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Exploring Nonlinear Fractional (2+1)-Dimensional KP–Burgers Model: Derivation and Ion Acoustic Solitary Wave Solution

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Abstract. The purpose of this study is to conduct a complete examination into the fractional (2+1)-dimensional nonlinear KP-Burgers (KP-B) model in the setting of quantum plasma. The main goal is to present a mathematical explanation and derivation for the fractional (2+1)-dimensional nonlinear KP-B model. We used the reductive perturbation technique to obtain the fractional (2+1)-dimensional ion acoustic solitary wave, which leads to the nonlinear KP-B model. To solve the fractional space-time KP-B model, we used the modified sub-equation technique and the extended hyperbolic function methodology. This methodology covers rational, periodic wave, and hyperbolic function solutions. Ion acoustic waves were explored in relation to the effects of ion pressure and an external electric field. The effects of density and fractional order on the properties of a single solution are examined. The plasma system is defined as an unmagnetized epi plasma system composed of relativistic ions, positrons, and nonextensive electrons that can be found in a range of astrophysical and cosmological environments. The solution variables include electron-positron and ion-electron temperature ratios, electron and positron nonextensivity strength, ion kinematic viscosity, positron concentration, and the weakly relativistic streaming factor. The fractional order and associated plasma properties have a considerable influence on the phase velocity of ion acoustic waves. When the fractional order equals one, the obtained results are consistent with known outcomes. This work makes a substantial contribution to our understanding of nonlinear events in quantum plasmas, particularly ion acoustic waves. It provides a solid approach for solution development and analysis, with implications for a variety of astrophysical and cosmological scenarios.

1. Introduction

Three well-known formulas used in mathematics relate to quantum plasma. The model of quantum hydrodynamics (QHD) is the first and most well-liked model. This model serves as

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a fundamental framework for understanding the behavior of quantum plasmas. QHD describes the collective behavior of charged particles in a quantum mechanical framework, incorporating effects such as quantum statistics, quantum pressure, and quantum potential into the dynamics of the plasma. It provides a macroscopic description of quantum plasmas by combining classical fluid equations with quantum mechanical principles [1- 5]. For classical plasma fluid, the transport equations extend the principles of classical fluid dynamics to plasma systems, considering the conservation of momentum and energy of the plasma particles. They form a basis for understanding the macroscopic behavior of classical plasmas and can be viewed as a special case or a generalization of the QHD model when quantum effects are negligible. Researchers frequently aim to formulate and scrutinize nonlinear fractional differential equations (FDEs) or partial differential equations (PDEs) to investigate wave propagation and structural dynamics within quantum plasmas. By combining the QHD model with appropriate mathematical techniques and physical considerations, such as nonlinearities and quantum effects, researchers aim to develop models that accurately capture the complex behavior of quantum plasma waves. The combination of these models and equations enables researchers to gain insights into the behavior of quantum plasmas across various scales and conditions, from microscopic quantum effects to macroscopic plasma dynamics [1-27].

In investigations centered on unmagnetized ion-electron-positron quantum plasmas, researchers explore both linear and nonlinear ion-acoustic (IA) waves. Khan and Mushtaq delved into the properties and stability of ion-acoustic waves (IAWs) within ultracold quantum plasmas composed of electrons, positrons, and ions. Utilizing the Quasi-Hydrodynamic (QHD) model, they derived the Kadomtsev–Petviashvili (KP) equation to scrutinize the dynamics of IAWs under the influence of transverse perturbations. Khan and Haque utilized the QHD model to derive the nonlinear weakly limit of the deformed KdV-B model. This model helps in understanding the nonlinear dynamics of ion-acoustic waves in quantum plasmas. Khan and Masood made a significant discovery related to the Zabusky-Kruskal (ZK) equation, which was initially suggested thirty years ago. This equation has been employed to study ion-acoustic solitary waves (IASWs) in magnetized plasma. Their discovery likely contributes to further advancements in understanding IASWs in unmagnetized quantum plasmas. These research efforts demonstrate the ongoing exploration of various mathematical models and equations, such as the QHD model and derived nonlinear equations, to deepen our understanding of ion-acoustic waves and related phenomena in quantum plasmas [1- 20].

The investigation into the quantum ZK equation, which arises in quantum magneto plasma, involved employing sophisticated mathematical techniques such as the Hirota bilinear approach and the auxiliary equation method. These methods facilitated the derivation of both single-wave and multiple-soliton solutions through symbolic computation. The quantum ZK equation plays a crucial role in understanding electrostatic wave propagation in nonlinear media characterized by both dispersive and dissipative properties. In one dimension, the dynamics of electrostatic wave propagation are governed by the KdV-B model, while in two dimensions, the KP-B model comes into play. These models incorporate both dispersive and dissipative effects, offering a comprehensive framework for studying nonlinear phenomena in quantum plasma systems. The dissipative factor present in the nonlinear KdV-B and KP-B models originates from the influence of kinematic viscosity on the plasma components. This factor accounts for how the motion of plasma particles is affected by internal friction or viscosity, leading to the dissipation of energy and the damping of wave oscillations. The extensive use of mathematical methods and models underscores the interdisciplinary nature of research in quantum plasma physics, where theoretical advancements are crucial for understanding complex plasma dynamics and phenomena.

The study of nonlinear waves such as solitons and solitary waves holds great significance in various fields, including technical and laboratory research, as well as astrophysical and space contexts. These waves have been observed in phenomena such as polar magnetospheres, solar wind, and Earth's magnetotail, highlighting their importance in understanding complex plasma dynamics. Solitons and solitary waves are intriguing aspects of nonlinear events in spatially expanded systems.

They represent locally optimized solutions to nonlinear partial differential equations and can exhibit positive or negative wave amplitudes. Their potential applications range from laboratory experiments to astrophysical observations. Mahmood et al. investigated quantum electron-ion (e-i) IASWs in plasma using the Sagdeev potential technique. Masood used a Quantum Magneto Hydrodynamics (QMHD) model to study both nonlinear and linear propagating magneto-acoustic waves in dissipative quantum magneto plasmas. Akhtar and Hussain investigated shock waves of quantum ion-acoustic waves in a negative ion quantum plasma using a QHD approach. Despite these advancements, most studies have been limited to planar geometry in one dimension, which may not accurately reflect the complexity of wave behavior observed in laboratory settings. Laboratory waves are typically not confined to one dimension, highlighting the need for further research to explore multi-dimensional wave phenomena. Continued investigation into multi-dimensional wave behavior in quantum plasma systems is essential for advancing our understanding of complex plasma dynamics and for potential applications in various fields. This may involve developing theoretical models, conducting numerical simulations, and performing experimental studies in laboratory settings [10- 27].

Fractional calculus indeed holds great promise as a significant development in mathematics, with far-reaching implications across various fields. Its applications have expanded rapidly, and it has become a distinct and exciting research topic in recent years. One of the key strengths of fractional calculus lies in its ability to provide more accurate and precise representations of real-life phenomena compared to traditional integer-order calculus models. This is particularly evident in situations where systems exhibit non-local or long-range memory effects, which are effectively captured by fractional differential equations. A notable aspect of the development of fractional calculus is the discovery and establishment of multiple definitions of fractional derivatives. Each definition offers unique advantages and is applicable in different contexts. Some of the well-known definitions include Riemann–Liouville fractional derivative, Chen's fractal definitions, Kolwankar–Gangal fractional derivatives, Cresson's fractional derivatives, Caputo fractional derivative and modified Riemann–Liouville fractional derivative. Each of these definitions has found applications in various fields, including physics, engineering, biology, finance, and more. Researchers continue to explore the properties and applications of fractional calculus, paving the way for new insights and discoveries [28- 43]. As fractional calculus continues to evolve, it is likely to become a cornerstone of mathematical modeling and analysis in the 21st century, offering a powerful tool for understanding and describing complex phenomena in the natural and applied sciences. In their manuscript [39], Khalil et al. introduced the conformable fractional derivative (CFD) by establishing its definition through the limits as

$$D^{\beta}\psi(\mathrm{y})=\lim_{\varepsilon\to 0}\frac{\psi(\mathrm{y}+\varepsilon y^{1-\beta})-\psi(\mathrm{y})}{\varepsilon},\quad \forall\, y>0,\quad \beta\in(0,1],$$

$$\psi^{(\beta)}(0)=\lim_{x\to 0^{+}}{}^{(\beta)}(\mathrm{y}).$$

Substituting $\alpha=1$ into the final equations, the noninteger differentials transition into the well-established integer differentials. The CFD adheres to the following axioms:

$$D^{\beta}y^{m}=m\,y^{m-\beta},\ \ m\in R,\qquad D^{\beta}c=0,\quad \forall\psi(\mathrm{y})=c,$$

$$D^{\beta}(c_1\psi+c_2\varphi)=c_1D^{\beta}\psi+c_2D^{\beta}\varphi,\qquad \forall\, c_1,c_2\in R,$$

$$D^{\beta}(\psi\varphi)=\psi D^{\beta}\varphi+\varphi D^{\beta}\psi,$$

$$D^{\alpha}\left(\frac{\psi}{\varphi}\right)=\frac{\varphi D^{\alpha}\psi-\psi D^{\alpha}\varphi}{\varphi^{2}},,$$

$$D^{\beta}\varphi(\psi)=\frac{d\varphi}{d\psi}D^{\beta}\psi,\qquad D^{\beta}\varphi(y)=y^{1-\beta}\frac{d\varphi}{dy},,$$

With φ,ψ represents two $\alpha-$differentiable functions of a dependent variable, the above relations are proved in reference [39].

