain parameters in a TP. However, either a priori predictable periodicity or a posterior frequency distribution is necessary for the stochastic distribution

1.1 Literature Review:

This sector presents a literature analysis of TP and fuzzy TP with stochastic. Singh [1] et al. presented a general construction of the multi-objective solid transportation problem (STP) with Gamma distribution and solved it using a solution practice based on the CCP technique with the uncertainty of the feasibility condition, which then extended the fuzzy programming approach (FPA). Biswas [2] et al. suggested solving a class of non-linear FCTP with multi-objective in classical and interval circumstances where NSGA-II was used to determine the FCTP with multi-objective in crisp and interval environments where NSGA-II was extended with interval order relations. Elsisy and El Sayed [3] created a bi-level multi-objective non-linear programming problem (BMNPP) with a fuzzy objective function and a rough set of constraints to transform BMNPP into two models, such as upper and lower approximation models. To solve such problems, the KKT and two models of the technique of order preference with a resemblance to the ideal solution approach are devised. Brikaa [4] et al. established an efficient multi-objective programming technique in fuzzy for solving constraint matrix games using payoffs of rough value in fuzzy, where a matrix game with fuzzy rough payoffs appears to be composed of a rough-type game value in fuzzy. Roy [5] et al. created a multi-objective, multi-item fixed-charge solid transportation problem (MOMIFCSTP) utilizing fuzzy-rough variables as objective functions and constraint coefficients. To convert fuzzy-rough MOMIFCSTP to deterministic MOMIFCSTP, the expected-value operator is used, and then solving the deterministic MOMIFCSTP using the procedure for order first choice by the alikeness towards the best solution and obtaining a non-dominated solution, distinct three approaches are used: weighted goal programming (WGP), extended TOPSIS, and fuzzy programming. Das [6] et al. provided a type-2 fuzzy parameterized safety-based restricted fixed charge STP that reduces together cost and time in which two models are created, firstly by reducing a type-2 to a type-1 fuzzy set using a critical value (CV)-based reduction mode and using a centroid process to reduce the fuzzy set to a crisp value, and in the next instance, the CV-based reduction method was used to create the CCP model based on generalized credibility, then solved the deterministic equivalent parametric programming problem by weighted mean programming approach and global criteria method using LINGO 13.0.

Jana and Jana [7] suggested random type-2 ambiguous variables with two types: triangular random type-2 and Gaussian random type-2 variables in fuzzy are used in 4D-TP, where the objective function is without random by an operator of the expected value and constraints are used in the CCP method, then the crisp problem is explained by the Generalized Reduced Gradient method using LINGO 14.0. Bera [8] et al. developed fixed-charge 4D-TP used for the breakable item formulated with triangular, Gaussian, and zigzag random type-2 variables in fuzzy, for de-randomized triangular by CV centroid method, Gaussian by CCP, and zigzag by CV method, then the deterministic reduced model solved by Generalized Reduced Gradient method using LINGO 18.0. Nasseri and Bavandi [9] presented a compromise solution for multi-objective TP with multi-choice parameters, where the parameters are interpolating polynomials that are converted into the classical model by CCP and then applied fuzzy programming tactics. Habiba and Quddoos [10] established the Pareto optimal solution for the multi-objective stochastic TP with interval, where the supply and demand are general distributions, which are converted into the classical problem and then solved by fuzzy programming. Agrawal and Ganesh [11] investigated the solution of fuzzy fractional TP with exponential parameters and triangular fuzzy numbers, which converted fuzzy constraints to deterministic ones by CCP and then applied fuzzy programming. Bera and Mondal [12] investigated the credit period policy in rough and bi-rough backgrounds for a two-stage multi-objective TP where independent parameters of transportation cost, requirements, and demand are rough and demand is bi-rough, and then the model was solved by the NAGA-II algorithm. Agrawal and Ganesh [13], the most effective approach to determining a solution for a multi-choice fractional TP is to employ random multi-choice parameters that follow a logistic distribution, with CCP converting the constraints to deterministic performance. Midya et al. [14], a method was developed to find the solution for multiple objective fractional FCTP in a rough environment where the parameters are fuzzy and converted to fuzzy CCP.

Kuiri [15] et al. built a stochastic solid TP and employed it to solve a lagrangian function under the Karush-Kuhn-Tucker conditions involving randomly distributed demands. Garg and Rizk-Allah [16] investigated the rough multi-objective TP solution, then used the advantages of the weighted sum method to identify Pareto optimal solutions and then to find the best compromise solution to multi-objective TP in difficult environments that produced the expected non-dominated value. Khalifa [17] et al. researched a way to use the weighted Tchebycheff approach with a type of stability set and trapezoidal fuzzy number penalties to produce the alpha Pareto solution and alpha compromise solution for the multi-objective, multi-item solid TP. Singh [18] et al. developed another way of CCP by the Essen inequality method to obtain the computational form of the problem by uniform, exponential, and gamma distributions, and this method has been employed to reduce the complexity of TP with random parameters in fuzzy. Kacher and Singh [19] elaborated and summarised the current forms of many different TP types and their efficient advancements in the direction of helping researchers in the future, categorizing which classes of problems should be addressed, and choosing the optimized criteria. Shivani [20] et al. investigated the unbalanced multi-objective FCTP with rough interval parameters, subsequently converting the unbalanced to the balanced multi-objective fixed-charge TP, then employing the three approaches of fuzzy programming, weighted sum method, and goal programming to find the Pareto optimal solution in rough. Devnath [21] et al. invented multiple items in a two-stage fixed-charge 4DTP in which all are fuzzy in nature with breakability, with and without flexible constraints. Converted to deterministic with and without flexible constraints using order relations of fuzzy numbers and modified graded mean integrated value methods, respectively, and then solved via the GRD method. Biswas [22] et al. developed the multi-objective FCTP with multiple items, in which the availabilities are multiple modes but the demands are classical and interval numbers. The NSGA-II and Strength Pareto Evolutionary Algorithm 2 were followed to solve the problem. Haque [23] et al. developed a non-linear STP with a fixed-charge multi-objective in which all parameters are closed intervals. The objective function is minimized within the budgetary constraints, and after being transformed to a crisp by multiple integrations, the solution is based on interval analysis. Buvaneshwari and Anuradha [24] et al. devised a method to deal with stochastic fuzzy transportation problems with mixed probabilistic constraints, in which an objective function is a fuzzy number that is converted into crisp by the alpha cut method and mixed probabilistic constraints are converted into deterministic by CCP. The resulting model is then solved by LINGO software.

Dutta and Kaur [25] developed a model for the multi-choice linear programming problem and solved it using the cubic spline interpolation method, where the parameters of the constraints are multi-choice. Niksirat [26] determined the Pareto optimal solution for the fully fuzzy multi-objective TP by adopting the nearest approximation method with uncertainty conditions. Agrawal [27] et al. implemented the Water Cycle Algorithm to determine the stochastic TP, at which supply and demand are random variables with Weibull distributions, and the stochastic is transformed into deterministic through a stochastic programming approach, and the results are compared with a neural network algorithm. Mardanya and Roy [28] developed Multi-Objective Multi-item STP (MMSTP) with uncertainty, where all the parameters are a trapezoidal fuzzy number that is converted into nearly interval number approximation and derived the updated rule for the converted nearly interval number approximation, then applied interval programming and fuzzy programming to the converted MMSTP and solved by MMSTP in rough variables. Roy and Midya [29] built an optimally compromised solution of the multi-objective FCTP, in which the objective function is a rough interval in random and supply and demand are rough intervals that are converted deterministic by the operator of expected, and then solved the resulting model through the approaches of fuzzy programming, global criteria, and €-constraint methods. Halder Jana [30] et al. established the fixed charge 4D multi-item TP with both space and budget constraints for these types of items in both crisp and rough circumstances where all the parameters are rough intervals except the items of the demand are nature of sustainability and complementary, which is converted to deterministic by expected value as well as lower and upper approximation value, then the resulting model is solved by GRG through LINGO 14.0 software. Midya and Roy [31] adopted fuzzy programming and the method of a linear weighted sum to determine a Pareto-optimal solution of the MOFCTP in rough parameters that are transformed into deterministic using the operator of the expected value.

1.2 Research Gap and Limitation:

In this circumstance, there are several TPs, such as two-dimensional TP, solid TP, 4D-TP, and multi-objective TP, with various types of factors such as budget factor, conveyance factor, etc. But to the best of our knowledge, till now, few researchers have investigated the multi-objective with probabilistic constraints. There are few papers in TP with cost, profit, and fixed charge in a rough environment, but they have not considered the fixed charge rough interval with probabilistic constraints, so a multi-objective investigation with rough intervals with a distribution of probabilistic constraints is proposed. The probability ‘p’ lies between $0<p<1$. In general, uncertainty has been defined using fuzzy or probability theory. Employing fuzzy theory or probability theory to flesh out indeterminacies might not always be appropriate due to a lack of the right information. So, we propose to characterize uncertainty using both fuzzy and probability parameters.

