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Optimizing Portfolio Efficiency in the Digital Era: A Data Envelopment Analysis of Range-Rebalanced Asset Investments

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Abstract: In the digital era, the advent of new asset classes like cryptocurrencies and the application of advanced analytical tools have significantly reshaped portfolio management. This study employs Data Envelopment Analysis (DEA) to assess the efficiency of range-rebalanced investment portfolios incorporating diverse assets such as cryptocurrencies, major currencies, technology securities, and commodities. The analysis spans from October 1, 2016, to June 30, 2022, evaluating various rebalancing strategies including Allowed Range, Threshold, Drifting Mix, and Tactical approaches during different market conditions, including pre-COVID-19, during COVID-19, and post-COVID-19 periods. The findings highlight the superiority of strategic rebalancing, particularly combining high-value cryptocurrencies with technology securities, in enhancing portfolio performance and risk management. This research provides valuable insights for optimizing asset allocation in the dynamic financial landscape, underscoring the importance of strategic rebalancing in maximizing returns while managing risk.

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1. INTRODUCTION

In the digital era, the landscape of portfolio management has evolved significantly, influenced by the advent of new asset classes and advanced analytical tools. The global market capitalization of cryptocurrencies alone reached approximately \$2 trillion by mid-2021, reflecting their growing importance in investment strategies [1]. Efficient portfolio management, particularly in the context of range-rebalanced asset investments, is critical for optimizing returns while managing risks. Recent studies have shown that portfolios incorporating a diverse mix of assets, including cryptocurrencies, can achieve a Sharpe ratio increase of up to 1.2 compared to traditional asset-only portfolios [2]. Additionally, the volatility of cryptocurrencies, which can be as high as 80%, necessitates sophisticated tools for balancing risk and reward [3]. Data Envelopment Analysis (DEA) has been increasingly recognized as a robust tool for evaluating the performance and efficiency of investment portfolios. [4] demonstrated this in their study on the Vietnam hospitality sector, where they applied the DEA model and Malmquist productivity index to evaluate 20 companies. Their research also used the GM(1,1) model for forecasting inputs and outputs, achieving an average Mean Absolute Percentage Error (MAPE) of 7.75, indicating the model's suitability for future value predictions of Decision-Making Units (DMUs).

Data Envelopment Analysis (DEA) is a powerful tool for assessing the efficiency of decisionmaking units, such as investment portfolios, by considering multiple input and output factors. DEA's ability to incorporate various performance metrics makes it particularly suitable for evaluating the financial efficiency of portfolios with mixed asset classes [5]. By applying DEA, investors can identify optimal asset allocation strategies that maximize returns relative to risk within predefined ranges, enhancing the overall efficiency of their investment decisions [6]. Furthermore, DEA's application in financial contexts has been shown to improve decision-making processes by providing clear efficiency benchmarks and performance insights [7].

Several studies have highlighted the effectiveness of DEA in financial applications, demonstrating its versatility and robust analytical capabilities. For instance, [4] applied DEA to measure the efficiency of the hospitality sector in Vietnam, showing how DEA can evaluate performance across various operational metrics. [6] reviewed robust optimization approaches within DEA, illustrating how DEA can handle uncertainty in financial data, enhancing its applicability in dynamic financial markets. [8] utilized DEA to evaluate socially responsible investments, providing insights into how DEA can be used to measure financial efficiency while

incorporating ethical considerations. [9] discussed the application of DEA in the context of industrial engineering, which, although not directly related to financial portfolios, underscores DEA's broad applicability in evaluating complex datasets.

Additionally, [10] applied DEA to assess the efficiency of Chinese commercial banks, showing significant differences in efficiency scores across different banks. [11] explored the application of sentiment analysis within DEA frameworks, highlighting its role in financial market analysis. [7] provided a thorough survey of DEA literature, showcasing its historical development and diverse applications in various sectors. [12] examined the efficiency of renewable energy investments using DEA, indicating the methodological flexibility of DEA. [13] examined Bayesian machine learning principles within DEA contexts, suggesting that integrating machine learning with DEA can enhance predictive analytics in finance.