This research paper has been arranged as follows. Section two derives the basic model of factional KP-B model. In section 3, we describe the procedure and construct explicit solitary wave solution for the desired model. We discuss the outcomes in section 4. Finally we conclude the manuscript and give some prospects and discussion.

2. The formulation problem of the derivation KP-B model

We investigate fractional nonlinear shock and IASW propagation in a totally ionized, unmagnetized, three-component plasma system composed of relativistic hot ions, positrons, and nonextensive electrons. It is anticipated that the spatial fractional speed of quantum ions in acoustics will be substantially greater than any spatial fractional speed associated with the plasma flow. It can be assumed that the neutrality of charges equilibrium criteria is $n_{e0}=n_{i0}+n_{p0}$ where n_{p0}, n_{i0} and n_{e0} stands for the concentrations of ions, positrons, and unperturbed electrons, respectively. It is also assumed that the electron and positron concentrations follow an equilibrium q-distribution function. It is possible to acquire the normalized non extensive concentrations of positron and electron [44] as

$$n_e=\frac{1}{1-a}\left[1+\varphi(q-1)\right]^{\frac{1+q}{2(q-1)}}$$
$$\left(1+\frac{1}{2}\varphi(q+1)-\frac{1}{8}\varphi^2(q+1)(q-3)+\frac{1}{48}\varphi^3(q+1)(q-3)(3q-5)+\dots\right),$$

$$n_p=\frac{a}{1-a}\left[1-\sigma\varphi(q-1)\right]^{\frac{1+q}{2(q-1)}}$$
$$=\frac{a}{1-a}\left(1-\frac{\sigma}{2}\varphi(q+1)-\frac{\sigma^2}{8}\varphi^2(q+1)(q-3)-\frac{\sigma^3}{48}\varphi^3(q+1)(q-3)(3q-5)+\dots\right),$$

where $a=n_{p0}/n_{e0}$ and $\sigma=T_e/T_p$. In n_e and n_p, we use q tends to 1 for isothermal electrons and positrons, $q>1$ for sub-thermal electrons and $-1<q<1$ for super-thermal electrons. The dynamics of two-dimensional IASWs in weakly relativistic plasma are given by the fractional continuity and motion equations for a normalized fluid. Additionally, closure for the model is provided by the fractional Poisson equation formulated in a two-dimensional fractional representation

$$D_t^{\alpha}n_i+D_x^{\alpha}(n_iu)+D_y^{\alpha}(n_iv)=0, \tag{1}$$

$$D_t^{\alpha}(\gamma u)+uD_x^{\alpha}(\gamma u)+vD_y^{\alpha}(\gamma u)+D_x^{\alpha}\varphi+\delta n_i^{-1}D_x^{\alpha}p_i-\mu(D_x^{\alpha\alpha}u+D_y^{\alpha\alpha}u)=0, \tag{2}$$

$$D_t^{\alpha}v+uD_x^{\alpha}v+vD_y^{\alpha}v+D_y^{\alpha}\varphi+\delta n_i^{-1}D_y^{\alpha}p_i-\mu(D_x^{\alpha\alpha}v+D_y^{\alpha\alpha}v)=0, \tag{3}$$

$$D_t^{\alpha}p_i+u\,D_x^{\alpha}p_i+vD_y^{\alpha}p_i+3p_i\left[D_x^{\alpha}(\gamma u)+D_y^{\alpha}v\right]=0, \tag{4}$$

$$\left(D_x^{\alpha\alpha}+D_y^{\alpha\alpha}\right)\varphi=\Omega n_e-(\Omega-1)n_p-n_i\,. \tag{5}$$

Note that equations (1) to (5) reduced to the well-known equations as obtained in [44]. In this case, n_i represents the concentration of ions normalized by the unperturbed electron concentration n_{e0}, the ion velocity is $\mathbf{u}$ and the electrostatic potential is φ, the ions flow velocities along the x and y directions are represented by u and v normalized by $c_s=\sqrt{T_e/m_i}$, the constant coefficient of dynamic viscosity is denoted by μ, T_i, T_p and T_e are the temperature ion, positron and electron plasma, the particles' masses are m_e for electrons and m_i for ions, P_i is pressure and D_x^α and D_y^α describes the conformable fractional differential in relation to x, $D_x^{\alpha\alpha}=D_x^\alpha D_x^\alpha$ and $D_y^{\alpha\alpha}=D_y^\alpha D_y^\alpha$ the twice conformable fractional differential in relation to x and y. Ions are thought to have a relatively small relativistic influence, which can be expanded to

$\gamma=1/\sqrt{1-u^2/c^2}\approx 1+u^2/2c^2$. In this collision less plasmas, the ion kinematic viscosity μ_i has an influence on dissipation. It is well understood that the ion kinematic viscosity for collisionless plasmas is affected by mass, ion-ion collision time, ion temperature, and ion gyro frequency. This information is applied in a wide range of astrophysical challenges, particularly in the solar wind [52].

Through the application of reductive perturbation techniques, the scale's new stretching coordinates (time and space) are provided as

$$\xi=\frac{\sqrt{\varepsilon}}{\alpha}\left(x^\alpha-\alpha V\,t^\alpha\right),\qquad ,\eta=\frac{\varepsilon}{\alpha}y^\alpha,\qquad \tau=\frac{\sqrt{\varepsilon^3}}{\alpha}t^\alpha, \tag{6}$$

with ε is the small expansion parameter, which is proportional to the magnitude of the perturbation and indicates the system's nonlinearity intensity. Here, V signifies the fractional spatial phase velocity of the wave propagating in the x-direction. The fractional operator can be represented in the following manner:

$$\begin{aligned}&D_x^\alpha=\sqrt{\varepsilon}\,D_\xi^\alpha,\quad D_y^\alpha=\varepsilon\,D_\eta^\alpha,\quad D_x^{\alpha\alpha}=\varepsilon\,D_\xi^{\alpha\alpha},\qquad D_y^{\alpha\alpha}=\varepsilon^2\,D_\eta^{\alpha\alpha},\\&D_t^\alpha=-V\,\sqrt{\varepsilon}\,D_\xi^\alpha+\sqrt{\varepsilon^3}\,D_\tau^\alpha.\end{aligned} \tag{7}$$

Substituting the operators in equation (7) into equations (1), (2) and (5) we have

$$-\alpha V\,\sqrt{\varepsilon}\,D_\xi^\alpha n_i+\sqrt{\varepsilon^3}\,D_\tau^\alpha n_i+\sqrt{\varepsilon}\,D_\xi^\alpha(n_i u)+\varepsilon\,D_\eta^\alpha(n_i v)=0, \tag{8}$$

$$\begin{aligned}&-\alpha V\,\sqrt{\varepsilon}\,D_\xi^\alpha(\gamma u)+\sqrt{\varepsilon^3}\,D_\tau^\alpha(\gamma u)+u\sqrt{\varepsilon}\,D_\xi^\alpha(\gamma u)+v\,\varepsilon\,D_\eta^\alpha(\gamma u)+\sqrt{\varepsilon}\,D_\xi^\alpha\varphi\\&+\delta n_i^{-1}\sqrt{\varepsilon}\,D_\xi^\alpha p_i-\mu(\varepsilon\,D_\xi^{\alpha\alpha}u+\varepsilon^2\,D_\eta^{\alpha\alpha}u)=0,\end{aligned} \tag{9}$$

$$-\alpha V\sqrt{\varepsilon}D_{\xi}^{\alpha}v+\sqrt{\varepsilon^{3}}D_{\tau}^{\alpha}v+u\sqrt{\varepsilon}D_{\xi}^{\alpha}v+v\,\varepsilon D_{\eta}^{\alpha}v+\varepsilon D_{\eta}^{\alpha}\varphi$$
$$+\delta n_i^{-1}\varepsilon D_{\eta}^{\alpha}p_i-\mu(\varepsilon D_{\xi}^{\alpha\alpha}v+\varepsilon^2D_{\eta}^{\alpha\alpha}v)=0, \quad (10)$$

$$-\alpha V\sqrt{\varepsilon}D_{\xi}^{\alpha}p_i+\sqrt{\varepsilon^{3}}D_{\tau}^{\alpha}p_i+u\sqrt{\varepsilon}D_{\xi}^{\alpha}p_i+v\,\varepsilon D_{\eta}^{\alpha}p_i$$
$$+3p_i\left[\sqrt{\varepsilon}D_{\xi}^{\alpha}(\gamma u)+\varepsilon D_{\eta}^{\alpha}v\right]=0, \quad (11)$$

$$\varepsilon D_{\xi}^{\alpha\alpha}\varphi+\varepsilon^2D_{\eta}^{\alpha\alpha}\varphi=\Omega\left[1+(q-1)\varphi\right]^{\frac{q+1}{2(q-1)}}-(\Omega-1)\left[1-\sigma(q-1)\varphi\right]^{\frac{q+1}{2(q-1)}}-n_i. \quad (12)$$

