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A Prey-Predator Mathematical Model With Diffusion and Home Ranges

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Abstract. A prey-predator model with diffusion and home ranges is considered. The model consists of partial differential and integral equations. The model incorporates complex mathematical expressions, which make it hard to analyze mathematically. Therefore, a numerical solution is provided in two cases. The first case considers the prey population growing logistically, while we consider the exponential growth of the prey population in the second case. We study the dynamic behavior of the two species for both cases. Special attention goes to the impact of home ranges and diffusion coefficients on the dynamics of prey and predator populations.

1. Introduction

The relationship between prey and predator has been widely described and examined in population dynamics. The prey-predator models are known as the Lotka-Volterra models because the nonlinear interaction between the two species was explained and discussed by Lotka [24] and Volterra [39]. The Lotka-Volterral equations have been generalized to include different kinds of mathematical models [3,4,8,16,19,34]. These models are described mainly by ordinary differential equations with different growth function shapes. Mathematical models allow the populations of prey or predator to grow according to the logistic growth functions as in [10, 11], Leslie-Gower functions as in [20, 27], and the modified Leslie-Gower functions as in [3, 34]. The exponential growth rate of the prey population is considered in [41].

Incorporating stage structure to define a system of prey and predator has gained considerable attention recently and in the past [14, 31, 38, 41, 42]. Stage-structure-Predator-prey models usually lead to models with time delay in prey differential equations [14, 31] or in predator differential equations [38, 41, 42]. The authors in [41] comprise stage structure into a predator-prey system

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to study the dynamical behavior among the two species and to compare the results to models without stage structure.

Spatial spread prey-predator models are also among the prevalent subjects in population dynamics. These models are usually partial differential equations known as reaction-diffusion systems, for example, [6,9–13,18,22,28–30,35,38,40]. Thieme and Zhao [38] describe a model with diffusion and non-local delay. [22, 28, 29] studied prey's influence on the asymptotic speed of predator's spreading. [9, 12, 18] analyzed a diffusive predator-prey model in order to show the existence of periodic or traveling wave solutions. Dunbar [10, 11] studied the following equation,

$$\begin{cases} \dfrac{\partial}{\partial t}P(x,t) &= d_P\nabla_x^2 P(x,t) + bP(x,t)\Big(1-\frac{P(x,t)}{K}\Big) - pP(x,t)T(x,t), \\ \dfrac{\partial}{\partial t}T(x,t) &= d_T\nabla_x^2 T(x,t) + qP(x,t)T(x,t) - mT(x,t), \end{cases} \tag{1.1}$$

where $P(x,t)$ and $T(x,t)$ are the density of prey and predator at time t and position $x \in \mathbb{R}$, respectively. The carrying capacity of preys K and the parameters b, p, q, m are positive constants. $\nabla_x^2 = \sum_{i=1}^n \partial_{x_i}^2$ is the Laplace operator, and d_P and d_Q are the diffusion coefficients. The system was employed to show the existence of traveling wave solutions. The model (1.1) was also considered and discussed by Murray [26].

The existence and uniqueness of limit cycles and the stability of equilibrium locally or globally have been extensively examined and analyzed [7, 16, 17, 19, 20, 30, 42, 44], and references therein.

The paper proposes a prey-predator mathematical model with diffusion and home ranges. The work aims to understand the influence of diffusion and home ranges on the dynamic behaviors of prey and predator populations. The mathematical model with diffusion and home ranges is introduced in Section 2. In Section 3, we solve the model numerically by considering two different growth functions of the prey population. For the first growth function, we assume the model grows logistically. In the second function, we assume the model grows exponentially. The impact of home ranges on prey and predator population dynamics is analyzed in Section 4. In Section 5, we study the influence of the diffusion coefficients on the dynamical behavior of the two species.

2. The prey-predator mathematical model with diffusion and home-ranges

Let $P(x,t)$ denote the density of prey at time t with home-range center at location $x \in \mathbb{R}$ and $T(x,t)$ the density of predator at time t and location $x \in \mathbb{R}$. Further, $\omega(x,y)$ is the rate at which a prey or a predator with home-range center x visits a location $y \in \mathbb{R}$. Then we have the following model,

$$\begin{cases} \dfrac{\partial}{\partial t}P(x,t) &= d_P\nabla_x^2 P(x,t) + \Big(f(P(x,t)) - m_1\Big)P(x,t) \\ & - \ pP(x,t)\displaystyle\int_{-\infty}^{\infty}\omega(x-z)T(z,t)dz, \\ \dfrac{\partial}{\partial t}T(x,t) &= d_T\nabla_x^2 T(x,t) - m_2 T(x,t) \\ & + \ qT(x,t)\displaystyle\int_{-\infty}^{\infty}\omega(x-z)P(z,t)dz, \end{cases} \tag{2.1}$$

$x \in \Omega,\ t \geq 0$, with the following given initial conditions

$$P(x,0) = P_0(x),\ T(x,0) = T_0(x),\ \ x \in \mathbb{R}.$$

We assume P and T satisfy the following boundary conditions

$$P(x,t) = P_b(x,t),\ \ x \in \partial\Omega \times (0,\infty),$$

$$T(x,t) = T_b(x,t),\ \ x \in \partial\Omega \times (0,\infty).$$

Here, $f(P)$ is the function growth rate of prey, $p \geq 0$ is the chance at which the meeting between a prey and a predator leads to the death of the prey by the predator. $q \geq 0$ is the rate of per unit predator density increase by killing one unit of prey. $m_1 > 0$ is the natural mortality rate of prey per unit of time, and $m_2 > 0$ is the natural mortality rate of predator per unit of time. d_P and d_T are the diffusion constants of prey and predator, respectively.