1.3 Managerial implication on research:

Uncontrollable circumstances may result in unidentified cost, availability, and demand quantities. Stochastic TP encompasses the usage of random variables with defined probability distributions to describe problematic parameters. Fuzziness and randomization strategies have the bonus of lacking prior identified regularities and are capable of handling imprecise input information, especially feelings, and emotions quantified based on the DM's subjective evaluations. Over the past decade, there has been growth in several industries, including transportation, economics, health care, agriculture, trade, army, engineering, and technology.

MORISFCTP, a newly created model with a rough interval multi-objective function and probabilistic constraints, is tackled in the present article. The expected value of the distribution operator, such as uniform, exponential, or gamma distributions, is used to convert the rough interval to classical in objective functions. The Weibull distribution is implemented to convert probabilistic constraints to deterministic constraints, and the resulting model is analyzed using three different methods: fuzzy programming, linear weighted sum, and €-constraints using the LINGO software. Sensitivity analysis is performed by considering various demand limitations and constant supply with a probability distribution. The superior complexity of the current investigation looks at a fuzzy TP with a rough interval in a stochastic environment, focusing on multi-objective transportation costs. Effective decision-making in complicated corporate situations is unable to depend on just one criterion. As a result, we must comprehend the presence of multiple factors that can help with multi-criteria decision-making. Some authors are researchers for multi-objectives with all the parameters being rough intervals and solving the TP but in this proposed multi with rough and probabilistic constraints.

Following is the outline of the paper. Section 2 explores the essentials of rough and random variables. Section 3 demonstrates the various distribution functions with an operator on the expected value of a rough and a random variable. The mathematical model of MORISFCTP is developed in Section 4. Section 5 discusses the recommended model's solution strategy. Section 6 contains a real-world example of an uncertain MORIFCSTP. Section 7 includes a discussion of the results and a comparison of the best solutions found using the three distinct methods. The proposed model (MORISFCTP) of sensitivity analysis is laid out in Section 8. Finally, Section 9 contains the conclusions.

1.4 Motivation and main contributions:

Many researchers have studied FCTPs in uncertain environments such as fuzzy, stochastic, and so on, but there are few research papers about TPs in rough backgrounds. Still, to our knowledge, no one has investigated MORISFCTP, with parameters (transport cost and fixed costs, traveling time, deterioration cost of items) being rough variables and supply and demand being probabilistic (Weibull distribution). The problem arises when the DM lacks particular data and the quantities of MOFCTP have predictable values. Because of this, the probable region of MOFCTP is not stable (i.e., variable). In suitable circumstances, introduce a rough interval and stochasticity in MOFCTP when the feasible region of MORISFCTP is more adaptable. Thus, MORISFCTP consumes an accurate occasion, which is the enthusiasm of developed research. In Table 1, we give an organized overview of some recent articles related to TP, stochastic TP, and STP with and without roughness.

The following are the main contributors:

- The MORISFCTP model is established.
- The MORIFCSTP is transformed into a classical form by an operator of expected-valued and Weibull distributions.
- The deterministic MORIFCSTP is explained by fuzzy programming and the method of a linear weighted-sum and €-constraint method.
- The comparison is made between the solutions of three methods for MORISFCTP.
- A sensitivity analysis of MORIFCSTP is presented.
- The case study problem is displayed using the recommended MORIFCSTP model, which is discussed.

Table 1: Some recent studies on transportation problem variants under various conditions

References (year)	No. of Objectives	Kind of TP	Fixed charge	Environment	Type of distribution	Programming Approach
Singh [1] et al. (2019)	Multi	Solid TP	No	Crisp	Gamma distribution	Fuzzy programming approach
Agrawal and Ganesh [13] (2020)	Single	Fractional TP	No	Fuzzy	Logistic distribution	Newton’s divided difference interpolating method.
Midya [14] et al. (2020)	Multi	Fractional TP	Yes	Rough	-	Rough approximation technique
Singh et al. [18] (2021)	Single	TP	No	Fuzzy	Uniform, exponential, and gamma distribution	Essen inequality approach
Buvaneshwari and Anuradha [24] (2022)	Single	Stochastic TP	No	Fuzzy	Weibull distribution	Three models of stochastic fuzzy TP with mixed constraints were solved.
Agrawal et al. [27] (2022)	Single	Stochastic TP	No	Crisp	Weibull distribution	Water cycle algorithm and compared to the neural network algorithm.
Shivani et al. [20] (2022)	Multi	TP	Yes	Rough	-	Fuzzy programming, goal programming, weighted sum method
Biswas et al. [22] (2022)	Multi	TP	Yes	Crisp and interval	-	Non-dominated soring genetic algorithm II and Strength pareto evolutionary algorithm
Proposed article	Multi	Stochastic TP	Yes	Rough	Weibull distribution	Fuzzy Programming, Weighted Sum method, €-constraint method by LINGO.

2. Preliminaries

This section provides rough space, rough intervals, and arithmetic operations on rough. In addition, we give some important definitions and theorems concerning a rough variable.

Definition: [29] Let $(\Lambda, \Delta, A, \pi)$ be is referred to as the rough space, where $\wedge$ as a set of non-empty, A is a σ-algebra of the subset $\wedge$, $\triangle$ is an element in A, and π is the real-valued function.

Definition: [4] The standard value S^R is called a rough interval (RI) when two closed intervals can be assigned S^L and S^U on a real number set R to it, where $S^L \subseteq S^R$. Furthermore,

$$(i) if\ t \in S^L\ then\ S^R\ surely\ take\ t\ (denoted\ by\ t \in S^R)$$

$$(ii) if\ t \in S^U\ then\ S^R\ possibly\ take\ t$$

$$(iii) if\ t \notin S^L\ then\ S^R\ does\ not\ surely\ take\ t\ (denoted\ by\ t \notin S^R)$$

Where $S^L and\ S^U$are denoted by the lower and upper approximation intervals S^Rrespectively. Supplementary S^R is denoted by $S^R = (S^L : S^U)$.

Definition: [31] Rough interval (RI) arithmetic procedures are identical to crisp interval arithmetic operations. For any two numbers $\delta^R = [\delta^{LL}, \delta^{LU}], [\delta^{UL}, \delta^{UU}]$ & $\rho^R = [\rho^{LL}, \rho^{LU}], [\rho^{UL}, \rho^{UU}]$ of rough intervals when $\delta^R \geq 0$, $\rho^R \geq 0$ and $* \in (\pm,\ \times,\ \div)$is an operation on a binary set of rough intervals. Then, we have:

Addition: $\delta^R * \rho^R = [\delta^{LL} * \rho^{LU}], [\delta^{LU} * \rho^{UU}]$ other arithmetic operations on rough intervals, such as subtraction and multiplication, are defined identically to addition.

Division: $\delta^R \div \rho^R = \{[\delta^{LL} \div \rho^{LU}], [\delta^{LU} \div \rho^{UU}]\}, \{[\delta^{UL} \div \rho^{UL}], [\delta^{UL} \div \rho^{UU}]$

If c is a scalar, then

$$c * \delta^R = \begin{cases} ([c\delta^{LL}, c\delta^{LU}], [c\delta^{UL}, c\delta^{UU}]), if c \geq 0 \\ ([c\delta^{LU}, c\delta^{LL}], [c\delta^{UU}, c\delta^{UL}]), if c < 0 \end{cases}$$

Where * denotes the scalar product of the rough interval.

Trust Measure: [12] The trust in rough set theory measures uncertainty. Trust is a measurable function from a rough space $(\wedge, \Delta, A, \pi)$ to [0, 1], where $\wedge$ as a non-empty set, Δ is a -algebra of the subsets of $\wedge$, A be an element of Δ, and π be a non-negative real-valued function represented by "Tr".

Definition: [29] Let $([p, q], [r, s])$ be a rough variable such that $r \leq p < q \leq s$ and it can be written as $Tr(\zeta \leq 0) = \begin{cases} 0, & if\ r \geq 0 \\ \frac{r}{2(r-s)} & if\ p \geq 0 \geq r \\ \frac{2pr-ps-qr}{2(q-p)(s-r)} & if\ q \geq 0 \geq p \\ \frac{s-2r}{2(s-r)} & if\ 0 \geq s \end{cases}$

The rough variable function "Tr" graphs are displayed in Fig 1.

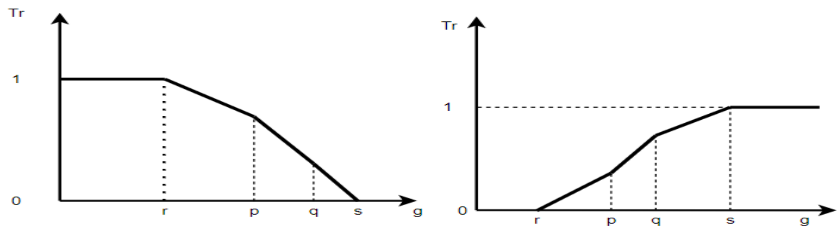

Fig.1 displays the two trust functions $Tr(\varepsilon \geq g)$ & $Tr(\varepsilon \leq g)$ diagram respectively.

3. Expected value on the rough interval:

An operator of expected value in a rough interval depends upon the parameters of the rough interval, which are identical to the probability model of an operator of expected.