The dependent variables n_i, p_i u, v and φ can be expanded in the manner described below within the power series of ε as

$$n_i=1+\varepsilon n_{i1}+\varepsilon^2n_{i2}+\varepsilon^3n_{i3}+...,$$
$$p_i=1+\varepsilon p_{i1}+\varepsilon^2p_{i2}+\varepsilon^3p_{i3}+...,$$
$$u=u_0+\varepsilon u_1+\varepsilon^2u_2+\varepsilon^3u_3+...,\quad (13)$$
$$\varphi=\varphi_1\varepsilon+\varphi_2\varepsilon^2+\varphi_3\varepsilon^3+...,$$
$$v=v_1\sqrt{\varepsilon^3}+v_2\sqrt{\varepsilon^5}+v_3\sqrt{\varepsilon^7}+...,$$

We assume the value of μ to be small, allowing us to set its value as $\mu=\mu_0\sqrt{\varepsilon}$, while η_{i0} maintains a finite value approximately of unity. Since $\gamma=1+u^2/2c^2$, therefore, equations (9) and (11) can be expressed in this way

$$-\alpha V\sqrt{\varepsilon}D_{\xi}^{\alpha}(u+u^3/2c^2)+\sqrt{\varepsilon^{3}}D_{\tau}^{\alpha}(u+u^3/2c^2)+u\sqrt{\varepsilon}D_{\xi}^{\alpha}(u+u^3/2c^2)$$
$$+v\,\varepsilon D_{\eta}^{\alpha}(u+u^3/2c^2)+\sqrt{\varepsilon}D_{\xi}^{\alpha}\varphi+\delta n_i^{-1}\sqrt{\varepsilon}D_{\xi}^{\alpha}p_i-\mu(\varepsilon D_{\xi}^{\alpha\alpha}u+\varepsilon^2D_{\eta}^{\alpha\alpha}u)=0, \quad (14)$$

$$-\alpha V\sqrt{\varepsilon}D_{\xi}^{\alpha}p_i+\sqrt{\varepsilon^{3}}D_{\tau}^{\alpha}p_i+u\sqrt{\varepsilon}D_{\xi}^{\alpha}p_i+v\,\varepsilon D_{\eta}^{\alpha}p_i$$
$$+3p_i\left[\sqrt{\varepsilon}D_{\xi}^{\alpha}(u+u^3/2c^2)+\varepsilon D_{\eta}^{\alpha}v\right]=0. \quad (15)$$

Using (13) in equation (8) and collecting terms of ε in the lowest order, from equation (8), one can obtain:

$$-\alpha V\sqrt{\varepsilon}D_{\xi}^{\alpha}\left(1+\varepsilon n_{i1}+\varepsilon^2n_{i2}+\varepsilon^3n_{i3}+...\right)+\sqrt{\varepsilon^3}D_{\tau}^{\alpha}\left(1+\varepsilon n_{i1}+\varepsilon^2n_{i2}+\varepsilon^3n_{i3}+...\right)$$
$$+\sqrt{\varepsilon}D_{\xi}^{\alpha}\left(\left(1+\varepsilon n_{i1}+\varepsilon^2n_{i2}+\varepsilon^3n_{i3}+...\right)\left(u_0+\varepsilon u_1+\varepsilon^2u_2+\varepsilon^3u_3+...\right)\right)$$
$$+\varepsilon D_{\eta}^{\alpha}\left(\left(1+\varepsilon n_{i1}+\varepsilon^2n_{i2}+\varepsilon^3n_{i3}+...\right)\left(v_1\sqrt{\varepsilon^3}+v_2\sqrt{\varepsilon^5}+v_3\sqrt{\varepsilon^7}+...\right)\right)=0,$$

$$\sqrt{\varepsilon}:\quad D_{\xi}^{\alpha}u_0=0,$$

$$\sqrt{\varepsilon^3}:\ -\alpha VD_{\xi}^{\alpha}n_{i1}+u_{i0}D_{\xi}^{\alpha}n_{i1}+u_{i1}D_{\xi}^{\alpha}n_{i0}+D_{\xi}^{\alpha}u_1=0,$$

$\sqrt{\varepsilon^5}: -\alpha V D_\xi^\alpha n_{i2} + D_\tau^\alpha n_{i1} + u_0 D_\xi^\alpha n_{i2} + u_2 D_\xi^\alpha n_{i0} + D_\xi^\alpha u_{i2} + u_1 D_\xi^\alpha n_{i1} + n_{i1} D_\xi^\alpha u_1 + D_\xi^\alpha v_1 = 0,$ Using (13) in equation (10) and collecting terms of ε in the lowest order, from equation (10), one can obtain:

$$-\alpha V \sqrt{\varepsilon} D_\xi^\alpha \left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right) + \sqrt{\varepsilon^3} D_\tau^\alpha \left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right)$$
$$+\sqrt{\varepsilon}\left(u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \varepsilon^3 u_3 + \dots\right) D_\xi^\alpha \left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right)$$
$$+\varepsilon\left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right) D_\eta^\alpha \left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right) + \varepsilon D_\eta^\alpha \left(\varphi_1\varepsilon + \varphi_2\varepsilon^2 + \varphi_3\varepsilon^3 + \dots\right)$$
$$+\delta\varepsilon\left(1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \varepsilon^3 n_{i3} + \dots\right)^{-1} D_\eta^\alpha \left(1 + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \varepsilon^3 p_{i3} + \dots\right)$$
$$-\mu\left(\varepsilon D_\xi^{\alpha\alpha}\left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right) + \varepsilon^2 D_\eta^{\alpha\alpha}\left(v_1\sqrt{\varepsilon^3} + v_2\sqrt{\varepsilon^5} + v_3\sqrt{\varepsilon^7} + \dots\right)\right) = 0,$$
$$\varepsilon^2: \quad -\alpha V\ D_\xi^\alpha v_1 + u_0 D_\xi^\alpha v_1 + D_\eta^\alpha \varphi_1 + \delta D_\eta^\alpha p_{i1} = 0,$$

Using (13) in equation (14) and collecting terms of ε in the lowest order, from equation (14), one can obtain:

$$-\alpha V \sqrt{\varepsilon} D_\xi^\alpha (u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots) - \frac{3\alpha V \sqrt{\varepsilon}}{2c^2}(u_0 + \varepsilon u_1 + \dots)^2 D_\xi^\alpha (u_0 + \varepsilon u_1 + \dots)$$
$$+\sqrt{\varepsilon^3} D_\tau^\alpha (u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots) + \frac{3\sqrt{\varepsilon^3}}{2c^2}(u_0 + \varepsilon u_1 + \dots)^2 D_\tau^\alpha (u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots)$$
$$+\sqrt{\varepsilon}(u_0 + \varepsilon u_1 + \dots) D_\xi^\alpha (u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots) + \frac{3\sqrt{\varepsilon}}{2c^2}(u_0 + \varepsilon u_1 + \dots)^3 D_\xi^\alpha (u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots)$$
$$+\varepsilon\left(v_1\sqrt{\varepsilon^3} + \dots\right) D_\eta^\alpha (u_0 + \varepsilon u_1 + \dots) + \frac{3\varepsilon}{2c^2}\left(v_1\sqrt{\varepsilon^3} + \dots\right)(u_0 + \varepsilon u_1 + \dots)^2 D_\eta^\alpha (u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots)$$
$$+\sqrt{\varepsilon} D_\xi^\alpha \left(\varphi_1\varepsilon + \varphi_2\varepsilon^2 + \dots\right) + \delta\sqrt{\varepsilon}\left(1 + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \dots\right)^{-1} D_\xi^\alpha \left(1 + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \varepsilon^3 p_{i3} + \dots\right)$$
$$-\mu_0\sqrt{\varepsilon^3} D_\xi^{\alpha\alpha}(u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots) - \mu_0\sqrt{\varepsilon^5} D_\eta^{\alpha\alpha}(u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \dots) = 0,$$
$$\sqrt{\varepsilon}: \quad -\alpha V\ D_\xi^\alpha u_0 - \frac{3\alpha V u_0^2}{2c^2} D_\xi^\alpha u_0 + u_0 D_\xi^\alpha u_0 + \frac{3u_0^2}{2c^2} D_\xi^\alpha u_0 = 0, \Rightarrow D_\xi^\alpha u_0 = 0,$$
$$\sqrt{\varepsilon^3}: \quad -\gamma_1\left(\alpha V - u_0\right) D_\xi^\alpha u_1 + D_\xi^\alpha \varphi_1 + \delta D_\xi^\alpha p_{i1} = 0, \qquad \gamma_1 = 1 + \frac{3u_0^2}{2c^2},$$
$$\sqrt{\varepsilon^5}: \quad -\gamma_1\left(\alpha V - u_0\right) D_\xi^\alpha u_2 + \gamma_1 D_\xi^\alpha u_1 + \left(\gamma_1 - \frac{2\gamma_2\left(\alpha V - u_0\right)}{u_0}\right) u_1 D_\xi^\alpha u_1 + D_\xi^\alpha \varphi_2 - \delta n_{i1} D_\xi^\alpha p_{i1}$$
$$+\delta D_\xi^\alpha p_{i2} - \mu_0 D_\xi^{\alpha\alpha} u_{i1} = 0, \qquad \gamma_2 = 1.5\beta^2, \quad \beta = \frac{u_0}{c}$$

Using (13) in equation (15) and collecting terms of ε in the lowest order, from equation (12), one can obtain:

$$-\alpha V\sqrt{\varepsilon}D_{\xi}^{\alpha}\left(1+\varepsilon p_{i1}+\varepsilon^{2}p_{i2}+...\right)+\sqrt{\varepsilon^{3}}D_{\tau}^{\alpha}\left(1+\varepsilon p_{i1}+\varepsilon^{2}p_{i2}+...\right)$$
$$+\sqrt{\varepsilon}(u_0+\varepsilon u_1+...)D_{\xi}^{\alpha}\left(1+\varepsilon p_{i1}+\varepsilon^{2}p_{i2}+...\right)+\varepsilon\left(v_1\sqrt{\varepsilon^{3}}+...\right)D_{\eta}^{\alpha}\left(1+\varepsilon p_{i1}+\varepsilon^{2}p_{i2}+...\right)$$
$$+3\sqrt{\varepsilon}\left(1+\varepsilon p_{i1}+...\right)D_{\xi}^{\alpha}(u_0+\varepsilon u_1+...)+\frac{9\sqrt{\varepsilon}}{2c^{2}}\left(1+\varepsilon p_{i1}+...\right)(u_0+\varepsilon u_1+...)^{2}D_{\xi}^{\alpha}(u_0+\varepsilon u_1+...)$$
$$+3\varepsilon\left(1+\varepsilon p_{i1}+\varepsilon^{2}p_{i2}+...\right)D_{\eta}^{\alpha}\left(v_1\sqrt{\varepsilon^{3}}+v_2\sqrt{\varepsilon^{5}}+v_3\sqrt{\varepsilon^{7}}+...\right)=0,$$
$$\sqrt{\varepsilon^{3}}:\ -(\alpha V-u_0)D_{\xi}^{\alpha}p_{i1}+3\gamma_1 D_{\xi}^{\alpha}u_1=0,$$
$$\sqrt{\varepsilon^{5}}:\ D_{\tau}^{\alpha}p_{i1}-(\alpha V-u_0)D_{\xi}^{\alpha}p_{i2}+3\gamma_1 D_{\xi}^{\alpha}u_2+u_1D_{\xi}^{\alpha}p_{i1}+3\gamma_1 p_{i1}D_{\xi}^{\alpha}u_1$$
$$+\frac{6\gamma_2}{u_0}u_1D_{\xi}^{\alpha}u_1+3D_{\eta}^{\alpha}v_1=0,$$

Using (13) in equation (12) and collecting terms of ε in the lowest order, from equation (15), one can obtain:

$$\varepsilon D_{\xi}^{\alpha\alpha}\left(\varphi_1\varepsilon+\varphi_2\varepsilon^{2}+...\right)+\varepsilon^{2}D_{\eta}^{\alpha\alpha}\left(\varphi_1\varepsilon+\varphi_2\varepsilon^{2}+...\right)=\Omega(1+\frac{1}{2}(q+1)(\varphi_1\varepsilon+\varphi_2\varepsilon^{2}+...)$$
$$-\frac{1}{8}(q+1)(q-3)(\varphi_1\varepsilon+\varphi_2\varepsilon^{2}+...)^{2}+...)-(\Omega-1)(1-\frac{\sigma}{2}(q+1)(\varphi_1\varepsilon+\varphi_2\varepsilon^{2}+...)$$
$$-\frac{\sigma^{2}}{8}(q+1)(q-3)(\varphi_1\varepsilon+\varphi_2\varepsilon^{2}+...)^{2}-...)-(1+\varepsilon n_{i1}+\varepsilon^{2}n_{i2}+\varepsilon^{3}n_{i3}+...),$$
$$\varepsilon^{0}:\qquad \Omega-(\Omega-1)-1=0,$$
$$\varepsilon:\qquad \frac{1}{2}(q+1)\Omega\varphi_1+\frac{\sigma}{2}(q+1)(\Omega-1)\varphi_1-n_{i1}=0,$$
$$\varepsilon^{2}:\qquad D_{\xi}^{\alpha\alpha}\varphi_1=\frac{(q+1)(1+a\sigma)}{2(1-a)}\varphi_2-\frac{(q+1)(q-3)(1-a\sigma^{2})}{8(1-a)}\varphi_1^{2}-n_{i2}.$$

Doing the conformable factional integration and using the boundary conditions,

$$n_{i1}=0,\qquad v_1=0,\quad p_{i1}=0,\quad u_1=0,\quad \varphi_1=0\quad \text{at}\quad \xi\to\infty, \tag{16}$$

to obtain the subsequent perturbed first-order quantities:

$$n_{i1}=\frac{1}{\gamma_1[\left(\alpha V-u_0\right)^{2}-3\delta]}\varphi_1=\frac{(q+1)(1+a\sigma)}{2(1-a)}\varphi_1,$$
$$u_1=-\frac{\alpha V-u_0}{\gamma_1[\left(\alpha V-u_0\right)^{2}-3\delta]}\varphi_1, \tag{17}$$
$$p_{i1}=\frac{3}{\left(\alpha V-u_0\right)^{2}-3\delta}\varphi_1,$$

Also, we get the (2+1) nonlinear fractional KP-B model may be created

$$D_{\xi}^{\alpha}\left(D_{\tau}^{\alpha}\varphi+A_1\varphi D_{\xi}^{\alpha}\varphi+A_2D_{\xi}^{\alpha\alpha\alpha}\varphi-A_3D_{\xi}^{\alpha\alpha}\varphi\right)+A_4D_{\eta}^{\alpha\alpha}\varphi=0, \tag{18}$$

The aforementioned formula, which reflects the two-dimensional space-time fractional KP-B equation, is extremely useful in studying the nonlinear propagation of weakly relativistic IA shock structures over the plasma system under discussion.The following forms are derived for equation (18) the nonlinearity A_1, dispersion A_2, dissipation A_3, and weakly transverse dispersion A_4 coefficients:

$$A_1 = \frac{1}{2K\gamma_1^2}\left(\gamma_1(\alpha V - u_0) + \frac{2\gamma_2(\alpha V - u_0)^2}{u_0} + \frac{6\delta\gamma_2}{u_0}\right) + \frac{(q-3)(1-a\sigma^2)K}{4(\alpha V - u_0)(1+a\sigma)} + \frac{1}{\alpha V - u_0}\left[\frac{9\delta}{2K} + \frac{1}{\gamma_1}\right], \tag{19}$$

$$A_2 = \frac{\gamma_1 K^2}{2(\alpha V - u_0)}, \quad A_3 = \frac{\mu_0}{2\gamma_1}, \quad A_4 = \frac{\gamma_1 K + 3\delta}{2(\alpha V - u_0)}, \quad K = (\alpha V - u_0)^2 - 3\delta. \tag{20}$$

From first equation of (17) we have

$$\frac{1}{\gamma_1[(\alpha V - u_0)^2 - 3\delta]} = \frac{(q+1)(1+a\sigma)}{2(1-a)}.$$

So the fractional phase velocity takes the formulae

$$V = \frac{1}{\alpha}\left[u_0 + \sqrt{3\delta + \frac{2(1-a)}{\gamma_1(q+1)(1+a\sigma)}}\right], \tag{21}$$

based on the explicit equations for the phase velocity V, we find an inverse relation between V and the fractional order α this means that when α increases the phase velocity V decreases and vice versa.