3. APPROXIMATED SOLUTIONS OF THE MODEL IN (2.1)

3.1. **Approximated schemes.** Let $[-a_1, a_1]$ for $a_1 > 0$ be a bounded domain of $\mathbb{R}$. Then (2.1) shall be

$$\begin{cases} \dfrac{\partial}{\partial t}P(x,t) &= d_P\nabla_x^2 P(x,t) + \Big(f(P(x,t)) - m_1\Big)P(x,t) \\ & - \ pP(x,t)\displaystyle\int_{-a_1}^{a_1}\omega(x-z)T(z,t)dz, \\ \dfrac{\partial}{\partial t}T(x,t) &= d_T\nabla_x^2 T(x,t) + qT(x,t)\displaystyle\int_{-a_1}^{a_1}\omega(x-z)P(z,t)dz \\ & - \ m_2 T(x,t), \end{cases} \tag{3.1}$$

$x \in [-a_1, a_1],\ t \geq 0$, with the following given initial conditions

$$P(x,0) = P_0(x),\ T(x,0) = T_0(x),\ \ x \in [-a_1, a_1]. \tag{3.2}$$

P and Q satisfy the following boundary conditions

$$P(x,t) = P_b(x,t),\ T(x,t) = T_b(x,t),\ \ x \in \partial\Omega \times [0, T_1],\ T_1 \geq 0. \tag{3.3}$$

The system in (3.1) will be approximated and replaced with algebraic expressions. For simplicity, we use the notations $P_i(t)$, and $Q_i(t)$ instead of $P(x_i, t)$, and $Q(x_i, t)$, respectively. Let $x_i = -a_1 + i\Delta x$, where $\Delta x = 2a_1/(N+1)$ is the spacing stepsize and $i = 0, 1, \ldots, N+1$ for $N > 0$. Then, the model in (3.1) is now taking the discrete form

$$\begin{cases} \dfrac{\partial}{\partial t}P_i(t) &= d_P\nabla_x^2 P_i(t) + \Big(f(P_i(t)) - m_1\Big)P_i(t) \\ &- pP_i(t)\displaystyle\int_{-a_1}^{a_1}\omega(x_i - z)T(z,t)dz, \\ \dfrac{\partial}{\partial t}T_i(t) &= d_T\nabla_x^2 T_i(t) + qT_i(t)\displaystyle\int_{-a_1}^{a_1}\omega(x_i - z)P(z,t)dz - m_2T_i(t), \end{cases} \tag{3.4}$$

for $i = 1, \ldots, N$. To approximate the integrals in (3.4), we use the composite trapezoidal rule

$$\begin{aligned} \int_{-a_1}^{a_1} & \omega(x_i - z)T(z,t)dz \approx W_T(x_i, t, \Delta x) \\ = & \frac{\Delta x}{2}\Big(\omega(x_i - x_0)T(x_0, t) + 2\sum_{k=1}^{N}\omega(x_i - x_k)T(x_k, t) \\ + & \omega(x_i - x_{N+1})T(x_{k+1}, t)\Big), \\ \int_{-a_1}^{a_1} & \omega(x_i - z)P(z,t)dz \approx W_P(x_i, t, \Delta x) \\ = & \frac{\Delta x}{2}\Big(\omega(x_i - x_0)P(x_0, t) + 2\sum_{k=1}^{N}\omega(x_i - x_k)P(x_k, t) \\ + & \omega(x_i - x_{N+1})P(x_{k+1}, t)\Big). \end{aligned}$$

We approximate $\nabla_x^2 P(x,t)$ and $\nabla_x^2 Q(x,t)$ by the central finite differences of the second order as follows,

$$\frac{\partial^2 P(x_i, t)}{\partial x^2} \approx \frac{P(x_{i-1}, t) - 2P(x_i, t) + P(x_{i+1}, t)}{\Delta x^2},$$

and

$$\frac{\partial^2 T(x_i, t)}{\partial x^2} \approx \frac{T(x_{i-1}, t) - 2T(x_i, t) + T(x_{i+1}, t)}{\Delta x^2}.$$

Now (3.4) takes the form

$$\begin{cases} P_i'(t) &= d_P\Big(\dfrac{P_{i-1}(t) - 2P_i(t) + P_{i+1}(t)}{\Delta x^2}\Big) + \Big(f(P_i(t)) - m_1\Big)P_i(t) \\ &- pP_i(t)W_T(x_i, t, \Delta x), \\ T_i'(t) &= d_T\Big(\dfrac{T_{i-1}(t) - 2T_i(t) + T_{i+1}(t)}{\Delta x^2}\Big) \\ &+ qT_i(t)W_P(x_i, t, \Delta x) - m_2T_i(t), \end{cases} \tag{3.5}$$

for $i = 1, \ldots, N$. The system in (3.5) with (3.2) and (3.3) are solved numerically by the continuous Runge-Kutta method of the fourth order and the discrete Runge-Kutta method of the third order. This numerical method is accurate and gives stable solutions. For more about the continuous Runge-Kutta method of the fourth order and the discrete Runge-Kutta method of the third order, we refer to the recent work by Alanazi [1] and Alanazi et al. [2]. We select the numerical values of the parameters as in Table 1.

Parameter	The parameter meaning	Values
K	The carrying capacity	25
b	The intrinsic growth rate	0.9
m_1	The natural mortality rate of preys	0.1
m_2	The natural mortality rate of predators	0.1
p	The rate of killing a prey by a predator	0.3
q	The rate of per unit predator population density increase	0.3
d_P	The diffusion constant of preys	0.3
d_T	The diffusion constant of predators	0.3
s	how far a prey or a predator could go from their home centers	0.5

Table 1. Numerical values of the parameters in (2.1).

3.2. **Approximated solutions.** In this section, we seek the numerical solutions of (3.5) with (3.2) and (3.3) for two different scenarios of $f(P)$. For the first choice, we define $f(P)$ to be

$$f(P) = bP(x,t)\Big(1 - \frac{P(x,t)}{K}\Big).$$

For the second choice, we let $f(P)$ be

$$f(P) = bP(x,t).$$

We assume ω is described by a normal distribution, i.e.,

$$\omega(x-z) = \frac{1}{\sqrt{4\pi s}} e^{-(x-z)^2/(4s)},$$

$s > 0$ describes how far a prey or a predator could go from the center of their homes. Let the initial conditions be