These studies collectively underscore the adaptability and effectiveness of DEA in various financial and operational contexts, reinforcing its value as a tool for optimizing investment strategies. Despite the extensive research on DEA and portfolio management, there is a notable gap in studies focusing on the integration of cryptocurrencies with traditional assets using range-rebalanced strategies. Previous research has primarily concentrated on traditional assets or treated cryptocurrencies as standalone investments. This paper addresses this gap by evaluating the efficiency of mixed-asset portfolios that include cryptocurrencies, major currencies, technology securities, and commodities, using DEA to optimize range-rebalanced strategies.

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The primary objective of this study is to employ DEA to assess the efficiency of rangerebalanced investment portfolios that incorporate a diverse set of assets. By analyzing daily closing prices from October 1, 2016, to June 30, 2022, sourced from reliable databases and verified for accuracy, this research aims to provide insights into optimal asset allocation strategies that balance risk and reward effectively.

This paper is structured as follows: The Literature Review section provides a detailed review

of previous studies related to DEA and portfolio management, highlighting the existing research gap. The Methodology section explains the data collection process, including sources and verification methods, and the DEA model used for analysis. The Empirical Analysis section presents the empirical. Rresults, including correlation, volatility, and return analyses, and the application of DEA to evaluate portfolio efficiency. The Discussion section interprets the results, comparing different rebalancing strategies and their implications for portfolio management. Finally, the Conclusion section summarizes key findings, contributions to the field, and suggestions for future research. By addressing the research gap and employing a comprehensive DEA approach, this study aims to enhance our understanding of efficient portfolio management in the digital financial era, providing valuable guidance for investors and financial professionals.

2. LITERATURE REVIEW

Background of Portfolio Theory

The principles of portfolio theory are central to managing investment portfolios, enabling investors and portfolio managers to analyze and select assets that align with their investment goals. The development of portfolio theory can be divided into different phases: Traditional Portfolio Theory (TPT), Modern Portfolio Theory (MPT), and Post-Modern Portfolio Theory (PMPT).

Traditional Portfolio Theory (TPT) involves investment analysis without a systematic approach or extensive numerical analysis to construct a portfolio. It lacks clear mathematical formulas and relies on heuristic methods and qualitative assessments. TPT is characterized by a focus on individual asset selection based on past performance, expert opinion, and market trends, without considering the interrelationships between assets.

In contrast, Modern Portfolio Theory (MPT), introduced by Harry Markowitz in the March 1952 issue of the Journal of Finance, provides a mathematical framework to optimize the risk-reward ratio. MPT represents a significant advancement by shifting the focus from individual asset analysis to portfolio analysis. It considers the variance among assets within a portfolio rather than the variance of individual asset returns [14]. MPT's key contribution is the concept of diversification, which reduces overall portfolio risk by combining assets that are not perfectly correlated. The efficient frontier, a core component of MPT, illustrates the set of optimal portfolios offering the highest expected return for a given level of risk [15].

Building on the foundation laid by MPT, Post Modern Portfolio Theory (PMPT) incorporates

additional measures to assess investment performance. As explained by [16], PMPT includes alpha and beta measurements, which are used to evaluate investment performance relative to market benchmarks and systemic risk. PMPT also emphasizes downside risk, addressing investor concerns about negative returns and providing a more comprehensive risk assessment. This approach aligns investment strategies with individual investor goals and risk tolerance, offering a tailored framework for portfolio management.

In addition to these theoretical frameworks, several empirical studies have furthered the understanding and application of portfolio theory. For example, [17] expanded on MPT by introducing the three-factor model, which includes size and value factors in addition to market risk, providing a more nuanced explanation of asset returns. This model has been widely adopted in academic research and practical applications, influencing investment strategies and portfolio construction.

Similarly, [18] contributed to the development of momentum investing strategies, which exploit the tendency of asset prices to continue moving in the same direction for a period. Their findings support the integration of momentum factors into portfolio management, enhancing returns through systematic trading rules.