Definition: [31] Let ε rough variable on the rough space $(\wedge, \Delta, A, \pi)$. The expected value of ε is defined as $E(\varepsilon) = \int_0^\infty Tr(\varepsilon \geq g)dg - \int_{-\infty}^0 Tr(\varepsilon \leq g)dg$ and given that at least one of the integrals occurs, where E is denoted by the operator of the expected value and "Tr" is denoted by the trust measure.

Definition: [29] Suppose that Ɛ is a random rough variable defined on the rough space $(\wedge, \Delta, A, \pi)$. The following represents the way its expected value is described:

$$E(\varepsilon) = \int_0^\infty Tr\{\eta \in \wedge : E[\epsilon(\eta)] \geq g\}dg - \int_{-\infty}^0 Tr\,\{\eta \in \wedge : E[\epsilon(\eta)] \leq g\}dg$$

Theorem 3.1: [31] Suppose that $\mathcal{E} = ([p,q],[r,s])$ is a rough interval then the expected value of Ɛ is defined by $E(\varepsilon) = \frac{1}{2}[\vartheta(p+q) + (1-\vartheta)(r+s)]\ where\ 0 < \vartheta < 1,$ the decision-maker-determined parameter.

Theorem 3.2: [31] Suppose that $\delta^R = [\delta^{LL}, \delta^{LU}], [\delta^{UL}, \delta^{UU}]$ & $\rho^R = [\rho^{LL}, \rho^{LU}], [\rho^{UL}, \rho^{UU}]$ rough intervals of expected values are finite values. Then, for any real number of g & h, we have $E[f\delta + g\rho] = fE[\delta] + gE[\rho]$.

3.1 The expected value of a random rough variable:

Definition: [29] Suppose that ε is a random rough variable that is defined in $(\wedge, \Delta, A, \pi)$ and $\varepsilon(\mu)$ is a random variable of the continuous distribution for any $\mu \epsilon \wedge$, and if it's the expected value of the rough variable is defined by $E[\varepsilon(\mu)] = \{[p,q],[r,s]\}, r \leq p < q \leq s,$ then ε called the continuous random rough variable.

Definition: [29] Suppose that ε is a random rough variable that is defined in $(\wedge, \Delta, A, \pi)$ and $\varepsilon(\mu)$ is a random variable, then the function $f(x, \varepsilon)$ is called the density function of $\varepsilon(\mu)$ is defined as follows: $\int_{x\in\psi} xf(x) = \{[p,q],[r,s]\}$where $p, q, r,$ and s are real number and finite, ψ is a specified region.

Definition: [29] The expected value of ε is defined as follows:

$$E(\varepsilon) = \int_0^{\infty} Tr\left[\int_{x\in\psi} xf(x)dx \geq r\right]dr - \int_{-\infty}^{0} Tr\left[\int_{x\in\psi}^{0} xf(x)dx \leq r\right]dr$$

Where ε is a random rough variable and its density function $f(x)$ and ψ is a specified region.

Theorem 3.3: [29] Suppose that $\zeta(x)$ is a uniform function with rough a random variable, which is provided below:

$$\zeta(x) = \begin{cases} \frac{1}{\theta_2-\theta_1}, & if\ \theta_1 \leq x \leq \theta_2 \\ 0, & otherwise \end{cases}$$

where $\theta_1 = \{[p_1,q_1],[r_1,s_1]\}, 0 < r_1 \leq p_1 < q_1 \leq s_1$ & $\theta_2 = \{[p_2,q_2],[r_2,s_2]\}, 0 < r_2 \leq p_2 < q_2 \leq s_2$ be rough variable parameters and $\theta_2 > \theta_1$, then the expected value can be determined by

$$E(\zeta) = \frac{1}{8}\left[[p_1+p_2]+[q_1+q_2]+[r_1+r_2]+[s_1+s_2]\right].$$

Theorem 3.4: [29] Suppose that $\zeta(x)$ is an exponential function with rough a random variable, which is provided below:

$$\zeta(x) = \begin{cases} \lambda e^{-\lambda x}, & if\quad x \geq 0 \\ 0, & otherwise \end{cases}$$

where $\lambda = \{[p,q],[r,s]\}, 0 < r \leq p < q \leq s$ is a random rough variable parameter, then the expected value can be determined by $E(\zeta) = \frac{1}{4}\left[\frac{1}{p}+\frac{1}{q}+\frac{1}{r}+\frac{1}{s}\right]$.

Theorem 3.5: [29] Suppose that $\zeta(x)$ is a gamma function with a rough random variable, which is provided below:

$$\zeta(x) = \begin{cases} \frac{\beta^{\alpha}x^{\alpha-1}e^{-\beta x}}{\Gamma\alpha}, & if\quad x \geq 0 \\ 0, & otherwise \end{cases}$$

where $\alpha = \{[p_1,q_1],[r_1,s_1]\}, 0 < r_1 \leq p_1 < q_1 \leq s_1$ & $\beta = \{[p_2,q_2],[r_2,s_2]\}, 0 < r_2 \leq p_2 < q_2 \leq s_2$ be a rough variable parameter, then the expected value can be determined by

$$E(\zeta) = \frac{1}{4}\left[\frac{p_1}{q_2}+\frac{q_1}{p_2}+\frac{r_1}{s_2}+\frac{s_1}{r_2}\right].$$

3.2 Conversion of stochastic supply and demand constraints:

This section covers the methods used to determine the optimal solution for the stochastic fixed-charge transportation problem. The problem has probabilistic constraints that must be converted into deterministic constraints.

Theorem 3.6: [27] Assume that $\omega_u, u = 1,2, \ldots m$ are independent Weibull distributions with random parameters φ_u, θ_u *and* γ_u for position, size, and shape. The probabilistic constraint (4.2) is subsequently transformed into deterministic behaviour as follows:

$$\sum_{u=1}^{m} x_{uv} \leq \sum_{v=1}^{n} \varphi_u + \theta_u[-log(\rho_u)]^{\frac{1}{\gamma_u}}, u = 1,2, \ldots m$$

Theorem 3.7: [27] Assume that $\varpi_v, v = 1,2, \ldots n$ are independent Weibull distributions with random parameters $\overline{\varphi_v}, \overline{\theta_v}$ *and* $\overline{\gamma_v}$for position, size, and shape. The probabilistic constraint (4.3) is subsequently transformed into probabilistic behavior as follows:

$$\sum_{v=1}^{n} x_{uv} \geq \sum_{u=1}^{m} \bar{\varphi}_v + \bar{\theta}_v[-log(1-\bar{\rho}_v)]^{\frac{1}{\bar{\gamma}_v}}, v = 1,2, \ldots n$$

Theorem 3.8: [31] Suppose x_{uv}^* for $u = 1,2, \ldots m$ & $v = 1,2, \ldots n$ is called the non-dominated (Pareto-optimal) solution of Model 2 if it satisfies the following conditions:

$$(i)\ Z_m(x) \leq Z_m(x^*), for\ m = 1,2,3$$
$$(ii)\ Z_m(x) < Z_m(x^*), for\ atleast\ one\ m$$

4. Mathematical model for MORISFCTP

Consider three objective functions: the first objective denotes the transportation cost with a fixed charge for every place of origin; the second objective represents the storage time of the products; and the third objective is the deterioration cost of goods. If there are 'm' sources ($u = 1,2, \ldots, m$, the product travels to 'n' destinations ($v = 1,2, \ldots, n$). The aim is to obtain the unknown capacity x_{uv} (decision variable) that is delivered from the u^{th} origin to v^{th} destination while minimizing the three objective function values. Assume that MORISFCTP parameters are treated as rough intervals in the objective functions and that supply and demand restrictions are modeled using the Weibull distribution. A mathematical form of the recommended method is described as follows:

Model 1:

$$Min\ \tilde{Z}_1 = \sum_u (\tilde{s}_{uv} x_{uv} + \tilde{\eta}_{uv} y_{uv})$$
$$Min\ \tilde{Z}_2 = max(\tilde{t}_{uv}: x_{uv} > 0, \forall u, v) \tag{4.1}$$
$$Min\ \tilde{Z}_3 = \sum_u \sum_v \tilde{d}_{uv} x_{uv}$$

Subject to the constraints,

$$P[\sum_v x_{uv} \leq \omega_u] \geq 1 - \rho_u, u = 1,2, \ldots, m \tag{4.2}$$
$$P[\sum_u x_{uv} \geq \varpi_v] \geq 1 - \bar{\delta}_v, v = 1,2, \ldots, n \tag{4.3}$$
$$x_{uv} \geq 0, u = 1,2, \ldots, m\ \&\ v = 1,2, \ldots, n$$

Model1is feasible if and only if

$$\sum_u \omega_u \geq \sum_v \varpi_v$$

The objective functions are denoted by equation (4.1), while the probabilistic constraints of supply and destination are denoted by equations (4.2) and (4.3), respectively.