$$P_0(x) = 0.5,\ T_0(x) = 0.2,\ \ x \in [-10, 10].$$

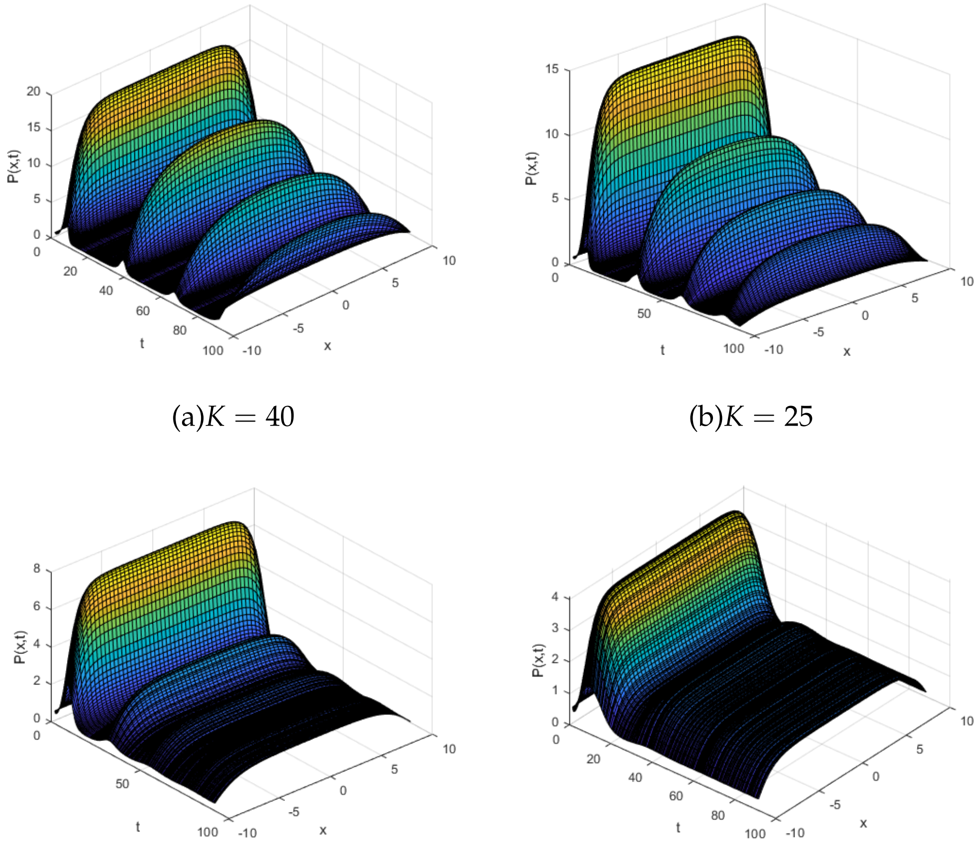
We assume the boundary conditions are

$$P_b(x,t) = 0,\ T_b(x,t) = 0,\ \ x \in \partial\Omega \times [0, T_1],\ T_1 \geq 0.$$

3.2.1. *Logistic growth of preys.* We assume the prey grows logistically such that

$$f(P) = bP(x,t)\Big(1 - \frac{P(x,t)}{K}\Big), \tag{3.6}$$

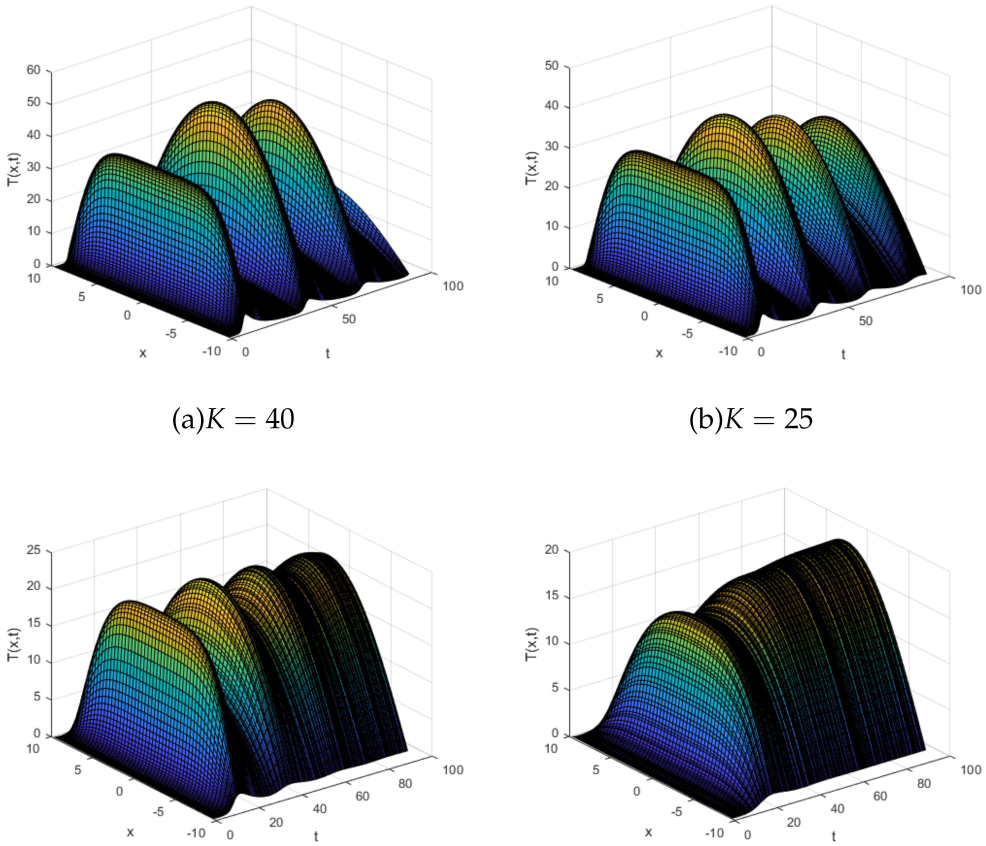
where K is the carrying capacity and b is the intrinsic growth rate. The numerical values of the parameters are chosen as in Table 1 or as what we choose underneath each figure. The numerical solutions of (2.1) when $f(P)$ is defined as in (3.6) are demonstrated in Fig. 3.1, Fig. 3.2, and Fig. 3.3. Fig. 3.1 demonstrates the prey densities $P(x,t)$ in time t and space x when the carrying capacity K equals $40(a)$, $25(b)$, $10(c)$, $5(d)$. Clearly, the amplitude of oscillation for the prey density $P(x,t)$ reduces as we decrease the value of K as shown in Fig. 3.1. The predator densities $T(x,t)$ in time t and space x when the carrying capacity K equals $40(a)$, $25(b)$, $10(c)$, $5(d)$ are given in Fig. 3.2. Choosing $b = 0$, the densities of prey $P(x,t)$ and predator $T(x,t)$ go extinct as in Fig. 3.3(a)(b). Fig. 3.3(c)(d) displays that the density of prey converges to zero faster when we choose $d_P = 0$ while this choice does not have a huge impact on the density of predator.



