Int. J. Anal. Appl. (2025), 23:70

International Journal of Analysis and Applications

Possibility Fermatean Interval Valued Fuzzy Soft Set and Their Application to Decision Making Framework

Ghulam Muhiuddin[1,*], Mohamed E. Elnair[1], ELsiddig Idriss Mohamed[2], Sivakumar Pushparaj[3], Hussein Eledum[2], M. Palanikumar[4]

[1]*Department of Mathematics, Faculty of Science, University of Tabuk, P.O. Box 741, Tabuk 71491, Saudi Arabia*

[2]*Department of Statistics, Faculty of Science, University of Tabuk, P.O. Box 741, Tabuk 71491, Saudi Arabia*

[3]*Department of Mathematics, Panimalar Engineering College, Chennai- 600 123, Tamilnadu, India*

[4]*Department of Mathematics, Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences, Chennai, 602105, India*

[*]*Corresponding author:* chishtygm@gmail.com

Abstract. In this paper, we introduce the theory of possibility Fermatean interval valued fuzzy soft (PFIVFS) set and its application to real life problems. The PFIVFS set is a generalization of Pythagorean fuzzy soft and soft set. We define some operations consist of complement, union, intersection, AND and OR. Notably, we show DeMorgan's laws and associative laws and distributive laws are valid in PFIVFS set theory. We discuss the need to buy a laptop and find several stages for consumer goes through before purchasing a product. We propose an algorithm to solve the decision making problem based on soft set method. To compare PFIVFS set and Fermatean interval valued fuzzy soft (FIVFS) set for dealing with decision making problems, we find a similarity measure. Finally, an illustrative example is discussed to prove that they can be effectively used to solve problems with uncertainties.

1. Introduction

In most real problems, uncertainty can be seen everywhere. In order to cope with the uncertainties, many uncertain theories are put forward such as fuzzy set [1], intuitionistic fuzzy set [2], Xiao et al initiated the concept of interval valued fuzzy soft sets [3] and Pythagorean fuzzy set [4]. Zadeh was introduced by fuzzy set suggests that decision makers are to be solving uncertain problems by considering membership degree. After, the concept of intuitionistic fuzzy set is

Received: Jun. 27, 2024.

2020 *Mathematics Subject Classification.* 03E72, 03B52, 90B50.

Key words and phrases. FIVFS set; PFIVFS set; decision making problem; soft set.

https://doi.org/10.28924/2291-8639-23-2025-70

ISSN: 2291-8639

introduced by Atanassov and is characterized by a degree of membership and non-membership satisfying the condition that the sum of its membership degree and non membership degree is not exceeding unity [2]. However, we may interact a problem in decision making events where the sum of the degree of membership and non-membership of a particular attribute is exceeding unity. So Yager was introduced by the concept of Pythagorean fuzzy sets characterized by the condition that the squares of sum of its degree of membership and non membership are not exceeding unity. In decision making problems, sometimes squares of sum of its degree of membership and non membership are exceeding unity. So Senapati et al. introduced by Fermatean fuzzy set [5]. It has been to extend the intuitionistic fuzzy sets and Pythagorean fuzzy sets and its characterized by the condition that the cubes of sum of its degree of membership and non membership is not exceeding unity.

The theory of soft sets was proposed by Molodtsov [6]. It is a tool of parameterization for coping with uncertainties. In comparison with other uncertain theories, soft sets more accurately reflect the objectivity and complexity of decision making during actual situations. It has been a great achievement both in theories and applications. Moreover, the combination of soft sets with other mathematical models is also a critical research area. For example, Maji et al. proposed the concept of fuzzy soft set [7] and intuitionistic fuzzy soft set [8]. These two theories are applied to solve various decision making problems. Alkhazaleh et al [9] defined the concept of possibility fuzzy soft sets. In recent years, Peng et al [10] has extended fuzzy soft set to Pythagorean fuzzy soft set. Also, Fermatean fuzzy soft set is a generalization of the Pythagorean fuzzy soft set. In general, the possibility degree of belongingness of the elements should be considered in multi attribute decision making problems. The purpose of this paper is to extend the concept of possibility Pythagorean interval valued soft set to parameterization of the possibility Fermatean interval valued fuzzy set. Recently, many authors discussed the extension of fuzzy logic such as [11–15].

The paper is organized into seven sections as follows. Section 1 is called an introduction. Section 2 brief descriptions of Pythagorean interval valued soft set and Fermatean interval valued soft set information are given. Section 3 discuss about for some operations for PFIVFS set. Section 4 discuss method for finding similarity measure under PFIVFS set set. Section 5 talks trough an application of PFIVFS set and FIVFS set information, an algorithm with a numerical examples. Section 6 deals with a comparison of the PFIVFS set and FIVFS set approaches. Finally, the conclusion is provided in Section 7. Throughout this paper, $\chi_A(u)$ and $\psi_A(u)$ represent the degree of membership and degree of non-membership of Fermatean fuzzy set A respectively, satisfying the condition that $\chi_A(u) + \psi_A(u) \nleq 1$ and $\chi_A^2(u) + \psi_A^2(u) \nleq 1$, but $\chi_A^3(u) + \psi_A^3(u) \leq 1$.

2. Preliminaries

In this section, we discuss some preliminary definition which are useful for this paper.

Definition 2.1. *[4, 15] Let $\mathbb{U}$ be a non-empty set of the universe, Pythagorean fuzzy set (PFS) A in $\mathbb{U}$ is an object having the following form : $A = \{u, \chi_A(u), \psi_A(u) | u \in \mathbb{U}\}$, where $\chi_A(u)$ and $\psi_A(u)$*

represent the membership grade and non-membership grade of A respectively. Consider the mapping $\chi_A : \mathbb{U} \to [0,1]$, $\psi_A : \mathbb{U} \to [0,1]$ *and* $0 \leq (\chi_A(u))^2 + (\psi_A(u))^2 \leq 1$. *The indeterminacy grade is determined as* $\pi_A(u) = \left[\sqrt{1-(\chi_A(u))^2-(\psi_A(u))^2}\right]$. *Since* $A = \langle \chi_A, \psi_A \rangle$ *is called a Pythagorean fuzzy number(PFN).*

Definition 2.2. *[12–14] Let* $\mathbb{U}$ *be a non-empty set of the universe, Pythagorean interval valued fuzzy set (PIVFS) A in* $\mathbb{U}$ *is an object having the following form :* $\widehat{A} = \{u, \widehat{\chi}_A(u), \widehat{\psi}_A(u) | u \in \mathbb{U}\}$, *where* $\widehat{\chi}_A(u) = [\chi_A^-(u), \chi_A^+(u)]$ *and* $\widehat{\psi}_A(u) = [\psi_A^-(u), \psi_A^+(u)]$ *represent the degree of membership and degree of non-membership of A respectively. Consider the mapping* $\widehat{\chi}_A : \mathbb{U} \to [0,1]$, $\widehat{\psi}_A : \mathbb{U} \to [0,1]$ *and* $0 \leq (\widehat{\chi}_A(u))^2 + (\widehat{\psi}_A(u))^2 \leq 1$ *means that* $0 \leq (\chi_A^+(u))^2 + (\psi_A^+(u))^2 \leq 1$. *The degree of indeterminacy is determined as* $\widehat{\pi}_A(u) = [\pi_A^-(u), \pi_A^+(u)] = \left[\sqrt{1-(\chi_A^+(u))^2-(\psi_A^+(u))^2},\ \sqrt{1-(\chi_A^-(u))^2-(\psi_A^-(u))^2}\right]$. *Since* $A = \langle [\chi_A^-, \chi_A^+], [\psi_A^-, \psi_A^+] \rangle$ *is called a Pythagorean interval valued fuzzy number(PIVFN).*

Definition 2.3. *[12] For any three PIVFNs* $\widehat{A_1}, \widehat{A_2}$ *and* $\widehat{A_3}$ *over* $(\mathbb{U}, \mathbb{E})$. *Then the following properties are holds:*

(1) $\widehat{A_1}^c = (\widehat{\psi}_{A_1}, \widehat{\chi}_{A_1})$
(2) $\widehat{A_2} \Cup \widehat{A_3} = \left(\max(\widehat{\chi}_{A_2}, \widehat{\chi}_{A_3}), \min(\widehat{\psi}_{A_2}, \widehat{\psi}_{A_3})\right)$
(3) $\widehat{A_2} \Cap \widehat{A_3} = \left(\min(\widehat{\chi}_{A_2}, \widehat{\chi}_{A_3}), \max(\widehat{\psi}_{A_2}, \widehat{\psi}_{A_3})\right)$
(4) $\widehat{A_2} \geq \widehat{A_3}$ *iff* $\widehat{\chi}_{A_2} \geq \widehat{\chi}_{A_3}$ *and* $\widehat{\psi}_{A_2} \leq \widehat{\psi}_{A_3}$
(5) $\widehat{A_2} = \widehat{A_3}$ *iff* $\widehat{\chi}_{A_2} = \widehat{\chi}_{A_3}$ *and* $\widehat{\psi}_{A_2} = \widehat{\psi}_{A_3}$.

Definition 2.4. *[5] Let* $\mathbb{U}$ *be a non-empty set of the universe, Fermatean fuzzy set (FFS) A in* $\mathbb{U}$ *is an object having the following form :* $A = \{u, \chi_A(u), \psi_A(u) | u \in \mathbb{U}\}$, *where* $\chi_A(u)$ *and* $\psi_A(u)$ *represent the degree of membership and degree of non-membership of A respectively. Consider the mapping* $\chi_A : \mathbb{U} \to [0,1]$, $\psi_A : \mathbb{U} \to [0,1]$ *and* $0 \leq (\chi_A(u))^3 + (\psi_A(u))^3 \leq 1$. *The degree of indeterminacy is determined as* $\pi_A(u) = \left[\sqrt[3]{1-(\chi_A(u))^3-(\psi_A(u))^3}\right]$. *Since* $A = \langle \chi_A, \psi_A \rangle$ *is called a Fermatean fuzzy number(FFN).*

Definition 2.5. *Let* $\mathbb{U}$ *be a non-empty set of the universe, Fermatean interval valued fuzzy set (FIVFS) A in* $\mathbb{U}$ *is an object having the following form :* $\widehat{A} = \{u, \widehat{\chi}_A(u), \widehat{\psi}_A(u) | u \in \mathbb{U}\}$, *where* $\widehat{\chi}_A(u) = [\chi_A^-(u), \chi_A^+(u)]$ *and* $\widehat{\psi}_A(u) = [\psi_A^-(u), \psi_A^+(u)]$ *represent the degree of membership and degree of non-membership of A respectively. Consider the mapping* $\widehat{\chi}_A : \mathbb{U} \to [0,1]$, $\widehat{\psi}_A : \mathbb{U} \to [0,1]$ *and* $0 \leq (\widehat{\chi}_A(u))^3 + (\widehat{\psi}_A(u))^3 \leq 1$ *means that* $0 \leq (\chi_A^+(u))^3 + (\psi_A^+(u))^3 \leq 1$. *The degree of indeterminacy is determined as* $\widehat{\pi}_A(u) = [\pi_A^-(u), \pi_A^+(u)] = \left[\sqrt[3]{1-(\chi_A^+(u))^3-(\psi_A^+(u))^3},\ \sqrt[3]{1-(\chi_A^-(u))^3-(\psi_A^-(u))^3}\right]$. *Since* $A = \langle [\chi_A^-, \chi_A^+], [\psi_A^-, \psi_A^+] \rangle$ *is called a Fermatean interval valued fuzzy number(FIVFN).*

Definition 2.6. *[11] Let* $\mathbb{U}$ *be a non-empty set of the universe and* $\mathbb{E}$ *be a set of parameter. The pair* $(\widehat{\mathscr{P}}, A)$ *is called an interval valued fuzzy soft (IVFS) set on* $\mathbb{U}$ *if* $A \sqsubseteq \mathbb{E}$ *and* $\widehat{\mathscr{P}} : A \to \widehat{\mathscr{P}(\mathbb{U})}$, *where* $\widehat{\mathscr{P}(\mathbb{U})}$ *is the set of all interval valued fuzzy subsets of* $\mathbb{U}$.

Definition 2.7. *[9] Let* $\mathbb{U}$ *be a non-empty set of the universe and* $\mathbb{E}$ *be a set of parameter. The pair* $(\mathbb{U}, \mathbb{E})$ *is a soft universe. Consider the mapping* $\mathscr{P} : \mathbb{E} \to \mathscr{P}(\mathbb{U})$ *and* χ *be a fuzzy subset of* $\mathbb{E}$, *ie.* $\chi : \mathbb{E} \to \mathscr{P}(\mathbb{U})$.