4.1 Deterministic Model for MORISFCTP:

The presence of a random rough variable, the suggested MORISFCTP idea, and the coefficient of the objective functions are random rough variables and probabilistic restrictions. The MORISFCTP could not be solved directly through random rough variables. Then, the value of the expected operator E is utilized in the objective function with probabilistic constraints for transforming Model 1 into Model 2 (classical mode) by employing in the objective function random rough variables and an operator of the expected rough along with Theorems 3.2, 3.3, 3.4, and 3.5 in Model 1. In Model 1, Theorems 3.6 and 3.7 transform the probabilistic constraints into deterministic constraints. The crisp MORISFCTP model is apparent below.

Model 2:

$$Min\ E[\tilde{Z}_1] = \sum_u E(\tilde{s}_{uv} x_{uv} + \tilde{\eta}_{uv} y_{uv})$$

$$\Leftrightarrow \sum_u (E(\tilde{s}_{uv}) x_{uv} + E(\tilde{\eta}_{uv}) y_{uv}) \tag{4.4}$$

$$Min\ E[\tilde{Z}_2] = max(E(\tilde{t}_{uv}): x_{uv} > 0,\ \forall u, v) \tag{4.5}$$

$$Min\ E[\tilde{Z}_3] = \sum_u \sum_v E(\tilde{d}_{uv}) x_{uv} \tag{4.6}$$

Subject to the constraints

$$\sum_u x_{uv} \leq \varphi_u + \theta_u [-log(\omega_u)]^{\frac{1}{\gamma_u}}, u = 1,2, \ldots m \tag{4.7}$$

$$\sum_v x_{uv} \geq \bar{\varphi}_v + \bar{\theta}_v [-log(1 - \varpi_v)]^{\frac{1}{\bar{\gamma}_v}}, v = 1,2, \ldots n \tag{4.8}$$

$$x_{uv} \geq 0, u = 1,2, \ldots, m\ \&\ v = 1,2, \ldots, n \tag{4.9}$$

$$y_{uv} = 0\ if\ x_{uv} = 0 \tag{4.10}$$

$$y_{uv} = 0\ if\ x_{uv} > 0 \tag{4.11}$$

Model 2 is said to be feasible if it satisfies the condition,

$$\sum_u \varphi_u + \theta_u [-log(\omega_u)]^{\frac{1}{\gamma_u}} \geq \sum_v \bar{\varphi}_v + \bar{\theta}_v [-log(1 - \varpi_v)]^{\frac{1}{\bar{\gamma}_v}}$$

Notations of MORISFCTP:

The suggested method using the notation is given below:

m number of origins.

n number of destinations.

x_{uv} represent the transport cost of the product per unit quantity from u^{th} origin to v^{th}destination.

$\tilde{s}_{uv}$ represent the transportation cost of the product per unit quantity from u^{th}origin to v^{th}destination

$\tilde{\eta}_{uv}$ represent a fixed cost from the u^{th}origin tov^{th}destination

$\tilde{t}_{uv}$ represent the rough transportation time from the random u^{th}origins of the product to a v^{th} destination, which is independent of the amount of commodity transported

$\tilde{d}_{uv}$ represent the deterioration cost with the random rough variable of the product from u^{th}origin tov^{th}destination

y_{uv} Binary variable taking the value '1' if the source $'i'$ is used, otherwise '0'

ω_u Weibull distribution of u^{th} source supply location

ϖ_v Demand location Weibull distribution for v^{th} destination

$\tilde{Z}_m$ Rough nature of objective functions, m = 1, 2, 3.

Z_m The classical nature of the objective functions (m = 1, 2, 3), where $Z_m = E[Z_m]$an 'E' is a value of an operator of the expected.

Assume, lacking the foregoing generality, the coefficient of random rough variables occurs in the suggested idea, with the probability distributions listed below.

- Cost of transportation ($\tilde{s}_{uv}$) and fixed-charge ($\tilde{\eta}_{uv}$) follows "Uniform distribution" with parameter $U(\chi_{uv}^1, \chi_{uv}^2)$, that is $\tilde{s}_{uv} \sim U(\chi_{uv}^1, \chi_{uv}^2)$ & $\tilde{\eta}_{uv} \sim U(\chi_{uv}^1, \chi_{uv}^2)$.
- Delivering time of transportation ($\tilde{t}_{uv}$) follows "Exponential distribution" with parameter μ_{uv} that is $\tilde{t}_{uv} \sim E(\mu_{uv})$.
- The deterioration cost of transportation $\left(\tilde{d}_{uv}\right)$ follows "Gamma distribution" with a parameter $\gamma(\vartheta, \phi)$ that is $\tilde{d}_{uv} \sim \gamma(\vartheta, \phi)$.

5. Solution approaches:

In multi-objectives, solution one is the best, and the others are the worst, due to a combination of incompatibility and conflict of the objective functions. As a result, in multi-objective circumstances, there is frequently a collection solution that can't be identified effectively compared to other objective functions. This part will go via the following methods of using and solving deterministic MORISFCTP, which are listed below:

- Fuzzy programming approach
- Method of a linear weighted sum
- €-constraint method

5.1 Fuzzy programming approach

In optimization, the multi-objective problem is solved through fuzzy programming. The fuzzy programming process gets started by determining the lower and upper bound of L^m and U^m for the m^{th}objective function $Z_m(m = 1,2,3)$ respectively, which $L^m$and U^mdenotes the desired levels of achievement and the greatest level of achievement for m^{th}objective function respectively, which is acceptable and $d_m = U^m - L^m$denotes the degradation budget of m^{th}objective function. After determining the determination level and degradation allowance for each required function, a model in fuzzy is generated and translated into a classical model. The following steps can be used to reach the MORISFCTP solution.

Stage 1: Solve the MORISFCTP by focusing on a distinct objective function at a time, while disregarding others. For the objective functions, repeat this method three times.

Stage 2: Using the outcomes of Stage 1, calculate the appropriate values for every objective function at each generated solution in Stage 2.

Step 3: Using the results of Stage 2, determine the best (L^m) and worst (U^m) values of every objective function that corresponds to established solutions. Beyond that, the first model in fuzzy can be expressed in terms of the desire levels of every objective function, as shown in the following description.

Obtain $x_{uv}\,(u = 1,2,\ldots,m, v = 1,2,\ldots,n)$, which meets $Z_m \leq L^m$ with m = 1, 2, 3 and if provided restrictions and required requirements. $\mu_m(x)$ is a membership function for MORISFCTP that corresponds to the m^{th} objective function and is expressed as below:

$$\mu_m(x) = \begin{cases} 1 \quad , & if\ Z_m \leq L^m \\ \dfrac{U^m - Z_m}{U^m - L^m}, & if\ L^m \leq Z_m \leq U^m \\ 0 \quad , & if\ Z_m \geq U^m \end{cases}$$

The following is a proposed formulation of the MORISFCTP equivalent linear programming problem.

Model 3:

$Max\ \lambda$

Subject to

$\lambda \leq \frac{U^m - Z_m}{U^m - L^m}$ $(m = 1,2,3)$

with constraints (4.7) to (4.11) and $\lambda \geq 0$ where $\lambda = min(\mu_m(x))$

The above linear programming problem is simplified, we get

$Max\ \lambda$

Subject to

$Z_m + \lambda(U^m - L^m) \leq\ U^m$ $(m = 1,2,3)$

Constraints (4.7) to (4.11) and $\lambda \geq 0$

5.2 Method of a linear weighted sum:

The weighted sum of the linear approach is used to reduce the multiple to the single objective optimization problem by pre-multiplying a specific weight for every objective function and combining multiple objective functions. We utilize weight $W_m(m = 1,2,3)$ for each objective function $Z_m(m = 1,2,3)$ which W_m symbolizes the corresponding weight of the objective function, which is compared to other objective functions. In further verses, we can realize the weight as indicative of our specialists over target functions. The greater weight of W_m is a high level of significance and the lower weight of W_m is a low level of significance in the target function Z_m. Utilizing the weight concept to transform multiple into a single objective function is described as $\sum_{m=1}^{3} W_m Z_m$ with $\sum_{m=1}^{3} W_m = 1$. Because of this aspect, this strategy is known as the weighted sum with a linear approach. The weighted sum with linear tackle (Athan and Papalamberos, 1996) can be summed up as below:

Stage 1: Initially, choose the coefficients of weights $W_1, W_{2,} and\ W_3$corresponding to the functions of objectives ($Z_{m,}\ m = 1,2,3$) by the pertinent nature to the target functions in Model 2. It is essential to $W_m > 0, m = 1,2,3,$ and $\sum_{m=1}^{3} W_m = 1$.

Stage 2: Solve the resultant objective problem, while all the objective functions occur in a weighted sum. Model 2 is a single objective problem represented by the following format:

Model 4:

$Minimize = \sum_{m=1}^{3} W_m E(Z_m(x_{ij}))$

Subject to constraints (4.7) to (4.11)

Theorem 3.9: Assume $W_m > 0$ & $m = 1,2,3$ If x* is a non-dominated solution of model 2, it is an optimal solution of model 4.