(a)$K = 40$ (b)$K = 25$

(c)$K = 10$ (d)$K = 5$

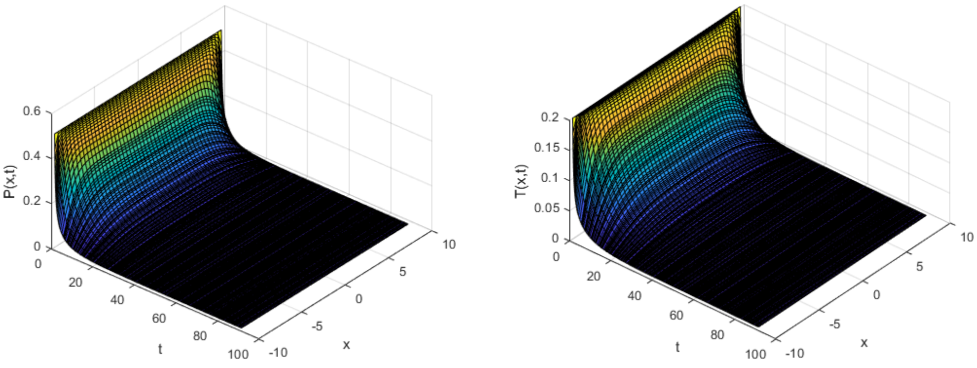
Figure 3.1. Illustration of how the carrying capacity K influences the solutions' shapes of $P(x,t)$.



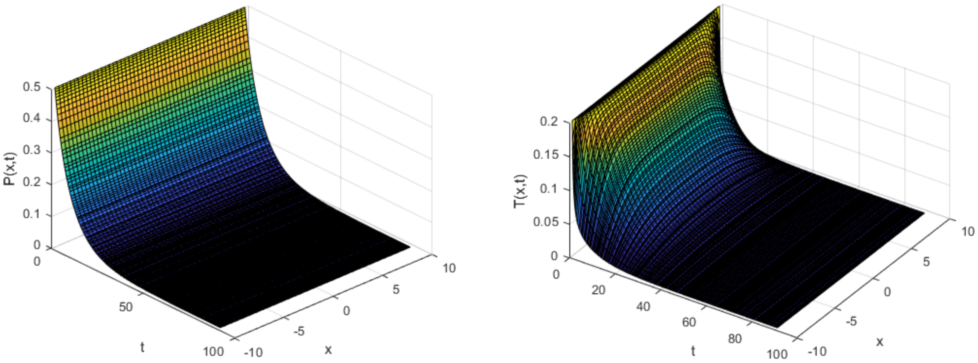
(a)$K = 40$ (b)$K = 25$

(c)$K = 10$ (d)$K = 5$

Figure 3.2. Illustration of the impact of the carrying capacity on the dynamics of $T(x,t)$.



(a)$b = 0$ (b)$b = 0$



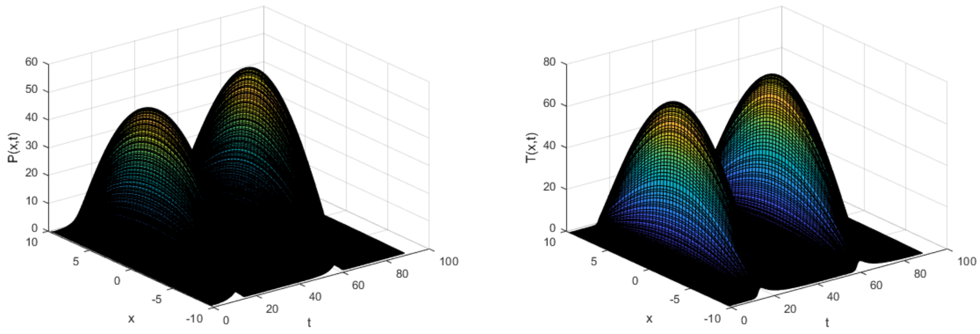
(c)$d_p = b = 0$ (d)$d_p = b = 0$

FIGURE 3.3. The approximated solutions of $P(x,t)$ and $T(x,t)$ when the intrinsic growth rate is set to zero. See Table 1 for other parameter values.

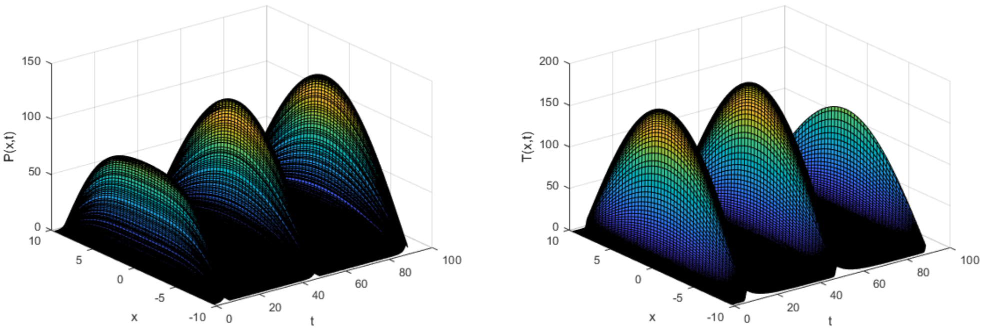
3.2.2. *Exponential growth of preys.* In this part, we find the numerical solutions of (3.5) with (3.2) and (3.3) when $f(P)$ is given by

$$f(P) = bP(x,t). \tag{3.7}$$

The numerical solutions of (3.5) when $f(P)$ is defined as in (3.7) are given in Fig. 3.4. Choosing $d_p = 1, d_T = 3$, $b = 0.5$, and $b = 1$ leads to the approximated solutions depicted in Fig. 3.4(a) (c) for the density of preys $P(x,t)$ and in Fig. 3.4(b) (d) for the density of predators $T(x,t)$.



(a)$b = 0.5$ (b)$b = 0.5$



(c)$b = 1$ (d)$b = 1$

FIGURE 3.4. Illustration of how the intrinsic growth rate b influences the solutions' shapes of $P(x,t)$ and $T(x,t)$ when $f(P) = bP(x,t)$, $d_p = 1$, and $d_T = 3$.