Furthermore, research by [19] explored the role of macroeconomic factors in asset pricing, highlighting the importance of incorporating economic indicators into portfolio analysis. Their work underscores the dynamic nature of financial markets and the need for adaptive portfolio strategies that respond to changing economic conditions.

More recent advancements include the use of ensemble machine learning and genetic algorithms for portfolio rebalancing, as discussed by [20]. These methods bridge the gap between modern portfolio theory and the Efficient Market Hypothesis (EMH), optimizing the selection and weighting of stocks in a portfolio. This integration of advanced computational techniques into portfolio management reflects the ongoing evolution and sophistication of investment analysis.

Overall, the evolution of portfolio theory from TPT to PMPT reflects a growing sophistication in investment analysis, incorporating advanced mathematical models, empirical research, and practical considerations to optimize portfolio performance and align with investor goals. This progression underscores the continuous improvement in methodologies and tools available to investors, allowing for more precise and effective management of investment portfolios.

Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is a powerful tool used to assess the efficiency of investment portfolios, particularly in financial contexts where transaction costs are relevant. It evaluates efficiency by comparing the input and output factors of Decision-Making Units (DMUs), favoring portfolios that generate higher outputs with lower inputs. Studies have demonstrated the effectiveness of DEA in financial performance evaluation, highlighting its robustness and applicability. [4] applied DEA to measure the efficiency of the hospitality sector in Vietnam, illustrating DEA's effectiveness across various operational metrics. [6] reviewed robust optimization approaches within DEA, illustrating its capability to handle uncertainty in financial data. This method helps identify the most efficient portfolios, providing valuable insights for investment decisions.

Rebalancing

Building on the evaluation insights provided by DEA, rebalancing investment portfolios is crucial to maintain a desired level of asset allocation and optimize returns. Rebalancing involves adjusting asset weights according to predetermined guidelines. [21] describe several rebalancing strategies that investors can employ. One common approach is Calendar Rebalancing, where asset allocation is adjusted at set intervals, such as monthly, quarterly, or annually. This method considers time constraints, transaction costs, and the proportions of assets in the portfolio. Regular rebalancing ensures that the portfolio remains aligned with the investor's objectives and risk tolerance, maintaining a balance between risk and return.

Range Rebalancing is another effective strategy, allowing asset weights to fluctuate within predefined limits before rebalancing to target levels. This method includes two sub-strategies: rebalancing to the allowed range and rebalancing when asset weights reach specified thresholds. By permitting some flexibility, this approach can reduce transaction costs and minimize the impact of frequent trading while ensuring that the portfolio does not deviate significantly from its desired allocation.

Moreover, Tactical Rebalancing involves custom strategies tailored by investors to optimize specific objectives. These strategies often involve dynamic adjustments based on market conditions, economic forecasts, or changes in the investor's circumstances [22]. Conversely, Drifting Mix allows asset weights to drift without regular rebalancing. However, [23] suggest that timely rebalancing is necessary to incorporate new assets and adjust weights effectively. This approach requires monitoring and adjustments to ensure that the portfolio remains aligned with

long-term investment goals, despite the lack of frequent rebalancing.

3. METHOD

Data Collection

This study focuses on evaluating the efficiency of mixed-asset investment portfolios that include cryptocurrencies, major currencies, high-value technology company securities, and commodities. The population for this research includes these diverse asset classes, which are crucial for constructing a robust and diversified portfolio. The sample comprises the top five highvalue or highly traded assets from each category, selected based on their significant market presence and trading volumes. This approach is supported by previous studies that have demonstrated the effectiveness of using high-value assets to reflect market prices and enhance portfolio performance [24-31].

The data for this study were collected from secondary sources, specifically focusing on high-value cryptocurrencies, major currencies, high-value technology company securities, and highly traded commodities. Data sources included Coinbase for cryptocurrencies, Investing for major currencies, Nasdaq for technology company securities, and Bloomberg for commodities. The data collection process involved retrieving daily closing prices of the selected assets from October 1, 2016, to June 30, 2022, using Python programming to ensure accuracy and efficiency in data handling.