Proof: If x^* is a non-dominated solution of model 2, then we get from Theorem 3.8

$E(\tilde{Z}_m(y) \leq E(\tilde{Z}_m(y^*),$ for $m = 1,2,3$ and

$E(\tilde{Z}_m(y) < E(\tilde{Z}_m(y^*)$, for atleast one m

As $W_m > 0,\ m = 1,2,3$ we can generate the following inequalities from the preceding.

$$W_m E(\tilde{Z}_m(y)) \leq W_m E(\tilde{Z}_m(y^*), \text{ for } m = 1,2,3 \text{ and} \quad \text{(a)}$$

$$W_m E(\tilde{Z}_m(y)) < W_m E(\tilde{Z}_m(y^*), \text{ for atleast one } m \quad \text{(b)}$$

We can describe the sum of inequalities (a) and (b) as

$$Min \sum_{m=1}^{3} W_m E\left(\tilde{Z}_m(y)\right) < Min \sum_{m=1}^{3} W_m E\left(\tilde{Z}_m(y^*)\right)$$

It contradicts the assumption that $x^*$is the ideal answer to model 4. As an illustration, x^* is a non-dominated solution for model 2. Hence the theorem.

5.3 €-constraint method:

Another tactic for solving the multi-objective optimization problem is the €-constrained method. To produce Pareto-optimal solutions, this approach was put forth by Roy and Midya [31]. A problem with multiple objectives is reduced to a single objective using this technique. As a result, separate Pareto-fronts corresponding to every objective function are produced by optimizing one of the objective functions while implementing the other objective functions as constraints.

$$Minimize\ [Z_1(x), Z_2(x), Z_3(x)]$$

Subject to the constraints (4.7) to (4.11).

The above problem solution is to obtain the following stages:

Stage 1: Only one objective function, $Z_{m_0}, m_0 = 1,2,3$ (let's assume) is chosen to be minimized, while the remaining functions are transformed into constraints. The model that comes out is the one below.

$$Minimize\ Z_{m_0}(x)$$

Subject to

$$Z_{m_0}(x) \leq \varepsilon_m,\ m = 1,2,3 \text{ where } m \neq m_0$$

Constraints (4.7) to (4.11)

where $\varepsilon_m, m = 1,2,3$ denote the upper value of m^{th} objective function.

Stage 2: Determine the values for the remained objective functions based on the findings of Stage 1.

Stage 3: Vary the values of 'm' along the Pareto-front for each objective function to obtain a subset of the Pareto-optimal set. For each new value of 'm' ($m = 1, 2, 3$), create a new optimization problem.

Stage 4: Determine the required optimal solution for Model 4 commencing the set of Pareto-optimal options in the above stages.

6. Numerical Illustration:

For petroleum products like gasoline, kerosene, liquefied petroleum gas, diesel, etc., a petroleum refinery processes crude oil. Three oil refineries and four depots owned by the firm are located throughout India. The corporation uses tankers and railroads to transfer refined oil from refineries to depots. The decision-maker wants to minimize the overall transporting cost (variable and fixed cost per unit), the amount of product deterioration, and the transit duration of the product. The transportation cost is given in dollars per barrel, along with a fixed charge also in dollars for an open route, the rate of deterioration is expressed in liters and the duration time is expressed in hours. To meet the overall demand, the decision-maker must determine how many barrels of petroleum product must be carried from u^{th}refineries (storage) to v^{th}depots.

The rough random data of transportation cost, fixed charge, transporting time, and rate of product deterioration are presented in Tables 2, 3, 4, and 5, accordingly. The estimated values for the availability and demand characteristics are shown in Table 6. Moreover, Tables 7 and 8 describe the values expected for the transportation cost and fixed charge in rough intervals, as well as the duration of the transport and the rate of product deterioration, respectively. S_u and D_v indicate corresponding to the u^{th} refinery and v^{th}depot, respectively, in Tables 2-5 and Tables 6. For a better comprehension of the suggested methodology, a numerical example is taken where supply and demand follow the Weibull distribution. The major goal is to reduce the overall transportation expense for petroleum products' availability and demand.

Table 2: Random rough interval of transportation cost $\tilde{s}_{uv} \sim U(\chi^1_{uv}, \chi^2_{uv})$

	$D1$	$D2$	$D3$	$D4$
S1	([2, 5], [1, 6]), ([3, 6], [2, 7])	([4, 7], [3, 8]), ([5, 8], [4, 9])	([6, 9], [5, 10]) ([7, 10], [6, 11])	([8, 11], [7, 12]), ([9,12],[8, 13])
S2	([10, 13], [9, 14]), ([11, 14], [10, 15])	([10, 15],[9, 16]), ([12, 15],[11, 16])	([3, 6], [2, 8]), ([4, 8], [3, 10])	([5, 9], [3, 11]), ([7, 11], [5, 14])
S2	([6, 9], [4, 12]), ([5, 10], [4, 14])	([2, 7], [1, 9]), ([8, 12], [6, 14])	([9, 12],[7, 14]) ([10, 14], [8, 16])	([11, 13], [9, 15]), ([12, 14], [10, 16])

Table 3: The random rough interval of the fixed charge $\tilde{\eta}_{uv} \sim U(\chi^1_{uv}, \chi^2_{uv})$

	$D1$	$D2$	$D3$	$D4$
S1	([22, 24], [20, 26]),	([22, 26], [20, 28]),	([31, 32], [30, 34])	([20, 22], [19, 23]),
	([23, 26], [21, 27])	([24, 28], [22, 30])	([31, 33], [30, 35])	([25, 26], [24, 28])
S2	([20, 23], [19, 24]),	([28, 29],[26, 30]),	([21, 22], [19, 23]),	([24, 26], [21, 29]),
	([27, 29], [26, 30])	([31, 33],[30, 37])	([32, 35], [30, 36])	([28, 32], [27, 33])
S3	([25, 27], [23, 29]),	([21, 25], [19, 27]),	([29, 33],[28, 34])	([27, 31], [25, 32]),
	([31, 33], [29, 37])	([23, 29], [21, 30])	([32, 34], [30, 36])	([31, 35], [30, 37])

Table 4: The random rough interval of the transportation time $\tilde{t}_{uv} \sim E(\mu_{uv})$

	$D1$	$D2$	$D3$	$D4$
S1	([.33, .40], [.30, .45])	([.25, .35],[.21, .39])	([.30, .40], [.28, .50])	([.26, .36], [.24, .48])
S2	([.18, .28], [.15, .32])	([.21, .31],[.18, .41])	([.22, .34], [.20, .36])	([.27, .37], [.25, .45])
S3	([.14, .24], [.12, .32])	([.34, .44], [.32, .48])	([.30, .35],[.28, .39])	([.35, .45], [.30, .50])

Table 5: The random rough interval of the deterioration ratio of goods $\tilde{d}_{uv} \sim \gamma(\vartheta, \phi)$

	$D1$	$D2$	$D3$	$D4$
S1	([.75, .85], [.72, 92]),	([.84, .88], [.80, .94]),	([.86, .94], [.84, .98])	([.84, .92], [.80, .96]),
	([.55, .65], [.52, .82])	([.56, .62], [.54, .72])	([.62, .68], [.58, .74])	([.52, .60], [.50, .66])
S2	([.78, .88], [.74, .94]),	([.77, .87],[.73, .93]),	([.84, .94], [.80, 1]),	([.81, .91], [.79, .97]),
	([.56, .66], [.52, .70])	([.57, .67],[.53, .73])	([.60, .68], [.50, .70])	([.59, .71], [.53, .77])
S3	([.90, .96], [.88, 1]),	([.84, .94], [.82, .98]),	([.86, .96],[.83, .97])	([.88, .92], [.86, .98]),
	([.54, .66], [.52, .72])	([.62, .72], [.60, .74])	([.52, .67], [.50, .70])	([.54, .62], [.52, .68])

Table 6: Data for the Weibull distribution of supply and destination locations

Parameters	Supply location			Destination locations			
Location	$\varphi_1 = 550$	$\varphi_2 = 650$	$\varphi_3 = 750$	$\bar{\varphi}_1 = 325$	$\bar{\varphi}_2 = 425$	$\bar{\varphi}_3 = 525$	$\bar{\varphi}_4 = 325$
Scale	$\theta_1 = 5.8$	$\theta_2 = 6.4$	$\theta_3 = 7.2$	$\bar{\theta}_1 = 6.2$	$\bar{\theta}_2 = 6.9$	$\bar{\theta}_3 = 7.3$	$\bar{\theta}_4 = 7.8$
Shape	$\gamma_1 = 650$	$\gamma_2 = 750$	$\gamma_3 = 850$	$\bar{\gamma}_1 = 725$	$\bar{\gamma}_2 = 825$	$\bar{\gamma}_3 = 925$	$\bar{\gamma}_4 = 1025$
Probability	$\omega_1 = 0.911$	$\omega_2 = 0.921$	$\omega_3 = 0.931$	$\bar{\omega}_1 = 0.933$	$\bar{\omega}_2 = 0.902$	$\bar{\omega}_3 = 0.912$	$\bar{\omega}_4 = 0.909$

Table 7: Transportation costs and fixed charges in crisp form $(\boldsymbol{c_{uv}}, \boldsymbol{f_{uv}})$

	D1	D2	D3	D4
S1	(4, 23.63)	(6, 25)	(8, 32)	(10, 23.38)
S2	(12, 24.75)	(13, 30.50)	(5.5, 27.25)	(8.13, 27.50)
S3	(8, 29.25)	(7.88, 24.38)	(11.25, 32)	(12.50, 31)