4. The influence of home ranges on the population dynamics

As before, $\omega(x,y)$ is the rate at which a prey or a predator with home-range center x visits a location $y \in \mathbb{R}$. We assume ω is described by a normal distribution, i.e.,

$$\omega(x-z) = \frac{1}{\sqrt{4\pi s}} e^{-(x-z)^2/(4s)},$$

where $s > 0$ describes how far a prey or a predator could go from the center of their homes. In this section, we discuss the influence of home ranges on population dynamics when the movement of the two species distributes normally, along with the logistic and exponential growth of prey.

When $f(P)$ grows logistically, we have

$$f(P) = bP(x,t)\Big(1 - \frac{P(x,t)}{K}\Big),$$

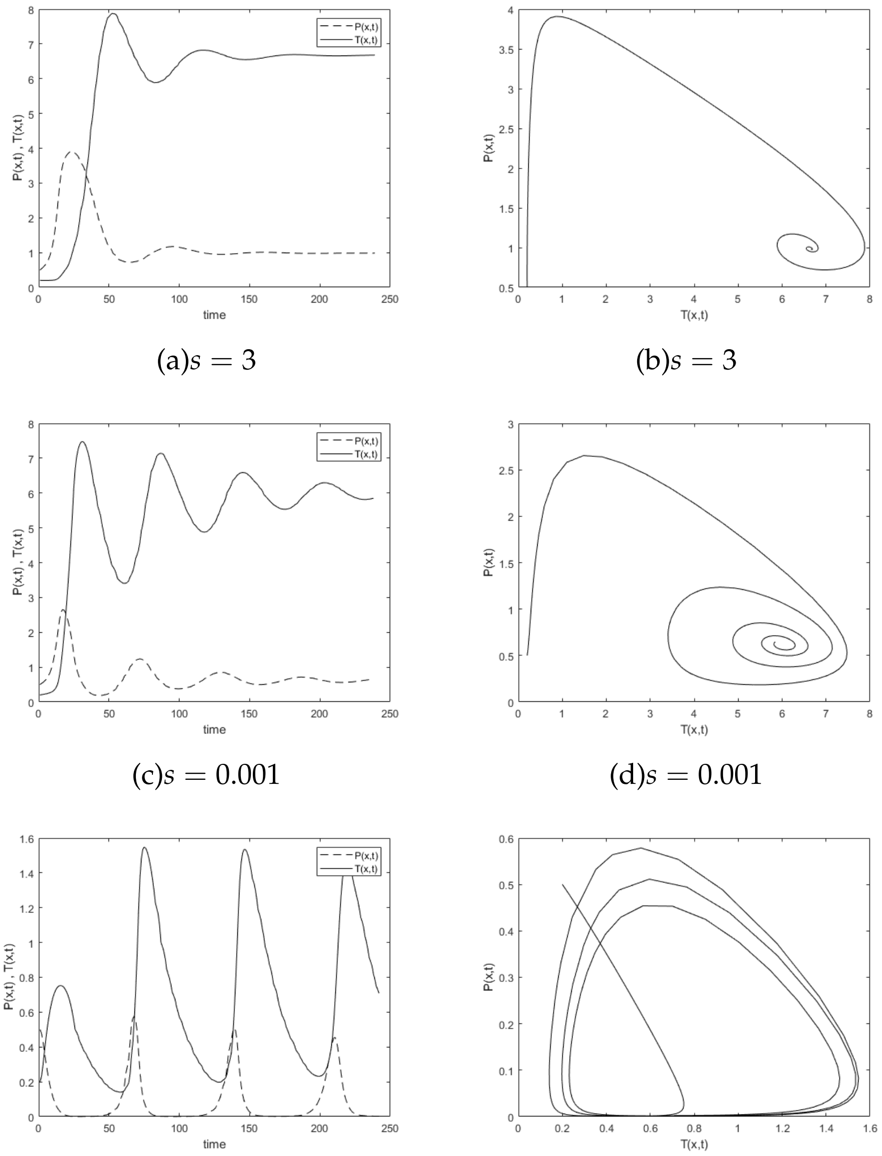
where K is the carrying capacity and b is the intrinsic growth rate. The results of this assumption are presented in Fig. 4.1. In Fig. 4.1, we depict the time-series plots of $P(x,t)$ and $T(x,t)$ in the first column and phase-plots of preys over predators in the second column at different values of s. When $s = 3$, the interior equilibria $E^* = (P_1, T_1) = (0.982, 6.676)$ is locally asymptotically stable as in Fig. 4.1(a)(b). The interior equilibria $E_1^* = (P_1, T_1)$ is still locally asymptotically stable even when we reduce the value of s to be equal 0.001. With this choice, the interior equilibria is $E^* = (P_1, T_1) = (0.64, 5.829)$. The system produces larger oscillations as we decrease the value of s as demonstrated in Fig. 4.1(e)(f).

If the prey population grows exponentially, we set

$$f(P) = bP(x,t).$$

The impact of the home-range size s on the dynamics of prey and predators is demonstrated in Fig. 4.2 and Fig. 4.3. Fig. 4.2 and Fig. 4.3 also display the time-series plots of $P(x,t)$ and $T(x,t)$ in the first column and phase-plots of prey over predators in the second column with different values of s. The dynamics of prey and predators produce periodic oscillations around the interior equilibria $E^* = (P_1, T_1)$ as depicted in Fig. 4.2 and Fig. 4.3. This suggests the interior equilibria $E^* = (P_1, T_1)$ is unstable. Also, the figures show that the oscillation amplitude increases with the value of s. In addition, the system with exponential prey growth exhibits limit cycles. The duration of these cycles decreases as we decrease the value of s as shown in Fig. 4.2 and Fig. 4.3. Another lesson from these figures is that the two species coexist in the environment. When $s = 0.00001$, preys and predators have oscillating populations as depicted in Fig. 4.2(f).

Letting $b = 0.5$ gives different dynamical behavior to the prey and predator population, as shown by Fig. 4.3. Choosing $b = 0.5$ and $s = 0.0001$ produces periodic solutions for both species as in Fig. 4.3(a)(b). This suggests that the populations of prey and predator will survive. Decreasing the value of s to 0.000001 and keeping $b = 0.5$ show that the populations of prey and predator struggle to survive and will go extinct as in Fig. 4.3(c)(d).