Let $\mathscr{P}_\chi : \mathbb{E} \to \mathscr{P}(\mathbb{U}) \times \mathscr{P}(\mathbb{U})$ be a function defined as $\mathscr{P}_\chi(e) = (\mathscr{P}(e)(u), \chi(e)(u)), \forall u \in \mathbb{U}$. Then $\mathscr{P}_\chi$ is called a possibility fuzzy soft (PFS) set on $(\mathbb{U}, \mathbb{E})$.

3. Operations for PFIVFS set

Definition 3.1. *Let $\mathbb{U}$ be a non-empty set of the universe and $\mathbb{E}$ be a set of parameter. The pair $(\mathbb{U}, \mathbb{E})$ is called a soft universe. Suppose that $\widehat{\mathscr{P}} : \mathbb{E} \to P\widehat{\mathscr{P}(\mathbb{U})}$, and $\widehat{p}$ is a Fermatean interval valued fuzzy subset of $\mathbb{E}$. That is $\widehat{p} : \mathbb{E} \to P\widehat{\mathscr{P}(\mathbb{U})}$, where $P\widehat{\mathscr{P}(\mathbb{U})}$ denotes the collection of all Fermatean interval valued fuzzy subsets of $\mathbb{U}$. If $\widehat{\mathscr{P}}_p : \mathbb{E} \to P\widehat{\mathscr{P}(\mathbb{U})} \times P\widehat{\mathscr{P}(\mathbb{U})}$ is a function defined as $\widehat{\mathscr{P}}_p(e) = (\widehat{\mathscr{P}}(e)(u), \widehat{p}(e)(u)), u \in \mathbb{U}$, then $\widehat{\mathscr{P}}_p$ is a Possibility Fermatean interval valued fuzzy soft (PFIVFS) set on $(\mathbb{U}, \mathbb{E})$. For each parameter e, $\widehat{\mathscr{P}}_p(e) = \left\{\left\langle u, \langle \chi_{\widehat{\mathscr{P}}(e)}(u), \psi_{\widehat{\mathscr{P}}(e)}(u)\rangle, \langle \chi_{\widehat{p}(e)}(u), \psi_{\widehat{p}(e)}(u)\rangle \right\rangle,\ u \in \mathbb{U}\right\}$*

Example 3.1. *Let $\mathbb{U} = \{u_1, u_2, u_3\}$ be a of three motorbike under consideration and parameters $E = \{e_1 = \text{Best design},\ e_2 = \text{Maximum durable},\ e_3 = \text{Maximum mileage},\ e_4 = \text{Best price}\}$. Suppose that $\widehat{\mathscr{P}}_p : \mathbb{E} \to P\widehat{\mathscr{P}(\mathbb{U})} \times P\widehat{\mathscr{P}(\mathbb{U})}$ is given by*

$$\widehat{\mathscr{P}}_p(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.55,0.75],[0.6,0.75]),([0.75,0.8],[0.2,0.7])\rangle} \\ \frac{x_2}{\langle([0.75,0.85],[0.55,0.6]),([0.65,0.8],[0.3,0.65])\rangle} \\ \frac{x_3}{\langle([0.5,0.7],[0.55,0.75]),([0.65,0.7],[0.6,0.8])\rangle} \end{array} \right\} ; \widehat{\mathscr{P}}_p(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.8,0.95],[0.35,0.45]),([0.5,0.6],[0.35,0.8])\rangle} \\ \frac{x_2}{\langle([0.75,0.85],[0.55,0.65]),([0.55,0.75],[0.45,0.75])\rangle} \\ \frac{x_3}{\langle([0.85,0.9],[0.45,0.6]),([0.65,0.75],[0.55,0.7])\rangle} \end{array} \right\}$$

$$\widehat{\mathscr{P}}_p(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.65,0.7],[0.6,0.8]),([0.55,0.85],[0.25,0.55])\rangle} \\ \frac{x_2}{\langle([0.8,0.85],[0.55,0.7]),([0.45,0.85],[0.35,0.7])\rangle} \\ \frac{x_3}{\langle([0.6,0.8],[0.55,0.65]),([0.55,0.75],[0.7,0.75])\rangle} \end{array} \right\}$$

Then, $\widehat{\mathscr{P}}_p$ is a PFIVFS.

Definition 3.2. *Let $\mathbb{U}$ be a non-empty set of the universe and $\mathbb{E}$ be a set of parameter. Suppose that $\widehat{\mathscr{P}}_p$ and $\widehat{\mathscr{Q}}_q$ are two PFIVFS sets on $(\mathbb{U}, \mathbb{E})$. Now $\widehat{\mathscr{Q}}_q$ is a possibility Fermatean interval valued fuzzy soft subset of $\widehat{\mathscr{P}}_p$ (denoted by $\widehat{\mathscr{Q}}_q \sqsubseteq \widehat{\mathscr{P}}_p$) if and only if*

(1) $\widehat{\mathscr{Q}}(e)(u) \sqsubseteq \widehat{\mathscr{P}}(e)(u)$ if $\chi_{\widehat{\mathscr{P}}(e)}(u) \geq \chi_{\widehat{\mathscr{Q}}(e)}(u)$, $\psi_{\widehat{\mathscr{P}}(e)}(u) \leq \psi_{\widehat{\mathscr{Q}}(e)}(u)$,

(2) $\widehat{q}(e)(u) \sqsubseteq \widehat{p}(e)(u)$ if $\chi_{\widehat{p}(e)}(u) \geq \chi_{\widehat{q}(e)}(u)$, $\psi_{\widehat{p}(e)}(u) \leq \psi_{\widehat{q}(e)}(u)$, $\forall e \in \mathbb{E}$.

Example 3.2. *Consider the PFIVFS set $\widehat{\mathscr{P}}_p$ over $(\mathbb{U}, \mathbb{E})$ in Example 3.1. Let $\widehat{\mathscr{Q}}_q$ be another PFIVFS set over $(\mathbb{U}, \mathbb{E})$ is defined as:*

$$\widehat{\mathscr{Q}}_q(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.5,0.65],[0.65,0.8]),\ ([0.7,0.75],[0.4,0.75])\rangle} \\ \frac{x_2}{\langle([0.6,0.75],[0.7,0.75]),\ ([0.6,0.65],[0.5,0.8])\rangle} \\ \frac{x_3}{\langle([0.45,0.6],[0.65,0.8]),\ ([0.55,0.6],[0.65,0.85])\rangle} \end{array} \right\} ; \widehat{\mathscr{Q}}_q(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.6,0.9],[0.45,0.55]),([0.45,0.55],[0.65,0.85])\rangle} \\ \frac{x_2}{\langle([0.65,0.75],[0.65,0.75]),([0.5,0.65],[0.5,0.8])\rangle} \\ \frac{x_3}{\langle([0.8,0.85],[0.55,0.65]),([0.35,0.45],[0.6,0.9])\rangle} \end{array} \right\}$$

$$\widehat{\mathscr{Q}}_q(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.45,0.55],[0.7,0.85]),\ ([0.5,0.65],[0.65,0.8])\rangle} \\ \frac{x_2}{\langle([0.7,0.75],[0.6,0.75]),\ ([0.25,0.45],[0.75,0.9])\rangle} \\ \frac{x_3}{\langle([0.55,0.65],[0.75,0.85]),\ ([0.35,0.55],[0.8,0.9])\rangle} \end{array} \right\}$$

Then $\widehat{\mathscr{Q}}_q \sqsubseteq \widehat{\mathscr{P}}_p$

Definition 3.3. *Let $\mathbb{U}$ be a non-empty set of the universe and $\mathbb{E}$ be a set of parameter. Suppose that $\widehat{\mathscr{P}_p}$ and $\widehat{\mathscr{Q}_q}$ are two PFIVFS sets on $(\mathbb{U},\mathbb{E})$. Now $\widehat{\mathscr{P}_p}$ and $\widehat{\mathscr{Q}_q}$ are possibility Fermatean interval valued fuzzy soft set equal(denoted by $\widehat{\mathscr{P}_p}=\widehat{\mathscr{Q}_q}$) if and only if*

(1) $\widehat{\mathscr{P}_p}\sqsubseteq\widehat{\mathscr{Q}_q}$
(2) $\widehat{\mathscr{P}_p}\sqsupseteq\widehat{\mathscr{Q}_q}$.

Definition 3.4. *Let $\mathbb{U}$ be a non-empty set of the universe and $\mathbb{E}$ be a set of parameter. Let $\widehat{\mathscr{P}_p}$ be a PFIVFS set on $(\mathbb{U},\mathbb{E})$. The complement of $\widehat{\mathscr{P}_p}$ is denoted by $\widehat{\mathscr{P}_p^c}$ and is defined by $\widehat{\mathscr{P}_p^c}=\langle\widehat{\mathscr{P}^c}(e)(u),\widehat{p^c}(e)(u)\rangle$, where $\widehat{\mathscr{P}^c}(e)(u)=\langle\psi_{\widehat{\mathscr{P}(e)}}(u),\chi_{\widehat{\mathscr{P}(e)}}(u)\rangle,\widehat{p^c}(e)(u)=\langle\psi_{\widehat{p(e)}}(u),\chi_{\widehat{p(e)}}(u)\rangle$.*
It is true that $(\widehat{\mathscr{P}_p^c})^c=\widehat{\mathscr{P}_p}$

Definition 3.5. *Let $\mathbb{U}$ be a non-empty set of the universe and $\mathbb{E}$ be a set of parameter. Let $\widehat{\mathscr{P}_p}$ and $\widehat{\mathscr{Q}_q}$ be two PFIVFS sets on $(\mathbb{U},\mathbb{E})$. The union and intersection of $\widehat{\mathscr{P}_p}$ and $\widehat{\mathscr{Q}_q}$ over $(\mathbb{U},\mathbb{E})$ are denoted by $\widehat{\mathscr{P}_p}\Cup\widehat{\mathscr{Q}_q}$ and $\widehat{\mathscr{P}_p}\Cap\widehat{\mathscr{Q}_q}$ respectively and are defined by $\widehat{B_j}:\mathbb{E}\to P\widehat{\mathscr{P}(\mathbb{U})}\times P\widehat{\mathscr{P}(\mathbb{U})}$, $\widehat{A_i}:\mathbb{E}\to P\widehat{\mathscr{P}(\mathbb{U})}\times P\widehat{\mathscr{P}(\mathbb{U})}$ such that $\widehat{B_j}(e)(u)=(\widehat{B}(e)(u),\widehat{j}(e)(u))$, $\widehat{A_i}(e)(u)=(\widehat{A}(e)(u),\widehat{i}(e)(u))$, where $\widehat{B}(e)(u)=\widehat{\mathscr{P}}(e)(u)\Cup\widehat{\mathscr{Q}}(e)(u)$, $\widehat{j}(e)(u)=\widehat{p}(e)(u)\Cup\widehat{q}(e)(u)$, $\widehat{A}(e)(u)=\widehat{\mathscr{P}}(e)(u)\Cap\widehat{\mathscr{Q}}(e)(u)$ and $\widehat{i}(e)(u)=\widehat{p}(e)(u)\Cap\widehat{q}(e)(u)$, for all $x\in\mathbb{U}$.*