Table 8: Transportation time and deterioration rate of goods in crisp form (t_{uv}, d_{uv})

	D1	D2	D3	D4
S1	(2.77, 1.34)	(3.55, 1.44)	(2.85, 1.40)	(3.22, 1.56)
S2	(4.73, 1.41)	(4, 1.36)	(3.82, 1.49)	(3.16, 1.39)
S3	(5.69, 1.57)	(2.61, 1.36)	(3.08, 1.57)	(2.60, 1.57)

Using the data, a mathematical description of the MORISFCTP utilizing the Weibull distribution is developed. An objective function and probabilistic restriction are formulated in the below manner, and a conversion technique is used for converting from a probabilistic form into a deterministic form, which is solved by fuzzy programming, the method of a linear weighted sum, €- constraint method.

$$Min: Z_1 = 4x_{11} + 6x_{12} + 8x_{13} + 10x_{14} + 12x_{21} + 13x_{22} + 5.5x_{23} + 8.13x_{24} + 8x_{31} + 7.88x_{32} + 11.25x_{33} + 12.50x_{34}$$

$$Min: Z_2 = 2.77x_{11} + 3.55x_{12} + 2.85x_{13} + 3.22x_{14} + 4.73x_{21} + 4x_{22} + 3.82x_{23} + 3.16x_{24} + 5.69x_{31} + 2.61x_{32} + 3.08x_{33} + 2.60x_{34}$$

$$Min: Z_3 = 1.34x_{11} + 1.44x_{12} + 1.40x_{13} + 1.56x_{14} + 1.41x_{21} + 1.36x_{22} + 1.49x_{23} + 1.39x_{24} + 1.57x_{31} + 1.36x_{32} + 1.57x_{33} + 1.57x_{34}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 555.7788653$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 656.3787239$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 757.1776883$$

$$x_{11} + x_{21} + x_{31} \geq 331.20851$$

$$x_{12} + x_{22} + x_{32} \geq 431.90705$$

$$x_{13} + x_{23} + x_{33} \geq 532.30701$$

$$x_{14} + x_{24} + x_{34} \geq 632.80666$$

$$x_{uv} \geq 0, u = 1,2,3 \ \& \ v = 1,2,3,4$$

7. Result and Analysis:

This part examines the optimum solutions to the corresponding classical Model 2, which are derived from the fuzzy programming approach in Section 5.1, the method of a linear weighted sum in Section 5.2, and the €-constrained method in Section 5.3.

- **Fuzzy programming approach**

The expected value of a rough random variable operator employed by Tables 2 and 3 is reduced to Table 7 by theorem 3.3, which is employed in Section 3. Theorems 3.4 and 3.5 in Tables 4 and 5 are then reduced to Table 8, which is accomplished as in Section 3. Using that crisp value established in Model 2 and the technique outlined in Subsection 5.1, as well as the Lingo program, our result is displayed below in Table 9.

Table 9: Non-dominated solution of proposed MORISFCTP using fuzzy programming

λ	Z_1	Z_2	Z_3
0.7289	14833	3.82	2730

Each objective function's desired value is $Z_1 = 14833$, $Z_2 = 3.82$, and $Z_3 = 2730$. Furthermore, for each of the objective functions Z_1, Z_2 *and* Z_3, we found the non-dominated optimal solutions (14619.6, 15378.714824, 15407.768049), (5932.149036, 5622.43, 5673.0283247), and (2793.278315, 2711.5156266, 2706) correspondingly.

- **Method of a linear weighted sum**

Calculate the value expected in every rough interval in Tables 2, 3, and 4. (DM option). Subsequently, using the crisp form defined in Model 2, and the process stated in Section 5.2, as well as the Lingo program, compute the outcomes shown in Table 10.

Table 10: The compromise solution of the proposed MORISFCTP using a weighted sum

Cases	W1	W2	W3	Z1	Z2	Z3
1	0.8	0.1	0.1	14624.05	3.82	2793.278
2	0.7	0.2	0.1	14624.05	3.82	2793.278
3	0.7	0.1	0.2	14624.05	3.82	2793.278
4	0.6	0.1	0.3	14624.05	3.82	2793.278
5	0.6	0.3	0.1	14624.05	3.82	2793.278
6	0.6	0.2	0.2	14624.05	3.82	2793.278
7	0.5	0.2	0.3	14624.05	3.82	2793.278
8	0.5	0.3	0.2	14765.53	3.82	2734.890
9	0.5	0.4	0.1	14765.53	3.82	2734.890
10	0.4	0.3	0.3	14765.53	3.82	2734.890
11	0.4	0.5	0.1	14765.53	3.82	2734.890
12	0.4	0.4	0.2	14765.53	3.82	2734.890
13	0.4	0.2	0.4	14765.53	3.82	2734.890
14	0.3	0.3	0.4	14765.53	3.82	2734.890
15	0.3	0.4	0.3	14765.53	3.82	2734.890
16	0.2	0.5	0.3	14765.53	3.82	2734.890
17	0.2	0.4	0.4	14765.53	3.82	2734.890
18	0.2	0.2	0.6	14765.53	3.82	2734.890
19	0.1	0.1	0.8	14765.53	3.82	2734.890

Depending on the above-discussed compromise solutions,

- We decide the high significance of the third objective function, which is compared to another two objective functions, and also that the most effective solution is better, and the same concept applies to the second objective function but not to the first objective function (see cases 1–7 of Table 10 in a comparison with Table 9).
- To conclude, reflecting the significance of the objective functions indicated in occurrences 8 through 19 of Table 10, the greatest response concerning Table 9 and any other circumstances in Table 10 is the optimal solution of all objective functions.

- **€-constraint method:**

Using the solution procedure accessible in Section 5.3 and LINGO software, to determine the Pareto-optimal solutions are calculated using the €-constraint method, which is revealed in Table 11. It deserves to be noted that the €-constraint technique provides the set of all optimal solutions for every distinct optimal solution $Z_{m_0}(x), m_0 = 1,2,3$ that can be obtained. Only three cases are studied in Table 11 to show the optimal solutions produced from the €-constraint technique for each Pareto-front, as illustrated in Fig. 4.

Table 11: Pareto-optimal solution for the proposed MORISFCTP by €-constraint Method

Case	Z_1	Z_2	Z_3
1	14619.6	3.82	2793.278
2	15365.34	3.82	2711.515
3	15400.63	3.82	2706.002

Subsequently, the Pareto-front is identified as being exactly the optimal solution. Based on Table 11, a Pareto-optimal exact solution occurs in the first case of the suggested MORISFCTP. The least values of objective function occur in Table 11, also known as the Pareto-front exact solution of the €-constraint approach.

Table 12: Pareto optimal solution for MORISFCTP by three distinct methods

Approach to applied	Z_1	Z_2	Z_3
Fuzzy Programming	14833	3.82	2730
Linear Weighted Sum Method	14765.53	3.82	2734.890
€-constraint Method	14619.6	3.82	2793.278

The Pareto optimal solutions for MORISFCTP are obtained using three distinct methods in Model 2, and our outcomes are outlined. (Table 12)

Comparison:

In Table 12, the compromise optimal solution for developing the algorithm MORISFCTP, which is more advisable for decision-makers to use the €-constraint method rather than fuzzy programming and the weighted sum method, occurs as the same ideal solution. A set of Pareto-optimal solutions is produced for each $Z_{m_0}(x), m_0 = 1,2,3$, consistent with distinct values $m, m = 1,2,3$, from which the decision-maker can select the minimum optimal solution.

- One of the most significant advantages of employing this method is that it generates Pareto-fronts, which yield an exact Pareto-front.
- In this technique, by determining a pair of optimal solutions, the decision-maker can select the best €-constraint method, proving that example 1 indicates that Z_1 is of greater significance than another two, and case 3 indicates that Z_3 is a higher-level preference than the other two methods.
- The €-constrained procedure contains fewer variables than another method.
- It ensures that an optimal solution is obtained.

In this aspect, the €-constrained procedure is stronger than the other methods. Finally, Fig. 1 displays a bar graph demonstration of the compromise optimal solutions of the distinct three methods of the proposed MORISFCTP. The article offers a comparison of the most beneficial solutions attained among the three methods.

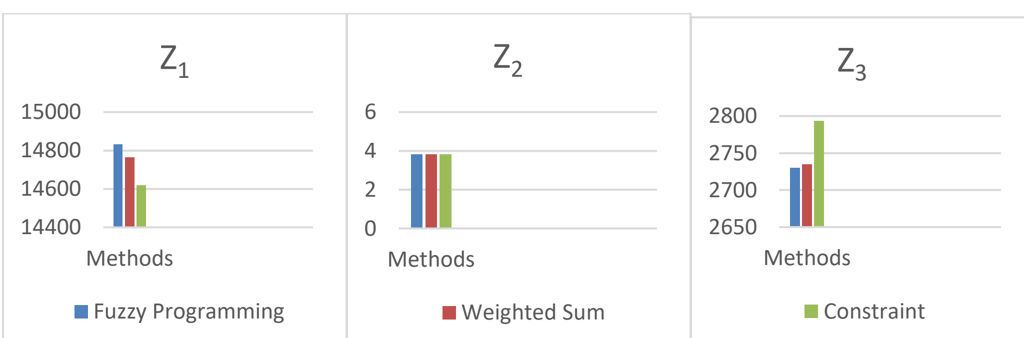

Fig.2 Optimal solution of three distinct methods for $Z_1, Z_{2,}$ and Z_3

Properly choosing the values $\varepsilon_m$of the €-constraint method is a must. Otherwise, the optimal solutions aren't adequately delivered to the objective functions.