Consider the solutions involving travelling waves as

$$\varphi(\tau, \eta, \xi) = \varphi(\zeta), \qquad \zeta = k\,\xi^\alpha + \ell\eta^\alpha + \omega\tau^\alpha, \tag{22}$$

with k, ℓ and ω are numbers and frequencies of the wave. Though Equation (18) turns into

$$(\omega k + A_4\ell^2)\varphi'' + A_1 k^2(\varphi\varphi'' + \varphi'^2) - \alpha A_3 k^3\varphi''' + \alpha^2 A_2 k^4\varphi^{(4)} = 0, \tag{23}$$

3 Explicit solitary wave solution for space time fractional KP-B equation

The extended sech-tanh function and extended hyperbolic function approaches were used to get the IASW solutions for the nonlinear (2+1) fractional KP-B equation. Many authors discuss the exact solution of nonlinear models see for example [40- 49]. Different electrostatic potential values result in various explicit solutions for Equation (18), that provide the subsequent cases:

3-I: Extended hyperbolic function approach

There is a generic solution in series for the nonlinear two-dimensional KP-B equation as

$$\varphi(\zeta)=a_0+\sum_{i=1}^{N}[a_i g^{i}(\zeta)+b_i g^{-i}(\zeta)]+\sum_{i=2}^{N}c_i g^{i-2}(\zeta)g'(\zeta)+\sum_{i=1}^{N}d_{-i}g^{-i}(\zeta)g'(\zeta), \quad (24)$$

By maintaining a homogenous balance between the predominant nonlinear components and the highest order derivatives of $\varphi(\zeta)$ in equation (23), N can be generated. Here, a_0, a_i, b_i, c_i and d_i represent constants that will be determined later. Additionally, we consider $g(\zeta)$ as the auxiliary ordinary differential equation.

$$\frac{dg}{d\zeta}=\sqrt{Ag^2+Bg^4}, \qquad \frac{d^2g}{d\zeta^2}=Ag+2Bg^3,$$
$$\frac{d^3g}{d\zeta^3}=(A+6Bg^2)\sqrt{Ag^2+Bg^4}, \quad (25)$$
$$\frac{d^4g}{d\zeta^4}=g(A^2+20ABg^2+24B^2g^4),$$

where $g(\zeta)$ determines the actual parameters that should be chosen, which are A and B. We obviously obtain new solutions by extending the mapping. The left side of equation (23) can be transformed into a polynomial about $g^m(\zeta),(m=\ldots,-2,-1,0,1,2,\ldots)$ by substituting equation (24) into equation (23) and applying equation (25). A group of formulas in algebra for $k,\ell,\omega,a_0,a_i,b_i,c_i$ and d_i can be obtained by making the diversified power $g^m(\zeta)$'s coefficients zero. Using Maple or Mathematica to solve the algebraic equations, we are able to express $k,\ell,\omega,a_0,a_i,b_i,c_i$ and d_i as A and B. By putting these results into equation (24) and applying the previously mentioned mapping, From equation (24), we may obtain the solutions for solitary waves.

Balancing $\left(\frac{d\varphi}{d\zeta}\right)^2$ or $\varphi\frac{d^2\varphi}{d\zeta^2}$ with $\frac{d^4\varphi}{d\zeta^4}$ gives $2N+2=N+4$, so the leading order $N=2$, hence, please accept the ansatz

$$\varphi(\zeta)=a_0+a_1g(\zeta)+a_2g^2(\zeta)+\frac{b_1}{g(\zeta)}+\frac{b_2}{g^2(\zeta)}+c_2g'(\zeta)+\frac{d_1g'(\zeta)}{g(\zeta)}+\frac{d_2g'(\zeta)}{g^2(\zeta)}, \quad (26)$$

with $\ell,\omega,a_0,a_1,a_2,b_1,b_2,c_2,d_1,d_2$ and k are constants that need to be calculated and $g(\zeta)$ is the solution to equation (25). We may get a set of nonlinear algebra system for $\ell,\omega,a_0,a_1,a_2,b_1,b_2,c_2,d_1,d_2$ and k by substituting equations (26) and (25) in equation (23) and taking the coefficients of $g^m(\zeta)$ to zero. When we solve this system in Mathematica or Maple, we obtain

$$a_o=\pm\frac{6A_3^2}{25A_1A_2},\qquad a_1=\pm\frac{6A_3^2}{25A_1A_2}\sqrt{\frac{B}{A}},\qquad a_2=\pm\frac{6A_3^2B}{25A_1A_2A},$$
$$c_2=\pm\frac{6A_3^2\sqrt{B}}{25A_1A_2A},\qquad d_1=\pm\frac{3A_3^2}{25A_1A_2\sqrt{A}},\qquad k=\pm\frac{A_3}{5\alpha A_2\sqrt{A}},\tag{27}$$
$$\omega=\pm\frac{3A_3^2}{125\alpha A_2^2\sqrt{A}}\pm\frac{5\alpha A_2A_4\ell^2\sqrt{A}}{A_3},\qquad b_1=b_2=d_2=0,$$

Equations (26) and (27) can be swapped to produce the following concentration formulas for traveling wave solutions of equation (19):

$$\varphi(\zeta)=\pm\frac{6A_3^2}{25A_1A_2}\left[1+\sqrt{\frac{B}{A}}g(\zeta)+\frac{B}{A}g^2(\zeta)+\frac{\sqrt{B}}{A}g'(\zeta)+\frac{1}{2\sqrt{A}}\frac{g'(\zeta)}{g(\zeta)}\right],$$
$$\zeta=\pm\frac{A_3}{5\alpha A_2\sqrt{A}}\xi^\alpha+\ell\eta^\alpha\pm\left(\frac{3A_3^2}{125\alpha A_2^2\sqrt{A}}+\frac{5\alpha A_2A_4\ell^2\sqrt{A}}{A_3}\right)\tau^\alpha,\tag{28}$$

1: If $A>0,\ B>0,$ so $g(\zeta)=\sqrt{\frac{A}{B}}\operatorname{csch}(\sqrt{A}\zeta)$. then we can obtain

$$\varphi_1(\zeta)=\pm\frac{6A_3^2}{25A_1A_2}[1+\operatorname{csch}(\sqrt{A}\zeta)+\operatorname{csch}^2(\sqrt{A}\zeta)-\operatorname{csch}(\sqrt{A}\zeta)\coth(\sqrt{A}\zeta)$$
$$-\frac{1}{2}\coth(\sqrt{A}\zeta)],\tag{29}$$

2: If $A<0,\ B>0,$ so $g(\zeta)=\sqrt{-\frac{A}{B}}\sec(\sqrt{-A}\zeta)$ or $g(\zeta)=\sqrt{-\frac{A}{B}}\csc(\sqrt{-A}\zeta)$ then we can obtain

$$\varphi_2(\zeta)=\pm\frac{6A_3^2}{25A_1A_2}[1+\sqrt{-1}\sec(\sqrt{-A}\zeta)-\sec^2(\sqrt{-A}\zeta)$$
$$-\sec(\sqrt{-A}\zeta)\tan(\sqrt{-A}\zeta)+\frac{1}{2}\sqrt{-1}\tan(\sqrt{-A}\zeta)],\tag{30}$$

$$\varphi_3(\zeta)=\pm\frac{6A_3^2}{25A_1A_2}[1+\sqrt{-1}\csc(\sqrt{-A}\zeta)-\csc^2(\sqrt{-A}\zeta)$$
$$-\csc(\sqrt{-A}\zeta)\cot(\sqrt{-A}\zeta)-\frac{1}{2}\cot(\sqrt{-A}\zeta)],\tag{31}$$

3: If $A>0,\ B>0,$ so $g(\zeta)=\sqrt{-\frac{A}{B}}\operatorname{sech}(\sqrt{A}\zeta)$, then we can obtain

$$\varphi_4(\zeta) = \pm \frac{6A_3^2}{25A_1A_2}[1+\sqrt{-1}\operatorname{sech}(\sqrt{A}\zeta) - \operatorname{sech}^2(\sqrt{A}\zeta) + \sqrt{-1}\operatorname{sech}(\sqrt{A}\zeta)\tanh(\sqrt{A}\zeta) - \frac{1}{2}\tanh(\sqrt{A}\zeta)], \tag{32}$$

Several elements influence the height of the shock structure, such as the nonlinear (A_1), dispersion (A_2), dissipative (A_3) coefficients , and fractional phase velocity (V). Moreover, the wave velocity relies on the interplay among the nonlinear coefficient (A_1), dispersion coefficient (A_2), dissipative coefficient (A_3), fractional order, and weakly transverse dispersion coefficients (A_4).