Example 3.3. *Let $\widehat{\mathscr{P}_p}$ and $\widehat{\mathscr{Q}_q}$ be the two PFIVFS sets on $(\mathbb{U},\mathbb{E})$ where $\mathbb{U}=\{x_1,x_2,x_3,x_4\}$ and $E=\{e_1,e_2,e_3\}$, we define*

$$\widehat{\mathscr{P}_p}(e_1)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.5,0.75],[0.6,0.75]),\ ([0.75,0.8],[0.2,0.7])\rangle}\\ \frac{x_2}{\langle([0.75,0.8],[0.65,0.75]),\ ([0.65,0.8],[0.3,0.65])\rangle}\\ \frac{x_3}{\langle([0.65,0.7],[0.8,0.85]),\ ([0.65,0.7],[0.6,0.8])\rangle}\\ \frac{x_4}{\langle([0.55,0.8],[0.5,0.75]),\ ([0.55,0.75],[0.3,0.7])\rangle}\end{array}\right\};\quad \widehat{\mathscr{P}_p}(e_2)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.55,0.9],[0.5,0.6]),([0.35,0.65],[0.45,0.85])\rangle}\\ \frac{x_2}{\langle([0.7,0.75],[0.6,0.75]),([0.45,0.85],[0.45,0.65])\rangle}\\ \frac{x_3}{\langle([0.8,0.85],[0.35,0.65]),([0.55,0.6],[0.55,0.9])\rangle}\\ \frac{x_4}{\langle([0.55,0.9],[0.5,0.6]),([0.35,0.65],[0.45,0.85])\rangle}\end{array}\right\};$$

$$\widehat{\mathscr{P}_p}(e_3)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.25,0.75],[0.7,0.8]),\ ([0.45,0.85],[0.45,0.55])\rangle}\\ \frac{x_2}{\langle([0.4,0.7],[0.65,0.85]),([0.5,0.8],[0.65,0.7])\rangle}\\ \frac{x_3}{\langle([0.65,0.8],[0.6,0.75]),([0.65,0.7],[0.7,0.85])\rangle}\\ \frac{x_4}{\langle([0.25,0.75],[0.7,0.8]),\ ([0.45,0.85],[0.45,0.55])\rangle}\end{array}\right\};$$

$$\widehat{\mathscr{Q}_q}(e_1)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.75,0.9],[0.4,0.55]),\ ([0.4,0.55],[0.6,0.85])\rangle}\\ \frac{x_2}{\langle([0.65,0.8],[0.5,0.75]),\ ([0.5,0.7],[0.4,0.8])\rangle}\\ \frac{x_3}{\langle([0.6,0.9],[0.3,0.6]),([0.4,0.75],[0.6,0.7])\rangle}\\ \frac{x_3}{\langle([0.75,0.9],[0.4,0.6]),([0.6,0.7],[0.5,0.85])\rangle}\end{array}\right\};\quad \widehat{\mathscr{Q}_q}(e_2)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.65,0.85],[0.5,0.6]),\ ([0.25,0.35],[0.5,0.95])\rangle}\\ \frac{x_2}{\langle([0.75,0.85],[0.6,0.7]),([0.55,0.6],[0.55,0.85])\rangle}\\ \frac{x_3}{\langle([0.85,0.9],[0.35,0.55]),([0.25,0.35],[0.7,0.95])\rangle}\\ \frac{x_3}{\langle([0.65,0.85],[0.5,0.6]),([0.25,0.35],[0.5,0.95])\rangle}\end{array}\right\};$$

$$\widehat{\mathscr{Q}}_q(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.55,0.65],[0.55,0.85]),\ ([0.45,0.6],[0.5,0.85])\rangle} \\ \frac{x_2}{\langle([0.75,0.8],[0.5,0.75]),([0.25,0.7],[0.75,0.85])\rangle} \\ \frac{x_3}{\langle([0.35,0.75],[0.65,0.8]),([0.55,0.65],[0.5,0.8])\rangle} \\ \frac{x_3}{\langle([0.55,0.65],[0.55,0.85]),([0.45,0.6],[0.5,0.85])\rangle} \end{array} \right\};$$

Then, $(\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q)$ and $(\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q)$ is calculated below.

$$(\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q)(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.75,0.9],[0.4,0.55]),([0.75,0.8],[0.2,0.7])\rangle} \\ \frac{x_2}{\langle([0.65,0.8],[0.5,0.75]),([0.65,0.8],[0.3,0.65])\rangle} \\ \frac{x_3}{\langle([0.6,0.7],[0.3,0.6]),([0.65,0.75],[0.6,0.7])\rangle} \\ \frac{x_4}{\langle([0.55,0.8],[0.4,0.6]),([0.6,0.75],[0.3,0.7])\rangle} \end{array} \right\}; \quad (\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q)(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.65,0.9],[0.5,0.6]),([0.35,0.65],[0.45,0.85])\rangle} \\ \frac{x_2}{\langle([0.7,0.75],[0.6,0.7]),([0.55,0.85],[0.45,0.65])\rangle} \\ \frac{x_3}{\langle([0.8,0.85],[0.35,0.55]),([0.55,0.6],[0.55,0.9])\rangle} \\ \frac{x_4}{\langle([0.55,0.85],[0.5,0.6]),([0.35,0.65],[0.45,0.85])\rangle} \end{array} \right\};$$

$$(\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q)(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.55,0.75],[0.5,0.8]),\ ([0.45,0.85],[0.45,0.55])\rangle} \\ \frac{x_2}{\langle([0.4,0.7],[0.5,0.75]),([0.5,0.8],[0.65,0.7])\rangle} \\ \frac{x_3}{\langle([0.35,0.75],[0.6,0.75]),([0.65,0.7],[0.5,0.8])\rangle} \\ \frac{x_4}{\langle([0.25,0.65],[0.55,0.8]),\ ([0.45,0.85],[0.45,0.55])\rangle} \end{array} \right\}; \quad (\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q)(e_1) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.5,0.75],[0.6,0.75]),\ ([0.4,0.55],[0.6,0.85])\rangle} \\ \frac{x_2}{\langle([0.65,0.8],[0.65,0.75]),\ ([0.5,0.7],[0.4,0.8])\rangle} \\ \frac{x_3}{\langle([0.6,0.7],[0.8,0.85]),\ ([0.4,0.7],[0.6,0.8])\rangle} \\ \frac{x_4}{\langle([0.55,0.8],[0.5,0.75]),\ ([0.55,0.7],[0.5,0.85])\rangle} \end{array} \right\};$$

$$(\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q)(e_2) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.55,0.85],[0.5,0.6]),([0.25,0.35],[0.5,0.95])\rangle} \\ \frac{x_2}{\langle([0.7,0.75],[0.6,0.75]),([0.45,0.6],[0.55,0.85])\rangle} \\ \frac{x_3}{\langle([0.8,0.85],[0.35,0.65]),([0.25,0.35],[0.7,0.95])\rangle} \\ \frac{x_4}{\langle([0.55,0.85],[0.5,0.6]),([0.25,0.35],[0.5,0.95])\rangle} \end{array} \right\}; \quad (\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q)(e_3) = \left\{ \begin{array}{c} \frac{x_1}{\langle([0.25,0.65],[0.7,0.85]),\ ([0.45,0.6],[0.5,0.85])\rangle} \\ \frac{x_2}{\langle([0.4,0.7],[0.65,0.85]),([0.25,0.7],[0.75,0.85])\rangle} \\ \frac{x_3}{\langle([0.35,0.75],[0.65,0.8]),([0.55,0.65],[0.7,0.85])\rangle} \\ \frac{x_4}{\langle([0.25,0.65],[0.7,0.85]),\ ([0.45,0.6],[0.5,0.85])\rangle} \end{array} \right\};$$

Definition 3.6. *A PFIVFS set $\widehat{\emptyset}_\theta(e)(u) = \langle \widehat{\emptyset}(e)(u), \widehat{\theta}(e)(u)\rangle$ is said to be a null PFIVFS set $\widehat{\emptyset}_\theta : \mathbb{E} \to P\widehat{\mathscr{P}(\mathbb{U})} \times P\widehat{\mathscr{P}(\mathbb{U})}$, where $\widehat{\emptyset}(e)(u) = ([0,0],[1,1])$ and $\widehat{\theta}(e)(u) = ([0,0],[1,1])$, $\forall x \in \mathbb{U}$.*

Definition 3.7. *A PFIVFS set $\widehat{\Omega}_\Lambda(e)(u) = \langle \widehat{\Omega}(e)(u), \widehat{\Lambda}(e)(u)\rangle$ is said to be absolute PFIVFS set $\widehat{\Omega}_\Lambda : \mathbb{E} \to P\widehat{\mathscr{P}(\mathbb{U})} \times P\widehat{\mathscr{P}(\mathbb{U})}$, where $\widehat{\Omega}(e)(u) = ([1,1],[0,0])$ and $\widehat{\Lambda}(e)(u) = ([1,1],[0,0])$, $\forall x \in \mathbb{U}$.*

Remark 3.1. *Let $\widehat{\mathscr{P}}_p$ be a PFIVFS set on $(\mathbb{U},\mathbb{E})$. If $\widehat{\mathscr{P}}_p \neq \widehat{\Omega}_\Lambda$ or $\widehat{\mathscr{P}}_p \neq \widehat{\emptyset}_\theta$, then $\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{P}}^c_p \neq \widehat{\Omega}_\Lambda$ and $\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{P}}^c_p \neq \widehat{\emptyset}_\theta$.*

Theorem 3.1. *Let $\widehat{\mathscr{P}}_p$ be a PFIVFS set on $(\mathbb{U},\mathbb{E})$. Then the following properties hold:*

(1) $\widehat{\mathscr{P}}_p = \widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{P}}_p$, $\widehat{\mathscr{P}}_p = \widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{P}}_p$
(2) $\widehat{\mathscr{P}}_p \Cup \widehat{\emptyset}_\theta = \widehat{\mathscr{P}}_p$, $\widehat{\mathscr{P}}_p \Cap \widehat{\emptyset}_\theta = \widehat{\emptyset}_\theta$
(3) $\widehat{\mathscr{P}}_p \Cup \widehat{\Omega}_\Lambda = \widehat{\Omega}_\Lambda$, $\widehat{\mathscr{P}}_p \Cap \widehat{\Omega}_\Lambda = \widehat{\mathscr{P}}_p$.

Theorem 3.2. *Let $\widehat{\mathscr{P}}_p$, $\widehat{\mathscr{Q}}_q$ and $\widehat{\mathscr{R}}_r$ are three PFIVFS sets over $(\mathbb{U},\mathbb{E})$, then the following properties hold:*

(1) $\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q = \widehat{\mathscr{Q}}_q \Cup \widehat{\mathscr{P}}_p$
(2) $\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q = \widehat{\mathscr{Q}}_q \Cap \widehat{\mathscr{P}}_p$
(3) $\widehat{\mathscr{P}}_p \Cup (\widehat{\mathscr{Q}}_q \Cup \widehat{\mathscr{R}}_r) = (\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q) \Cup \widehat{\mathscr{R}}_r$

(4) $\widehat{\mathscr{P}}_p \Cap (\widehat{\mathscr{Q}}_q \Cap \widehat{\mathscr{R}}_r) = (\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q) \Cap \widehat{\mathscr{R}}_r$
(5) $(\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q) \Cap \widehat{\mathscr{P}}_p = \widehat{\mathscr{P}}_p$
(6) $(\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q) \Cup \widehat{\mathscr{P}}_p = \widehat{\mathscr{P}}_p$.

Theorem 3.3. *Let $\widehat{\mathscr{P}}_p$ and $\widehat{\mathscr{Q}}_q$ are two PFIVFS sets over $(\mathbb{U}, \mathbb{E})$, then the following properties are hold:*

(1) $(\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q)^c = \widehat{\mathscr{P}}_p^c \Cap \widehat{\mathscr{Q}}_q^c$
(2) $(\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q)^c = \widehat{\mathscr{P}}_p^c \Cup \widehat{\mathscr{Q}}_q^c$.

Theorem 3.4. *Let $\widehat{\mathscr{P}}_p$, $\widehat{\mathscr{Q}}_q$ and $\widehat{\mathscr{R}}_r$ are three PFIVFS sets over $(\mathbb{U}, \mathbb{E})$, then the following properties are hold:*

(1) $\widehat{\mathscr{P}}_p \Cup (\widehat{\mathscr{Q}}_q \Cap \widehat{\mathscr{R}}_r) = (\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{Q}}_q) \Cap (\widehat{\mathscr{P}}_p \Cup \widehat{\mathscr{R}}_r)$
(2) $\widehat{\mathscr{P}}_p \Cap (\widehat{\mathscr{Q}}_q \Cup \widehat{\mathscr{R}}_r) = (\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{Q}}_q) \Cup (\widehat{\mathscr{P}}_p \Cap \widehat{\mathscr{R}}_r)$.