8. Sensitivity analysis of MORISFCTP and discussion:

Sensitivity analysis (SA) is a fascinating and engrossing optimization study. This analysis investigates the impact of variations in the coefficients in the objective functions and the impact of variations in the right-side restrictions, as well as the effects on the range of availabilities and demand. The MORISFCTP, determining the demand probabilistic constraint following slight modifications in particular demand probabilities and remaining supplies, is problematic. Many articles exploring the TP problem with SA and problems associated with fixed-charge TP and Weibull distribution have been published, including those by Midya and Roy (2020), Buvaneshwari and Anuradha (2022), and others. In this part, MORISFCTP executed an SA of optimality in terms of fluctuations in probabilities on uncertain parameters such as source and demand. We used Model 2 for the SA problem and changed the probability from $0 < p < 1$, where P is the probability on ω_u or ϖ_v. We analyzed the problem by holding one probability parameter$\left(P_{\omega_u}\ or\ P_{\varpi_v}\right)$ stable at 0.5 and changing the value of the other probability factor$\left(P_{\omega_u}\ or\ P_{\varpi_v}\right)$. The transportation cost was obtained and listed below for Model 2, which contains every stochastic optimal solution.

To overcome this optimization difficulty, the Lingo program was used to manage multiple objective functions in this SA. Table 13 shows the SA outcomes for the probability of demand ϖ_v. Figures 3, 4, and 5 exhibit graphical representations of transport cost, time, and deterioration cost concerning the probability of ϖ_v. As illustrated in Figures 3 and 5, transportation costs and deterioration costs gradually increase in probability ϖ_v. This substance mentions that transportation costs and deterioration costs are variations in probability demand that need an analysis of sensitivity. Figure 4 shows the exact time constant value as the probability of demand requirements varies. Similarly, we follow the identical procedure in $P_{\omega_u}$for SA. By investigating the sensitive probability patterns for uncertain parameters, DMs gain an understanding as well as the ability to develop the transportation system.

Table 13: Sensitivity analysis of MORISFCTP for the different probability of demands

Probability value of P_{ϖ_v}	Probability value of P_{ω_u}		Optimal transportation cost		
		(λ)	Z_1	Z_2	Z_3
0.9		0.7294539	14832.84	3.82	2729.613
0.8		0.7296328	14832.69	3.82	2729.597
0.7		0.7297778	14832.58	3.82	2729.585
0.6		0.7299140	14832.47	3.82	2729.573
0.5	0.5	0.7300532	14832.36	3.82	2729.561
0.4		0.7302055	14832.24	3.82	2729.547
0.3		0.7303844	14832.10	3.82	2729.532
0.2		0.7306182	14831.92	3.82	2729.511
0.1		0.7309924	14831.62	3.82	2729.479

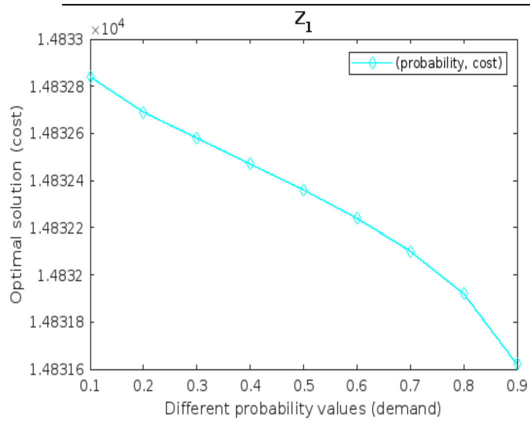

Fig 3: SA for transportation cost with various probabilities

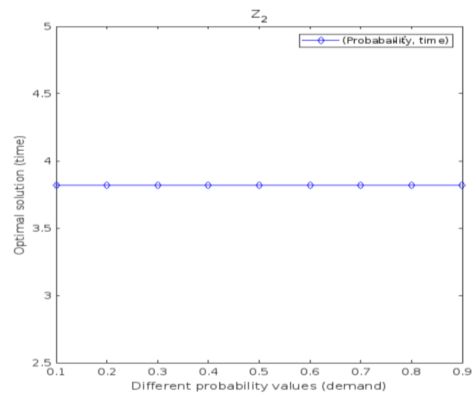

Fig 4: SA for delivered time with various probability

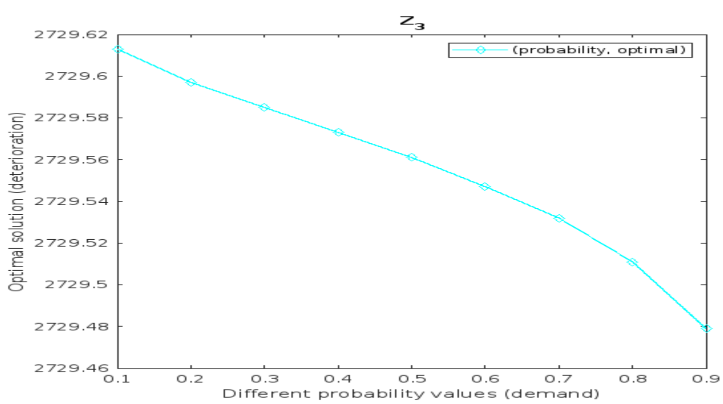

Fig 5: SA for the deterioration cost with various probability

This study found that the proposed MORISFCTP models provide optimal answers in uncertain situations. In very uncertain situations, it's tempting to make less risky decisions. The results show that the various probabilities for ω_u or ϖ_v correspond to different amounts of transportation cost, delivered time, and deterioration in this scenario. When the optimization problem is undetermined conservative solutions are frequently adopted. When modeling MORIFCTP with unpredictability from ω_u as well as ϖ_v probabilities, conservative solutions are decided on as optimal. This investigation highlights the importance of understanding the sensitivity of probabilistic constraints in an atmosphere of increased uncertainty. It assists in making choices by establishing the appropriate level of uncertainty for uncertain factors.

When a company's decision-maker sees new traders from unfamiliar cities, it is impractical for the decision-maker to acquire statistical data on the parameters for MOFCTP, which are the unit costs of transportation, fixed charge for an open route, and other objective functions linked to MOFCTP. The science of probability is unsuitable for tackling such scenarios. Furthermore, in everyday circumstances, the supply as well as demand factors that govern MOFCTP are both deterministic and insufficient. The present investigation treats demand as well as supply elements as a rough interval.

9. Conclusion and Future Scope:

The present study introduces the impression of rough random variables in a MORISFCTP, for the first time in the literature. The rough random variable is built, realistically in the real situation of MOFCTP, to throw out the ambiguity concerning probability and expected value with distributions. The probability distributions are applied to the objective functions of variables, such as costs (cost of transport and fixed fee), the duration for transporting the product, and the deterioration ratio of items, in which three distinct possibility distributions, specifically the exponential distribution, uniform distribution, and gamma distribution, have been chosen, respectively. In the objective functions, the parameters of the product, such as cost of transportation and fixed rate, duration of transporting the product, and deterioration ratio of the item, are consumed as rough random variables in the proposed MORISFCTP, while the probabilistic nature of supply and demand restrictions is represented by a Weibull distribution. An operator of expected value has been used to transform MORISFCTP with rough and probabilistic parameters into deterministic MORISFCTP. Fuzzy programming, weighted sum, and €-constrained approaches have been used to provide optimal compromise solutions to the deterministic MORISFCTP. A €-constrained method yielded a stronger compromise optimum solution among the methods. The focal improvement of the €-constrained method over the other methods has been specified. A real-life example is included to show the practicality of implementing rough intervals with random probabilistics for the proposed MORISFCTP. Because of this model, future studies will focus on the model's rough interval multi-objective with non-linear fixed-cost transportation with probabilistic diverse conditions such as budget and conveyance restrictions. In the upcoming research, we will acquire practical data from credible sources and then employ statistical consistency factors to calculate its probability distribution.

Data Availability: The data used to support the findings of this study are available from the corresponding author upon request.

Acknowledgement: The authors wish to thank the management of Vellore Institute of Technology, Vellore-632014 for their continuous support and encouragement to carry out this research work.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] S. Singh, A. Pradhan, M.P. Biswal, Multi-Objective Solid Transportation Problem Under Stochastic Environment, Sādhanā 44 (2019), 105. https://doi.org/10.1007/s12046-019-1094-0.

[2] A. Biswas, A.A. Shaikh, S.T.A. Niaki, Multi-objective non-linear fixed charge transportation problem with multiple modes of transportation in crisp and interval environments, Appl. Soft Comp. 80 (2019), 628–649. https://doi.org/10.1016/j.asoc.2019.04.011.