3-2 The modified sub-equation technique

There is a general solution in series for the two-dimensional nonlinear KP-B model as

$$\varphi(\zeta) = a_0 + \sum_{i=1}^{N}[a_i F^i(\zeta) + b_i F^{-i}(\zeta)] + \sum_{i=2}^{N} c_i F^{i-2}(\zeta) F'(\zeta) + \sum_{i=1}^{N} d_{-i} F^{-i}(\zeta) F'(\zeta), \tag{33}$$

with a_0, a_i, b_i, c_i and d_i are constants to be calculated later. In equation (23), the uniform equilibrium between the highest-order derivatives of $\varphi(\zeta)$ and the controlling nonlinear terms can be used to find the positive integer N and $F(\zeta)$ explains how to solve the auxiliary ordinary differential equation that follows:

$$\frac{dF}{d\zeta} = \sqrt{AF^2 + BF^3 + CF^4}, \qquad \frac{d^2g}{d\zeta^2} = AF + \frac{3}{2}BF^2 + 2CF^3,$$
$$\frac{d^3g}{d\zeta^3} = (A + 3BF + 6BCF^2)\sqrt{AF^2 + BF^3 + CF^4}, \tag{34}$$
$$\frac{d^4g}{d\zeta^4} = \frac{1}{2}(2A^2F + 15ABF^2 + 5(3B^2 + 8AC)F^3 + 60ACF^4 + 48C^2F^5),$$

with A, B and C are real values that $F(\zeta)$ will be selected. Obviously, in order to get new solutions, we have to modify the sub-equation procedure. The equation's left-hand side of (23) able could be changed into a polynomial around $F^m(\zeta), (m = \ldots, -2, -1, 0, 1, 2, \ldots)$ by substituting equation (33) into equation (23) and applying equation (43). An algebraic system of equations for $k, \ell, \omega, a_0, a_i, b_i, c_i$ and d_i can be obtained by putting the coefficients of the distinct power of $F^m(\zeta)$ to zero. Using Maple or Mathematica to solve the algebraic equations, we obtain $k, \ell, \omega, a_0, a_i, b_i, c_i$ and d_i expressed by A and B. Equation (26) can be solved for the solitary wave solutions of equation (23) by substituting these results and applying the previously mentioned mapping.

Balancing $\varphi\frac{d^2\varphi}{d\zeta^2}$ or $\left(\frac{d\varphi}{d\zeta}\right)^2$ with $\frac{d^4\varphi}{d\zeta^4}$ gives $2N+2=N+4$, so the leading order $N=2$, hence, please accept the ansatz

$$\varphi(\zeta)=a_0+a_1F(\zeta)+a_2F^2(\zeta)+\frac{b_1}{F(\zeta)}+\frac{b_2}{F^2(\zeta)}+c_2F'(\zeta)+\frac{d_1F'(\zeta)}{F(\zeta)}+\frac{d_2F'(\zeta)}{F^2(\zeta)}, \quad (35)$$

where $F(\zeta)$ is the answer to equation (35), $\ell,\omega,a_0,a_1,a_2,b_1,b_2,c_2,d_1,d_2$ and k are constants that must be computed. Equations (35) and (34) can be substituted into equation (23) to get a system of nonlinear algebraic equations for and, where the coefficients of $F^m(\zeta)$ are set to zero. When we use Maple or Mathematica to solve this system, we gain

$$\begin{gathered}a_o=\pm\frac{6A_3^2}{25A_1A_2},\qquad a_1=\pm\frac{6A_3^2(B+2\sqrt{AC})}{25A_1A_2A},\qquad a_2=\pm\frac{6A_3^2C}{25A_1A_2A},\\ c_2=\pm\frac{6A_3^2\sqrt{C}}{25A_1A_2A},\qquad d_1=\pm\frac{3A_3^2}{25A_1A_2\sqrt{A}},\qquad k=\pm\frac{A_3}{5\alpha A_2\sqrt{A}},\\ \omega=\pm\frac{3A_3^2}{125\alpha A_2^2\sqrt{A}}\pm\frac{5\alpha A_2A_4\ell^2\sqrt{A}}{A_3},\qquad b_1=b_2=d_2=0,\end{gathered} \quad (36)$$

The subsequent concentration formulas for traveling wave solutions of equation (18) are given by substituting equations (29) into (28)

$$\begin{gathered}\varphi(\zeta)=\frac{6A_3^2}{25A_1A_2}\left[1+\frac{(B+2\sqrt{AC})}{A}F(\zeta)+\frac{C}{A}F^2(\zeta)+\frac{\sqrt{C}}{A}F'(\zeta)+\frac{1}{2\sqrt{A}}\frac{F'(\zeta)}{F(\zeta)}\right],\\ \zeta=\frac{A_3}{5\alpha A_2\sqrt{A}}\xi^\alpha+\ell\eta^\alpha+\left(\frac{3A_3^2}{125\alpha A_2^2\sqrt{A}}+\frac{5\alpha A_2A_4\ell^2\sqrt{A}}{A_3}\right)\tau^\alpha.\end{gathered} \quad (37)$$

It is noted that the nonlinear coefficient (A_1), dispersion coefficient (A_2), and the dissipative coefficient (A_3) all affect the height of the shock structure. In addition, the wave speed ω depends on the dispersion coefficient (A_2), dissipative coefficient (A_3), the fractional order α and weakly transverse dispersion A_4 coefficients.

3-3 Extended technique of Sech-tanh

The two-dimensional nonlinear KPB problem can be solved as a sequence of sech and tanh functions

$$\varphi(\zeta)=a_0+\sum_{j=1}^{N}\left(a_j\operatorname{sech}(\zeta)+b_j\tanh(\zeta)\right)\operatorname{sech}^{j-1}(\zeta), \quad (38)$$

with $a_0,a_1,...,a_N,b_1,...,b_n$ are arbitrary coefficients in this case. *N*= *2* is produced by equating the highest-order linear differential and nonlinear terms in Equation (23). Equation (23) has a solution that looks like this

$$\varphi(\zeta)=a_0+a_1\operatorname{sech}(\zeta)+a_2\operatorname{sech}^2(\zeta)+b_1\tanh(\zeta)+b_2\operatorname{sech}(\zeta)\tanh(\zeta), \tag{39}$$

Replacing Equation (42) in Equation (23) yields an algebraic system of equations. By collecting the coefficients of $\operatorname{sech}(\zeta)$, $\operatorname{sech}(\zeta)\tanh(\zeta)$, $\operatorname{sech}^2(\zeta)$, $\tanh(\zeta)$,...and set them equivalent to zero, we gain this system. The constraints $\ell,\omega,a_0,a_1,a_2,b_1,b_2$ and k, solving this system yields

$$a_0=b_1=2a_2=\frac{12A_3\alpha k}{5A_1},\quad a_1=b_2=0,\quad \omega=\frac{12A_3\alpha^3k^3-5A_4\ell^2}{5\alpha^2k},\quad k=\frac{A_3}{10\alpha A_2}, \tag{40}$$

$$a_0=b_1=a_2=\frac{a_1}{\sqrt{-1}}=\frac{b_2}{\sqrt{-1}}=\frac{6A_3\alpha k}{5A_1},\quad \omega=\frac{6A_3\alpha^3k^3-5A_4\ell^2}{5\alpha^2k},\quad k=\frac{A_3}{5\alpha A_2}, \tag{41}$$

Equation (18)'s electrostatic potential can be determined using IASW solution by substituting from Equations (41) and (40) into (39)

$$\begin{aligned}\varphi_7(\zeta)&=\frac{6A_3^2}{25A_1A_2}\left[2+\operatorname{sech}^2(\zeta)-2\tanh(\zeta)\right],\\ \zeta&=\frac{A_3}{10\alpha A_2}\xi^\alpha+\ell\eta^\alpha+\left(\frac{12A_3^4-5000A_4A_2^3\ell^2}{500\alpha A_2^2A_3}\right)\tau^\alpha.\end{aligned} \tag{42}$$

$$\begin{aligned}\varphi_8(\zeta)&=\frac{6A_3^2}{25A_1A_2}\left[-1+\operatorname{sech}^2(\zeta)-\tanh(\zeta)+\sqrt{-1}\operatorname{sech}(\zeta)(1+\tanh(\zeta))\right],\\ \zeta&=\frac{A_3}{5\alpha A_2}\xi^\alpha+\ell\eta^\alpha+\left(\frac{6A_3^4-625A_4A_2^3\ell^2}{125\alpha A_2^2A_3}\right)\tau^\alpha.\end{aligned} \tag{43}$$

The obtained results when $\alpha=1$ are the same obtained in references [44, 45].