Definition 3.8. *Let $(\widehat{\mathscr{P}}_p, X)$ and $(\widehat{\mathscr{Q}}_q, Y)$ be two PFIVFS sets on $(\mathbb{U}, \mathbb{E})$. Then the operations "$(\widehat{\mathscr{P}}_p, X)$ AND $(\widehat{\mathscr{Q}}_q, Y)$" is denoted by $(\widehat{\mathscr{P}}_p, X) \barwedge (\widehat{\mathscr{Q}}_q, Y)$ and is defined by $(\widehat{\mathscr{P}}_p, X) \barwedge (\widehat{\mathscr{Q}}_q, Y) = (\widehat{\mathscr{R}}_r, X \times Y)$, where $\widehat{\mathscr{R}}_r(x, y) = (\widehat{\mathscr{R}}(x, y)(u), \widehat{r}(x, y)(u))$ such that $\widehat{\mathscr{R}}(x, y) = \widehat{\mathscr{P}}(x) \Cap \widehat{\mathscr{Q}}(y)$ and $\widehat{r}(x, y) = \widehat{p}(x) \Cap \widehat{q}(y)$, for all $(x, y) \in X \times Y$.*

Example 3.4. *By the Example 3.3, the values of $\widehat{\mathscr{P}}_p(e_1)$, $\widehat{\mathscr{P}}_p(e_2)$ and $\widehat{\mathscr{Q}}_q(e_1)$, $\widehat{\mathscr{Q}}_q(e_2)$ are stated above. we apple to Definition 3.8, we have*

$$\widehat{\mathscr{R}}_r(e_1, e_1) = \left\{\begin{matrix} \frac{x_1}{\langle([0.5,0.75],[0.6,0.75]),\ ([0.4,0.55],[0.6,0.85])\rangle} \\ \frac{x_2}{\langle([0.65,0.8],[0.65,0.75]),\ ([0.5,0.7],[0.4,0.8])\rangle} \\ \frac{x_3}{\langle([0.6,0.7],[0.8,0.85]),\ ([0.4,0.7],[0.6,0.8])\rangle} \\ \frac{x_4}{\langle([0.55,0.8],[0.5,0.75]),\ ([0.55,0.7],[0.5,0.85])\rangle} \end{matrix}\right\}; \quad \widehat{\mathscr{R}}_r(e_1, e_2) = \left\{\begin{matrix} \frac{x_1}{\langle([0.5,0.75],[0.5,0.6]),([0.25,0.35],[0.2,0.7])\rangle} \\ \frac{x_2}{\langle([0.75,0.8],[0.6,0.7]),([0.55,0.6],[0.3,0.65])\rangle} \\ \frac{x_3}{\langle([0.65,0.7],[0.35,0.55]),([0.25,0.35],[0.6,0.8])\rangle} \\ \frac{x_4}{\langle([0.55,0.8],[0.5,0.6]),([0.25,0.35],[0.3,0.7])\rangle} \end{matrix}\right\};$$

$$\widehat{\mathscr{R}}_r(e_2, e_1) = \left\{\begin{matrix} \frac{x_1}{\langle([0.55,0.9],[0.4,0.55]),([0.35,0.55],[0.45,0.85])\rangle} \\ \frac{x_2}{\langle([0.65,0.75],[0.5,0.75]),([0.45,0.7],[0.4,0.65])\rangle} \\ \frac{x_3}{\langle([0.6,0.85],[0.3,0.6]),([0.4,0.6],[0.55,0.7])\rangle} \\ \frac{x_4}{\langle([0.55,0.9],[0.4,0.6]),([0.35,0.65],[0.45,0.85])\rangle} \end{matrix}\right\}; \quad \widehat{\mathscr{R}}_r(e_2, e_2) = \left\{\begin{matrix} \frac{x_1}{\langle([0.55,0.85],[0.5,0.6]),([0.25,0.35],[0.5,0.95])\rangle} \\ \frac{x_2}{\langle([0.7,0.75],[0.6,0.75]),([0.45,0.6],[0.55,0.85])\rangle} \\ \frac{x_3}{\langle([0.8,0.85],[0.35,0.65]),([0.25,0.35],[0.7,0.95])\rangle} \\ \frac{x_4}{\langle([0.55,0.85],[0.5,0.6]),([0.25,0.35],[0.5,0.95])\rangle} \end{matrix}\right\};$$

Definition 3.9. *Let $(\widehat{\mathscr{P}}_p, X)$ and $(\widehat{\mathscr{Q}}_q, Y)$ be two PFIVFS sets on $(\mathbb{U}, \mathbb{E})$. Then the operations "$(\widehat{\mathscr{P}}_p, X)$ OR $(\widehat{\mathscr{Q}}_q, Y)$" is denoted by $(\widehat{\mathscr{P}}_p, X) \veebar (\widehat{\mathscr{Q}}_q, Y)$ and is defined by $(\widehat{\mathscr{P}}_p, X) \veebar (\widehat{\mathscr{Q}}_q, Y) = (\widehat{\mathscr{R}}_r, X \times Y)$, where $\widehat{\mathscr{R}}_r(x, y) = (\widehat{\mathscr{R}}(x, y)(u), \widehat{r}(x, y)(u))$ such that $\widehat{\mathscr{R}}(x, y) = \widehat{\mathscr{P}}(x) \Cup \widehat{\mathscr{Q}}(y)$ and $\widehat{r}(x, y) = \widehat{p}(x) \Cup \widehat{q}(y)$, for all $(x, y) \in X \times Y$.*

Example 3.5. *By the Example 3.3, the values of $\widehat{\mathscr{P}}_p(e_1)$, $\widehat{\mathscr{P}}_p(e_2)$ and $\widehat{\mathscr{Q}}_q(e_1)$, $\widehat{\mathscr{Q}}_q(e_2)$ are stated above. we apple to Definition 3.9, we have*

$$\widehat{\mathscr{R}}_r(e_1, e_1) = \left\{\begin{matrix} \frac{x_1}{\langle([0.75,0.9],[0.4,0.55]),([0.75,0.8],[0.2,0.7])\rangle} \\ \frac{x_2}{\langle([0.65,0.8],[0.5,0.75]),([0.65,0.8],[0.3,0.65])\rangle} \\ \frac{x_3}{\langle([0.6,0.7],[0.3,0.6]),([0.65,0.75],[0.6,0.7])\rangle} \\ \frac{x_4}{\langle([0.55,0.8],[0.4,0.6]),([0.6,0.75],[0.3,0.7])\rangle} \end{matrix}\right\}; \quad \widehat{\mathscr{R}}_r(e_1, e_2) = \left\{\begin{matrix} \frac{x_1}{\langle([0.65,0.85],[0.6,0.75]),([0.75,0.8],[0.5,0.95])\rangle} \\ \frac{x_2}{\langle([0.75,0.85],[0.65,0.75]),([0.65,0.8],[0.55,0.85])\rangle} \\ \frac{x_3}{\langle([0.85,0.9],[0.8,0.85]),([0.65,0.7],[0.7,0.95])\rangle} \\ \frac{x_4}{\langle([0.65,0.85],[0.5,0.75]),([0.55,0.75],[0.5,0.95])\rangle} \end{matrix}\right\};$$

$$\widehat{\mathscr{R}}_r(e_2,e_1)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.75,0.9],[0.5,0.6]),([0.4,0.65],[0.6,0.85])\rangle}\\ \frac{x_2}{\langle([0.7,0.8],[0.6,0.75]),([0.5,0.85],[0.45,0.8])\rangle}\\ \frac{x_3}{\langle([0.8,0.9],[0.35,0.65]),([0.55,0.75],[0.6,0.9])\rangle}\\ \frac{x_4}{\langle([0.75,0.9],[0.5,0.6]),([0.6,0.7],[0.5,0.85])\rangle}\end{array}\right\};\quad \widehat{\mathscr{R}}_r(e_2,e_2)=\left\{\begin{array}{c}\frac{x_1}{\langle([0.65,0.9],[0.5,0.6]),([0.35,0.65],[0.45,0.85])\rangle}\\ \frac{x_2}{\langle([0.7,0.75],[0.6,0.7]),([0.55,0.85],[0.45,0.65])\rangle}\\ \frac{x_3}{\langle([0.8,0.85],[0.35,0.55]),([0.55,0.6],[0.55,0.9])\rangle}\\ \frac{x_4}{\langle([0.55,0.85],[0.5,0.6]),([0.35,0.65],[0.45,0.85])\rangle}\end{array}\right\};$$

Remark 3.2. *Let $(\widehat{\mathscr{P}}_p, X)$ and $(\widehat{\mathscr{Q}}_q, Y)$ be two PFIVFS sets on $(\mathbb{U},\mathbb{E})$. For all $(x,y)\in X\times Y$, if $x\neq y$. Then $(\widehat{\mathscr{P}}_p, X)\veebar(\widehat{\mathscr{Q}}_q, Y)\neq(\widehat{\mathscr{Q}}_q, Y)\veebar(\widehat{\mathscr{P}}_p, X)$ and $(\widehat{\mathscr{P}}_p, X)\barwedge(\widehat{\mathscr{Q}}_q, Y)\neq(\widehat{\mathscr{Q}}_q, Y)\barwedge(\widehat{\mathscr{P}}_p, X)$.*

Theorem 3.5. *Let $(\widehat{\mathscr{P}}_p, X)$ and $(\widehat{\mathscr{Q}}_q, Y)$ be two PFIVFS sets on $(\mathbb{U},\mathbb{E})$. Then*
(i) $((\widehat{\mathscr{P}}_p, X)\barwedge(\widehat{\mathscr{Q}}_q, Y))^c=(\widehat{\mathscr{P}}_p, X)^c\veebar(\widehat{\mathscr{Q}}_q, Y)^c$
(ii) $((\widehat{\mathscr{P}}_p, X)\veebar(\widehat{\mathscr{Q}}_q, Y))^c=(\widehat{\mathscr{P}}_p, X)^c\barwedge(\widehat{\mathscr{Q}}_q, Y)^c$.

Proof. (i) Suppose that $(\widehat{\mathscr{P}}_p, X)\barwedge(\widehat{\mathscr{Q}}_q, Y)=(\widehat{\mathscr{R}}_r, X\times Y)$. Now, $\widehat{\mathscr{R}}^c_r(x,y)=(\widehat{\mathscr{R}}^c(x,y)(u),\widehat{r^c}(x,y)(u))$, for all $(x,y)\in X\times Y$. By Theorem 3.3 and Definition 3.8, $\widehat{\mathscr{R}}^c(x,y)=(\widehat{\mathscr{P}}(x)\Cap\widehat{\mathscr{Q}}(y))^c=\widehat{\mathscr{P}}^c(x)\Cup\widehat{\mathscr{Q}}^c(y)$ and $\widehat{r^c}(x,y)=(\widehat{p}(x)\Cap\widehat{q}(y))^c=\widehat{p^c}(x)\Cup\widehat{q^c}(y)$. On the other hand, given that $(\widehat{\mathscr{P}}_p, X)^c\veebar(\widehat{\mathscr{Q}}_q, Y)^c=(\widehat{\Lambda}_o, X\times Y)$, where $\widehat{\Lambda}_o(x,y)=(\widehat{\Lambda}(x,y)(u),\widehat{o}(x,y)(u))$ such that $\widehat{\Lambda}(x,y)=\widehat{\mathscr{P}}^c(x)\Cup\widehat{\mathscr{Q}}^c(y)$ and $\widehat{o}(x,y)=\widehat{p^c}(x)\Cup\widehat{q^c}(y)$ for all $(x,y)\in X\times Y$. Thus, $\widehat{\mathscr{R}}^c_r=\widehat{\Lambda}_o$. Hence, $((\widehat{\mathscr{P}}_p, X)\barwedge(\widehat{\mathscr{Q}}_q, Y))^c=(\widehat{\mathscr{P}}_p, X)^c\veebar(\widehat{\mathscr{Q}}_q, Y)^c$.
(ii) Suppose that $(\widehat{\mathscr{P}}_p, X)\veebar(\widehat{\mathscr{Q}}_q, Y)=(\widehat{\mathscr{R}}_r, X\times Y)$. Now, $\widehat{\mathscr{R}}^c_r(x,y)=(\widehat{\mathscr{R}}^c(x,y)(u),\widehat{r^c}(x,y)(u))$, for all $(x,y)\in X\times Y$. By Theorem 3.3 and Definition 3.9, $\widehat{\mathscr{R}}^c(x,y)=(\widehat{\mathscr{P}}(x)\Cup\widehat{\mathscr{Q}}(y))^c=\widehat{\mathscr{P}}^c(x)\Cap\widehat{\mathscr{Q}}^c(y)$ and $\widehat{r^c}(x,y)=(\widehat{p}(x)\Cup\widehat{q}(y))^c=\widehat{p^c}(x)\Cap\widehat{q^c}(y)$. On the other hand, given that $(\widehat{\mathscr{P}}_p, X)^c\barwedge(\widehat{\mathscr{Q}}_q, Y)^c=(\widehat{\Lambda}_o, X\times Y)$, where $\widehat{\Lambda}_o(x,y)=(\widehat{\Lambda}(x,y)(u),\widehat{o}(x,y)(u))$ such that $\widehat{\Lambda}(x,y)=\widehat{\mathscr{P}}^c(x)\Cap\widehat{\mathscr{Q}}^c(y)$ and $\widehat{o}(x,y)=\widehat{p^c}(x)\Cap\widehat{q^c}(y)$ for all $(x,y)\in X\times Y$. Thus, $\widehat{\mathscr{R}}^c_r=\widehat{\Lambda}_o$. Hence, $((\widehat{\mathscr{P}}_p, X)\veebar(\widehat{\mathscr{Q}}_q, Y))^c=(\widehat{\mathscr{P}}_p, X)^c\barwedge(\widehat{\mathscr{Q}}_q, Y)^c$. □

4. Similarity measure between two PFIVFS sets

In this section, we discuss similarity measure between two PFIVFS sets.