[3] M.A. Elsisy, M.A. El Sayed, Fuzzy Rough Bi-Level Multi-Objective Nonlinear Programming Problems, Alex. Eng. J. 58 (2019), 1471–1482. https://doi.org/10.1016/j.aej.2019.12.002.

[4] M.G. Brikaa, Z. Zheng, E.-S. Ammar, Fuzzy Multi-objective Programming Approach for Constrained Matrix Games with Payoffs of Fuzzy Rough Numbers, Symmetry 11 (2019), 702. https://doi.org/10.3390/sym11050702.

[5] S.K. Roy, S. Midya, G.W. Weber, Multi-Objective Multi-Item Fixed-Charge Solid Transportation Problem Under Twofold Uncertainty, Neural Comp. Appl. 31 (2019), 8593–8613. https://doi.org/10.1007/s00521-019-04431-2.

[6] A. Das, U.K. Bera, M. Maiti, A Solid Transportation Problem in Uncertain Environment Involving Type-2 Fuzzy Variable, Neural Comp. Appl. 31 (2019), 4903–4927. https://doi.org/10.1007/s00521-018-03988-8.

[7] S.H. Jana, B. Jana, Application of Random Triangular and Gaussian Type-2 Fuzzy Variable to Solve Fixed Charge Multi-Item Four Dimensional Transportation Problem, Appl. Soft Comp. 96 (2020), 106589. https://doi.org/10.1016/j.asoc.2020.106589.

[8] S. Bera, P.K. Giri, D.K. Jana, et al. Fixed charge 4D-TP for a Breakable Item Under Hybrid Random Type-2 Uncertain Environments, Inf. Sci. 527 (2020), 128–158. https://doi.org/10.1016/j.ins.2020.03.050.

[9] S.H. Nasseri, S. Bavandi, Solving Multi-objective Multi-choice Stochastic Transportation Problem with Fuzzy Programming Approach, in: 2020 8th Iranian Joint Congress on Fuzzy and Intelligent Systems (CFIS), IEEE, Mashhad, Iran, 2020: pp. 207–210. https://doi.org/10.1109/CFIS49607.2020.9238695.

[10] U. Habiba, A. Quddoos, Multiobjective Stochastic Interval Transportation Problem Involving General Form of Distributions, Adv. Math., Sci. J. 9 (2020), 3213--3219. https://doi.org/10.37418/amsj.9.6.1.

[11] P. Agrawal, T. Ganesh, Fuzzy Fractional Stochastic Transportation Problem Involving Exponential Distribution, OPSEARCH 57 (2020), 1093–1114. https://doi.org/10.1007/s12597-020-00458-5.

[12] R.K. Bera, S.K. Mondal, Credit Linked Two-Stage Multi-Objective Transportation Problem in Rough and Bi-Rough Environments, Soft Comp. 24 (2020), 18129–18154. https://doi.org/10.1007/s00500-020-05066-5.

[13] P. Agrawal, T. Ganesh, Solving Multi-choice Fractional Stochastic Transportation Problem Involving Newton’s Divided Difference Interpolation, in: D. Dutta, B. Mahanty (Eds.), Numerical Optimization in Engineering and Sciences, Springer Singapore, Singapore, 2020: pp. 289–298. https://doi.org/10.1007/978-981-15-3215-3_28.

[14] S. Midya, S. Kumar Roy, G. Wilhelm Weber, Fuzzy Multiple Objective Fractional Optimization in Rough Approximation and Its Aptness to the Fixed-Charge Transportation Problem, RAIRO-Oper. Res. 55 (2021), 1715–1741. https://doi.org/10.1051/ro/2021078.

[15] A. Kuiri, B. Das, S.K. Mahato, An Optimization of Solid Transportation Problem With Stochastic Demand by Lagrangian Function and KKT Conditions, RAIRO-Oper. Res. 55 (2021), S2969–S2982. https://doi.org/10.1051/ro/2020136.

[16] H. Garg, R.M. Rizk-Allah, A Novel Approach for Solving Rough Multi-Objective Transportation Problem: Development and Prospects, Comp. Appl. Math. 40 (2021), 149. https://doi.org/10.1007/s40314-021-01507-5.

[17] H. Khalifa, M. Elhenawy, M. Masoud, H. Bhuiyan, N.R. Sabar, On Multi-Objective Multi-Item Solid Transportation Problem in Fuzzy Environment, Int. J. Appl. Comp. Math 7 (2021), 24. https://doi.org/10.1007/s40819-021-00961-3.

[18] S. Singh, A. Pradhan, M.P. Biswal, Computation of Some Stochastic Transportation Problems Using Essen Inequality, Int. J. Appl. Comp. Math. 7 (2021), 241. https://doi.org/10.1007/s40819-021-01131-1.

[19] Y. Kacher, P. Singh, A Comprehensive Literature Review on Transportation Problems, Int. J. Appl. Comp. Math. 7 (2021), 206. https://doi.org/10.1007/s40819-021-01134-y.

[20] Shivani, D. Rani, A. Ebrahimnejad, An Approach to Solve an Unbalanced Fully Rough Multi-Objective Fixed-Charge Transportation Problem, Comp. Appl. Math. 41 (2022), 129. https://doi.org/10.1007/s40314-022-01830-5.

[21] S. Devnath, P.K. Giri, S.S. Mondal, et al. Fully Fuzzy Multi-Item Two-Stage Fixed Charge Four-Dimensional Transportation Problems with Flexible Constraints, Granul. Comp. 7 (2021), 779–797. https://doi.org/10.1007/s41066-021-00295-x.

[22] A. Biswas, L.E. Cárdenas-Barrón, A.A. Shaikh, et al. A Study of Multi-Objective Restricted Multi-Item Fixed Charge Transportation Problem Considering Different Types of Demands, Appl. Soft Comp. 118 (2022), 108501. https://doi.org/10.1016/j.asoc.2022.108501.

[23] S. Haque, A.K. Bhurjee, P. Kumar, Multi-Objective Non-Linear Solid Transportation Problem with Fixed Charge, Budget Constraints Under Uncertain Environments, Syst. Sci. Control Eng. 10 (2022), 899–909. https://doi.org/10.1080/21642583.2022.2137707.

[24] T.K. Buvaneshwari, D. Anuradha, Solving Stochastic Fuzzy Transportation Problem with Mixed Constraints Using the Weibull Distribution, J. Math. 2022 (2022), 6892342. https://doi.org/10.1155/2022/6892342.

[25] S. Dutta, A. Kaur, Cubic Spline Interpolation Approach to Solve Multi-Choice Programming Problem, Int. J. Appl. Comp. Math 9 (2022), 6. https://doi.org/10.1007/s40819-022-01483-2.

[26] M. Niksirat, A New Approach to Solve Fully Fuzzy Multi-Objective Transportation Problem, Fuzzy Information and Engineering 14 (2022) 456–467. https://doi.org/10.1080/16168658.2022.2152836.

[27] P. Agrawal, K. Alnowibet, T. Ganesh, et al. An Artificial Intelligence Approach for Solving Stochastic Transportation Problems, Comp. Mater. Cont. 70 (2022), 817–829. https://doi.org/10.32604/cmc.2022.019685.

[28] D. Mardanya, S.K. Roy, New Approach to Solve Fuzzy Multi-Objective Multi-Item Solid Transportation Problem, RAIRO-Oper. Res. 57 (2023), 99–120. https://doi.org/10.1051/ro/2022211.

[29] S.K. Roy, S. Midya, V.F. Yu, Multi-Objective Fixed-Charge Transportation Problem with Random Rough Variables, Int. J. Uncertain. Fuzziness Knowl.-Based Syst. 26 (2018), 971–996. https://doi.org/10.1142/s0218488518500435.

[30] S. Halder Jana, B. Jana, B. Das, G. Panigrahi, M. Maiti, Constrained FC 4D MITPs for Damageable Substitutable and Complementary Items in Rough Environments, Mathematics 7 (2019), 281. https://doi.org/10.3390/math7030281.

[31] S. Midya, S.K. Roy, Multi-Objective Fixed-Charge Transportation Problem Using Rough Programming, Int. J. Oper. Res. 37 (2020), 377. https://doi.org/10.1504/ijor.2020.105444.

[32] M.Y.N. Attari, A.A. Moslemi Beirami, A. Ala, et al. Resolving the Practical Factors in the Healthcare System Management by Considering a Combine Approach of AHP and ANP Methods, Eval. Program Plan. 100 (2023), 102339. https://doi.org/10.1016/j.evalprogplan.2023.102339.

[33] A. Ala, A. Goli, S. Mirjalili, et al. A Fuzzy Multi-Objective Optimization Model for Sustainable Healthcare Supply Chain Network Design, Appl. Soft Comp. 150 (2024), 111012. https://doi.org/10.1016/j.asoc.2023.111012.

[34] A. Ala, V. Simic, D. Pamucar, et al. Appointment Scheduling Problem under Fairness Policy in Healthcare Services: Fuzzy Ant Lion Optimizer, Expert Syst. Appl. 207 (2022), 117949. https://doi.org/10.1016/j.eswa.2022.117949.