The statistical analysis methods employed in this study include the correlation coefficient to analyze relationships between different assets in the portfolio, percentage returns to calculate the returns of the investment portfolio, standard deviation to measure the volatility (risk) of the portfolio, and the mean return to assess the average return relative to the risk.

Statistical Analysis

The study tests various trading strategies, including the buy-and-hold strategy, where assets are purchased on October 1, 2016, and held until June 30, 2022. It also tests periodic buyand-sell strategies at monthly, quarterly, semi-annual, and annual intervals. Furthermore, it evaluates buy-and-sell strategies during crisis periods such as pre-COVID-19, during COVID-19, post-COVID-19, and during the Russia-Ukraine conflict. Additionally, the study tests the SMA 50-200 strategy, where assets are bought when the 50-day Simple Moving Average (SMA) exceeds the 200-day SMA and sold when the 50-day SMA falls below the 200-day SMA. The rebalancing strategies tested include periodic rebalancing, where equal weights are assigned to each asset at the beginning of each period, with rebalancing occurring at set intervals (monthly, quarterly, semi-annually, annually, biannually). Range rebalancing allows asset weights to fluctuate within a specified range, with adjustments made to maintain the desired allocation. Threshold rebalancing involves rebalancing when asset weights move beyond predefined thresholds. The drifting mix strategy allows asset weights to drift naturally without regular rebalancing. Tactical rebalancing involves custom strategies based on criteria such as best returns, maximum Sharpe ratio, and minimum drawdown [32].

Finally, the study employs Data Envelopment Analysis (DEA) to evaluate the efficiency of the portfolios under various trading and rebalancing strategies, providing a comprehensive analysis of the efficiency of mixed-asset portfolios in dynamic financial markets.

4. RESULTS

This study evaluates the performance of investment portfolios using range-rebalanced strategies between two asset classes: cryptocurrencies and other traditional assets. The results are presented for two main rebalancing approaches: Allowed Range and Threshold Rebalancing.

Allowed Range Rebalancing

In the Allowed Range rebalancing strategy, asset weights are maintained within a set range, allowing them to increase up to 60% and decrease to 40% of the total portfolio weight. The rebalancing occurs when weights exceed these limits, ensuring the portfolio stays within the desired range.

The results indicate that the portfolio comprising six high-value cryptocurrencies and five high-value technology company securities yielded the highest return of 45,141.84% throughout the study period. This was followed by the portfolio of five high-value cryptocurrencies and five major currencies, which returned 39,552.59%. The portfolio combining seven high-value cryptocurrencies with five highly traded commodities showed the lowest return of 31,465.82%.

In terms of the Sharpe ratio, which measures the risk-adjusted return, the portfolio with high-value cryptocurrencies and technology company securities again ranked highest with a Sharpe ratio of 188.57. The portfolio of cryptocurrencies and major currencies followed with a Sharpe ratio of 172.78, while the portfolio with commodities had the lowest Sharpe ratio of 131.39. These findings are summarized in Table 1.

Portfolio	Return (%)	Standard Deviation (SD)	Sharpe Ratio
Portfolio 5	39,552.59	228.91	172.78
Portfolio 6	45,141.84	239.37	188.57
Portfolio 7	31,465.82	239.47	131.39

 Table 1 Allowed Range Rebalancing Results

The DEA efficiency scores for portfolios rebalanced using the Allowed Range method during different crisis periods, including pre-COVID-19, during COVID-19, post-COVID-19, and the Russia-Ukraine conflict, were also analyzed. The results indicated that Portfolio 7, combining five high-value cryptocurrencies with five high-trade-volume commodities, had the highest efficiency score of 1.0544 post-COVID-19. Portfolio 6, with five high-value cryptocurrencies and five high-value technology company securities, had an efficiency score of 1.0204 pre-COVID-19 and 0.9776 post-COVID-19. Portfolio 5, consisting of five high-value cryptocurrencies and five major currencies, had an efficiency score of 0.9744 post-COVID-19.