Definition 4.1. *Let $\mathbb{U}$ be a non-empty set of the universe and $\mathbb{E}$ be a set of parameter. Suppose that $\widehat{\mathscr{P}}_p$ and $\widehat{\mathscr{Q}}_q$ are two PFIVFS sets on $(\mathbb{U},\mathbb{E})$. Then the similarity measure between two PFIVFS sets $\widehat{\mathscr{P}}_p$ and $\widehat{\mathscr{Q}}_q$ is denoted by $Sim(\widehat{\mathscr{P}}_p,\widehat{\mathscr{Q}}_q)$ and is defined as:*

$$Sim(\widehat{\mathscr{P}}_p,\widehat{\mathscr{Q}}_q)=\Big[Sim(\mathscr{P}^-_p,\mathscr{Q}^-_q),\ Sim(\mathscr{P}^+_p,\mathscr{Q}^+_q)\Big]$$
$$=\Big[(\Upsilon(\mathscr{P}^-,\mathscr{Q}^-)\cdot\Psi(p^-,q^-)),\ (\Upsilon(\mathscr{P}^+,\mathscr{Q}^+)\cdot\Psi(p^+,q^+))\Big]$$

such that $\Upsilon(\widehat{\mathscr{P}},\widehat{\mathscr{Q}})=\Big[\Upsilon(\mathscr{P}^-,\mathscr{Q}^-),\ \Upsilon(\mathscr{P}^+,\mathscr{Q}^+)\Big]=$

$$\left[\frac{\Gamma(\mathscr{P}^-(e)(u),\mathscr{Q}^-(e)(u))+\Delta(\mathscr{P}^-(e)(u),\mathscr{Q}^-(e)(u))}{2},\frac{\Gamma(\mathscr{P}^+(e)(u),\mathscr{Q}^+(e)(u))+\Delta(\mathscr{P}^+(e)(u),\mathscr{Q}^+(e)(u))}{2}\right]$$ *and*

$$\Psi(\widehat{p},\widehat{q}) = \Big[\Psi(p^-,q^-),\ \Psi(p^+,q^+)\Big] = \left[1 - \frac{\sum_{j=1}^{m}|\delta_j^- - \kappa_j^-|}{\sum_{j=1}^{m}|\delta_j^- + \kappa_j^-|}\ ,\ 1 - \frac{\sum_{j=1}^{m}|\delta_j^+ - \kappa_j^+|}{\sum_{j=1}^{m}|\delta_j^+ + \kappa_j^+|}\right].$$

since, $m = |E|$, *where* $\Big[\Gamma(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)),\ \Gamma(\mathscr{P}^+(e)(u), \mathscr{Q}^+(e)(u))\Big] =$

$$\left[\frac{\sum_{j=1}^{m}\left(\chi^{3/2-}_{\mathscr{P}(e_j)}(u)\ \cdot\ \chi^{3/2-}_{\mathscr{Q}(e_j)}(u)\right)}{\sum_{j=1}^{m}\left(1 - \sqrt[3]{\left((1-\chi^{3-}_{\mathscr{P}(e_j)}(u))\ \cdot\ (1-\chi^{3-}_{\mathscr{Q}(e_j)}(u))\right)^{3/2}}\right)}\ ,\ \frac{\sum_{j=1}^{m}\left(\chi^{3/2+}_{\mathscr{P}(e_j)}(u)\ \cdot\ \chi^{3/2+}_{\mathscr{Q}(e_j)}(u)\right)}{\sum_{j=1}^{m}\left(1 - \sqrt[3]{\left((1-\chi^{3+}_{\mathscr{P}(e_j)}(u))\ \cdot\ (1-\chi^{3+}_{\mathscr{Q}(e_j)}(u))\right)^{3/2}}\right)}\right]$$

and $\Big[(\Delta(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)))\ ,(\Delta(\mathscr{P}^+(e)(u), \mathscr{Q}^+(e)(u)))\Big] =$

$$\left[\sqrt[3]{1 - \frac{\sum_{j=1}^{m}\left|\psi^{3-}_{\mathscr{P}(e_j)}(u)\ -\ \psi^{3-}_{\mathscr{Q}(e_j)}(u)\right|}{\sum_{j=1}^{m}1 + \left(\psi^{3-}_{\mathscr{P}(e_j)}(u)\ \cdot\ \psi^{3-}_{\mathscr{Q}(e_j)}(u)\right)}}\ ,\ \sqrt[3]{1 - \frac{\sum_{j=1}^{m}\left|\psi^{3+}_{\mathscr{P}(e_j)}(u)\ -\ \psi^{3+}_{\mathscr{Q}(e_j)}(u)\right|}{\sum_{j=1}^{m}1 + \left(\psi^{3+}_{\mathscr{P}(e_j)}(u)\ \cdot\ \psi^{3+}_{\mathscr{Q}(e_j)}(u)\right)}}\right]$$

and $\delta_j^- = \dfrac{\chi^{3-}_{p(e_j)}(u)}{\chi^{3-}_{p(e_j)}(u) + \psi^{3-}_{p(e_j)}(u)}$, $\delta_j^+ = \dfrac{\chi^{3+}_{p(e_j)}(u)}{\chi^{3+}_{p(e_j)}(u) + \psi^{3+}_{p(e_j)}(u)}$ *and*
$\kappa_j^- = \dfrac{\chi^{3-}_{q(e_j)}(u)}{\chi^{3-}_{q(e_j)}(u) + \psi^{3-}_{q(e_j)}(u)}$, $\kappa_j^+ = \dfrac{\chi^{3+}_{q(e_j)}(u)}{\chi^{3+}_{q(e_j)}(u) + \psi^{3+}_{q(e_j)}(u)}$.

Theorem 4.1. *Let* $\widehat{\mathscr{P}}_p$, $\widehat{\mathscr{Q}}_q$ *and* $\widehat{\mathscr{R}}_r$ *be three PFIVFS sets over* $(\mathbb{U}, \mathbb{E})$*. Then, the following properties hold:*

(1) $Sim(\widehat{\mathscr{P}}_p, \widehat{\mathscr{Q}}_q) = Sim(\widehat{\mathscr{Q}}_q, \widehat{\mathscr{P}}_p)$
(2) $0 \leq Sim(\widehat{\mathscr{P}}_p, \widehat{\mathscr{Q}}_q) \leq= 1$
(3) $\widehat{\mathscr{P}}_p = \widehat{\mathscr{Q}}_q \implies Sim(\widehat{\mathscr{P}}_p, \widehat{\mathscr{Q}}_q) = 1$
(4) $\widehat{\mathscr{P}}_p \sqsubseteq \widehat{\mathscr{Q}}_q \sqsubseteq \widehat{\mathscr{R}}_r \implies Sim(\widehat{\mathscr{P}}_p, \widehat{\mathscr{R}}_r) \leq Sim(\widehat{\mathscr{Q}}_q, \widehat{\mathscr{R}}_r)$.

Proof. The proof (1), (2) and are trivial. Now we proof (3). Suppose that $\widehat{\mathscr{P}}_p = \widehat{\mathscr{Q}}_q$ implies that $\chi_{\widehat{\mathscr{P}}(e)}(u) = \chi_{\widehat{\mathscr{Q}}(e)}(u), \psi_{\widehat{\mathscr{P}}(e)}(u) = \psi_{\widehat{\mathscr{Q}}(e)}(u), \chi_{\widehat{p}(e)}(u) = \chi_{\widehat{q}(e)}(u)$ and $\psi_{\widehat{p}(e)}(u) = \psi_{\widehat{q}(e)}(u)$.
Thus,

$$[\chi^-_{\mathscr{P}(e)}(u), \chi^+_{\mathscr{P}(e)}(u)] = [\chi^-_{\mathscr{Q}(e)}(u), \chi^+_{\mathscr{Q}(e)}(u)],$$

$$[\psi^-_{\mathscr{P}(e)}(u), \psi^+_{\mathscr{P}(e)}(u)] = [\psi^-_{\mathscr{Q}(e)}(u), \psi^+_{\mathscr{Q}(e)}(u)],$$

$$[\chi^-_{p(e)}(u), \chi^+_{p(e)}(u)] = [\chi^-_{q(e)}(u), \chi^+_{q(e)}(u)],$$

$$[\psi^-_{p(e)}(u), \psi^+_{p(e)}(u)] = [\psi^-_{q(e)}(u), \psi^+_{q(e)}(u)]$$

. Now, $\Big[\Gamma(\mathscr{P}^{-}(e)(u),\mathscr{Q}^{-}(e)(u)),\ \Gamma(\mathscr{P}^{+}(e)(u),\mathscr{Q}^{+}(e)(u))\Big]$

$$
\begin{aligned}
&= \left[\frac{\sum_{j=1}^{m}(\chi^{-}_{\mathscr{P}(e_j)}(u))^{\frac{3}{2}+\frac{3}{2}}}{\sum_{j=1}^{m}\Big(1-1+(\chi^{-}_{\mathscr{P}(e_j)}(u))^{3}\Big)},\ \frac{\sum_{j=1}^{m}(\chi^{+}_{\mathscr{P}(e_j)}(u))^{\frac{3}{2}+\frac{3}{2}}}{\sum_{j=1}^{m}\Big(1-1+(\chi^{+}_{\mathscr{P}(e_j)}(u))^{3}\Big)}\right]\\
&= \left[\frac{\sum_{j=1}^{m}(\chi^{-}_{\mathscr{P}(e_j)}(u))^{3}}{\sum_{j=1}^{m}(\chi^{-}_{\mathscr{P}(e_j)}(u))^{3}},\ \frac{\sum_{j=1}^{m}(\chi^{+}_{\mathscr{P}(e_j)}(u))^{3}}{\sum_{j=1}^{m}(\chi^{+}_{\mathscr{P}(e_j)}(u))^{3}}\right]\\
&= 1
\end{aligned}
$$

and $\Big[\Delta(\mathscr{P}^{-}(e)(u),\mathscr{Q}^{-}(e)(u)),\Delta(\mathscr{P}^{+}(e)(u),\mathscr{Q}^{+}(e)(u))\Big]=\Big[\sqrt[3]{(1-0)},\ \sqrt[3]{(1-0)}\Big]=1.$
Thus, $\Upsilon(\widehat{\mathscr{P}},\widehat{\mathscr{Q}})=\Big[\Upsilon(\mathscr{P}^{-},\mathscr{Q}^{-}),\ \Upsilon(\mathscr{P}^{+},\mathscr{Q}^{+})\Big]=\Big[\frac{1+1}{2},\frac{1+1}{2}\Big]=1$ and
$\Psi(\widehat{p},\widehat{q})=\Big[\Psi(p^{-},q^{-}),\ \Psi(p^{+},q^{+})\Big]=1.$
Hence, $Sim(\widehat{\mathscr{P}}_p,\widehat{\mathscr{Q}}_q)=\Big[Sim(\mathscr{P}^{-}_p,\mathscr{Q}^{-}_q),\ Sim(\mathscr{P}^{+}_p,\mathscr{Q}^{+}_q)\Big]=1$. Thus. (3) proved.
(4) Given that

$$
\begin{cases}
\widehat{\mathscr{P}}_p\sqsubseteq\widehat{\mathscr{Q}}_q \implies \chi_{\widehat{\mathscr{P}}(e)}(u)\le\chi_{\widehat{\mathscr{Q}}(e)}(u),\quad \psi_{\widehat{\mathscr{P}}(e)}(u)\ge\psi_{\widehat{\mathscr{Q}}(e)}(u)\\
\chi_{\widehat{p}(e)}(u)\le\chi_{\widehat{q}(e)}(u),\quad \psi_{\widehat{p}(e)}(u)\ge\psi_{\widehat{q}(e)}(u)\\
\widehat{\mathscr{P}}_p\sqsubseteq\widehat{\mathscr{R}}_r \implies \chi_{\widehat{\mathscr{P}}(e)}(u)\le\chi_{\widehat{\mathscr{R}}(e)}(u),\quad \psi_{\widehat{\mathscr{P}}(e)}(u)\ge\psi_{\widehat{\mathscr{R}}(e)}(u)\\
\chi_{\widehat{p}(e)}(u)\le\chi_{\widehat{r}(e)}(u),\quad \psi_{\widehat{p}(e)}(u)\ge\psi_{\widehat{r}(e)}(u)\\
\widehat{\mathscr{Q}}_q\sqsubseteq\widehat{\mathscr{R}}_r \implies \chi_{\widehat{\mathscr{Q}}(e)}(u)\le\chi_{\widehat{\mathscr{R}}(e)}(u),\quad \psi_{\widehat{\mathscr{Q}}(e)}(u)\ge\psi_{\widehat{\mathscr{R}}(e)}(u)\\
\chi_{\widehat{q}(e)}(u)\le\chi_{\widehat{r}(e)}(u),\quad \psi_{\widehat{q}(e)}(u)\ge\psi_{\widehat{r}(e)}(u)
\end{cases}
\tag{4.1}
$$