It is observed that the coefficients (A_1) for nonlinearity, (A_2) for dispersion term, and (A_3) for the dissipative term are all affect the height of the shock structure. In addition, the wave speed ω depends on the dispersion coefficient (A_2), dissipative coefficient (A_3), the fractional order α and weakly transverse dispersion A_4 coefficients.

The scalar function's gradient, φ, known as the potential electrostatically (which is likewise referred to as the voltage), can be used to express the electric field because it is irrotational. An electric field, denoted technically as the field of electricity $\overrightarrow{E}$, is a vector that extends from regions with varying electric potential, from high to low, can be expressed as $\overrightarrow{E}=-\nabla_\alpha\varphi=-D_\xi^\alpha\varphi\bar{e}_\xi-D_\eta^\alpha\varphi\bar{e}_\eta$,

$$\begin{aligned}\overrightarrow{E_1}(\zeta)&=\frac{12A_3^2}{25A_1A_2}\operatorname{sech}^2(\zeta)(1+\tanh(\zeta))\left(\frac{A_3}{10A_2}\bar{e}_\xi+\alpha\ell\bar{e}_\eta\right),\\ \zeta&=\frac{A_3}{10\alpha A_2}\xi^\alpha+\ell\eta^\alpha+\left(\frac{12A_3^4-5000A_4A_2^3\ell^2}{500\alpha A_2^2A_3}\right)\tau^\alpha.\end{aligned} \tag{44}$$

$$\overrightarrow{E_2}(\zeta)=\frac{6A_3^2}{25A_1A_2}\frac{(1-\sqrt{-1})(\cosh(\zeta/2)+\sinh(\zeta/2))}{(\cosh(\zeta/2)-\sqrt{-1}\sinh(\zeta/2))^3}\left(\frac{A_3}{5A_2}\bar{e}_\xi+\alpha\ell\bar{e}_\eta\right),$$
$$\zeta=\frac{A_3}{5\alpha A_2}\xi^\alpha+\ell\eta^\alpha+\left(\frac{6A_3^4-625A_4A_2^3\ell^2}{125\alpha A_2^2A_3}\right)\tau^\alpha. \tag{45}$$

A Hamiltonian scheme provides fraction momentum of the potentials electrostatic reported in the earlier states

$$M_\alpha=\lim_{\rho\to\infty}\int_0^\rho\varphi_j(\zeta)d\zeta^\alpha, \tag{46}$$

where $\int_0^\zeta(.)d\zeta^\alpha$, is the conformable fractional integration and *j=1,2,…,8* . the solitary Dust acoustic wave solution must be stable under the specific situation

$$\frac{\partial M_\alpha}{\partial\omega}>0. \tag{47}$$

4. Results and discussion

Numerical investigations have explored the impact of the non integer order α on several parameters, including the phase fractional velocity V, wave amplitude, wave speed ω, and electrostatic potential φ. These investigations examine the unperturbed ratio of positron-electron density $a=n_{p0}/n_{e0}$, electron-positron temperatures $\sigma=T_e/T_p$, ion-electron temperatures $\delta=T_i/T_e$, normalized coefficient of ion kinematic viscosity (μ_0), relativistic streaming coefficient (β), and degree of electron- positron nonextensivity q. The purpose is to comprehend the nonlinear propagation of electrostatic IA waves in relativistic plasmas.

Figure (1) represented the influence of the non-integer order α to the fractional phase velocity V obtained by equation (21) via the initial ion flow velocity u_0 along the x axes with different values of the fractional order $\alpha=1,\ 0.9,\ 0.7,\ 0,5$, when $\delta=0.01$, $q=0.8$, $a=0.01$ and $\sigma=0.01$, the lower curve (red curve) when $\alpha=1$ and the upper curve (black curve) when $\alpha=0.5$. As shown in Fig. (1) the variation of initial velocity of the ions u_0 has an affection on the phase velocity V . For $\alpha=1$ the increase of the u_0 leads to increase of V, also this affect applies for all other values of α. The variation of the fractional order α from 1 to 0.5 which gives by decreasing the fractional order increases the fractional wave phase velocity.

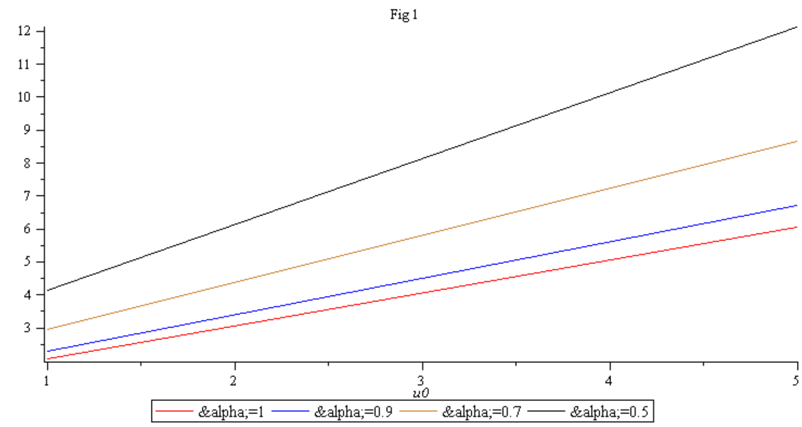

FIG.1 The fractional phase velocity V is depicted versus the initial ion flow velocity u_0 with the variation of fractional order $\alpha = 1,\ 0.9,\ 0.7,\ 0,5$ at $\delta = 0.01,\ q = 0.8,\ a = 0.01$ and $\sigma = 0.01$.

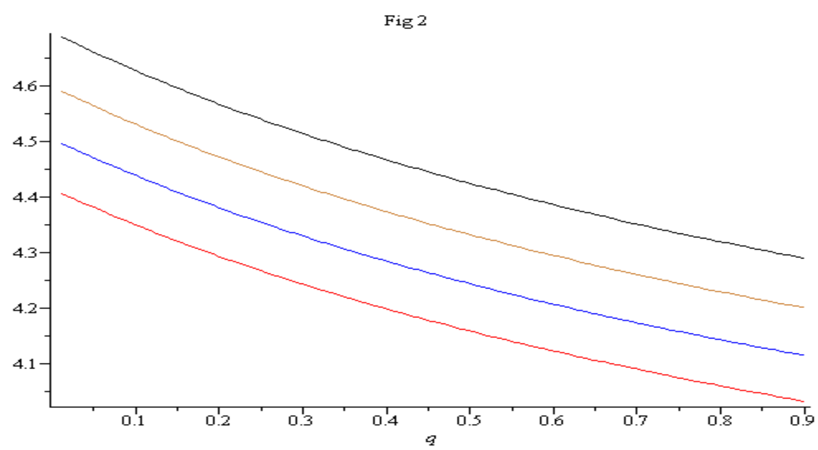

FIG.2 The phase velocity V is depicted versus the positrons nonextensivity q with the variation of fractional order $\alpha = 1,\ 0.98,\ 0.96,\ 0,94$ at $\delta = 0.01,\ \beta = 0.1,\ a = 0.01$ and $\sigma = 0.02$.

Figure (2) represented the effects of the fractional order α to the fractional phase velocity V via the positrons nonextensivity q with different values of $\alpha = 1,\ 0.98,\ 0.96,\ 0,94$, when $\delta = 0.01,\ \beta = 0.1,\ a = 0.01$ and $\sigma = 0.02$, the lower curve (red curve) when $\alpha = 1$ and the upper curve (black curve) when $\alpha = 0.94$. As shown in Fig. (2) the variation of positrons nonextensivity q has an affection on the phase velocity V. For $\alpha = 1$ the increase of the q leads to decrease of V, also this affect applies for all other values of α. The variation of the fractional order α from 1 to 0.94 which gives by decreasing the fractional order increases the phase velocity of the wave.

Figure (3) represented the effects of the fractional order α to the fractional phase velocity V via the ion-electron temperature ratio $\delta = T_i / T_e$, with different values of $\alpha = 1,\ 0.98,\ 0.96,\ 0,94$, when $q = 0.7,\ \beta = 0.1,\ a = 0.01$ and $\sigma = 0.02$, the lower curve (red curve) when $\alpha=1$ and the upper curve (black curve) when $\alpha=0.94$. As shown in Fig. (3) the variation of ion-electron temperature ratio $\delta=T_i/T_e$, has an affection on the phase velocity V. For $\alpha=1$ the increase of the δ leads to increase of V, also this affect applies for all other values of α. The variation of the fractional order α from 1 to 0.94 which gives by decreasing the fractional order increases the phase velocity of the wave.