Period	Portfolio5	Portfolio6	Portfolio7
1/11/2018 - 31/10/2019 (Pre-COVID-19)	0.7412	1.0204	0.6547
1/11/2019 - 31/10/2020 (During-COVID-19)	0.7480	0.7541	0.2121
1/11/2020 - 31/10/2021 (Post-COVID-19)	0.9744	0.9776	1.0544
24/2/2022 – 24/6/2022 (Russia Vs Ukraine)	0.0000	0.0000	0.0000

 Table 2 Efficiency Scores for Allowed Range Rebalancing

Threshold Rebalancing

In the Threshold Rebalancing strategy, assets are rebalanced back to equal weights when they exceed set thresholds, which in this study are set at 60% for the highest performing assets and 40% for the lowest performing ones.

The results show that the portfolio with high-value cryptocurrencies and technology company securities also performed best under this strategy, with a return of 22,076.67%. This portfolio was followed by the one combining high-value cryptocurrencies and major currencies, which returned 16,649.55%. The portfolio mixing high-value cryptocurrencies with commodities returned the lowest at 15,317.37%.

The Sharpe ratio analysis for this strategy shows that the portfolio of high-value cryptocurrencies and technology company securities had the highest Sharpe ratio of 95.83. The next best Sharpe ratio was 80.41 for the portfolio of high-value cryptocurrencies and major

currencies. The portfolio including commodities had the lowest Sharpe ratio of 66.49. These findings are summarized in Table 3.

Portfolio	Return (%)	Standard Deviation (SD)	Sharpe Ratio
Portfolio 5	16,649.55	207.04	80.41
Portfolio 6	22,076.67	230.36	95.83
Portfolio 7	15,317.37	230.34	66.49

Table 3 Threshold Rebalancing Results

These results demonstrate that portfolios with high-value cryptocurrencies and technology company securities generally provide higher returns and better risk-adjusted performance compared to portfolios that include major currencies or commodities. This highlights the potential benefits of incorporating a range-rebalanced strategy in managing investment portfolios in the dynamic financial market.

The DEA efficiency scores for portfolios rebalanced using the Threshold method during different crisis periods were evaluated. Post-COVID-19, Portfolio 7 had the highest efficiency score of 1.0869. Portfolio 6 scored 0.9769, and Portfolio 5 scored 0.9740 for the same period. During COVID-19, Portfolio 6 had an efficiency score of 0.7357, and Portfolio 5 scored 0.7256.

Period	Portfolio5	Portfolio6	Portfolio7
1/11/2018 - 31/10/2019 (Pre-COVID-19)	0.9390	0.6228	0.6143
1/11/2019 - 31/10/2020 (During-COVID-19)	0.7256	0.7357	0.0000
1/11/2020 - 31/10/2021 (Post-COVID-19)	0.9740	0.9769	1.0869
24/2/2022 - 24/6/2022 (Russia Vs Ukraine)	0.0000	0.0000	0.0000

Table 4 Efficiency Scores for Threshold Rebalancing

Drifting Mix Rebalancing

The rebalancing of investment portfolios using the Drifting Mix approach, which adjusts asset weights based on their performance, was analyzed across two asset categories. The study found that Portfolio 6, consisting of five high-value cryptocurrencies and five high-value technology company securities, yielded the highest return of 82,374.68%. This was followed by Portfolio 7, comprising five high-value cryptocurrencies and five high-trade-volume commodities, with a return of 57,444.15%. Portfolio 5, which included five high-value cryptocurrencies and five high-value

Regarding the Sharpe ratio, which measures risk-adjusted return, Portfolio 6 also performed the best with a Sharpe ratio of 295.37. Portfolio 7 had a Sharpe ratio of 206.40, while Portfolio 5 had the lowest Sharpe ratio of 171.19. These results are summarized in Table 5 below. **Table 5** Results of Drifting Mix Rebalancing Between Two Asset Categories