Thus,

$$
\begin{cases}
\Big[\chi^{-}_{\mathscr{P}(e)}(u),\chi^{+}_{\mathscr{P}(e)}(u)\Big]=\chi_{\widehat{\mathscr{P}}(e)}(u),\ \Big[\psi^{-}_{\mathscr{P}(e)}(u),\psi^{+}_{\mathscr{P}(e)}(u)\Big]=\psi_{\widehat{\mathscr{P}}(e)}(u)\\
\Big[\chi^{-}_{p(e)}(u),\chi^{+}_{p(e)}(u)\Big]=\chi_{\widehat{p}(e)}(u),\quad \Big[\psi^{-}_{p(e)}(u),\psi^{+}_{p(e)}(u)\Big]=\psi_{\widehat{p}(e)}(u)\\
\Big[\chi^{-}_{\mathscr{Q}(e)}(u),\chi^{+}_{\mathscr{Q}(e)}(u)\Big]=\chi_{\widehat{\mathscr{Q}}(e)}(u),\ \Big[\psi^{-}_{\mathscr{Q}(e)}(u),\psi^{+}_{\mathscr{Q}(e)}(u)\Big]=\psi_{\widehat{\mathscr{Q}}(e)}(u)\\
\Big[\chi^{-}_{q(e)}(u),\chi^{+}_{q(e)}(u)\Big]=\chi_{\widehat{q}(e)}(u),\quad \Big[\psi^{-}_{q(e)}(u),\psi^{+}_{q(e)}(u)\Big]=\psi_{\widehat{q}(e)}(u)\\
\Big[\chi^{-}_{\mathscr{R}(e)}(u),\chi^{+}_{\mathscr{R}(e)}(u)\Big]=\chi_{\widehat{\mathscr{R}}(e)}(u),\ \Big[\psi^{-}_{\mathscr{R}(e)}(u),\psi^{+}_{\mathscr{R}(e)}(u)\Big]=\psi_{\widehat{\mathscr{R}}(e)}(u)\\
\Big[\chi^{-}_{r(e)}(u),\chi^{+}_{r(e)}(u)\Big]=\chi_{\widehat{r}(e)}(u),\quad \Big[\psi^{-}_{r(e)}(u),\psi^{+}_{r(e)}(u)\Big]=\psi_{\widehat{r}(e)}(u).
\end{cases}
$$

By Equation 4.1, Clearly $\chi^{3/2}_{\widehat{\mathscr{P}}(e)}(u)\cdot\chi^{3/2}_{\widehat{\mathscr{R}}(e)}(u)\leq\chi^{3/2}_{\widehat{\mathscr{Q}}(e)}(u)\cdot\chi^{3/2}_{\widehat{\mathscr{R}}(e)}(u)$
which implies

$$\sum_{j=1}^{m}\left(\chi^{3/2}_{\widehat{\mathscr{P}}(e_j)}(u)\cdot\chi^{3/2}_{\widehat{\mathscr{R}}(e_j)}(u)\right)\leq\sum_{j=1}^{m}\left(\chi^{3/2}_{\widehat{\mathscr{Q}}(e_j)}(u)\cdot\chi^{3/2}_{\widehat{\mathscr{R}}(e_j)}(u)\right)\tag{4.2}$$

By Equation 4.1, Clearly, $(\chi_{\widehat{\mathscr{P}}(e)}(u))^3\leq(\chi_{\widehat{\mathscr{Q}}(e)}(u))^3$
which implies $(1-(\chi_{\widehat{\mathscr{P}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\geq(1-(\chi_{\widehat{\mathscr{Q}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)$ and

$$\left((1-(\chi_{\widehat{\mathscr{P}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}\geq\left((1-(\chi_{\widehat{\mathscr{Q}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}$$ and

$$1-\sqrt[3]{\left((1-(\chi_{\widehat{\mathscr{P}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}}\leq1-\sqrt[3]{\left((1-(\chi_{\widehat{\mathscr{Q}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}}$$
and

$$\sum_{j=1}^{m}\left\{1-\sqrt[3]{\left((1-(\chi_{\widehat{\mathscr{P}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}}\right\}\leq\sum_{j=1}^{m}\left\{1-\sqrt[3]{\left((1-(\chi_{\widehat{\mathscr{Q}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}}\right\}\tag{4.3}$$

Equation 4.2 is divided by 4.3,

$$\frac{\sum_{j=1}^{m}\left(\chi^{3/2}_{\widehat{\mathscr{P}}(e_j)}(u)\cdot\chi^{3/2}_{\widehat{\mathscr{R}}(e_j)}(u)\right)}{\sum_{j=1}^{m}\left\{1-\sqrt[3]{\left((1-(\chi_{\widehat{\mathscr{P}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}}\right\}}\leq\frac{\sum_{j=1}^{m}\left(\chi^{3/2}_{\widehat{\mathscr{Q}}(e_j)}(u)\cdot\chi^{3/2}_{\widehat{\mathscr{R}}(e_j)}(u)\right)}{\sum_{j=1}^{m}\left\{1-\sqrt[3]{\left((1-(\chi_{\widehat{\mathscr{Q}}(e)}(u))^3)\cdot(1-(\chi_{\widehat{\mathscr{R}}(e)}(u))^3)\right)^{3/2}}\right\}}\tag{4.4}$$

By Equation 4.1, Clearly, $\psi^3_{\widehat{\mathscr{P}}(e)}(u)\geq\psi^3_{\widehat{\mathscr{Q}}(e)}(u)$ and $\psi^3_{\widehat{\mathscr{P}}(e)}(u)-\psi^3_{\widehat{\mathscr{R}}(e)}(u)\geq\psi^3_{\widehat{\mathscr{Q}}(e)}(u)-\psi^3_{\widehat{\mathscr{R}}(e)}(u)$.
Hence

$$\sum_{j=1}^{m}\left|\psi^3_{\widehat{\mathscr{P}}(e_j)}(u)-\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right|\geq\sum_{j=1}^{m}\left|\psi^3_{\widehat{\mathscr{Q}}(e_j)}(u)-\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right|\tag{4.5}$$

Also, $\left(\psi^3_{\widehat{\mathscr{P}}(e)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e)}(u)\right)\geq\left(\psi^3_{\widehat{\mathscr{Q}}(e)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e)}(u)\right)$ implies

$$\sum_{j=1}^{m}\left\{1+\left(\psi^3_{\widehat{\mathscr{P}}(e_j)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}\geq\sum_{j=1}^{m}\left\{1+\left(\psi^3_{\widehat{\mathscr{Q}}(e_j)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}\tag{4.6}$$

Equation 4.5 is divided by 4.6, we get

$$\frac{\sum_{j=1}^{m}\left|\psi^3_{\widehat{\mathscr{P}}(e_j)}(u)-\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right|}{\sum_{j=1}^{m}\left\{1+\left(\psi^3_{\widehat{\mathscr{P}}(e_j)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}}\geq\frac{\sum_{j=1}^{m}\left|\psi^3_{\widehat{\mathscr{Q}}(e_j)}(u)-\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right|}{\sum_{j=1}^{m}\left\{1+\left(\psi^3_{\widehat{\mathscr{Q}}(e_j)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}}$$

and

$$1-\frac{\sum_{j=1}^{m}\left|\psi^3_{\widehat{\mathscr{P}}(e_j)}(u)-\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right|}{\sum_{j=1}^{m}\left\{1+\left(\psi^3_{\widehat{\mathscr{P}}(e_j)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}}\leq1-\frac{\sum_{j=1}^{m}\left|\psi^3_{\widehat{\mathscr{Q}}(e_j)}(u)-\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right|}{\sum_{j=1}^{m}\left\{1+\left(\psi^3_{\widehat{\mathscr{Q}}(e_j)}(u)\cdot\psi^3_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}}$$

and

$$\sqrt[3]{1-\frac{\sum_{j=1}^{m}\left|\psi^{3}_{\widehat{\mathscr{P}}(e_j)}(u)-\psi^{3}_{\widehat{\mathscr{R}}(e_j)}(u)\right|}{\sum_{j=1}^{m}\left\{1+\left(\psi^{3}_{\widehat{\mathscr{P}}(e_j)}(u)\cdot\psi^{3}_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}}}\leq\sqrt[3]{1-\frac{\sum_{j=1}^{m}\left|\psi^{3}_{\widehat{\mathscr{Q}}(e_j)}(u)-\psi^{3}_{\widehat{\mathscr{R}}(e_j)}(u)\right|}{\sum_{j=1}^{m}\left\{1+\left(\psi^{3}_{\widehat{\mathscr{Q}}(e_j)}(u)\cdot\psi^{3}_{\widehat{\mathscr{R}}(e_j)}(u)\right)\right\}}} \tag{4.7}$$

Adding Equation 4.4, 4.7 and divided by 2,

$$\Upsilon(\widehat{\mathscr{P}},\widehat{\mathscr{R}})\leq\Upsilon(\widehat{\mathscr{Q}},\widehat{\mathscr{R}}) \tag{4.8}$$

By Equation 4.1, Clearly $\delta_j^-\leq\kappa_j^-\leq\rho_j^-$ and $\delta_j^+\leq\kappa_j^+\leq\rho_j^+$,
where

$$\left[\delta_j^-,\delta_j^+\right]=\left[\frac{\chi^{3-}_{p(e_j)}(u)}{\chi^{3-}_{p(e_j)}(u)\ +\ \psi^{3-}_{p(e_j)}(u)},\frac{\chi^{3+}_{p(e_j)}(u)}{\chi^{3+}_{p(e_j)}(u)\ +\ \psi^{3+}_{p(e_j)}(u)}\right]$$

and

$$\left[\kappa_j^-,\kappa_j^+\right]=\left[\frac{\chi^{3-}_{q(e_j)}(u)}{\chi^{3-}_{q(e_j)}(u)\ +\ \psi^{3-}_{q(e_j)}(u)},\frac{\chi^{3+}_{q(e_j)}(u)}{\chi^{3+}_{q(e_j)}(u)\ +\ \psi^{3+}_{q(e_j)}(u)}\right]$$

and

$$\left[\rho_j^-,\rho_j^+\right]=\left[\frac{\chi^{3-}_{r(e_j)}(u)}{\chi^{3-}_{r(e_j)}(u)\ +\ \psi^{3-}_{r(e_j)}(u)},\frac{\chi^{3+}_{r(e_j)}(u)}{\chi^{3+}_{r(e_j)}(u)\ +\ \psi^{3+}_{r(e_j)}(u)}\right].$$

Hence $\left[|\kappa_j^-|-|\rho_j^-|,|\kappa_j^+|-|\rho_j^+|\right]\leq\left[|\delta_j^-|-|\rho_j^-|,|\delta_j^+|-|\rho_j^+|\right]$ and

$$-\left[|\delta_j^-|-|\rho_j^-|,|\delta_j^+|-|\rho_j^+|\right]\leq-\left[|\kappa_j^-|-|\rho_j^-|,|\kappa_j^+|-|\rho_j^+|\right]$$

and

$$\left[|\delta_j^-|+|\rho_j^-|,|\delta_j^+|+|\rho_j^+|\right]\leq\left[|\kappa_j^-|+|\rho_j^-|,|\kappa_j^+|+|\rho_j^+|\right]$$

and hence,

$$-\sum_{j=1}^{m}\left[|\delta_j^-|-|\rho_j^-|,|\delta_j^+|-|\rho_j^+|\right]\leq-\sum_{j=1}^{m}\left[|\kappa_j^-|-|\rho_j^-|,|\kappa_j^+|-|\rho_j^+|\right] \tag{4.9}$$

and

$$\sum_{j=1}^{m}\left[|\delta_j^-|+|\rho_j^-|,|\delta_j^+|+|\rho_j^+|\right]\leq\sum_{j=1}^{m}\left[|\kappa_j^-|+|\rho_j^-|,|\kappa_j^+|+|\rho_j^+|\right] \tag{4.10}$$