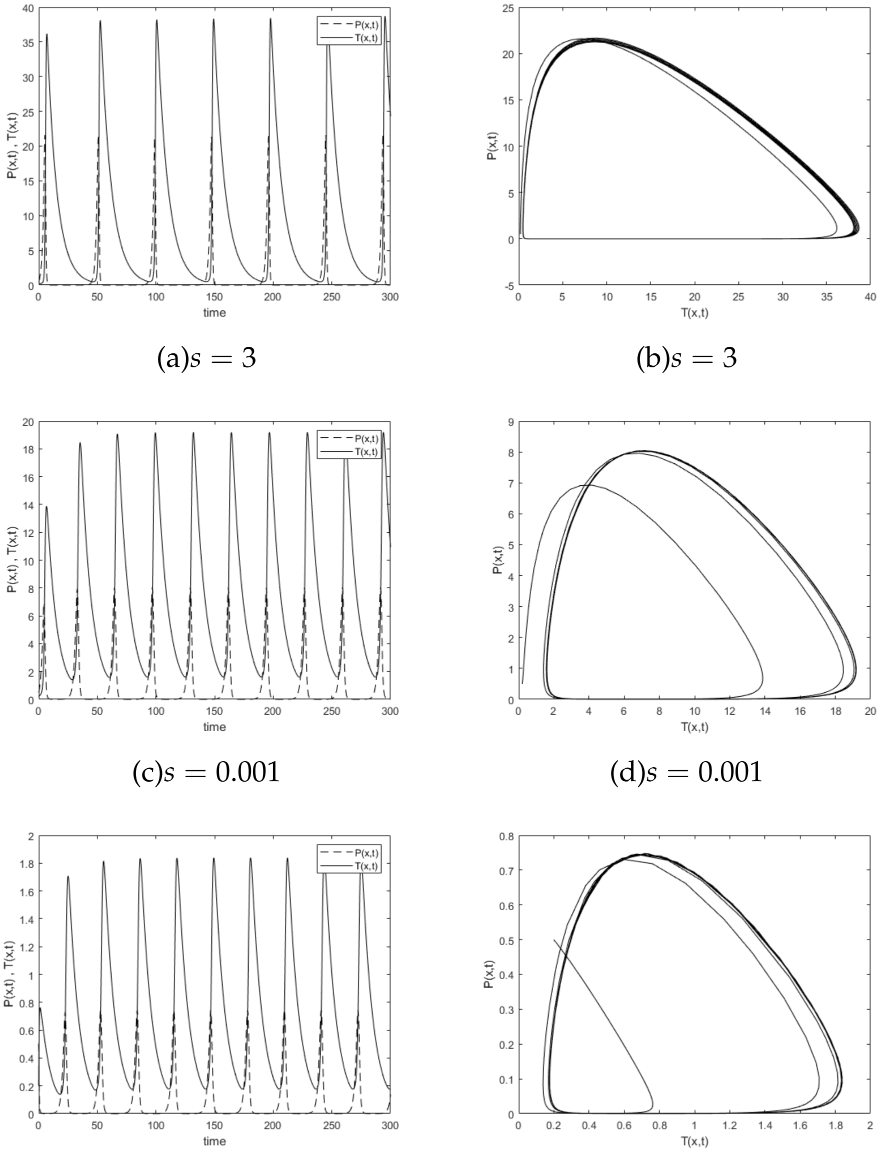


(a)$s = 3$ (b)$s = 3$

(c)$s = 0.001$ (d)$s = 0.001$

(e)$s = 0.00001$ (f)$s = 0.00001$

FIGURE 4.1. The effects of the home-range size s on the dynamics of prey and predators. The first column displays the time-series plots of $P(x,t)$ and $T(x,t)$. The prey population is denoted by (dashed line), while the population of predators is denoted by (solid line). The second column shows phase-plots of prey over predators. Here $f(P) = bP(x,t)\left(1 - \frac{P(x,t)}{K}\right)$. Other parameter values are in Table 1.

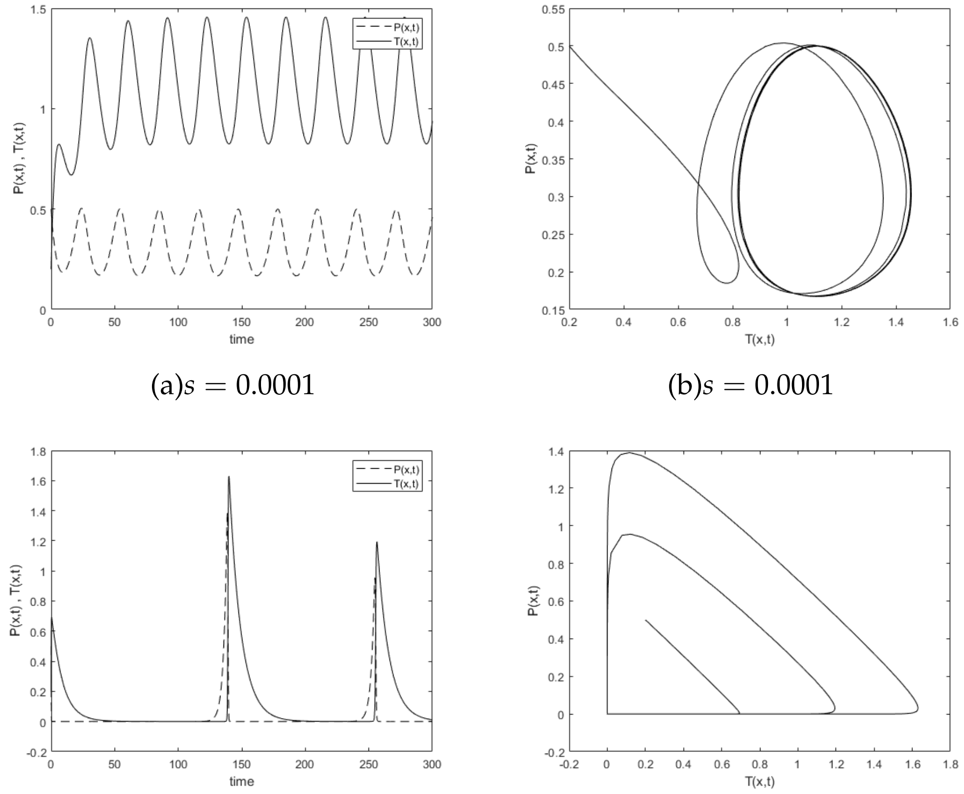


(a)$s = 3$ (b)$s = 3$

(c)$s = 0.001$ (d)$s = 0.001$

(e)$s = 0.00001$ (f)$s = 0.00001$

FIGURE 4.2. The influence of the home-range size s on the dynamics of prey and predators. The first column displays the time-series plots of $P(x,t)$ and $T(x,t)$. The population of prey is denoted by (dashed line), while the population of predators is denoted by (solid line). The second column shows phase-plots of prey over predators. Here $f(P) = bP(x,t)$. Other parameter values are in Table 1.