Rebalancing Method	Portfolio 5	Portfolio 6	Portfolio 7
Return (%)	39,182.63	82,374.68	57,444.15
Standard Deviation	228.87	278.88	278.31
Sharpe Ratio	171.19	295.37	206.40

These findings indicate that the Drifting Mix rebalancing strategy, particularly when combining high-value cryptocurrencies with high-value technology company securities, can significantly enhance portfolio returns and improve the Sharpe ratio. This approach highlights the importance of strategic asset selection and rebalancing to maximize investment performance in the digital financial era.

The DEA efficiency scores for portfolios rebalanced using the Drifting Mix method during different crisis periods were analyzed. Post-COVID-19, Portfolio 7 had the highest efficiency score of 1.0582, followed by Portfolio 6 with 0.9772 and Portfolio 5 with 0.9746. During COVID-19, Portfolio 6 had an efficiency score of 0.7722, and Portfolio 5 scored 0.7688.

Table 6 Efficiency Scores for Drifting Mix Rebalancing

Period	Portfolio5	Portfolio6	Portfolio7
1/11/2018 - 31/10/2019 (Pre-COVID-19)	0.6278	0.6700	0.6659
1/11/2019 - 31/10/2020 (During-COVID-19)	0.7688	0.7722	0.2267
1/11/2020 – 31/10/2021 (Post-COVID-19)	0.9746	0.9772	1.0582
24/2/2022 – 24/6/2022 (Russia Vs Ukraine)	0.0000	0.0000	0.0000

Data Envelopment Analysis (DEA) for Buy-and-Hold Strategy

The efficiency of various investment portfolios was analyzed using the Data Envelopment Analysis (DEA) method under a buy-and-hold strategy throughout the study period. The findings indicated that Portfolio 1, which consists of five high-value cryptocurrencies, exhibited the highest efficiency score of 2.1550 from the start of the study period until the 60th month. This was followed by Portfolio 1 (the same five high-value cryptocurrencies) evaluated up to the 69th month, with an efficiency score of 0.4833. Portfolio 6, combining five high-value cryptocurrencies with five high-value technology company securities, showed an efficiency score of 0.2660 up to the 60th month. Other notable portfolios include:

Portfolio 5: Five high-value cryptocurrencies and five major currencies up to the 60th month (Efficiency score: 0.1413).

Portfolio 6: Five high-value cryptocurrencies and five high-value technology company securities up to the 69th month (Efficiency score: 0.1385).

Portfolio 7: Five high-value cryptocurrencies and five high-trade-volume commodities up to the 60th month (Efficiency score: 0.1106).

The efficiency scores for the remaining portfolios and time periods varied, with the lowest efficiency score observed for Portfolio 4, which included five high-trade-volume commodities, with a score of 0.0005 during the initial study month.

Period	Portfolio1	Portfolio2	Portfolio3	Portfolio4	Portfolio5	Portfolio6	Portfolio7
1/10/2016-	0.0000	0.0000	0.0005	0.0000	0.0000	0.0000	0.0000
31/10/2016							
1/10/2016-	0.0000	0.0009	0.0000	0.0000	0.0000	0.0000	0.0000
31/12/2016							
1/10/2016-	0.0153	0.0021	0.0060	0.0005	0.0132	0.0150	0.0123
31/3/2017	0.0100	0100_1	010000	010000	010101	010100	010120
1/10/2016-	0.0553	0.0063	0.0088	0.0000	0.0289	0.0324	0.0276
30/9/2017	0.0000	0.0000	0.0000	0.0000	0.0209	0.0021	0.0270
1/10/2016-	0.1072	0.0029	0.0146	0.0000	0.0412	0.0551	0.0402
30/9/2018	0.1072	0.002	0.0110	0.0000	0.0112	0.0001	0.0102
1/10/2016-	0.0744	0.0000	0.0108	0.0000	0.0328	0.0455	0.0326
30/9/2019	0.07 11	0.0000	0.0100	0.0000	0.0020	0.0100	0.0020
1/10/2016-	0.0944	0.0010	0.0175	0.0000	0.0386	0.0643	0.0221
30/9/2020	0.0711	0.0010	0.0175	0.0000	0.0000	0.0040	0.0221
1/10/2016-	0.1550	0.0014	0.0240	0.0000	0.1413	0.2660	0.1106
30/9/2021	0.1000	0.0014	0.0240	0.0000	0.1413	0.2000	0.1100
1/10/2016-	0.4833	0.0030	0.0202	0.0000	0.0965	0.1385	0.0830
30/6/2022	0.1000	0.0000	0.0202	0.0000	0.0705	0.1000	0.0000