Equation 4.9 is divided by 4.10, we get

$$\left[-\frac{\sum_{j=1}^{m}|\delta_j^- - \rho_j^-|}{\sum_{j=1}^{m}|\delta_j^- + \rho_j^-|}, -\frac{\sum_{j=1}^{m}|\delta_j^+ - \rho_j^+|}{\sum_{j=1}^{m}|\delta_j^+ + \rho_j^+|}\right] \leq \left[-\frac{\sum_{j=1}^{m}|\kappa_j^- - \rho_j^-|}{\sum_{j=1}^{m}|\kappa_j^- + \rho_j^-|}, -\frac{\sum_{j=1}^{m}|\kappa_j^+ - \rho_j^+|}{\sum_{j=1}^{m}|\kappa_j^+ + \rho_j^+|}\right] \text{and}$$

$$\left[1-\frac{\sum_{j=1}^{m}|\delta_j^- - \rho_j^-|}{\sum_{j=1}^{m}|\delta_j^- + \rho_j^-|}, 1-\frac{\sum_{j=1}^{m}|\delta_j^+ - \rho_j^+|}{\sum_{j=1}^{m}|\delta_j^+ + \rho_j^+|}\right] \leq \left[1-\frac{\sum_{j=1}^{m}|\kappa_j^- - \rho_j^-|}{\sum_{j=1}^{m}|\kappa_j^- + \rho_j^-|}, 1-\frac{\sum_{j=1}^{m}|\kappa_j^+ - \rho_j^+|}{\sum_{j=1}^{m}|\kappa_j^+ + \rho_j^+|}\right].$$

Hence

$$\Psi(\widehat{p},\widehat{r}) \leq \Psi(\widehat{q},\widehat{r}) \tag{4.11}$$

Multiply by Equation 4.8 and 4.11, $\Upsilon(\widehat{\mathscr{P}},\widehat{\mathscr{R}}) \cdot \Psi(\widehat{p},\widehat{r}) \leq \Upsilon(\widehat{\mathscr{Q}},\widehat{\mathscr{R}}) \cdot \Psi(\widehat{q},\widehat{r})$.
Hence, $Sim(\widehat{\mathscr{P}}_p,\widehat{\mathscr{R}}_r) \leq Sim(\widehat{\mathscr{Q}}_q,\widehat{\mathscr{R}}_r)$. This proves (4). □

Example 4.1. *We calculate the similarity measure between the two PFIVFS sets namely $\widehat{\mathscr{P}}_p$ and $\widehat{\mathscr{Q}}_q$. We choose the first sample of $\widehat{\mathscr{P}}_p$ and $\widehat{\mathscr{Q}}_q$, $\mathbb{E} = \{e_1, e_2, e_3, e_4\}$ can be defined as below:*

$\widehat{\mathscr{P}}_p(e)$	e_1	e_2	e_3	e_4
$\widehat{\mathscr{P}}(e)(u)$	$\langle[0.5, 0.75], [0.6, 0.75]\rangle$	$\langle[0.75, 0.8], [0.65, 0.75]\rangle$	$\langle[0.65, 0.7], [0.8, 0.85]\rangle$	$\langle[0.55, 0.8], [0.5, 0.75]\rangle$
$\widehat{p}(e)(u)$	$\langle[0.75, 0.8], [0.2, 0.7]\rangle$	$\langle[0.65, 0.8], [0.3, 0.65]\rangle$	$\langle[0.65, 0.7], [0.6, 0.8]\rangle$	$\langle[0.55, 0.75], [0.3, 0.7]\rangle$

$\widehat{\mathscr{Q}}_q(e)$	e_1	e_2	e_3	e_4
$\widehat{\mathscr{Q}}(e)(u)$	$\langle[0.75, 0.9], [0.4, 0.55]\rangle$	$\langle[0.65, 0.8], [0.5, 0.75]\rangle$	$\langle[0.6, 0.9], [0.3, 0.6]\rangle$	$\langle[0.75, 0.9], [0.4, 0.6]\rangle$
$\widehat{q}(e)(u)$	$\langle[0.4, 0.55], [0.6, 0.85]\rangle$	$\langle[0.5, 0.7], [0.4, 0.8]\rangle$	$\langle[0.4, 0.75], [0.6, 0.7]\rangle$	$\langle[0.6, 0.7], [0.5, 0.85]\rangle$

Now, $\Gamma(\widehat{\mathscr{P}}(e)(u), \widehat{\mathscr{Q}}(e)(u)) = \Big[\Gamma(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)),\ \Gamma(\mathscr{P}^+(e)(u), \mathscr{Q}^+(e)(u))\Big]$.
where, $\Gamma(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)) = \frac{X_1}{Y_1}$,
$X_1 = \left(0.5^{3/2} \times 0.75^{3/2}\right) + \left(0.75^{3/2} \times 0.65^{3/2}\right) + \left(0.65^{3/2} \times 0.6^{3/2}\right) + \left(0.55^{3/2} \times 0.75^{3/2}\right) = 1.078506$ *and*

$$Y_1 = \left\{1 - \left(\left((1-0.5^3) \times (1-0.75^3)\right)^{3/2}\right)^{1/3}\right\} + \left\{1 - \left(\left((1-0.75^3) \times (1-0.65^3)\right)^{3/2}\right)^{1/3}\right\}$$
$$+ \left\{1 - \left(\left((1-0.65^3) \times (1-0.6^3)\right)^{3/2}\right)^{1/3}\right\} + \left\{1 - \left(\left((1-0.55^3) \times (1-0.75^3)\right)^{3/2}\right)^{1/3}\right\} = 1.192847.$$

$\Gamma(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)) = \frac{1.078506}{1.192847} = 0.904144.$
Similarly, $\Gamma(\mathscr{P}^+(e)(u), \mathscr{Q}^+(e)(u)) = \frac{X_2}{Y_2} = \frac{2.177556}{2.330567} = 0.934346.$
$\Gamma(\widehat{\mathscr{P}}(e)(u), \widehat{\mathscr{Q}}(e)(u)) = \Big[0.904144, 0.934346\Big].$
$\Delta(\widehat{\mathscr{P}}(e)(u), \widehat{\mathscr{Q}}(e)(u)) = \Big[\Delta(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)),\ \Delta(\mathscr{P}^+(e)(u), \mathscr{Q}^+(e)(u))\Big].$
$\Delta(\mathscr{P}^-(e)(u), \mathscr{Q}^-(e)(u)) = \left(1 - \frac{X_3}{Y_3}\right)^{1/3}$,
$X_3 = \left|0.6^3 - 0.4^3\right| + \left|0.65^3 - 0.5^3\right| + \left|0.8^3 - 0.3^3\right| + \left|0.5^3 - 0.4^3\right| = 0.847625$ *and*
$Y_3 = \left(1 + (|0.6^3 \times 0.4^3|)\right) + \left(1 + (|0.65^3 \times 0.5^3|)\right) + \left(1 + (|0.8^3 \times 0.3^3|)\right) + \left(1 + (|0.5^3 \times 0.4^3|)\right) = 4.069976.$

$\Delta(\mathscr{P}^-(e)(u),\mathscr{Q}^-(e)(u)) = \left(1-\frac{0.847625}{4.069976}\right)^{1/3} = 0.925111.$

$$\begin{aligned}\Delta(\mathscr{P}^+(e)(u),\mathscr{Q}^+(e)(u)) &= \left(1-\frac{X_4}{Y_4}\right)^{1/3}\\ &= \left(1-\frac{0.859500}{4.471944}\right)^{1/3}\\ &= 0.931326.\end{aligned}$$

$\Delta(\widehat{\mathscr{P}}(e)(u),\widehat{\mathscr{Q}}(e)(u)) = \Big[0.925111, 0.931326\Big].$

$$\begin{aligned}\Upsilon(\widehat{\mathscr{P}},\widehat{\mathscr{Q}}) &= \Big[\Upsilon(\mathscr{P}^-,\mathscr{Q}^-),\ \Upsilon(\mathscr{P}^+,\mathscr{Q}^+)\Big]\\ &= \Big[0.925111, 0.931326\Big]\times\Big[0.904144, 0.934346\Big]\\ &= \Big[0.914627, 0.932836\Big].\end{aligned}$$

$$\begin{aligned}\Psi(\widehat{p},\widehat{q}) &= \Big[\Psi(p^-,q^-),\ \Psi(p^+,q^+)\Big]\\ &= \left[1-\frac{1.560045}{5.063945}, 1-\frac{0.978965}{3.726702}\right]\\ &= \Big[0.691931, 0.737311\Big].\end{aligned}$$

$$\begin{aligned}Sim(\widehat{\mathscr{P}}_p,\widehat{\mathscr{Q}}_q) &= \Big[Sim(\mathscr{P}_p^-,\mathscr{Q}_q^-),\ Sim(\mathscr{P}_p^+,\mathscr{Q}_q^+)\Big]\\ &= \Big[0.914627, 0.932836\Big]\times\Big[0.691931, 0.737311\Big]\\ &= \Big[0.632859, 0.687790\Big].\end{aligned}$$

5. Application of PFIVFS set in decision making

The personal computer is what the majority of people have in their homes at present. With personal computers, people can use them at home, school, or business. These computers can store abundant memory and space. Computers themselves have a glass monitor, like a television screen, which enables people to see more colors. It also has a higher resolution rate so people can see more clearly. A personal computer can have some remarkable features added to it. People can add printers, bigger speakers, desktop scanner beds, and best of all,a hard drive of bigger energy. Nowadays, the laptop is a computer that is lightweight and portable for easy transportation, which makes life easier to take on business trips, vacations, and anywhere people want to take it. A laptop simply means that people can set the computer down on their lap, desk, or on any flat surface. Laptop computers themselves have a plastic screen that reduces the resolution rate. This is why people have such a hard time seeing things on the computer. No matter where people sit in front of the computer screen, it will always produce different colors; therefore, making it harder to read the screen. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria. The Need to buy a laptop can be due to various reasons. A consumer goes through several stages before purchasing a product or service.

5.1. **Algorithm for PFIVFS set model.**

(1) Input the values for PFIVFS sets in tabular form.
(2) Input the set of choice parameters $A \sqsubseteq E$.
(3) Compute the values for Γ and Δ.
(4) Calculate the Υ value by taking $\frac{\Gamma+\Delta}{2}$.
(5) Determine the value $\Psi(\widehat{p},\widehat{q}) = \left[1 - \frac{\sum_{j=1}^{m} |\delta_j^- - \kappa_j^-|}{\sum_{j=1}^{m} |\delta_j^- + \kappa_j^-|}, 1 - \frac{\sum_{j=1}^{m} |\delta_j^+ - \kappa_j^+|}{\sum_{j=1}^{m} |\delta_j^+ + \kappa_j^+|} \right]$.
(6) Compute the similarity measure by taking the product of Υ and Ψ.
(7) Determine maximum similarity measure = $Max\{similarity \ \ measure^i\}$ and $1 \leq i \leq m$.
(8) Optimal output solution yields to the problem.

5.2. **Decision making during laptop purchase.** Let a customer decides to purchase a laptop form the analyses five laptop brands namely A, B, C, D and E. The differentiates attributes of the laptop evaluated by the experts is represented by $\mathbb{E} = \{e_1$ = battery life, e_2 = operating system, e_3 = storage capacity, e_4= speed of the processor, e_5 = overall cost$\}$. Now, we have following five PFIVFS sets for five laptop brands representatives along write PFIVFS source for the ideal laptop.