(a)$s = 0.0001$ (b)$s = 0.0001$

(c)$s = 0.000001$ (d)$s = 0.000001$

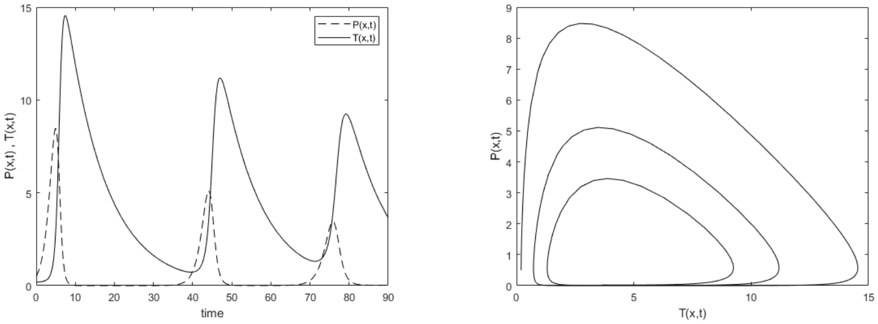
FIGURE 4.3. More plots showing the impact of s on the dynamical behavior of the PDE system. The first column displays the time-series plots of $P(x,t)$ and $T(x,t)$. The prey population is denoted by (dashed line), while the population of predators is denoted by (solid line). The second column shows phase-plots of prey over predators. Here $f(P) = bP(x,t)$ and $b = 0.5$. Other parameter values are in Table 1.

5. The impact of diffusion coefficients on the dynamical behavior of prey and predators

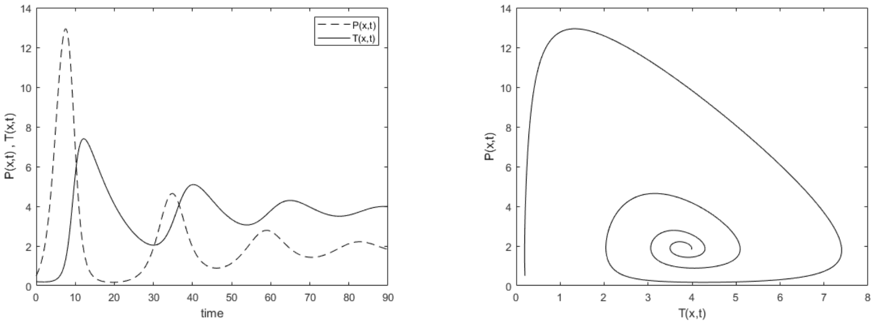
In this part, we examine the influence of the diffusion coefficients d_p and d_T on the dynamical behavior of the densities of prey and predator when $f(P) = bP(x,t)\left(1 - \frac{P(x,t)}{K}\right)$ and $f(P) = bP(x,t)$ for three different scenarios.

We first discuss the case when $f(P) = bP(x,t)\left(1 - \frac{P(x,t)}{K}\right)$. In the first scenario, we assume $d_p = d_T = 0$. This assumption demonstrates that the population densities of prey and predators decrease over time and go to zero for a long time. In this case, the equilibria $E^* = (0,0)$ is asymptotically stable as in Fig. 5.1(a)(b). In the second scenario, we let $d_p = 2 \geq d_T = 0$. This scenario shows that population densities of prey and predator will survive and both populations will reach the equilibria $E^* = (P_1, T_1) = (1.9, 3.9)$, which is locally asymptotically stable as in Fig. 5.1(c)(d). In the last scenario, we assume $d_p = 0 \leq d_T = 2$. This choice also leads to the extinction of prey much faster than the first scenario when both diffusion coefficients are zeros. As a result of the extinction of the prey population, predator density will also go extinct. Therefore, both population densities will reach the the equilibria $E^* = (0,0)$ over time as shown in Fig. 5.1(e)(f).

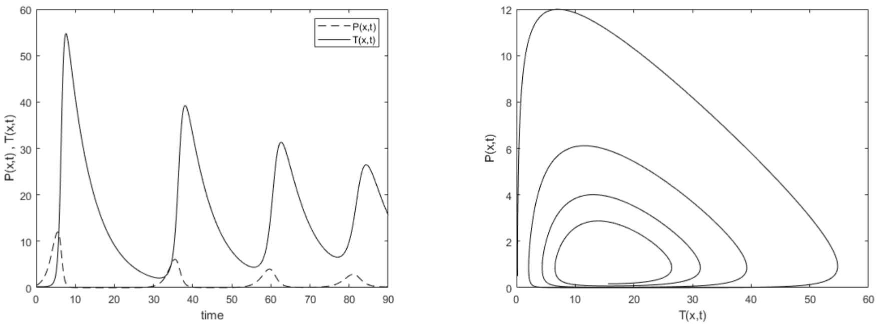
In the second case, we consider $f(P) = bP(x,t)$ for three scenarios. The amplitude of oscillation of the predator density $T(x,t)$ is higher than the amplitude of oscillation of the prey density $P(x,t)$ when $d_p = d_T = 0$ and $d_p = 0 \leq d_T = 2$ as shown in Fig. 5.2(a)(e). When $d_p = 2 \geq d_T = 0$, the amplitude of oscillation of the prey density $P(x,t)$ is higher than the amplitude of oscillation of the predator density $T(x,t)$ as reflected in Fig. 5.2(c). However, stable limit cycles arise for all the scenarios we consider, as demonstrated in Fig. 5.2(b)(d)(f).