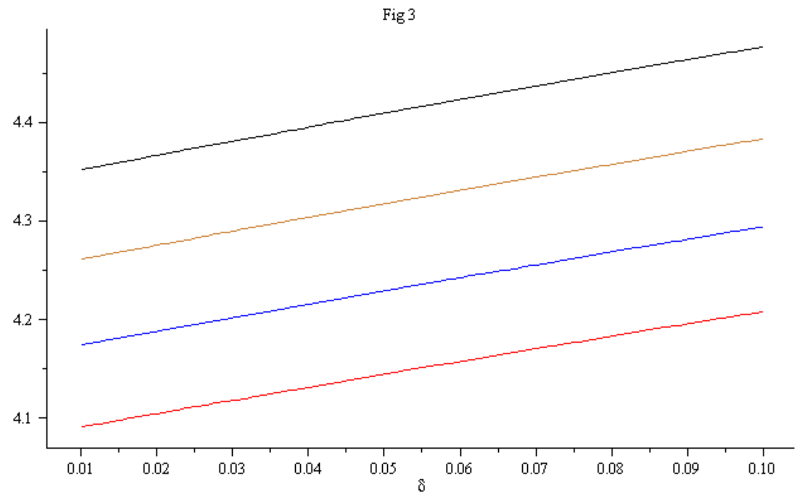

FIG.3 The phase velocity V is depicted versus the ion-electron temperature ratio δ, with the variation of fractional order $\alpha=1,\ 0.98,\ 0.96,\ 0,94$ at $q=0.7,\ \beta=0.1,\ a=0.01$ and $\sigma=0.02$.

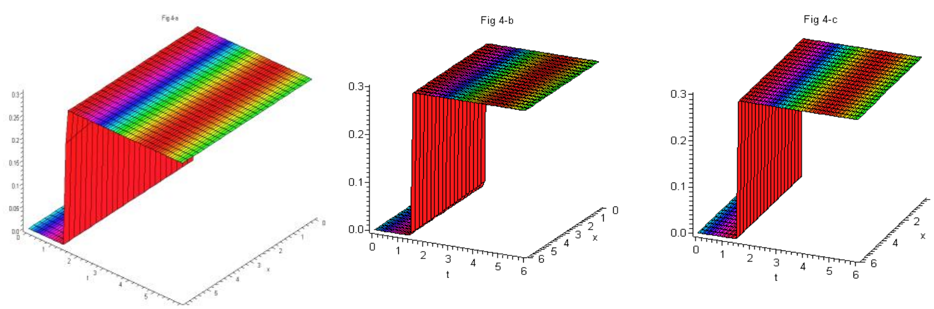

Figure 4 cross section behavior of soliton solution (42) when $\sigma=0.02, q=0.7,\ \beta=0.1,$ $a=0.01, \ell=1, \mu_0=0.8, p=0.7, \delta=0.01$ and $\eta=5$ with different values of fractional order $\alpha=1,\ 0.95,\ 0.90.$

Figure 4 represents the 3-dimentional cross section behavior of soliton solution (42) with $\sigma=0.02, q=0.7,\ \beta=0.1,\ a=0.01, \ell=1, \mu_0=0.8, p=0.7, \delta=0.01$ and $\eta=5$, fig (4-a) when $\alpha=1$, fig (4-b) when $\alpha=0.95$, and fig (4-c) when $\alpha=0.90$. We see that the amplitude peak moves apart with the variation of fractional order. This means that the soliton solution depended on the fractional order.

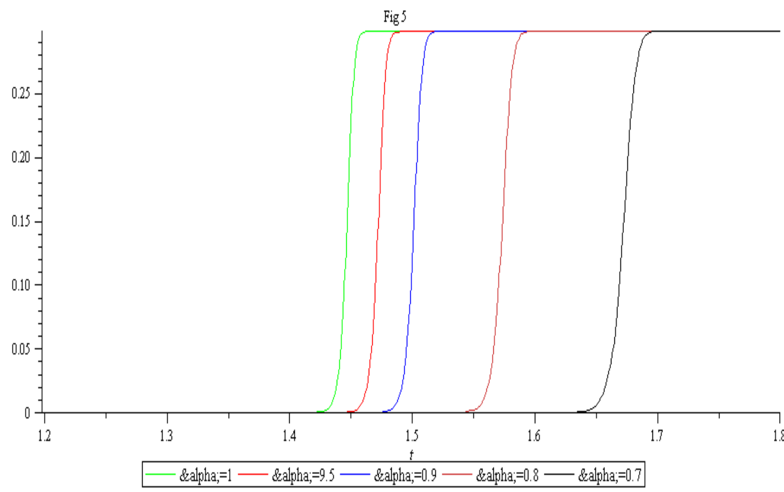

Figure 5 cross section behavior of soliton solution (42) when $\sigma = 0.02, q = 0.7, \beta = 0.1,$ $a = 0.01, \ell = 1, \mu_0 = 0.8, p = 0.7, \delta = 0.01, \xi = 2$ and $\eta = 5$ with different values of fractional order $\alpha = 1, 0.95, 0.90, 0.80, 0.70.$

Figure 5 represents the 2-dimensional evolution behavior of soliton solution (29) with $\sigma = 0.02, q = 0.7, \beta = 0.1, a = 0.01, \ell = 1, \mu_0 = 0.8, p = 0.7, \delta = 0.01, \xi = 2$ and $\eta = 5$, green line when $\alpha = 1$, red line when $\alpha = 0.95$, the blue line when $\alpha = 0.90$, the orange line when $\alpha = 0.80$, and the black line when $\alpha = 0.70$. We see that the amplitude peak moves apart with the variation of fractional order. This means that the soliton solution depended on the fractional order.

5. Conclusion

The emergence of a two-dimensional nonlinear KP-B equation was stemmed from the study of shocks and the propagation of nonlinear IASWs within dissipation quantum plasma employing the reductive perturbation theory. This equation holds significant physical relevance across various applications. This investigation employing several methodologies including the extended sech-tanh, modification of extended mapping, and extended sech function approaches. These approaches were instrumental in addressing the nonlinear two-dimensional KP-B problem, specifically corresponding ions within solitary traveling waves.

Investigating the interesting world of un-magnetized epi plasma systems, which contain non-extensive electrons, relativistic ions, and positrons, provides important insights into astrophysical and cosmological phenomena. In this context, our study is dedicated to unraveling the complexities of 2-dim nonlinear electrostatic IA structures.

We derive solutions to the governing equations using detailed analysis, revealing complex dependencies on key parameters like positron concentration, electron-positron and ion-electron temperature ratios, ion kinematic viscosity, electron and positron non-extensivity, and the weakly relativistic streaming factor. Notably, our investigation underscores the significant influence of the fractional order and associated plasma parameters on the phase fractional velocity of IA waves, while also highlighting the limited impact of the ion kinematic viscosity coefficient. This comprehensive study offers fresh insights into the dynamics of ion acoustic waves within complex plasma environments, thereby enhancing our understanding of fundamental astrophysical processes.

In our research, we delved into the intricate dynamics of nonlinear wave phenomena within dissipative quantum plasmas, leading to the formulation of a comprehensive two-dimensional KP-B equation. By leveraging the powerful tools of reductive perturbation theory, we unraveled the underlying mechanisms governing the propagation of shocks and IASWs. The derived KP-B equation unveils a rich tapestry of physical phenomena with wide-ranging applications. To tackle this complex problem, we employed a multifaceted approach, integrating techniques such as the extended sech-tanh, modification of extended mapping direct algebraic, and extended direct sech methods. Through meticulous analysis and computational simulations, we gained valuable insights into the behavior of ions within solitary traveling waves, paving the way for advancements in plasma physics and related disciplines.

This research focuses on small yet finite amplitude ion acoustic structures within a planar geometry, however it is critical to understand its limitations. We focus on the nonlinear propagation of shocks and solitons in our plasma model, which includes nonextensive electrons, positrons, and relativistic thermal ions. However, it is critical to recognize the broad range of future research potential.

Future research could look at the nonlinear analysis of other phenomena, including shock waves, solitary waves, vortices, solitons, and double layers, utilizing the same plasma model with nonextensive electrons, positrons, and relativistic thermal ions. Although these subjects are beyond the purview of our current research, they are critical to improving our understanding of plasma physics and discovering the features of astrophysical compact objects. As a result, they provide intriguing opportunities for future research and investigation in the sector.

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