Table 7 Efficiency Scores for Portfolios Over Time Using DEA

The DEA analysis for time-based buy and sell strategies revealed varying efficiency scores across different portfolios and periods. The analysis indicated that certain portfolios performed exceptionally well under specific market conditions and time frames. Notably, portfolios consisting of a mix of high-value cryptocurrencies and high-value technology securities consistently demonstrated high efficiency scores compared to other asset combinations. These results underscore the potential benefits of strategic asset selection and rebalancing to optimize investment performance in a dynamic financial environment.

Tactical Rebalancing

The DEA efficiency scores for portfolios rebalanced using the Tactical method during different crisis periods were also examined. Post-COVID-19, Portfolio 5 had the highest efficiency score of 1.1799, followed by Portfolio 6 with 0.9379. During COVID-19, Portfolio 7 had the highest efficiency score of 2.3865.

Period	Portfolio5	Portfolio6	Portfolio7
1/11/2018 - 31/10/2019 (Pre-COVID-19)	0.0006	0.0006	0.0020
1/11/2019 - 31/10/2020 (During-COVID-19)	0.0000	0.0000	0.0001
1/11/2020 - 31/10/2021 (Post-COVID-19)	1.1799	0.9190	0.0027
24/2/2022 – 24/6/2022 (Russia Vs Ukraine)	0.0000	0.0000	0.0000

Table 8 Efficiency Scores for Tactical Rebalancing

Crisis Period Analysis

The efficiency scores of investment portfolios were analyzed across different rebalancing strategies and crisis periods. Tactical rebalancing strategies generally showed the highest efficiency scores, particularly during and post-COVID-19. Portfolios consisting of high-value cryptocurrencies combined with major currencies, technology company securities, and hightrade-volume commodities performed variably under different market conditions, highlighting the importance of strategic rebalancing and asset selection to optimize investment performance.

5. DISCUSSION

The results of this study underscore the significant impact of rebalancing strategies on the performance of investment portfolios that include both cryptocurrencies and traditional assets. By applying range-rebalanced strategies, we observed substantial improvements in portfolio returns while maintaining acceptable levels of risk. These findings offer valuable insights into the

effectiveness of different rebalancing approaches and their implications for investment management.

The Allowed Range rebalancing strategy demonstrated the highest effectiveness among the studied approaches. Portfolios combining high-value cryptocurrencies with high-value technology company securities yielded the most substantial returns and Sharpe ratios, indicating superior risk-adjusted performance. For instance, the portfolio that included both high-value cryptocurrencies and technology company securities achieved a return of 45,141.84% and a Sharpe ratio of 188.57. This suggests that the combination of these asset classes can significantly enhance portfolio performance, leveraging the high growth potential of cryptocurrencies and the stability of technology stocks.

Our findings align with those of [2], who highlighted the benefits of incorporating highgrowth assets like cryptocurrencies into diversified portfolios to enhance returns. Similarly, [8] demonstrated the effectiveness of Data Envelopment Analysis (DEA) in evaluating the performance of socially responsible investments, providing a methodological framework that supports our approach to optimizing mixed-asset portfolios.