(a)$d_p = d_T = 0$ (b)$d_p = d_T = 0$

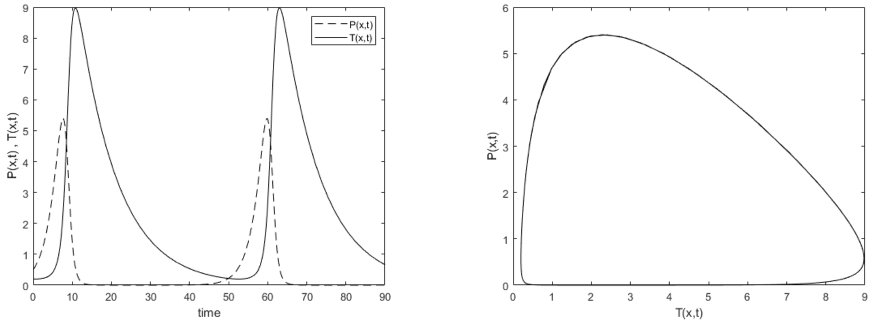


(c)$d_p = 2, d_T = 0$ (d)$d_p = 2, d_T = 0$

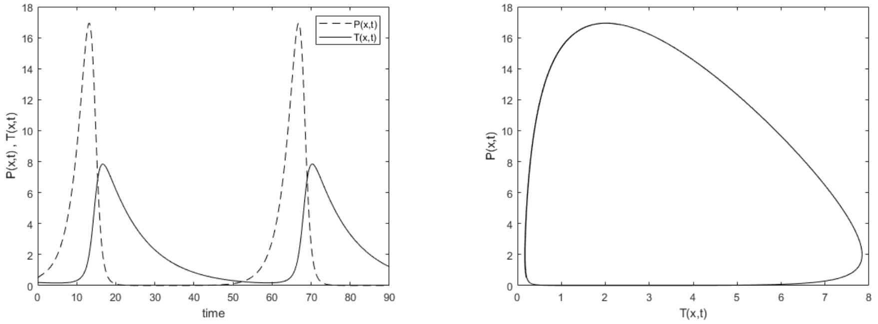


(e)$d_p = 0, d_T = 2$ (f)$d_p = 0, d_T = 2$

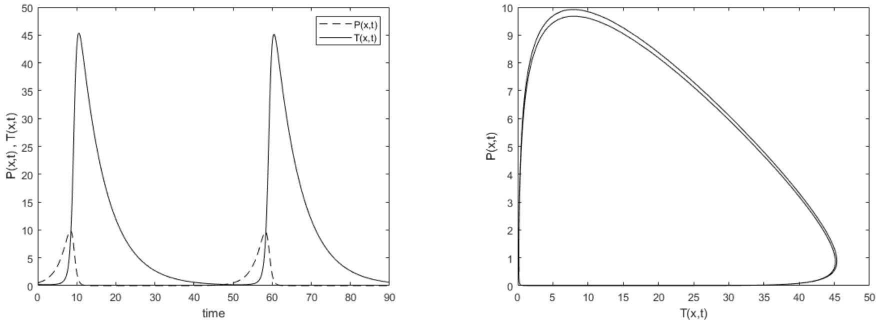
FIGURE 5.1. The influence of the diffusion coefficients d_p and d_T on the dynamical behaviors of preys P and predators T. The first column displays the time-series plots of $P(x,t)$ and $T(x,t)$. The prey population is denoted by (dashed line), while the population of predators is denoted by (solid line). The second column shows phase-plots of prey over predators. Here $f(P) = bP(x,t)\left(1 - \frac{P(x,t)}{K}\right)$. Other parameter values are in Table 1.



(a)$d_p = d_T = 0$ (b)$d_p = d_T = 0$



(c)$d_p = 2, d_T = 0$ (d)$d_p = 2, d_T = 0$



(e)$d_p = 0, d_T = 2$ (f)$d_p = 0, d_T = 2$

FIGURE 5.2. The influence of the diffusion coefficients d_p and d_T on the dynamical behaviors of preys P and predators T. The first column displays the time-series plots of $P(x,t)$ and $T(x,t)$. The prey population is denoted by (dashed line), while the population of predators is denoted by (solid line). The second column shows phase-plots of prey over predators. Here $f(P) = bP(x,t)$. Other parameter values are in Table 1.

6. Conclusion

In this paper, we discussed a prey-predator model that incorporates constant diffusion coefficients and home ranges. This work aims to understand the impact of diffusion and home ranges on the dynamic behavior of prey and predator populations. We solve the model numerically to fully understand the dynamics of the two species. The first solution assumes the prey population grows according to the logistic growth function. As we increase the carrying capacity K, the oscillation amplitude increases for the prey and predator densities. When we reduce the value of the carrying capacity K, the oscillatory approaches to a steady state for the prey and predator populations. The second numerical solution assumes the prey model grows exponentially. This case shows that the prey and predator have oscillating populations. When the prey growth rate is set to zero, the solution approaches the steady state $E* = (0,0)$ for both cases due to the rarity of the prey.

We also examine the impact of home ranges on the population dynamics when the movement of the two species distribute normally, i.e.,

$$\omega(x-z) = \frac{1}{\sqrt{4\pi s}} e^{-(x-z)^2/(4s)}.$$

$s > 0$ describes how far a prey or a predator could go from the center of their homes. The results demonstrate that the amplitude of oscillation decreases as we increase the value of s when $f(P) = bP(x,t)\Big(1 - \frac{P(x,t)}{K}\Big)$. Therefore, the stability of oscillatory coexistence of the populations increases as we increase s. On the other hand, the amplitude of oscillation increases as we increase s when $f(P) = bP(x,t)$ and $b = 0.9$. The two species would go extinct for a very small value of s as suggested by Fig. 4.3(e)(f).

The last section discusses the impact of diffusion coefficients on the dynamical behavior of prey and predator populations when $f(P) = bP(x,t)\Big(1 - \frac{P(x,t)}{K}\Big)$ and $f(P) = bP(x,t)$. The values of the diffusion coefficients have a major influence on the dynamic behaviors of prey and predator population densities. The two species coexist for both cases when $d_p = 2 \geq d_T = 0$, while the prey population struggles to survive when $d_p = d_T = 0$ or $d_p = 0 \leq d_T = 2$.

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