Table 1
PFIVFS set for the ideal laptop

$\widehat{\mathscr{I}_p}(e)$	e_1	e_2	e_3
$\widehat{\mathscr{I}}(e)$	$\langle[0.9, 0.92], [0.35, 0.45]\rangle$	$\langle[0.85, 0.95], [0.3, 0.35]\rangle$	$\langle[0.9, 0.95], [0.25, 0.4]\rangle$
$\widehat{p}(e)$	$\langle[1 \ \ 1], [0 \ \ 0]\rangle$	$\langle[1 \ \ 1], [0 \ \ 0]\rangle$	$\langle[1 \ \ 1], [0 \ \ 0]\rangle$

$\widehat{\mathscr{I}_p}(e)$	e_4	e_5
$\widehat{\mathscr{I}}(e)$	$\langle[0.8, 0.9], [0.4, 0.5]\rangle$	$\langle[0.85, 0.9], [0.35, 0.45]\rangle$
$\widehat{p}(e)$	$\langle[1 \ \ 1], [0 \ \ 0]\rangle$	$\langle[1 \ \ 1], [0 \ \ 0]\rangle$

Table 2
PFIVFS set for the first laptop

$\widehat{\mathscr{A}_{p_1}}(e)$	e_1	e_2	e_3
$\widehat{\mathscr{A}}(e)$	$\langle[0.65, 0.85], [0.6, 0.63]\rangle$	$\langle[0.7, 0.8], [0.7, 0.72]\rangle$	$\langle[0.8, 0.85], [0.6, 0.62]\rangle$
$\widehat{p_1}(e)$	$\langle[0.6, 0.82], [0.65, 0.7]\rangle$	$\langle[0.6, 0.78], [0.75, 0.78]\rangle$	$\langle[0.7, 0.81], [0.65, 0.7]\rangle$

$\widehat{\mathscr{A}_{p_1}}(e)$	e_4	e_5
$\widehat{\mathscr{A}}(e)$	$\langle[0.75, 0.8], [0.65, 0.7]\rangle$	$\langle[0.7, 0.85], [0.45, 0.55]\rangle$
$\widehat{p_1}(e)$	$\langle[0.7, 0.75], [0.7, 0.75]\rangle$	$\langle[0.6, 0.7], [0.75, 0.8]\rangle$

Table 3

PFIVFS set for the second laptop

$\widehat{\mathscr{B}_{p_2}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{B}}(e)$	$\langle[0.85, 0.9], [0.36, 0.47]\rangle$	$\langle[0.6, 0.85], [0.5, 0.53]\rangle$	$\langle[0.55, 0.7], [0.75, 0.8]\rangle$	$\langle[0.6, 0.88], [0.5, 0.52]\rangle$	$\langle[0.65, 0.85], [0.55, 0.58]\rangle$
$\widehat{p_2}(e)$	$\langle[0.5, 0.6], [0.7, 0.8]\rangle$	$\langle[0.4, 0.5], [0.6, 0.88]\rangle$	$\langle[0.5, 0.6], [0.75, 0.85]\rangle$	$\langle[0.5, 0.65], [0.65, 0.8]\rangle$	$\langle[0.6, 0.7], [0.7, 0.75]\rangle$

Table 4

PFIVFS set for the third laptop

$\widehat{\mathscr{C}_{p_3}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{C}}(e)$	$\langle[0.75, 0.8], [0.65, 0.68]\rangle$	$\langle[0.65, 0.7], [0.72, 0.75]\rangle$	$\langle[0.68, 0.73], [0.65, 0.7]\rangle$	$\langle[0.65, 0.8], [0.6, 0.63]\rangle$	$\langle[0.55, 0.7], [0.73, 0.75]\rangle$
$\widehat{p_3}(e)$	$\langle[0.7, 0.75], [0.7, 0.75]\rangle$	$\langle[0.6, 0.65], [0.75, 0.8]\rangle$	$\langle[0.5, 0.6], [0.7, 0.8]\rangle$	$\langle[0.6, 0.7], [0.65, 0.75]\rangle$	$\langle[0.5, 0.7], [0.75, 0.78]\rangle$

Table 5

PFIVFS set for the fourth laptop

$\widehat{\mathscr{D}_{p_4}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{D}}(e)$	$\langle[0.8, 0.85], [0.65, 0.68]\rangle$	$\langle[0.7, 0.75], [0.72, 0.75]\rangle$	$\langle[0.7, 0.73], [0.75, 0.78]\rangle$	$\langle[0.6, 0.85], [0.67, 0.69]\rangle$	$\langle[0.6, 0.75], [0.8, 0.83]\rangle$
$\widehat{p_4}(e)$	$\langle[0.7, 0.75], [0.8, 0.85]\rangle$	$\langle[0.6, 0.65], [0.75, 0.78]\rangle$	$\langle[0.5, 0.6], [0.78, 0.85]\rangle$	$\langle[0.6, 0.7], [0.7, 0.75]\rangle$	$\langle[0.5, 0.7], [0.8, 0.85]\rangle$

Table 6

PFIVFS set for the fifth laptop

$\widehat{\mathscr{E}_{p_5}}(e)$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{E}}(e)$	$\langle[0.6, 0.71], [0.63, 0.73]\rangle$	$\langle[0.75, 0.85], [0.56, 0.58]\rangle$	$\langle[0.85, 0.92], [0.42, 0.45]\rangle$	$\langle[0.8, 0.85], [0.53, 0.55]\rangle$	$\langle[0.75, 0.8], [0.64, 0.65]\rangle$
$\widehat{p_5}(e)$	$\langle[0.6, 0.8], [0.65, 0.75]\rangle$	$\langle[0.5, 0.75], [0.65, 0.7]\rangle$	$\langle[0.8, 0.85], [0.45, 0.55]\rangle$	$\langle[0.75, 0.8], [0.55, 0.7]\rangle$	$\langle[0.7, 0.75], [0.65, 0.7]\rangle$

To find a laptop which is closest to the ideal laptop of the consumer. We should calculate the similarity measures of PFIVFS sets as shown in Tables 2-6 with the PFIVFS set in Table 1. Similarity measures between the 1-5 laptops and ideal laptop are given below in table 7. Table 7 represents the relative calculation of Γ, Δ and Υ for the similarity measures.

Table 7

	Γ	Δ	Υ
$\widehat{(\mathscr{I},\mathscr{A})}$	[0.919488, 0.953847]	[0.93292, 0.933251]	[0.926204, 0.943549]
$\widehat{(\mathscr{I},\mathscr{B})}$	[0.848667, 0.943867]	[0.951794, 0.952811]	[0.900231, 0.948339]
$\widehat{(\mathscr{I},\mathscr{C})}$	[0.861673, 0.871448]	[0.902856, 0.904415]	[0.882265, 0.887932]
$\widehat{(\mathscr{I},\mathscr{D})}$	[0.891595, 0.908378]	[0.873559, 0.875805]	[0.882577, 0.892091]
$\widehat{(\mathscr{I},\mathscr{E})}$	[0.932762, 0.950886]	[0.950004, 0.951781]	[0.941383, 0.951333]

	Ψ	*Similarity*
$\widehat{(\mathscr{I},\mathscr{A})}$	[0.605855, 0.688595]	[0.561146, 0.649723]
$\widehat{(\mathscr{I},\mathscr{B})}$	[0.443200, 0.463774]	[0.398983, 0.439815]
$\widehat{(\mathscr{I},\mathscr{C})}$	[0.523886, 0.574241]	[0.462206, 0.509886]
$\widehat{(\mathscr{I},\mathscr{D})}$	[0.468822, 0.538181]	[0.413772, 0.480106]
$\widehat{(\mathscr{I},\mathscr{E})}$	[0.739695, 0.755761]	[0.696336, 0.718981]

From the above results, we infer that the laptops have similarity measures in order $B \leq D \leq C \leq A \leq E$. Hence, we find that the fifth laptop is closest to the ideal laptop due to having the highest value of the similarity measure.

5.3. **Algorithm for Fermatean Interval valued fuzzy soft set (FIVFS set) model.**

(1) Input the values for FIVFS set in tabular form.
(2) Input the set of choice parameters $A \sqsubseteq E$.
(3) Compute the values for Γ and Δ.
(4) Calculate the similarity measure = $\frac{\Gamma+\Delta}{2}$.
(5) Determine maximum similarity = $Max\{similarity\ measure^i\}$ and $1 \leq i \leq m$.
(6) Optimal output yields solution to the problem.

We investigate the above mentioned decision making problem during laptop purchase using the FIVFS set approach to consider the effect of the possibility parameter. Calculating the similarity measures for the above mention five laptops, we have the following table.

Table 8

	Γ	Δ	*Similarity*
$\widehat{(\mathscr{I},\mathscr{A})}$	[0.919488, 0.953847]	[0.93292, 0.933251]	[0.926204, 0.943549]
$\widehat{(\mathscr{I},\mathscr{B})}$	[0.848667, 0.943867]	[0.951794, 0.952811]	[0.900231, 0.948339]
$\widehat{(\mathscr{I},\mathscr{C})}$	[0.861673, 0.871448]	[0.902856, 0.904415]	[0.882265, 0.887932]
$\widehat{(\mathscr{I},\mathscr{D})}$	[0.891595, 0.908378]	[0.873559, 0.875805]	[0.882577, 0.892091]
$\widehat{(\mathscr{I},\mathscr{E})}$	[0.932762, 0.950886]	[0.950004, 0.951781]	[0.941383, 0.951333]

From the above results, it can be understood the parameter has a significant impact on similarity measure of PFIVFS sets. It is observed that the first four laptops from the perspective of similarity measure are quite away from the ideal laptop resource. If the fifth laptop it has one unit chooses the threshold $[\mathbf{0.60}, \mathbf{0.71}]$, we should choose the fifth laptop resource as a potential laptop.

6. Comparison of approach of PFIVFS set and FIVFS set

On the contrary, when using the FIVFS set approach without the generalization parameter, we cannot distinguish which laptop resource is the best one. So the possibility parameter has an important influence on the similarity measure of the fifth laptop resource. Therefore, the PFIVFS set approach is more scientific and reasonable than the FIVFS set approach without the generalization parameter in the process of decision making.

7. Conclusion

The main goal of this work is to present a PFIVFS set to solve the phenomena related to decision making in which cubes of the sum of its membership and non membership is not exceeding unity. To illustrate the validity of this similarity measure, the PFIVFS set is applied to decision making problems. Therefore, the PFIVFS set approach is more scientific and reasonable than the FIVFS set approach without the generalization parameter in the process of decision making.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] L.A. Zadeh, Fuzzy Sets, Inf. Control 8 (1965), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X.

[2] K.T. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets Syst. 20 (1986), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3.

[3] Z. Xiao, W. Chen, L. Li, A Method Based on Interval-Valued Fuzzy Soft Set for Multi-Attribute Group Decision-Making Problems under Uncertain Environment, Knowl. Inf. Syst. 34 (2013), 653–669. https://doi.org/10.1007/s10115-012-0496-7.

[4] R.R. Yager, Pythagorean Membership Grades in Multicriteria Decision Making, IEEE Trans. Fuzzy Syst. 22 (2014), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989.

[5] T. Senapati, R.R. Yager, Fermatean Fuzzy Sets, J. Ambient Intell. Humaniz. Comput. 11 (2020), 663–674. https://doi.org/10.1007/s12652-019-01377-0.

[6] D. Molodtsov, Soft Set Theory—First Results, Comput. Math. Appl. 37 (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5.

[7] P.K. Maji, R. Biswas, A.R. Roy, Fuzzy Soft Set, J. Fuzzy Math. 9 (2001), 589-602.

[8] P.K. Maji, R. Biswas, A.R. Roy, On Intuitionistic Fuzzy Soft Set, J. Fuzzy Math. 9 (2001), 677-692.

[9] S. Alkhazaleh, A.R. Salleh, N. Hassan, Possibility Fuzzy Soft Set, Adv. Decis. Sci. 2011 (2011), 479756. https://doi.org/10.1155/2011/479756.

[10] X.D. Peng, Y. Yang, J.P. Song, Pythagorean Fuzzy Soft Set and Its Application, Comput. Eng. 41 (2015), 224-229. https://doi.org/10.3969/j.issn.1000-3428.2015.07.043.

[11] S. Alkhazaleh, A.R. Salleh, Generalised Interval-Valued Fuzzy Soft Set, J. Appl. Math. 2012 (2012), 870504. https://doi.org/10.1155/2012/870504.

[12] X. Peng, Y. Yang, Fundamental Properties of Interval-Valued Pythagorean Fuzzy Aggregation Operators: Interval-Valued Pythagorean Fuzzy Aggregation Operators, Int. J. Intell. Syst. 31 (2016), 444–487. https://doi.org/10.1002/int.21790.

[13] T.-Y. Chen, An Interval-Valued Pythagorean Fuzzy Compromise Approach with Correlation-Based Closeness Indices for Multiple-Criteria Decision Analysis of Bridge Construction Methods, Complexity 2018 (2018), 6463039. https://doi.org/10.1155/2018/6463039.

[14] X. Zhang, Multicriteria Pythagorean Fuzzy Decision Analysis: A Hierarchical QUALIFLEX Approach with the Closeness Index-Based Ranking Methods, Inf. Sci. 330 (2016), 104–124. https://doi.org/10.1016/j.ins.2015.10.012.

[15] R.R. Yager, A.M. Abbasov, Pythagorean Membership Grades, Complex Numbers, and Decision Making: Pythagorean Membership Grades and Fuzzy Subsets, Int. J. Intell. Syst. 28 (2013), 436–452. https://doi.org/10.1002/int.21584.