However, our results also present contrasts with previous studies. [21] emphasized the benefits of Calendar Rebalancing, which involves adjusting asset allocations at set intervals. While this method ensures alignment with investment objectives and risk tolerance, our study found that Range Rebalancing strategies, which allow asset weights to fluctuate within predefined limits, provided better returns and risk management. This contrast highlights the importance of flexibility in rebalancing strategies, particularly in volatile markets.

Threshold Rebalancing Strategy

The Threshold Rebalancing strategy, while effective, showed relatively lower returns and Sharpe ratios compared to the Allowed Range strategy. For example, the portfolio that combined high-value cryptocurrencies with technology company securities under the Threshold Rebalancing strategy achieved a return of 22,076.67% and a Sharpe ratio of 95.83. This suggests that while Threshold Rebalancing provides a structured approach to maintaining portfolio balance, it may not fully capitalize on the growth potential of high-performing assets due to its more conservative nature.

Practical Implications and Future Research

The practical implications of these findings are substantial for portfolio managers and investors. The study underscores the importance of strategic rebalancing in managing diversified

portfolios, particularly those including volatile assets like cryptocurrencies. By carefully selecting and adjusting asset weights, investors can optimize returns while effectively managing risk.

This study also highlights the potential of combining traditional assets with modern, highgrowth assets to achieve superior portfolio performance. This approach aligns with contemporary investment strategies advocating for diversification across a broad range of asset classes to enhance portfolio resilience and growth [14].

Limitations and Future Research Directions

Despite the valuable insights provided, this study has limitations. The analysis focused on a specific set of high-value assets and a defined study period, which may not fully capture the long-term dynamics of asset performance. Additionally, transaction costs and tax implications were not accounted for, which can significantly impact net returns.

Future research should explore a broader range of assets and longer study periods to validate the findings. Moreover, incorporating transaction costs and tax considerations into the analysis would provide a more comprehensive understanding of the practical implications of different rebalancing strategies.

6. CONCLUSION

The analysis of investment portfolios using Data Envelopment Analysis (DEA) under various rebalancing strategies during different crisis periods has provided valuable insights into the effectiveness of each strategy. The findings highlight the importance of strategic asset rebalancing in optimizing investment performance, particularly in volatile market conditions.

Time-based rebalancing proved to be highly effective, particularly for Portfolio 5, which combined high-value cryptocurrencies with major currencies. This portfolio achieved the highest efficiency score of 1.2829 from months 61 to 69, indicating that this method can capitalize on the strong growth periods of certain asset classes. The success of this strategy underscores the potential of time-based approaches to harness the growth potential of volatile assets like cryptocurrencies over specific periods.

The Allowed Range rebalancing strategy also demonstrated significant effectiveness, especially in the post-COVID-19 period. Portfolio 7, which included high-value cryptocurrencies and high-trade-volume commodities, achieved an efficiency score of 1.0544. This strategy, which sets upper and lower bounds for asset weights, helps maintain portfolio balance and performance during recovery phases. Similarly, the Threshold rebalancing method proved beneficial, with

Portfolio 7 achieving an efficiency score of 1.0869 post-COVID-19, highlighting the strategy's effectiveness in maintaining portfolio performance when asset weights deviate significantly from their targets.

Drifting Mix rebalancing showed varied results, performing well post-COVID-19 for Portfolio 7, which scored 1.0582. This strategy allows asset weights to drift based on performance, which can be effective in certain recovery scenarios. Tactical rebalancing emerged as a highly effective strategy, particularly during and after the COVID-19 pandemic. Portfolio 5 achieved an efficiency score of 1.1799 post-COVID-19, while Portfolio 7 reached 2.3865 during the pandemic, showcasing the strategy's flexibility and adaptability in volatile markets.

Overall, these findings suggest that strategic rebalancing is crucial for maximizing investment performance, especially during periods of market volatility and economic crises. By carefully selecting and implementing appropriate rebalancing strategies, investors can better navigate market uncertainties and achieve their investment objectives. Future research could further explore the application of these strategies to other asset classes and market conditions, potentially incorporating advanced techniques such as machine learning for predictive rebalancing to enhance portfolio performance optimization.

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